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ON THE MODELING OF DEVELOPABLE HERMITE PATCHES

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Abstract: The developable surfaces are useful in plat-metal and plywood sheet installation. For this reason, the paper discusses designing the developable Hermite patches with its boundary curves placed in different parallel planes. The steps used are in this way. Using the boundary curves data, and a developable patches' criteria equation, we evaluate this equation to find some geometric elements of both curves that can be practically used to construct the developable pieces. Then the study makes a procedure to design the pieces. As a result, we obtain some equations and methods to model many shapes of the developable Hermite patches that can be constructed by its endpoints, intermediate points, the tangent, and acceleration vectors at the endpoints of the patches' boundary curves. The endpoints and the intermediate control points can be applied to raise and lower the surface shape along the pieces. Moving the intermediate control points between the endpoints, and determining the tangent, and acceleration vectors are used to change the patches' surface slopes in a different form. Using this method, the surface shape of the pieces can be curved and fluctuate into more than three arches. We also numerically calculate the connection G^1 between two developable Hermite pieces adjacent.

Keywords: developable Hermite patches; control point; tangent vector; acceleration vector.

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1. INTRODUCTION

The techniques for modeling the developable surfaces have already been introduced. Zhao and Wang [1] constructed the developable surface via the surface pencil passing through a given curve. This technique presented the surface with the combination of the curve, and the vectors that are defined by the Serret-Frenet frame. Meanwhile, Al-Ghefari and Abdel-Baky [2] proposed a method for modeling the developable surface and classified it in a cylinder, cone, and tangent surface. A new technique for constructing the minimal surface has been reported by Xu et al. [3]. From a given boundary of surface, they apply quasi-harmonic Bézier approximation and quasi-harmonic mask methods to define the minimal surface. Then, Xu et al. [4] presented the construction of IGA-suitable planar B-spline parameterizations from a certain complex CAD boundary. This technique requires many steps to model the surfaces. It needs a process of Bézier extraction and subdivision, a partition framework of planar domain, quadrangulation, and a optimization process. Hu et al. [5] introduced the developable Bézier-like surfaces with Bernstein-like basis functions of the multiple shape parameters. Kusno [6,7,8] discussed the construction of regular developable Bézier patches defined by their boundary curves of four, five, and six degrees. These pieces are also modeled by using Hermite spline interpolation curves, and simulated the method for cutting and adjusting the shapes of the surfaces. In recent times, the method of the developable surface construction bounded by two rational or NURBS curves is presented by Fernándes and Pérez [9]. This method imposes that the endpoints of patch' boundary curves must be coplanar, and at those endpoints position, it has to fulfill the developability condition. Evaluating these methods introduced, we get some limitations. Applying the low degree planar curves is efficient for constructing the simple developable patches or surfaces, but it is not practical to design the developable surfaces of complex shapes. Using the derivatives and developability condition at the endpoints of the boundary curves can locally control the form and continuity of a patch in the nearest area of the endpoints, but, it cannot straightforwardly control to create various slopes and fluctuate shapes in the middle surface part of the patch. In other words, the presented methods just can construct a surface shape of the patches not more than two arches. To find a solution of these restrictions, this paper proposes a new approach to design the developable pieces of many arches by arranging its endpoints, intermediate points, the tangent, and acceleration vectors at the endpoints of the patches' boundary curves.

2. MATHEMATICAL EQUATIONS OF DEVELOPABLE SURFACES AND HERMITE CURVES

Mathematical equations of regular developable surfaces in the algebraic representation are defined as follows [6,10].

Definition 2.1. A regular ruled surface $\mathbf{S}(\mathrm{u},\mathrm{v})$ is a surface generated by a one parameter of line in the form $\mathbf{S}(\mathrm{u},\mathrm{v}) = \mathbf{f}(\mathrm{u}) + \mathrm{v}\,\mathbf{g}(\mathrm{u})$ with **f** and **g** of class C^n and $[(\mathbf{f}'(\mathrm{u}) + \mathrm{v}\,\mathbf{g}'(\mathrm{u})) \wedge \mathbf{g}(\mathrm{u})] \neq \mathbf{0}$. The curves **f**(u) and **g**(u) are respectively called directrix curve and generatrix (ruling).

Definition 2.2. The regular ruled surface $\mathbf{S}(\mathrm{u,v}) = \mathbf{f}(\mathrm{u}) + \mathrm{v}\,\mathbf{g}$ is developable, if the tangent plane is constant along each generatrix, that is the vectors $[\mathbf{g}'(\mathrm{u}),\mathbf{f}'(\mathrm{u}),\mathbf{g}]$ are coplanar.

Consider, in Definition 2.2, the ruling $\mathbf{g}(u) = \mathbf{q}(u) - \mathbf{P}(u)$ and the curve $\mathbf{f}(u) = \mathbf{P}(u)$. Then, the surface $\mathbf{S}(u,v)$ can be noted in the form of their boundary curves $\mathbf{P}(u)$ and $\mathbf{q}(u)$ as follows

$$\mathbf{S}(u,v) = \mathbf{f}(u) + v\,\mathbf{g}(u) = \mathbf{P}(u) + v\,[\mathbf{q}(u) - \mathbf{P}(u)] = (1-v)\,\mathbf{P}(u) + v\,\mathbf{q}(u). \tag{2.1}$$

When the surface $\mathbf{S}(u,v)$ is developable, then three vectors $[\mathbf{g}'(u),\mathbf{P}'(u),\mathbf{g}(u)]$ are coplanar. This means that the vector $\mathbf{g}'(u)$ can be represented in linear combination of two vectors $\mathbf{P}'(u)$ and $[\mathbf{q}(u) - \mathbf{P}(u)]$ such that $\mathbf{q}'(u) = \rho(u)\,\mathbf{P}'(u) + \sigma(u)\,[\mathbf{q}(u) - \mathbf{P}(u)]$ with $\rho(u)$ and $\sigma(u)$ two real scalars. Because of application purposes, in this paper, we constrain that the curves $\mathbf{P}(u)$ and $\mathbf{q}(u)$ must be in two parallel planes $\Upsilon_1 // \Upsilon_2$, correspondingly, and the criteria equation of the developable surfaces will be

$$\mathbf{q}'(u) = \rho(u)\,\mathbf{P}'(u). \tag{2.2}$$

In this discussion, due to two curves $[\mathbf{P}(u),\mathbf{q}(u)]$ are regular in the same degrees and orientation (direction), we limit that the choice of the real scalar $\rho(u)$ is positive constant ρ for all values u. The fixed criteria ρ classify two types of developable surfaces. If the value $\rho = 1$, then it defines a cylinder type, if not, a cone.

The geometric representation of cubic and quintic polynomial Hermite curves $\mathbf{P}_3(u)$ and $\mathbf{P}_5(u)$ that are formulated by endpoints $[\mathbf{P}_o, \mathbf{P}_1]$, tangent vectors $[\mathbf{P}_o^u, \mathbf{P}_1^u]$ and acceleration vectors $[\mathbf{P}_o^{uu}, \mathbf{P}_1^{uu}]$ of the curve is [11,12]

(2.3a) $$\mathbf{P}_3(u) = H_{31}(u)\,\mathbf{P}_o + H_{32}(u)\,\mathbf{P}_1 + H_{33}(u)\,\mathbf{P}_o^u + H_{34}(u)\,\mathbf{P}_1^u$$

(2.3b) $$\mathbf{P}_5(u) = H_{51}(u)\,\mathbf{P}_o + H_{52}(u)\,\mathbf{P}_1 + H_{53}(u)\,\mathbf{P}_o^u + H_{54}(u)\,\mathbf{P}_1^u + H_{55}(u)\,\mathbf{P}_o^{uu} + H_{56}(u)\,\mathbf{P}_1^{uu}$$

with

$H_{31} = 2u^3 - 3u^2 + 1$; $H_{32} = -2u^3 + 3u^2$; $H_{33} = u^3 - 2u^2 + u$; $H_{34} = u^3 - u^2$;

$H_{51} = -6u + 15u^4 - 10u^3 + 1$; $H_{52} = 6u^5 - 15u^4 + 10u^3$; $H_{53} = -3u^5 + 8u^4 - 6u^3 + u$;

$H_{54} = -3u^5 + 7u^4 - 4u^3$; $H_{55} = -\frac{1}{2}u^5 + 3/2.u^4 - 3/2.u^3 + \frac{1}{2}u^2$; $H_{56} = \frac{1}{2}u^5 - u^4 + \frac{1}{2}u^3$.

If we give a tension value k_1 and k_2 to the tangent vectors $\mathbf{P}_o^u$ and $\mathbf{P}_1^u$ in that order, the Hermite curve formulations of Equations (2.3) will be

(2.4a) $$\mathbf{P}_3(u) = H_{31}(u)\,\mathbf{P}_o + H_{32}(u)\,\mathbf{P}_1 + H_{33}(u)\,\mathbf{P}_o^{u*} + H_{34}(u)\,\mathbf{P}_1^{u*}$$

(2.4b) $$\mathbf{P}_5(u) = H_{51}(u)\,\mathbf{P}_o + H_{52}(u)\,\mathbf{P}_1 + H_{53}(u)\,\mathbf{P}_o^{u*} + H_{54}(u)\,\mathbf{P}_1^{u*} + H_{55}(u)\,\mathbf{P}_o^{uu} + H_{56}(u)\,\mathrm{P}_1^{\mathrm{uu}}$$

with $\mathbf{P}_o^{u*} = k_1\,\mathbf{P}_o^u$ and $\mathbf{P}_1^{u*} = k_2\,\mathbf{P}_1^u$. Therefore, it can formulate these cubic and quintic Hermite curves of Equations (2.3) in the algebraic representation

(2.5) $$\mathbf{P}_3(u) = \mathbf{a}_3 u^3 + \mathbf{b}_3 u^2 + \mathbf{c}_3 u + \mathbf{d}_3$$

(2.6) $$\mathbf{P}_5(u) = \mathbf{a}_5 u^5 + \mathbf{b}_5 u^4 + \mathbf{c}_5 u^3 + \mathbf{d}_5 u^2 + \mathbf{e}_5 u + \mathbf{f}_5$$

with

$\mathbf{a}_3 = -2(\mathbf{P}_1 - \mathbf{P}_o) + \mathbf{P}_o^u + \mathbf{P}_1^u$; $\mathbf{b}_3 = 3(\mathbf{P}_1 - \mathbf{P}_o) - 2\mathbf{P}_o^u - \mathbf{P}_1^u$; $\mathbf{c}_3 = \mathbf{P}_o^u$; $\mathbf{d}_3 = \mathbf{P}_o$;

$\mathbf{a}_5 = 6(\mathbf{P}_1 - \mathbf{P}_o) - 3\mathbf{P}_o^u - 3\mathbf{P}_1^u - \frac{1}{2}.\mathbf{P}_o^{uu} + \frac{1}{2}\mathbf{P}_1^{uu}$; $\mathbf{b}_5 = -15(\mathbf{P}_1 - \mathbf{P}_o) + 8\mathbf{P}_o^u + 7\mathbf{P}_1^u + 3/2.\mathbf{P}_o^{uu} - \mathbf{P}_1^{uu}$;

$\mathbf{c}_5 = 10(\mathbf{P}_1 - \mathbf{P}_o) - 6\mathbf{P}_o^u - 4\mathbf{P}_1^u - 3/2.\,\mathbf{P}_o^{uu} + \frac{1}{2}\mathbf{P}_1^{uu}$; $\mathbf{d}_5 = \frac{1}{2}\mathbf{P}_o^{uu}$; $\mathbf{e}_5 = \mathbf{P}_o^u$; $\mathbf{f}_5 = \mathbf{P}_o$.

Base on the developable surfaces criteria of Equation (2.2) and the patches' boundary curves $\mathbf{P}(u)$ and $\mathbf{q}(u)$ of Equations (2.5), and (2.6), in the next section, this paper will introduce a new method to design the developable Hermite pieces in the form of Equation (2.1). In these patches modeling, to apply many parameters of the equations, we use the algebraic representations rather than the geometric representations.

3. MAIN RESULTS OF DEVELOPABLE HERMITE PATCHES CONSTRUCTION

The developable surfaces have distinctive properties among the surface types, i.e. it can be laid flat on a plane without stretching and tearing. These surfaces are handy in plat-metal-based industries and plywood sheet installation, for example, aircrafts industries, ship hulls, and trains [13,14,15]. In the applications, those object surfaces, generally, are formed by some small surface sections (pieces) that are bounded by two curves $\mathbf{P}(u)$ and $\mathbf{q}(u)$ laid in the plane Υ_1 parallel to the plane Υ_2, respectively. For this reason, this paper discusses the construction of the developable patches via the boundary curves in form of the Hermite polynomials.

3.1. Patches Construction with Boundary Curves Defined by Endpoints, Tangent and Acceleration Vectors

Given the Hermite curves $\mathbf{P}_3(u)$ and $\mathbf{P}_5(u)$ in the Plane Υ_1, and the Hermite curves $\mathbf{q}_3(u)$ and $\mathbf{q}_5(u)$ in the plane Υ_2, correspondingly. Both planes are parallel to Cartesian coordinate' plane *YOZ*, i.e., $\Upsilon_1 // \Upsilon_2 // YOZ$. Using Equation (2.1), (2.5) and (2.6), we will define the developable Hermite pieces in the forms

(3.1a) $$\mathbf{D}_3(u, v) = (1-v)\,\mathbf{P}_3(u) + v\,\mathbf{q}_3(u)$$

(3.1b) $$\mathbf{D}_5(u, v) = (1-v)\,\mathbf{P}_5(u) + v\,\mathbf{q}_5(u)$$

with

$\mathbf{P}_3(u) = \mathbf{a}_{31}u^3 + \mathbf{b}_{31}u^2 + \mathbf{c}_{31}u + \mathbf{d}_{31}$; $\quad \mathbf{q}_3(u) = \mathbf{a}_{32}u^3 + \mathbf{b}_{32}u^2 + \mathbf{c}_{32}u + \mathbf{d}_{32}$;

$\mathbf{P}_5(u) = \mathbf{a}_{51}u^5 + \mathbf{b}_{51}u^4 + \mathbf{c}_{51}u^3 + \mathbf{d}_{51}u^2 + \mathbf{e}_{51}u + \mathbf{f}_{51}$; $\quad \mathbf{q}_5(u) = \mathbf{a}_{52}u^5 + \mathbf{b}_{52}u^4 + \mathbf{c}_{52}u^3 + \mathbf{d}_{52}u^2 + \mathbf{e}_{52}u + \mathbf{f}_{52}$;

and u, v in interval $0 \le u, v \le 1$. Henceforward, the patches $\mathbf{D}_3(u,v)$ and $\mathbf{D}_5(u,v)$ are, in that order, called the cubic and quintic developable Hermite pieces.

If the cubic developable patches' criteria $\mathbf{D}_3(u,v)$ is $\mathbf{q}_3^u(u) = \rho\ \mathbf{P}_3^u(u)$ with $\mathbf{P}_3^u(u) = 3\mathbf{a}_{31}u^2 + 2\mathbf{b}_{31}u + \mathbf{c}_{31}$, and $\mathbf{q}_3^u(u) = 3\mathbf{a}_{32}u^2 + 2\mathbf{b}_{32}u + \mathbf{c}_{32}$, meanwhile, the criteria of the quintic developable patch $\mathbf{D}_5(u,v)$ is $\mathbf{q}_5^u(u) = \rho\ \mathbf{P}_5^u(u)$ with $\mathbf{P}_5^u(u) = 5\mathbf{a}_{51}u^4 + 4\mathbf{b}_{51}u^3 + 3\mathbf{c}_{51}u^2 + 2\mathbf{d}_{51}u + \mathbf{e}_{51}$ and $\mathbf{q}_5^u(u) = 5\mathbf{a}_{52}u^4 + 4\mathbf{b}_{52}u^3 + \mathbf{c}_{52}u^2 + 2\mathbf{d}_{52}u + \mathbf{e}_{52}$, then

$$3(\mathbf{a}_{32} - \rho\mathbf{a}_{31})u^2 + 2(\mathbf{b}_{32} - \rho\mathbf{b}_{31})u + (\mathbf{c}_{32} - \rho\mathbf{c}_{31}) = \mathbf{0};$$

and

$$5(\mathbf{a}_{52} - \rho\mathbf{a}_{51})u^4 + 4(\mathbf{b}_{52} - \rho\mathbf{b}_{51})u^3 + 3(\mathbf{c}_{52} - \rho\mathbf{c}_{51})u^2 + 2(\mathbf{d}_{52} - \rho\mathbf{d}_{51})u + (\mathbf{e}_{52} - \rho\mathbf{e}_{51}) = \mathbf{0}.$$

Due to the canonical basis $[u^2, u, 1]$ and $[u^4, u^3, u^2, u, 1]$ are not zero, then, the equation coefficients of the patch $\mathbf{D}_3(u,v)$ must be $(\mathbf{a}_{32} - \rho\mathbf{a}_{31}) = \mathbf{0}$, $(\mathbf{b}_{32} - \rho\mathbf{b}_{31}) = \mathbf{0}$, $(\mathbf{c}_{32} - \rho\mathbf{c}_{31}) = \mathbf{0}$, and the equation coefficients of the patch $\mathbf{D}_5(u,v)$ have to fulfill $(\mathbf{a}_{52} - \rho\mathbf{a}_{51}) = \mathbf{0}$, $(\mathbf{b}_{52} - \rho\mathbf{b}_{51}) = \mathbf{0}$, $(\mathbf{c}_{52} - \rho\mathbf{c}_{51}) = \mathbf{0}$, $(\mathbf{d}_{52} - \rho\mathbf{d}_{51}) = \mathbf{0}$, and $(\mathbf{e}_{52} - \rho\mathbf{e}_{51}) = \mathbf{0}$. These mean that, for the patch $\mathbf{D}_3(u,v)$, it must meet

$$-2(\mathbf{q}_1 - \mathbf{q}_o) + \mathbf{q}_o^u + \mathbf{q}_1^u = \rho[-2(\mathbf{P}_1 - \mathbf{P}_o) + \mathbf{P}_o^u + \mathbf{P}_1^u]$$
$$3(\mathbf{q}_1 - \mathbf{q}_o) - 2\mathbf{q}_o^u - \mathbf{q}_1^u = \rho[3(\mathbf{P}_1 - \mathbf{P}_o) - 2\mathbf{P}_o^u - \mathbf{P}_1^u]$$
$$\mathbf{q}_o^u = \rho\mathbf{P}_o^u$$

and, for the patch $\mathbf{D}_5(u,v)$, it have to fulfill the conditions

$$6(\mathbf{q}_1-\mathbf{q}_o) - 3\mathbf{q}_o^u - 3\mathbf{q}_1^u - 1/2.\mathbf{q}_o^{uu} + 1/2.\mathbf{q}_1^{uu} = \rho[6(\mathbf{P}_1-\mathbf{P}_o) - 3\mathbf{P}_o^u - 3\mathbf{P}_1^u - 1/2.\mathbf{P}_o^{uu} + 1/2.\mathbf{P}_1^{uu}]$$
$$-15(\mathbf{q}_1-\mathbf{q}_o) + 8\mathbf{q}_o^u + 7\mathbf{q}_1^u + 3/4.\mathbf{q}_o^{uu} - \mathbf{q}_1^{uu} = \rho[-15(\mathbf{P}_1-\mathbf{P}_o) + 8\mathbf{P}_o^u + 7\mathbf{P}_1^u + 3/4.\mathbf{P}_o^{uu} - \mathbf{P}_1^{uu}]$$
$$10(\mathbf{q}_1-\mathbf{q}_o) - 6\mathbf{q}_o^u - 4\mathbf{q}_1^u - 3/2.\mathbf{q}_o^{uu} + 1/2.\mathbf{q}_1^{uu} = \rho[10(\mathbf{P}_1-\mathbf{P}_o) - 6\mathbf{P}_o^u - 4\mathbf{P}_1^u - 3/2.\mathbf{P}_o^{uu} + 1/2.\mathbf{P}_1^{uu}]$$
$$\mathbf{q}_o^{uu} = \rho\mathbf{P}_o^{uu}$$
$$\mathbf{q}_o^u = \rho\mathbf{P}_o^u.$$

Thus, it can deduce that the patch $\mathbf{D}_3(u,v)$ will be developable when their control points and its tangent vectors are in conditions

(3.2) $$\mathbf{q}_o^u = \rho\mathbf{P}_o^u; \quad (\mathbf{q}_1-\mathbf{q}_o) = \rho(\mathbf{P}_1-\mathbf{P}_o); \quad \mathbf{q}_1^u = \rho\mathbf{P}_1^u.$$

The equations (3.2) means that $\mathbf{q}_o^u // \mathbf{P}_o^u$, $(\mathbf{P}_1-\mathbf{P}_o)//(\mathbf{q}_1-\mathbf{q}_o)$, $\mathbf{P}_1^u // \mathbf{q}_1^u$, and $|\mathbf{q}_o^u|/|\mathbf{P}_o^u| = |\mathbf{q}_1-\mathbf{q}_o|/|\mathbf{P}_1\text{-}\mathbf{P}_o| = |\mathbf{q}_1^u|/|\mathbf{P}_1^u| = \rho$. It is shown in Figure 1 that, from the points data $[\mathbf{P}_o, \mathbf{P}_1, \mathbf{P}_o^u, \mathbf{P}_1^u]$, we can calculate the control points $[\mathbf{q}_o, \mathbf{q}_1, \mathbf{q}_o^u, \mathbf{q}_1^u]$ relative to the center point *O*' with the ratio value ρ by applying the triangle similarity theorem. The calculated surface shape of the cubic developable Hermite patch in Figure 1 is presented in Figure 2. In another side, the quintic Hermite developable patch $\mathbf{D}_5(u,v)$ have to meet

(3.3) $$\mathbf{q}_o^u = \rho\mathbf{P}_o^u; \quad \mathbf{q}_o^{uu} = \rho\mathbf{P}_o^{uu}; \quad (\mathbf{q}_1-\mathbf{q}_o) = \rho(\mathbf{P}_1-\mathbf{P}_o); \quad \mathbf{q}_1^u = \rho\mathbf{P}_1^u; \quad \mathbf{q}_1^{uu} = \rho\mathbf{P}_1^{uu}.$$

Thus, designing the cubic and quintic developable Hermite patches $\mathbf{D}_3(u,v)$ and $\mathbf{D}_5(u,v)$ can be undertaken in the following steps:

1. Determine two parallel vectors $(\mathbf{P}_1-\mathbf{P}_o)//(\mathbf{q}_1-\mathbf{q}_o)$ in the planes $\Upsilon_1 // \Upsilon_2$, correspondingly;

2. Calculate $\rho = |\mathbf{q}_1 - \mathbf{q}_o)|/|\mathbf{P}_1 - \mathbf{P}_o|$;
3. Determine the tangent and acceleration vectors data $[\mathbf{P}_o^u, \mathbf{P}_1^u, \mathbf{P}_o^{uu}, \mathbf{P}_1^{uu}]$ of the boundary curve $\mathbf{P}(u)$ and compute the tangent and acceleration vectors $[\mathbf{q}_o^u, \mathbf{q}_1^u, \mathbf{q}_o^{uu}, \mathbf{q}_1^{uu}]$ of the boundary curve $\mathbf{q}(u)$ by computing Equation (3.2) for $\mathbf{D}_3(u,v)$, and Equation (3.3) for $\mathbf{D}_5(u,v)$.
4. Substitute, one-to-one, both the existed control points, the tangent and the acceleration vectors into Equation (3.1a), and (3.1b) to construct the developable Hermite pieces $\mathbf{D}_3(u,v)$ and $\mathbf{D}_5(u,v)$.

As a validation of this method, we illustrate the numerical calculation to design the developable Hermite patches $\mathbf{D}_3(u,v)$ as follows (Figure 1(a)). Let the control points $\mathbf{P}_o = <10,-40,10>$, $\mathbf{P}_1 = <10,40,25>$, $\mathbf{q}_o = <-20,-90,15>$, and $\mathbf{q}_1 = <-20,70,45>$ such that $(\mathbf{P}_1 - \mathbf{P}_o)//(\mathbf{q}_1 - \mathbf{q}_o)$ and $\rho = 2$. Furthermore, choosing $\mathbf{P}_o^u = <0,10,40>$, and $\mathbf{P}_1^u = <0,10,-30>$ and evaluating Equation (3.2) will find $\mathbf{q}_o^u = <0,20,80>$, and $\mathbf{q}_1^u = <0,20,-60>$. So, the cubic Hermite curves $\mathbf{P}_3(u)$, and $\mathbf{q}_3(u)$ are

$$\mathbf{P}_3(u) = <10, -140u^3 + 210u^2 + 10u - 40, -20u^3 - 5u^2 + 40u + 10>$$

$$\mathbf{q}_3(u) = <-20, -280u^3 + 420u^2 + 20u - 90, -40u^3 - 10u^2 + 80u + 15>.$$

Thus, via Equation (3.1a), it can construct the patch $\mathbf{D}_3(u,v)$ as presented in Figure 1 (b).

From Equation (3.2), when the real scalar ρ is positive and $\rho \neq 1$, it will find a cone patch and their generatrixes intersect at one point O' (Figure 1(a), 1(b), 1(d)). If $\rho = 1$, it will obtain a cylinder patch, and all generatrixes are parallel (Figure 1(c)). We can locally model the patch shapes $\mathbf{D}_3(u,v)$ by arranging the directions of both tangent vector pairs $[\mathbf{P}_o^u, \mathbf{q}_o^u]$ and $[\mathbf{P}_1^u, \mathbf{q}_1^u]$. In Figure 1(b), the patch $\mathbf{D}_3(u,v)$ will be convex up when the direction $[\mathbf{P}_o^u, \mathbf{q}_o^u]$ goes up and the direction $[\mathbf{P}_1^u, \mathbf{q}_1^u]$ goes down. Meanwhile when the direction $[\mathbf{P}_o^u, \mathbf{q}_o^u]$ and $[\mathbf{P}_1^u, \mathbf{q}_1^u]$ go down, the form of patch will be oscillate (Figure1(c)). Their profiles can also be modified by giving the different values tension k_1 and k_2 for tangent vectors $\mathbf{P}_o^{u*} = k_1 \mathbf{P}_o^u$, and $\mathbf{P}_1^{u*} = k_2 \mathbf{P}_1^u$ as formulated in Equation 2.4a. Figure 1(d) shows that the boundary curves will change the surface forms when its tangent vectors at their endpoints are given some different values k_1 and k_2.

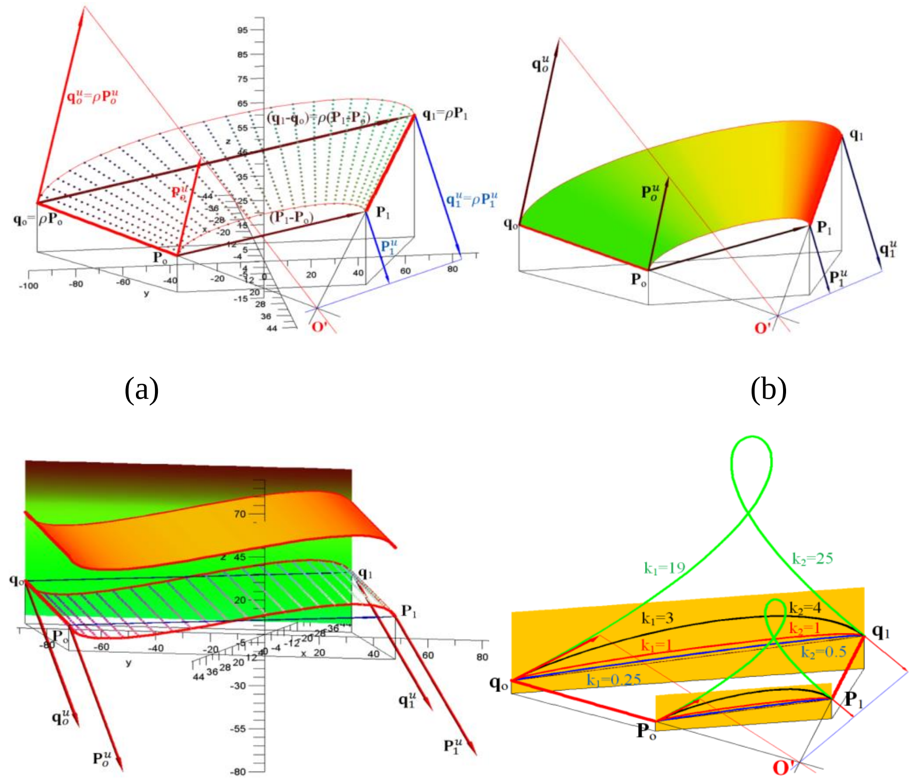


FIGURE 1. Construction of cubic and quintic developable Hermite patches.

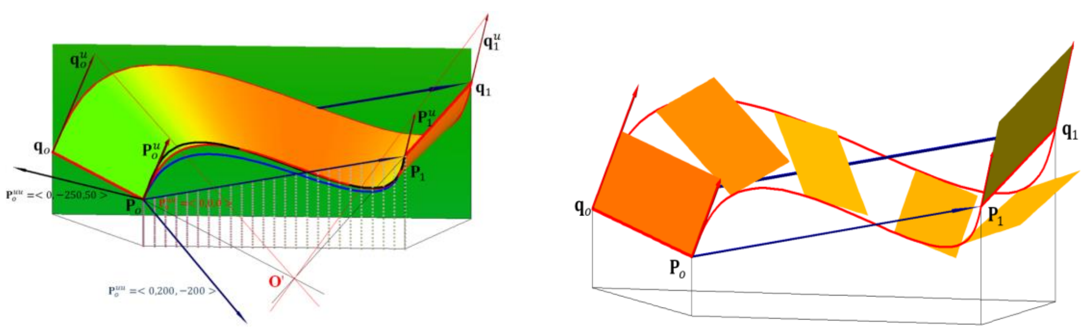
Evaluating the design method $\mathbf{D}_5(u,v)$ with Equation (3.3), this paper use the data as follows. Given the control points $\mathbf{P}_o$ = <20,−50,10>, $\mathbf{P}_1$ = <20,50,25>, $\mathbf{q}_o$ = <−20,−90,15>, and $\mathbf{q}_1$ = <−20,70,39> such that $(\mathbf{P}_1-\mathbf{P}_o)//(\mathbf{q}_1-\mathbf{q}_o)$ and ρ = 8/5. We elect $\mathbf{P}_o^u$ = <0,50,100>, $\mathbf{P}_1^u$ = <0,20,70>, $\mathbf{P}_o^{uu}$ = <0,0,0>, $\mathbf{P}_1^{uu}$ = <0,0,0>, and after computing Equation (33), it obtain $\mathbf{q}_o^u$ = <0,80,160>, $\mathbf{q}_1^u$ = <0,32,112>, $\mathbf{q}_o^{uu}$ = <0,0,0>, and $\mathbf{q}_1^{uu}$ = <0,0,0>. So, the curves $\mathbf{P}_5(u)$, and $\mathbf{q}_5(u)$ are

$$\mathbf{P}_5(u) = <20,\ 390u^5 - 960u^4 + 620u^3 + 50u - 50,\ -20u^5 + 1065u^4 - 730u^3 + 100u + 10>$$

$$\mathbf{q}_5(u) = <-20,\ 624u^5 - 1536u^4 + 992u^3 + 80u - 90,\ -672u^5 + 1704u^4 - 1168u^3 + 160u + 15>.$$

When $\mathbf{P}_5(u)$ and $\mathbf{q}_5(u)$ are inserted in Equation (3.1b), the form of patch $\mathbf{D}_5(u,v)$ is represented in Figure 2(a). In this case, when at $\mathbf{P}_o$ we change the value of the acceleration vector $\mathbf{P}_o^{uu}$ = <0,−250,50>, then in around of the control point $\mathbf{P}_o$ the curve $\mathbf{P}_5$(u) will be moved to tend up position (black color curve in Figure 2(a)), but, if $\mathbf{P}_o^{uu}$ = <0,200,−200>, the curve shape will change in the down position (curve in blue color). Figure 2(b) shows that along the parameter u of the curves $\mathbf{P}_5(u)$, and $\mathbf{q}_5(u)$, the tangent vectors and the generatrixes of the patch $\mathbf{D}_5(u,v)$ are coplanar.

We have presented the method to construct the cubic and quintic developable Hermite patches with their boundary curves defined by the endpoints, tangent, and acceleration vectors. Both pieces type provide some advantages for defining the developable surfaces. Using the data of two endpoints, tangent, and acceleration vectors of the boundary curves can straightforward design various patches shapes in the area near their endpoints of the curves. They also can locally form the patches with two surface arches (Figure 1(c), 2(a)). To develop the different patch types, we need to evaluate the boundary curves shapes that are not only controlled by their endpoints, tangent, and acceleration vectors but also they are determined by its intermediate control points.



(a) (b)

FIGURE 2. Different tension values effect (Figure 2(**a**)) and tangent planes (Figure 2(**b**)) of a quintic developable Hermite patch.

3.2. Patches Construction with Boundary Curves Interpolated by One Intermediate Control Point

Let the cubic curve $\mathbf{P}_{x3}(u) = \mathbf{a}_{x3}u^3 + \mathbf{b}_{x3}u^2 + \mathbf{c}_{x3}u + \mathbf{d}_{x3}$ with the given data at $\mathbf{P}_{x3}(0) = \mathbf{P}_o$, $\mathbf{P}_{x3}(1) = \mathbf{P}_1$, $\mathbf{P}^u_{x3}(0) = \mathbf{P}^u_o$, and at a parameter value $u = x$ in interval $0 < u < 1$ the intermediate control point $\mathbf{P}_{x3}(x) = \mathbf{P}_x$. Computing the data will get four equations for determining four coefficient vectors $\mathbf{a}_{x3}$, $\mathbf{b}_{x3}$, $\mathbf{c}_{x3}$, and $\mathbf{d}_{x3}$ as follows

$$\mathbf{P}_{x3}(0) = \mathbf{a}_{x3}.0^3 + \mathbf{b}_{x3}.0^2 + \mathbf{c}_{x3}.0 + \mathbf{d}_{x3}; \quad \mathbf{P}_{x3}(1) = \mathbf{a}_{x3}.1^3 + \mathbf{b}_{x3}.1^2 + \mathbf{c}_{x3}.1 + \mathbf{d}_{x3};$$

$$\mathbf{P}^{u}\ (0) = 3.\mathbf{a}_{x3}.0^2 + 2.\mathbf{b}_{x3}.0 + \mathbf{c}_{x3}; \quad \mathbf{P}_{x3}(x) = \mathbf{a}_{x3}.x^3 + \mathbf{b}_{x3}.x^2 + \mathbf{c}_{x3}.x + \mathbf{d}_{x3}.$$

The solution of these equations is

$\mathbf{a}_{x3} = \frac{1}{(x^3-x^2)}\,[-x^2(\mathbf{P}_1-\mathbf{P}_o)+(\mathbf{P}_x-\mathbf{P}_o)+(x^2-x)\;\mathbf{P}_o^u]$; $\mathbf{b}_{x3} = \frac{1}{(x^3-x^2)}\,[x^3(\mathbf{P}_1-\mathbf{P}_o)-(\mathbf{P}_x-\mathbf{P}_o)+(x-x^3)\;\mathbf{P}_o^u]$;

$\mathbf{c}_{x3} = \mathbf{P}_o^u$; $\mathbf{d}_{x3} = \mathbf{P}_o$.

Therefore, the cubic Hermite polynomial with the intermediate point $\mathbf{P}_x$ can be wrote

(3.4) $$\mathbf{P}_{x3}(u) = \frac{1}{(x^3-x^2)}\,[-x^2(\mathbf{P}_1-\mathbf{P}_o)+(\mathbf{P}_x-\mathbf{P}_o)+(x^2-x)\;\mathbf{P}_o^u].\,u^3 + \frac{1}{(x^3-x^2)}\,[-x^3(\mathbf{P}_1-\mathbf{P}_o)-(\mathbf{P}_x-\mathbf{P}_o)+(x-x^3)\;\mathbf{P}_o^u].\,u^2 + \mathbf{P}_o^u\,.\,u+\mathbf{P}_o.$$

After rearranging, the geometric representation of the curve is

(3.5) $$\mathbf{P}_{x3}(u) = H_{x31}(u)\mathbf{P}_o + H_{x32}(u)\mathbf{P}_x + H_{x33}(u)\mathbf{P}_1 + H_{x34}(u)\;\mathbf{P}_o^u$$

and

$H_{x31} = \frac{1}{(x^3-x^2)}\,[(x^2-1)u^3+(1-x^3)u^2+x^2(-1+x)]$; $H_{x32} = \frac{1}{(x^3-x^2)}\,[u^3-u^2]$;

$H_{x33} = \frac{1}{(x^3-x^2)}\,[-x^2u^3+x^3u^2)]$; $H_{x34} = \frac{1}{(x^3-x^2)}\,[(-x+x^2)u^3+(x-x^3)u^2+(x^2(-1+x))u]$.

Consider a quartic curve $\mathbf{P}_{x4}(u)$ in the conditions $\mathbf{P}_{x4}(0) = \mathbf{P}_o$, $\mathbf{P}_{x4}(1) = \mathbf{P}_1$, $\mathbf{P}_{x4}^u(0) = \mathbf{P}_o^u$, $\mathbf{P}_{x4}^u(1) = \mathbf{P}_1^u$, and at a parameter value $u = x$ in interval $0 < u < 1$ the intermediate control point $\mathbf{P}_{x4}(x) = \mathbf{P}_x$. Applying the same method of the cubic Hermite curve' calculation to find the coefficient vectors of canonical basis $[u^4, u^3, u^2, u, 1]$ of the curve $\mathbf{P}_{x4}(u)$, it can define the algebraic representation of quartic Hermite curve $\mathbf{P}_{x4}(u)$ with the coefficients $\mathbf{a}_{x4}$, $\mathbf{b}_{x4}$, $\mathbf{c}_{x4}$, $\mathbf{d}_{x4}$ and $\mathbf{e}_{x4}$ as follows

(3.6) $$\mathbf{P}_{x4}(u) = \mathbf{a}_{x4}\,u^4+\mathbf{b}_{x4}\,u^3+\mathbf{c}_{x4}\,u^2+\mathbf{d}_{x4}\,u+\mathbf{e}_{x4}$$

with

$\mathbf{a}_{x4} = \frac{1}{(x^4-2x^3+x^2)}\,[(+2x^3-3x^2)(\mathbf{P}_1-\mathbf{P}_o)+(\mathbf{P}_x-\mathbf{P}_o)-x^3(\mathbf{P}_1^u-\mathbf{P}_o^u)+x^2(\mathbf{P}_1^u+2\mathbf{P}_o^u)-x\mathbf{P}_1^u]$;

$\mathbf{b}_{x4} = \frac{1}{(x^4-2x^3+x^2)}\,[(-2x^4+4x^2)(\mathbf{P}_1-\mathbf{P}_o)-2(\mathbf{P}_x-\mathbf{P}_o)+x^4(\mathbf{P}_1^u-\mathbf{P}_o^u)-x^2(\mathbf{P}_1^u+3\mathbf{P}_o^u)+2x\mathbf{P}_1^u]$;

$\mathbf{c}_{x4} = \frac{1}{(x^4-2x^3+x^2)}\,[(+3x^4-4x^3)(\mathbf{P}_1-\mathbf{P}_o)+(\mathbf{P}_x-\mathbf{P}_o)-x^4(\mathbf{P}_1^u-\mathbf{P}_o^u)+x^2(\mathbf{P}_1^u+3\mathbf{P}_o^u)-x\mathbf{P}_1^u]$;

$\mathbf{d}_{x4} = \mathbf{P}_o^u$; $\mathbf{e}_{x4} = \mathbf{P}_o$.

Let two curves $\mathbf{P}_{x31}(u)$, and $\mathbf{q}_{x32}(u)$ in the form of Equation (3.4) in the planes $\Upsilon_1 // \Upsilon_2$, respectively. From the curves, we will formulate the cubic developable Hermite patches $\mathbf{D}_{x3}(u, v)$ in the equation $\mathbf{D}_{x3}(u, v) = (1-v)\,\mathbf{P}_{x31}(u) + v\,\mathbf{q}_{x32}(u)$ with $\mathbf{P}_{x31}(u) = \mathbf{a}_{x31}u^3 + \mathbf{b}_{x31}u^2 + \mathbf{c}_{x31}u + \mathbf{d}_{x31}$

and $\mathbf{q}_{x32}(u) = \mathbf{a}_{x32}u^3 + \mathbf{b}_{x32}u^2 + \mathbf{c}_{x32}u + \mathbf{d}_{x32}$ for u, and v in interval $0 \le u, v \le 1$. Using the formula (2.2), these boundary curves must apply $\mathbf{q}^u_{x32}(u) = \rho\ \mathbf{P}^u_{x31}(u)$ or $3(\mathbf{a}_{x32} - \rho\mathbf{a}_{x31})\ u^2 + 2(\mathbf{b}_{x32} - \rho\mathbf{b}_{x31})\ u + (\mathbf{c}_{x32} - \rho\mathbf{c}_{x31}) = \mathbf{0}$. Due to the values $[1, u, u^2]$ are different from zero, then $(\mathbf{a}_{x32} - \rho\mathbf{a}_{x31}) = \mathbf{0}$, $(\mathbf{b}_{x32} - \rho\mathbf{b}_{x31}) = \mathbf{0}$, and $(\mathbf{c}_{x32} - \rho\mathbf{c}_{x31}) = \mathbf{0}$. These mean that

$$-x^2(\mathbf{q}_1 - \mathbf{q}_o) + (\mathbf{q}_x - \mathbf{q}_o) + (x^2 - x)\ \mathbf{q}^u_o = \rho[-x^2(\mathbf{P}_1 - \mathbf{P}_o) + (\mathbf{P}_x - \mathbf{P}_o) + (x^2 - x)\ \mathbf{P}^u_o];$$
$$x^3(\mathbf{q}_1 - \mathbf{q}_o) - (\mathbf{q}_x - \mathbf{q}_o) - (x^3 - x)\ \mathbf{q}^u_o = \rho[\quad x^3(\mathbf{P}_1 - \mathbf{P}_o) - (\mathbf{P}_x - \mathbf{P}_o) - (x^3 - x)\ \mathbf{P}^u_o]$$
$$\mathbf{q}^u_o = \rho\mathbf{P}^u_o.$$

Thus, it must meet

$$\mathbf{q}^u_o = \rho\ \mathbf{P}^u_o; \quad (\mathbf{q}_1 - \mathbf{q}_o) = \rho\ (\mathbf{P}_1 - \mathbf{P}_o);\ (\mathbf{q}_x - \mathbf{q}_o) = \rho\ (\mathbf{P}_x - \mathbf{P}_o). \tag{3.7}$$

On the other hand, when the restriction $\mathbf{D}_{x4}(u,v) = (1-v)\ \mathbf{P}_{x41}(u) + v\ \mathbf{q}_{x42}(u)$ is $\mathrm{q}^{\mathrm{u}}_{\mathrm{x42}}(u) = \rho\ \mathbf{P}^u_{x41}(u)$, then, from Equation (3.6), the patches have to fulfill

$$\mathbf{q}^u_o = \rho\ \mathbf{P}^u_o; \quad \mathbf{q}^u_1 = \rho\ \mathbf{P}^u_1;\ (\mathbf{q}_1 - \mathbf{q}_o) = \rho\ (\mathbf{P}_1 - \mathbf{P}_o); \quad (\mathbf{q}_x - \mathbf{q}_o) = \rho\ (\mathbf{P}_x - \mathbf{P}_o). \tag{3.8}$$

Therefore, the cubic and quartic developable Hermite pieces $\mathbf{D}_{x3}(u,v)$ and $\mathbf{D}_{x4}(u,v)$, as shown in Figure 3(a) and Figure 3(c), can be designed in the following procedure.

1. Determine the vectors $(\mathbf{P}_1 - \mathbf{P}_o)//(\mathbf{q}_1 - \mathbf{q}_o)$, respectively, in the planes $\Upsilon_1//\Upsilon_2$, and elect the control point $\mathbf{P}_x$. In this case, to make a simple calculation, it can state $\mathbf{P}_x$ in the form $\mathbf{P}_x = [(1-x)\ \mathbf{P}_o + \ x\ \mathbf{P}_1] + < 0, 0, z >$ with $0 < x < 1$.
2. Calculate $\rho = |(\mathbf{q}_1 - \mathbf{q}_o)|/|(\mathbf{P}_1 - \mathbf{P}_o)|$ and $\mathbf{q}_x = \rho\ (\mathbf{P}_x - \mathbf{P}_o) + \mathbf{q}_o$ such that $(\mathbf{P}_x - \mathbf{P}_o)//(\mathbf{q}_x - \mathbf{q}_o)$;
3. Determine the tangent vectors data $[\mathbf{P}^u_o, \mathbf{P}^u_1]$ of the boundary curve $\mathbf{P}_x(u)$ and compute the tangent vectors $[\mathbf{q}^u_o, \mathbf{q}^u_1]$ of the boundary curve $\mathbf{q}_x(u)$ via Equation (3.7) for $\mathbf{D}_{x3}(u,v)$, and Equation (3.8) for $\mathbf{D}_{x4}(u,v)$;
4. Substitute, correspondingly, the calculated curve results $\mathbf{P}_x(u)$ and $\mathbf{q}_x(u)$ into the equation of the developable Hermite patches $\mathbf{D}_{x3}(u,v)$ and $\mathbf{D}_{x4}(u,v)$.

To implement the procedure, this study simulates the data as follows. Let the control points $\mathbf{P}_o = <10,-40, 10>$, $\mathbf{P}_x = <10,24,38>$ at $u = x = 0.8$, $\mathbf{P}_1 = <10,40,25>$, $\mathbf{q}_o = <-20,-90,15>$, and $\mathbf{q}_1 = <-20,70,45>$ such that $(\mathbf{P}_1 - \mathbf{P}_o)//(\mathbf{q}_1 - \mathbf{q}_o)$, $\rho = 2$, and $\mathbf{q}_x = <-20,38,71>$. Determining $\mathbf{P}^u_o = <0,10,40>$ and applying Equation (3.7) will find $\mathbf{q}^u_o = <0,20,80>$. The cubic Hermite curves $\mathbf{P}_{x3}(u)$, and $\mathbf{q}_{x3}(u)$ are, therefore,

$\mathbf{P}_{x3}(u) = <10, -87.5u^3 + 157.5u^2 + 10u - 40, -93.75u^3 + 68.75u^2 + 40u + 10>;$

$\mathbf{q}_{x3}(u) = <-20, -75u^3 + 315u^2 + 20u - 90, -187.5u^3 + 137.5u^2 + 80u + 15>.$

Thus, with both curves, we can formulate the developable patch $\mathbf{D}_{x3}(u,v)$, and its graph presented in Figure 3(a). When $\mathbf{P}_x$ =<10,16,15.5> at $u = x = 0.7$, the graph shape will change as shown in Figure 3(b). In case the patch $\mathbf{D}_{x4}(u,v)$, if the control points data are fixed $\mathbf{P}_o$ = <10,−40,10>, $\mathbf{P}_x$ = <10,−16, 5.5> at $u = x = 0.3$, $\mathbf{P}_1$ = <10,40,25>, $\mathbf{q}_o$ = <−20,−90,15>, $\mathbf{q}_1$ = <−20,70,45 >, and the determined tangent vectors $\mathbf{P}_o^u$ = <0,100,100>, and $\mathbf{P}_1^u$ = <0,120,−100>, then it will obtain Figure 3(d).

This method offers a simple construction that can apply the intermediate control point for raising and lowering the patches' surface shape (Figure 3(a), 3(b), 3(c), 3(d)). Changing the value of the parameter x in interval $0 < x < 1$ or modifying the vector value of the intermediate control point $\mathbf{P}_x$ along the line segment $\mathbf{P}_o\mathbf{P}_1$ can be used to change the patches' slope (Figure 3(a), 3(c)). Applying the quartic developable Hermite $\mathbf{D}_{x4}(u,v)$, we can form the pieces in three surface arches (Figure 3(d)).

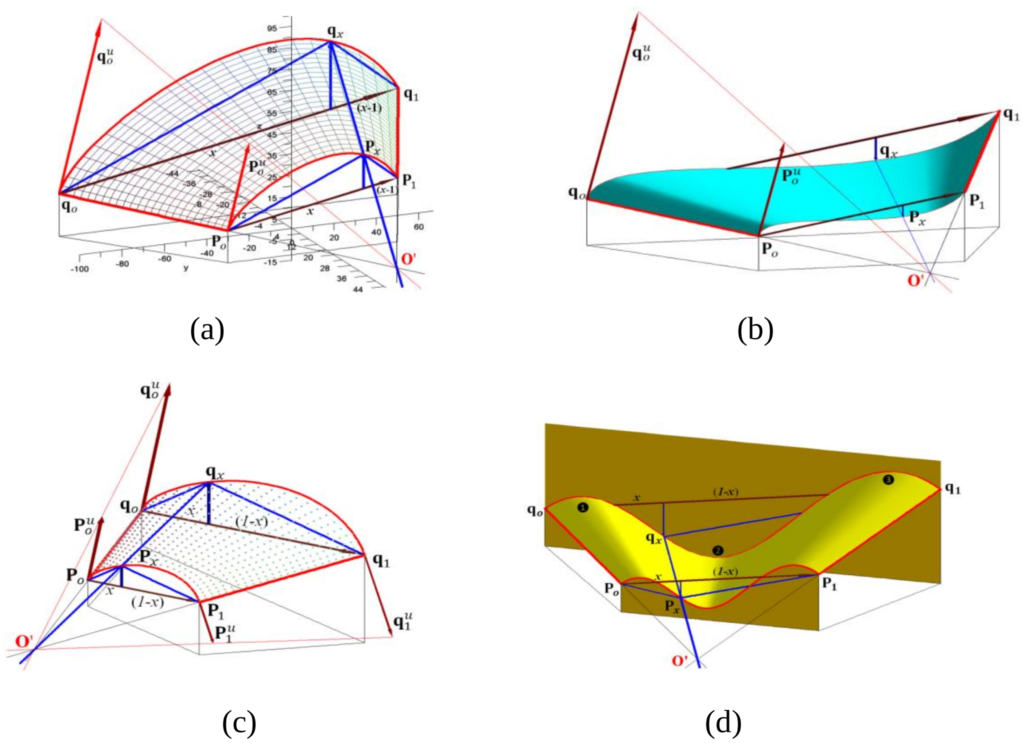


FIGURE 3. Cubic and quartic developable Hermite patches with boundary curves interpolated by one intermediate control point $\mathbf{P}_x$.

3.3. Patches Construction with Boundary Curves Interpolated by Two Intermediate Control Points and Tangent Vectors

To handle the plate sheets need the developable patches properties that can be adapted to the object's surface fluctuations and, locally, can be formed into many surface arches. For this purpose, in this section, first, it will introduce the formulations of quintic Hermite curves that are restricted by two intermediate points; after that, one tangent vector is posted at these intermediate points. Second, we will present, via these curves, a method to construct the developable patches that can have these adaptive properties.

Given a quintic polynomial curve $\mathbf{P}_{xy5}(u)$ with the data conditions at $\mathbf{P}_{xy5}(0) = \mathbf{P}_o$, $\mathbf{P}_{xy5}(1) = \mathbf{P}_1$, $\mathbf{P}^u_{xy5}(0) = \mathbf{P}^u_o$, $\mathbf{P}^u_{xy5}(1) = \mathbf{P}^u_1$, and at two intermediate control points of values $u = x$, and $u = y$ with $0 < x < y < 1$, the control points $\mathbf{P}_{xy5}(x) = \mathbf{P}_x$, $\mathbf{P}_{xy5}(y) = \mathbf{P}_y$. Using six equations of these conditions, and after calculating six unknown coefficients of curve $\mathbf{P}_{xy5}(u)$, it can be defined the algebraic representation of the quintic Hermite curve $\mathbf{P}_{xy5}(u)$ as follows

$$\mathbf{P}_{xy5}(u) = \mathbf{a}_{xy5}\, u^5 + \mathbf{b}_{xy5}\, u^4 + \mathbf{c}_{xy5}\, u^3 + \mathbf{d}_{xy5}\, u^2 + \mathbf{e}_{xy5}\, u + \mathbf{f}_{xy5}; \tag{3.9}$$

$\mathbf{a}_{xy5}=1/w[m_1(\mathbf{P}_1-\mathbf{P}_o)+m_2(\mathbf{P}_x-\mathbf{P}_o)+m_3(\mathbf{P}_y-\mathbf{P}_o)+m_4\mathbf{P}^u_o+m_5(3\mathbf{P}^u_o+\mathbf{P}^u_1)+m_6(2\mathbf{P}^u_o+\mathbf{P}^u_1)+m_7(\mathbf{P}^u_o+\mathbf{P}^u_1)]$;

$\mathbf{b}_{xy5}=1/w[n_1(\mathbf{P}_1-\mathbf{P}_o)+n_2(\mathbf{P}_x-\mathbf{P}_o)+n_3(\mathbf{P}_y-\mathbf{P}_o)+n_4\mathbf{P}^u_o+ n_5(\mathbf{P}^u_o+\mathbf{P}^u_1)+ n_6(2\mathbf{P}^u_o+\mathbf{P}^u_1)+ n_7(4\mathbf{P}^u_o+\mathbf{P}^u_1)]$;

$\mathbf{c}_{xy5}=1/w[r_1(\mathbf{P}_1-\mathbf{P}_o)+ r_2(\mathbf{P}_x-\mathbf{P}_o)+ r_3(\mathbf{P}_y-\mathbf{P}_o)+ r_4\mathbf{P}^u_o+ r_5(4\mathbf{P}^u_o+\mathbf{P}^u_1)+ r_6(\mathbf{P}^u_o+\mathbf{P}^u_1)+ r_7(3\mathbf{P}^u_o+\mathbf{P}^u_1)]$;

$\mathbf{d}_{xy5}=1/w[s_1(\mathbf{P}_1-\mathbf{P}_o)+ s_2(\mathbf{P}_x-\mathbf{P}_o)+ s_3(\mathbf{P}_y-\mathbf{P}_o)+ s_4\mathbf{P}^u_o+ s_5(4\mathbf{P}^u_o+\mathbf{P}^u_1)+ s_6(2\mathbf{P}^u_o+\mathbf{P}^u_1)+ s_7(3\mathbf{P}^u_o+\mathbf{P}^u_1)]$;

$\mathbf{e}_{xy5} = \mathbf{P}^u_o$; $\mathbf{f}_{xy5} = \mathbf{P}_o$;

and

$w = (x^2y^2).(x + y^2x - 2xy - y^3 - y + 2y^2).(1 + x^2 - 2x)$;

$m_1 = 3y^2x^4 - 3x^2y^4 + 4x^2y^3 - 4y^2x^3 + 2y^4x^3 - 2y^3x^4$; $m_2 = y^2 - 2y^3 + y^4$;

$m_3 = -x^2 - x^4 + 2x^3$; $m_4 = -xy^2 + yx^4 - 2yx^3 - y^4x + 2y^3x + yx^2$;

$m_5 = x^3y^2 - x^2y^3$; $m_6 = -x^4y^2 + x^2y^4$; $m_7 = y^3x^4 - y^4x^3$;

$n_1 = 2y^3x^5 - 3y^2x^5 - 2y^5x^3 + 5x^3y^2 + 3y^5x^2 - 5x^2y^3$; $n_2 = -y^5 + 3y^3 - 2y^2$;

$n_3 = -3x^3 + 2x^2 + x^5$; $n_4 = 2xy^2 - yx^5 + 3yx^3 + y^5x - 3y^3x - 2yx^2$;

$n_5 = -y^3x^5 + y^5x^3$; $n_6 = -y^5x^2 + x^5y^2$; $n_7 = -x^3y^2 + x^2y^3$;

$r_1 = -4y^5x^2 + 5y^4x^2 + 2y^5x^4 - 2y^4x^5 + 4y^2x^5 - 5y^2x^4$; $r_2 = y^2 + 2y^5 - 3y^4$;

$r_3 = -x^2 + 3x^4 - 2x^5$; $r_4 = yx^2 + 3y^4x - 2y^5x - xy^2 - 3yx^4 + 2yx^5$;

$r_5 = -x^2y^4 + x^4y^2$; $r_6 = -y^5x^4 + y^4x^5$; $r_7 = y^5x^2 - y^2x^5$;

$s_1 = -3y^5x^4 + 5y^3x^4 + 3y^4x^5 + 4y^5x^3 - 4y^3x^5 - 5y^4x^3$; $s_2 = -y^5 + 2y^4 - y^3$;

$s_3 = -2x^4 + x^5 + x^3$; $s_4 = -yx^3 - 2y^4x + y^5x + 2yx^4 - yx^5 + y^3x$;

$s_5 = -y^3x^4 + y^4x^3$; $s_6 = y^5x^4 - y^4x^5$; $s_7 = y^3x^5 - y^5x^3$.

In this part, we will define the quintic developable Hermite patches $\mathbf{D}_{xy5}(u,v) = (1-v)\,\mathbf{P}_{xy51}(u) + v\,\mathbf{q}_{xy52}(u)$ for u, and v in interval $0 \le u,v \le 1$. The boundary curves $\mathbf{P}_{xy51}(u)$ and $\mathbf{q}_{xy52}(u)$ of Equation (3.9) must be laid in the planes $\Upsilon_1 // \Upsilon_2$, respectively, and

$$\mathbf{P}_{xy51}(u) = \mathbf{a}_{xy51}\,u^5 + \mathbf{b}_{xy51}\,u^4 + \mathbf{c}_{xy51}\,u^3 + \mathbf{d}_{xy51}\,u^2 + \mathbf{e}_{xy51}\,u + \mathbf{f}_{xy51};$$

$$\mathbf{q}_{xy52}(\mathrm{u}) = \mathbf{a}_{xy52}\,u^5 + \mathbf{b}_{xy52}\,u^4 + \mathbf{c}_{xy52}\,u^3 + \mathbf{d}_{xy52}\,u^2 + \mathbf{e}_{xy52}\,u + \mathbf{f}_{xy52};$$

with

$\mathbf{a}_{xy51} = 1/w\,[m_1(\mathbf{P}_1-\mathbf{P}_o) + m_2(\mathbf{P}_x-\mathbf{P}_o) + m_3(\mathbf{P}_y-\mathbf{P}_o) + m_4\mathbf{P}_o^u + m_5(3\mathbf{P}_o^u+\mathbf{P}_1^u) + m_6(2\mathbf{P}_o^u+\mathbf{P}_1^u) + m_7(\mathbf{P}_o^u+\mathbf{P}_1^u)]$;

$\mathbf{b}_{xy51} = 1/w\,[n_1(\mathbf{P}_1-\mathbf{P}_o) + n_2(\mathbf{P}_x-\mathbf{P}_o) + n_3(\mathbf{P}_y-\mathbf{P}_o) + n_4\mathbf{P}_o^u + n_5(\mathbf{P}_o^u+\mathbf{P}_1^u) + n_6(2\mathbf{P}_o^u+\mathbf{P}_1^u) + n_7(4\mathbf{P}_o^u+\mathbf{P}_1^u)]$;

$\mathbf{c}_{xy51} = 1/w\,[r_1(\mathbf{P}_1-\mathbf{P}_o) + r_2(\mathbf{P}_x-\mathbf{P}_o) + r_3(\mathbf{P}_y-\mathbf{P}_o) + r_4\mathbf{P}_o^u + r_5(4\mathbf{P}_o^u+\mathbf{P}_1^u) + r_6(\mathbf{P}_o^u+\mathbf{P}_1^u) + r_7(3\mathbf{P}_o^u+\mathbf{P}_1^u)]$;

$\mathbf{d}_{xy51} = 1/w\,[s_1(\mathbf{P}_1-\mathbf{P}_o) + s_2(\mathbf{P}_x-\mathbf{P}_o) + s_3(\mathbf{P}_y-\mathbf{P}_o) + s_4\mathbf{P}_o^u + s_5(4\mathbf{P}_o^u+\mathbf{P}_1^u) + s_6(2\mathbf{P}_o^u+\mathbf{P}_1^u) + s_7(3\mathbf{P}_o^u+\mathbf{P}_1^u)]$;

$\mathbf{e}_{xy51} = \mathbf{P}_o^u$; $\mathbf{f}_{xy51} = \mathbf{P}_O$;

$\mathbf{a}_{xy52} = 1/w.\,[m_1(\mathbf{q}_1-\mathbf{q}_o) + m_2(\mathbf{q}_x-\mathbf{q}_o) + m_3(\mathbf{q}_y-\mathbf{q}_o) + m_4\mathbf{q}_o^u + m_5(3\mathbf{q}_o^u+\mathbf{q}_1^u) + m_6(2\mathbf{q}_o^u+\mathbf{q}_1^u) + m_7(\mathbf{q}_o^u+\mathbf{q}_1^u)]$;

$\mathbf{b}_{xy52} = 1/w.\,[n_1(\mathbf{q}_1-\mathbf{q}_o) + n_2(\mathbf{q}_x-\mathbf{q}_o) + n_3(\mathbf{q}_y-\mathbf{q}_o) + n_4\mathbf{q}_o^u + n_5(\mathbf{q}_o^u+\mathbf{q}_1^u) + n_6(2\mathbf{q}_o^u+\mathbf{q}_1^u) + n_7(4\mathbf{q}_o^u+\mathbf{q}_1^u)]$;

$\mathbf{c}_{xy52} = 1/w.\,[r_1(\mathbf{q}_1-\mathbf{q}_o) + r_2(\mathbf{q}_x-\mathbf{q}_o) + r_3(\mathbf{q}_y-\mathbf{q}_o) + r_4\mathbf{q}_o^u + r_5(4\mathbf{q}_o^u+\mathbf{q}_1^u) + r_6(\mathbf{q}_o^u+\mathbf{q}_1^u) + r_7(3\mathbf{q}_o^u+\mathbf{q}_1^u)]$;

$\mathbf{d}_{xy52} = 1/w.\,[s_1(\mathbf{q}_1-\mathbf{q}_o) + s_2(\mathbf{q}_x-\mathbf{q}_o) + s_3(\mathbf{q}_y-\mathbf{q}_o) + s_4\mathbf{q}_o^u + s_5(4\mathbf{q}_o^u+\mathbf{q}_1^u) + s_6(2\mathbf{q}_o^u+\mathbf{q}_1^u) + s_7(3\mathbf{q}_o^u+\mathbf{q}_1^u)]$;

$\mathbf{e}_{xy52} = \mathbf{q}_o^u$; $\mathbf{f}_{xy52} = \mathbf{q}_O$.

Due to the patch $\mathbf{D}_{xy5}(u,v)$ is developable, then, from the criteria equation (2.2), their boundary curves must meet $\mathbf{q}_{xy52}^u(u) = \rho\;\mathbf{P}_{xy51}^u$, that is

$$5(\mathbf{a}_{xy52} - \rho\, \mathbf{a}_{xy51})\, u^4 + 4(\mathbf{b}_{xy52} - \rho\, \mathbf{b}_{xy51})\, u^3 + 3(\mathbf{c}_{xy52} - \rho\, \mathbf{c}_{xy51})\, u^2 + 2(\mathbf{d}_{xy52} - \rho\, \mathbf{d}_{xy51})\, u + (\mathbf{e}_{xy52} - \rho\, \mathbf{e}_{xy51}) = \mathbf{0}.$$

Consequently, the coefficients of this equation must be zero, or

$$\text{(3.10)} \quad \mathbf{q}_o^u = \rho \mathbf{P}_o^u;\ \mathbf{q}_1^u = \rho \mathbf{P}_1^u;\ (\mathbf{q}_1 - \mathbf{q}_o) = \rho(\mathbf{P}_1 - \mathbf{P}_o);\ (\mathbf{q}_x - \mathbf{q}_o) = \rho(\mathbf{P}_x - \mathbf{P}_o);\ (\mathbf{q}_y - \mathbf{q}_o) = \rho(\mathbf{P}_y - \mathbf{P}_o).$$

These mean that $\mathbf{P}_o^u // \mathbf{q}_o^u$; $\mathbf{P}_1^u // \mathbf{q}_1^u$; $(\mathbf{P}_1 - \mathbf{P}_o)//(\mathbf{q}_1 - \mathbf{q}_o)$; $(\mathbf{P}_x - \mathbf{P}_o)//(\mathbf{q}_x - \mathbf{q}_o)$; $(\mathbf{P}_y - \mathbf{P}_o)//(\mathbf{q}_y - \mathbf{q}_o)$; and the value $|\mathbf{q}_o^u|/|\mathbf{P}_o^u| = |\mathbf{q}_1^u|/|\mathbf{P}_1^u| = |(\mathbf{q}_1 - \mathbf{q}_o)|/|(\mathbf{P}_1 - \mathbf{P}_o)| = |(\mathbf{q}_x - \mathbf{q}_o)|/|(\mathbf{P}_x - \mathbf{P}_o)| = |(\mathbf{q}_y - \mathbf{q}_o)| / |(\mathbf{P}_y - \mathbf{P}_o)| = \rho$. Following the presented method and the criteria of Equation (3.10), this section will demonstrate the construction of the developable patches in this way.

Let the data $\mathbf{P}_o = \langle 20,-60,10 \rangle$, for $u = x = 0.4$ and $u = y = 0.6$ the control points $\mathbf{P}_x = \langle 20,-16,21 \rangle$ and $\mathbf{P}_y$ $\langle 20,6,9 \rangle$, $\mathbf{P}_1 = \langle 20,50,25 \rangle$, $\mathbf{q}_o = \langle -20,-90,15 \rangle$, and $\mathbf{q}_1 = \langle -20,70,36.8 \rangle$ such that $(\mathbf{P}_1 - \mathbf{P}_o)//(\mathbf{q}_1 - \mathbf{q}_o)$, $\rho = 1.45$, $\mathbf{q}_x = \langle -20,-26,31 \rangle$, and $\mathbf{q}_y = \langle -20,6,13.6 \rangle$. The tangent vectors are elected $\mathbf{P}_o^u = \langle 0,90,-90 \rangle$, $\mathbf{P}_1^u = \langle 0,90,-100 \rangle$, and after calculating Equation (3.10) will find $\mathbf{q}_o^u = \langle 0,130.5,-130.5 \rangle$, and $\mathbf{q}_1^u = \langle 0,130.5,-145 \rangle$ that are shown in Figure 4(a). From Equation (3.9), it can obtain the quintic Hermite curves

$$\mathbf{P}_{xy51}(u) = \langle 20, -166.67\, u^5 + 416.67\, u^4 - 373.33\, u^3 + 143.33\, u^2 + 90\, u - 60, -2218.75\, u^5 + 5482.64\, u^4 - 4529.03\, u^3 + 1370.14\, u^2 - 90\, u + 10 \rangle;$$

$$\mathbf{q}_{xy52}(u) = \langle -20, -245.83\, u^5 + 614.58\, u^4 - 550.67\, u^3 + 211.42\, u^2 + 130.5\, u - 90, -3217.77\, u^5 + 7951.28\, u^4 - 6568.40\, u^3 + 1984.20\, u^2 - 130.5\, u + 15 \rangle.$$

As a result, the quintic developable Hermite patch can be shown in Figure 4(b). While, Figure 4(c) present that along the boundary curves $\mathbf{P}_{xy51}(u)$, and $\mathbf{q}_{xy52}(u)$, its tangent vectors, and the generatrixes of the patch are coplanar.

These developable patches type provide some advantages. Changing the vector values and position of the intermediate control points $\mathbf{P}_x$ and $\mathbf{P}_y$ or modifying the values x and y in interval $0 < x < y < 1$ can modify the patch shapes more than one slope and fluctuation (Figure 4(b), 4(d)). Consequently, it can design the pieces in four arches of surface oscillation variation (Figure 4(b)).

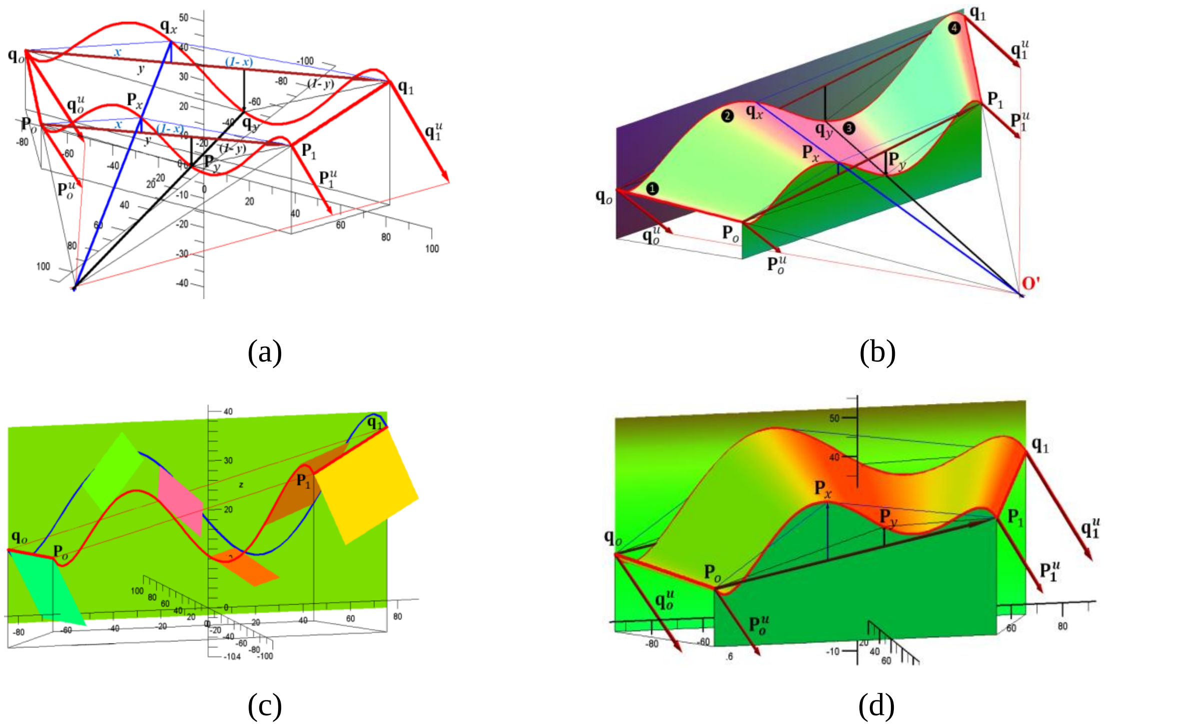


FIGURE 4. Quintic developable Hermite patches with boundary curves interpolated by two intermediate control points $\mathbf{P}_x$ and $\mathbf{P}_y$.

Consider a quintic polynomial curve $\mathbf{P}_{x5}(u)$ in which at $\mathbf{P}_{x5}(0) = \mathbf{P}_o$, $\mathbf{P}_{x5}(1) = \mathbf{P}_1$, $\mathbf{P}^u_{x5}(0) = \mathbf{P}^u_o$, $\mathbf{P}^u_{x5}(1) = \mathbf{P}^u_1$, at values $u = x$ with $0 < x < 1$ the control points $\mathbf{P}_{x5}(x) = \mathbf{P}_x$ and $\mathbf{P}^u_{x5}(x) = \mathbf{P}^u_x$. Using these six equations, and after calculating six unknown coefficients of the curve $\mathbf{P}_{x5}(u)$, it can discover the algebraic representation of this curve as follows

(3.11) $$\mathbf{P}_{x5}(u) = \mathbf{a}_{x5}\,u^5 + \mathbf{b}_{x5}\,u^4 + \mathbf{c}_{x5}\,u^3 + \mathbf{d}_{x5}\,u^2 + \mathbf{e}_{x5}\,u + \mathbf{f}_{x5};$$

with

$$\mathbf{a}_{x5} = k_1[(4x^3-2x^4)(\mathbf{P}_1-\mathbf{P}_o)+(2-4x)(\mathbf{P}_x-\mathbf{P}_o)+(x^2-x)\mathbf{P}^u_x+(3x^2-x)\mathbf{P}^u_1-x^3(3\mathbf{P}^u_o+\mathbf{P}^u_1)+x^4(\mathbf{P}^u_o+\mathbf{P}^u_1)];$$

$$\mathbf{b}_{x5} = k_1[(-5x^3-5x^4+4x^5)(\mathbf{P}_1-\mathbf{P}_o)+(5x^2+5x-4)(\mathbf{P}_x-\mathbf{P}_o)+(2x-x^2-x^3)\mathbf{P}^u_1-2x^5(\mathbf{P}^u_o+\mathbf{P}^u_1)+ x^4(4\mathbf{P}^u_o+\mathbf{P}^u_1)+(2x-4x^2)\mathbf{P}^u_o+x^3\mathbf{P}^u_1)];$$

$$\mathbf{c}_{x5} = k_1[(-2x^5+10x^4-2x^6)(\mathbf{P}_1-\mathbf{P}_o)-(10x^2-2x-2)(\mathbf{P}_x-\mathbf{P}_o)+(2x^3-x^2-x)\mathbf{P}^u_x+(8x^3-x^2-x)\mathbf{P}^u_o+ (x^6+x^5)(\mathbf{P}^u_1+\mathbf{P}^u_o)-2x^4(\mathbf{P}^u_1+4\mathbf{P}^u_o)];$$

$$\mathbf{d}_{x5} = k_2[-(5x^4-3x^5)(\mathbf{P}_1-\mathbf{P}_o)-(3-5x)(\mathbf{P}_x-\mathbf{P}_o)-(x^2-x)\mathbf{P}^u_x+(2x-4x^2)\mathbf{P}^u_o-x^5(\mathbf{P}^u_1+2\mathbf{P}^u_o)+ x^4(\mathbf{P}^u_1+4\mathbf{P}^u_o)];$$

$\mathbf{e}_{x5} = \mathbf{P}_o^u$; $\mathbf{f}_{x5} = \mathbf{P}_o$; $k_1 = \frac{1}{(x^3(x-1)(1-2x+x^2))}$; $k_2 = \frac{1}{(x^2(x-1)(1-2x+x^2))}$.

Let the Hermite curves $\mathbf{P}_{x51}(u)$, and $\mathbf{q}_{x52}(u)$ of Equation (3.11). We will construct the quintic developable Hermite patches $\mathbf{D}_{x5}(u,v) = (1-v)\,\mathbf{P}_{x51}(u) + v\,\mathbf{q}_{x52}(u)$ with u, v in interval $0 \leq u, v \leq 1$. The curves $\mathbf{P}_{x51}$(u), and $\mathbf{q}_{x52}$(u) are in the planes $\Upsilon_1 // \Upsilon_2$, respectively, with $\mathbf{P}_{x51}(u) = \mathbf{a}_{x51}\,u^5 + \mathbf{b}_{x51}\,u^4 + \mathbf{c}_{x51}\,u^3 + \mathbf{d}_{x51}\,u^2 + \mathbf{e}_{x51}\,u + \mathbf{f}_{x51}$ and $\mathbf{q}_{x52}(u) = \mathbf{a}_{x52}\,u^5 + \mathbf{b}_{x52}\,u^4 + \mathbf{c}_{x52}\,u^3 + \mathbf{d}_{x52}\,u^2 + \mathbf{e}_{x52}\,u + \mathbf{f}_{x52}$. From Equation (2.2), the criteria of the developable pieces $\mathbf{D}_{x5}(u,v)$ is $\mathbf{q}_{x52}^u(u) = \rho\mathbf{P}_{x51}^u(u)$. As a result, it can state the equation

$$5(\mathbf{a}_{x52} - \rho\mathbf{a}_{x51})\,u^4 + 4(\mathbf{b}_{x52} - \rho\mathbf{b}_{x51})\,u^3 + 3(\mathbf{c}_{x52} - \rho\mathbf{c}_{x51})\,u^2 + 2(\mathbf{d}_{x52} - \rho\mathbf{d}_{x51})\,u + (\mathbf{e}_{x52} - \rho\mathbf{e}_{x51}) = \mathbf{0}.$$

Consequently, this condition can be simplified in this manner

$$(3.12) \qquad \mathbf{q}_o^u = \rho\mathbf{P}_o^u; \quad \mathbf{q}_x^u = \rho\mathbf{P}_x^u; \quad \mathbf{q}_1^u = \rho\mathbf{P}_1^u; \quad (\mathbf{q}_1 - \mathbf{q}_o) = \rho(\mathbf{P}_1 - \mathbf{P}_o); \ (\mathbf{q}_x - \mathbf{q}_o) = \rho(\mathbf{P}_x - \mathbf{P}_o).$$

Thus, it must fulfill $\mathbf{P}_o^u // \mathbf{q}_o^u$, $\mathbf{P}_x^u // \mathbf{q}_x^u$, $\mathbf{P}_1^u // \mathbf{q}_1^u$, $(\mathbf{P}_1 - \mathbf{P}_o) // (\mathbf{q}_1 - \mathbf{q}_o)$, $(\mathbf{P}_x - \mathbf{P}_o) // (\mathbf{q}_x - \mathbf{q}_o)$ and $|\mathbf{q}_o^u|/|\mathbf{P}_o^u| = |\mathbf{q}_x^u|/|\mathbf{P}_x^u| = |\mathbf{q}_1^u|/|\mathbf{P}_1^u| = |(\mathbf{q}_1 - \mathbf{q}_o)|/|(\mathbf{P}_1 - \mathbf{P}_o)| = |(\mathbf{q}_x - \mathbf{q}_o)|/|(\mathbf{P}_x - \mathbf{P}_o)| = \rho$. If in Equation (3.12) we give a tension value k_1, k, and k_2 to the tangent vectors $\mathbf{P}_o^u$, $\mathbf{P}_x^u$, and $\mathbf{P}_1^u$, correspondingly, then, the criteria equation will be

$$(3.13) \qquad \mathbf{q}_o^u = \rho\mathbf{P}_o^{u*}; \quad \mathbf{q}_x^u = \rho\mathbf{P}_x^{u*}; \ \mathbf{q}_1^u = \rho\mathbf{P}_1^{u*}; (\mathbf{q}_1 - \mathbf{q}_o) = \rho(\mathbf{P}_1 - \mathbf{P}_o); (\mathbf{q}_x - \mathbf{q}_o) = \rho(\mathbf{P}_x - \mathbf{P}_o)$$

with $\mathbf{P}_o^{u*} = k_1\mathbf{P}^u$, $\mathbf{P}_x^{u*} = k\mathbf{P}_x^{u*}$, and $\mathbf{P}_1^{u*} = k_2\mathbf{P}^u$.

Some application examples of the equations (3.12) and (3.13) in designing the quintic developable Hermite patches $\mathbf{D}_{x5}(u,v)$ are presented as follows. Given the data $\mathbf{P}_o$ = <20,−60,10>, $\mathbf{P}_1$ =<20,50,25>, $\mathbf{q}_o$ =<−20,−90,15>, $\mathbf{q}_1$ =<−20,70,36.8>, and at $u = x = 0.5$ the control points $\mathbf{P}_x$ =<20,−5,27.5> such that $(\mathbf{P}_1 - \mathbf{P}_o) // (\mathbf{q}_1 - \mathbf{q}_o)$, ρ = 1.45, and $\mathbf{q}_x^u$ =<−20,−10,40.4>. The tangent vectors are determined $\mathbf{P}_o^u$ =< 0,90,90>, $\mathbf{P}_x^u$ =<0,45,45>, $\mathbf{P}_1^u$ =< 0,90,−100>, and after computing Equation (3.12), it can find $\mathbf{q}_o^u$ =< 0,130.5,130.5>, $\mathbf{q}_x^u$ =<0,65.3,65.3>, and $\mathbf{q}_1^u$ =<0,130.5,−145>. When these results are inserted in Equation (3.11), the curve equations will be

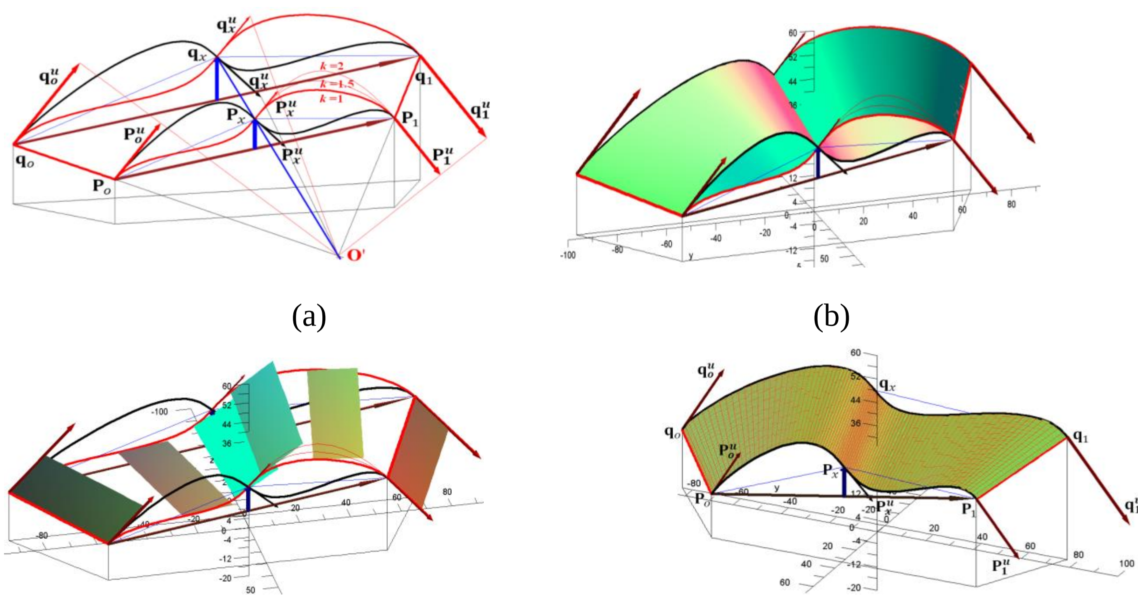
$$\mathbf{P}_{x51}(u) = \langle 20, -1200u^5+3000u^4-2440u^3+660u^2+90u, -60u^5-1020u^4+1040u^3-415u^2+90u+10 \rangle$$

and

$$\mathbf{q}_{x52}(u) = \langle -20, -1752u^5+4380u^4-3563u^3+965u^2+131u, -90u^5-1474u^4+1505u^3-601u^2+131u+15 \rangle$$

that are shown in red color curves on Figure 5(a). In this case, when the value $\mathbf{P}_x^u$ is be replaced by the vector $\mathbf{P}_x^u$ =<0,60,−50>, the boundary curves $\mathbf{P}_{x51}(u)$ and $\mathbf{q}_{x52}(u)$ will change as exposed in black color curves in the Figure 5(a). Both data calculations build the developable patches graph, as represented in Figure 5(b). On the other hand, if at the control point $\mathbf{P}_x$, the vector tangent $\mathbf{P}_x^u$ is defined from the different tension values k, then the founded curve shapes at $\mathbf{P}_x$ will be vary (Figure 5(a), 5(b), 5(c)). To ensure the designed pieces are developable, we show in Figure 5(c) that, along the parameter u of the curves $\mathbf{P}_{x51}(u)$ and $\mathbf{q}_{x52}(u)$, its tangent vectors and the generatrixes are coplanar.

The patches offer some advantages. The intermediate control point $\mathbf{P}_x$ can be used to modify the slope and the surface shape in the middle position of the pieces (Figure 5(a), 5(b), 5(c), 5(d)). Changing the tension tangent vector at $\mathbf{P}_x$ can create various shapes of the pieces. Moreover, the patches can be modeled in three surface arches (Figure 5(d)).



(a) (b) (c) (d)

FIGURE 5. Quintic developable Hermite patches interpolated by one intermediate control point $\mathbf{P}_x$ with one tangent vector $\mathbf{P}_x^u$.

3.4. Connection between Two Developable Hermite Patches

In plat-metal-based industries and plywood sheet installation, the modeled object surfaces are, generally, composed of some developable patches. To obtain the best connections, continuity among these patches depends on their used continuity level. For this purpose, this section will discuss the continuous connection between two developable pieces adjacent.

Let two developable Hermite pieces $\mathbf{D}_1(u,v) = (1-v)\,\mathbf{P}_1(u) + v\,\mathbf{q}_1(u)$ and $\mathbf{D}_2(u,v) = (1-v)\mathbf{P}_2(u) + v\,\mathbf{q}_2(u)$ with u, v in interval $0 \le u, v \le 1$, and their boundary curves $\mathbf{P}(u)$, and $\mathbf{q}(u)$ in the planes $\Upsilon_1 // \Upsilon_2$, respectively. Two patches adjacent will be continuous connection geometrically G^0, if along the value $0 \le v \le 1$, they support in the common generatrix $\mathbf{D}_1(1,v) = \mathbf{D}_2(0,v) = \mathbf{g}(v)$, that is $\mathbf{P}_{11} + v\,(\mathbf{q}_{11} - \mathbf{P}_{11}) = \mathbf{P}_{o2} + v\,(\mathbf{q}_{o2} - \mathbf{P}_{o2}) = \mathbf{g}(v)$ with $\mathbf{P}_{11} = \mathbf{P}_1(1)$, $\mathbf{q}_{11} = \mathbf{q}_1(1)$, and $\mathbf{P}_{02} = \mathbf{P}_2(0)$, $\mathbf{q}_{02} = \mathbf{q}_2(0)$. Thus, they must meet

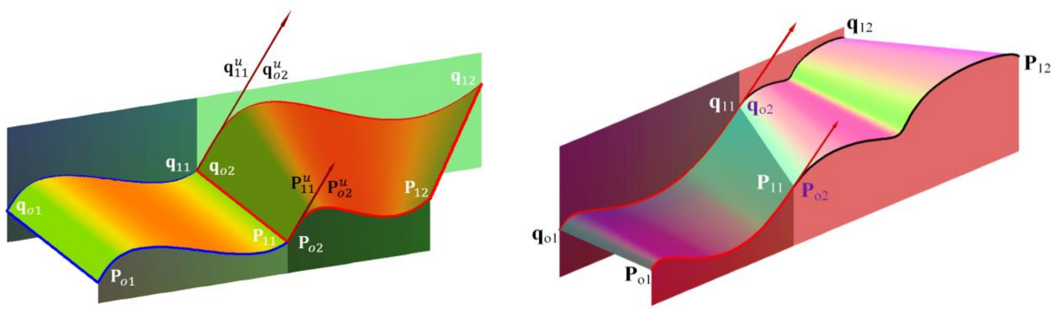
(3.14) $$\mathbf{P}_{11} = \mathbf{P}_{o2}; \quad \mathbf{q}_{11} = \mathbf{q}_{o2}.$$

Both pieces connection will also be continuous geometrically G^1, if at the points along the common generatrix $\mathbf{g}$(v), their tangent planes are equal (identical) for all values $0 \le v \le 1$. If we state $\mathbf{D}_1^u(1,0) = \mathbf{P}_{11}^u$, $\mathbf{D}_1^u(1,1) = \mathbf{q}_{11}^u$, and $\mathbf{D}_2^u(0,0) = \mathbf{P}_{o2}^u$, $\mathbf{D}_2^u(0,1) = \mathbf{q}_{o2}^u$, then, this condition can be simplified that, at the points along the common generatrix $\mathbf{g}(v)$, three vectors $[\mathbf{P}_{11}^u, \mathbf{q}_{11}^u, \mathbf{g}'(v)]$ and $[\mathbf{P}_{o2}^u, \mathbf{q}_{o2}^u, \mathbf{g}'(v)]$ are coplanar. Because of the vectors $\mathbf{P}_{11}^u // \mathbf{q}_{11}^u$, $\mathbf{P}_{o2}^u // \mathbf{q}_{o2}^u$, the vectors $[\mathbf{P}_{11}^u, \mathbf{P}_{o2}^u]$ in the plane Υ_1, and the vectors $[\mathbf{q}_{11}^u, \mathbf{q}_{o2}^u]$ in the plane Υ_1, it can deduce that the vectors $[\mathbf{P}_{11}^u, \mathbf{P}_{o2}^u]$ and $[\mathbf{q}_{11}^u, \mathbf{q}_{o2}^u]$ must be align. Thus, the condition G^1 is

(3.15) $$\mathbf{P}_{o2}^u = \alpha\ \mathbf{P}_{11}^u; \quad \mathbf{q}_{o2}^u = \beta\ \mathbf{q}_{11}^u$$

with α and $\beta = [(\rho_2/\rho_1).\alpha]$ the positive real scalars. To justify the implementation of these equations (3.14, 3.15), it will demonstrate in the following connection of patches.

Given the cubic developable Hermite patches adjacent $\mathbf{D}_1(u,v)$ and $\mathbf{D}_2(u,v)$ of cylinder and cone types respecvively. The first patch $\mathbf{D}_1(u,v)$ is constructed by the control points $\mathbf{P}_{o1}$=<30,−125,5>, $\mathbf{P}_{11}$=<30,−45,10>, $\mathbf{q}_{o1}$=<−20,−150,10>, and $\mathbf{q}_{11}$=<−20,−70,15> such that $(\mathbf{P}_{11} - \mathbf{P}_{o1}) // (\mathbf{q}_{11} - \mathbf{q}_{o1})$ and $\rho_1 = 1$. The tangent vectors are selected $\mathbf{P}_{o1}^u$ = <0,40,70>, $\mathbf{P}_{11}^u$=<0,40,70>, and using Equation (3.2) will find $\mathbf{q}_{o1}^u$=<0,40,70>, $\mathbf{q}_{11}^u$=<0,40,70>. The second patch $\mathbf{D}_2(u,v)$ is determined by the control points $\mathbf{P}_{o2}$ =<30,−45,10>, $\mathbf{P}_{12}$ =<30,15,20>, $\mathbf{q}_{o2}$ =<−20,−70,15>, and $\mathbf{q}_{12}$ =<−20,50,35> such that $(\mathbf{P}_{12}-\mathbf{P}_{o2})//(\mathbf{q}_{12}-\mathbf{q}_{o2})$ and $\rho_2 = 2$. The tangent vectors are selected $\mathbf{P}^u_{o2}$ =<0,40,70>, $\mathbf{P}^u_{12}$=<0,60, 40>, and calculating Equation (3.2) will obtain $\mathbf{q}^u_{o2}$ =<0,80,140>, $\mathbf{q}^u_{12}$ =<0,120,80>. As represented in Figure 6(a), the connection of both pieces are continuous G^0 with $\mathbf{P}_{11} = \mathbf{P}_{02}$ =<30,−45,10>, and $\mathbf{q}_{11} = \mathbf{q}_{o2}$ =<−20,−70,15>. They are also G^1 that fulfill $\mathbf{P}^u_{11} = \mathbf{P}^u_{o2}$= <0,40,70> with $\alpha = 1$, and [$\mathbf{q}^u_{o2}$=<0,80,140> = β $\mathbf{q}^u_{11}$ = 2. < 0, 40, 70 >]. Meanwhile, Figure 6(b) is other example of continuous connection G^1 between two cone pieces.



(a) (b)

FIGURE 6. Connection between two developable Hermite patches.

4. CONCLUSIONS

This paper introduced the formulas and procedures to design the developable Hermite patches. The surface shapes can be modeled by the endpoints, intermediate points, the tangent, and acceleration vectors at the endpoints of the patches' boundary curves. These methods provide some advantages for creating the various forms of developable pieces. First, the endpoints and the intermediate control points can be applied to raise and lower the patches' surface shape. Second, moving the intermediate control points, the tangent, and acceleration vectors are used to change the patches' different surface slopes. They can also be used to modify their shapes in two, three, or four surface arches. Finally, the connection G^1 between two developable Hermite pieces adjacent is determined by the existence of collinear tangent vectors at the joint points of their boundary curves. The exciting thing to discuss in the future is how to implement the developable pieces in the operation of plat-metal and plywood sheet installation, for example, in cutting and fitting the plates, also, in making a hole on the plates.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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