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NUMERICAL SOLUTION OF THE STERILE INSECT TECHNOLOGY MODEL FOR THE CONTROL OF ZIKA VIRUS VECTOR POPULATION

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Abstract. This work is aimed at determining a numerical solution to the Sterile Insect Technology (SIT) model for the control of zika virus vector population. SIT is an environmental friendly method which depends on the release of sterile male mosquitoes, that compete with the wild male mosquitoes and mate with wild female mosquitoes, which leads to production of no offspring and as such reduces the population of zika virus vector population over time. Differential Transformation Method (DTM) is employed to determine an approximate solution to the model. DTM has the capability of reducing computational size while maintaining solution accuracy at the same time providing a fast convergence rate. This method can be considered as a viable alternative mathematical tool for solving non-linear and linear problems in sciences and engineering.

Keywords: zika; technology; sterile; wild; vector; transformation.

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1. INTRODUCTION

Zika virus was first isolated in 1948 from a monkey found in the zika forest of Uganda[1].It belongs to the family of Flaviviridae and genus Flavivirus that is closed to yellow fever, dengue, West Nile Virus[1,2].The virus is usually transmitted in the day by the active Aedes Mosquitoes which are; Aedes aegypti and the Aedes albopictus[3]. Reports from the Centre for Disease Prevention and Control[4], shows that the disease is always accompanied by slight symptoms that are related to that of Dengue fever, Yellow fever, Chikungunya, Japanese encephalitis and West Nile[1,5,6,7].The virus was also reported to be transmitted sexually either through sexual activities, blood transmission and by vertical transmission, that is from pregnant woman to her baby during child birth[8,9,10].Vector population management and control remain the best way to control and prevent vector borne related diseases. Zika virus disease symptoms include fever, popular rash, typically low grade arthralgia.

Non-purulentConjunctivitis, myalgia, tiredness and headache[3].Oadema, Sore throat, vomiting, Uveitis and Lymphadenopathy are also considered by[3,4] as minor symptoms of the disease. The Sterile Insect Technique (SIT) is a kind of environmental vector control that is friendly and pleasant which is not harmful to the environment.Chemicals such as Insecticides, pesticides have been widely used over the years to control the population of mosquitoes, apart from the harmful effect of these chemicals to the surrounding environment, many vectors are becoming too resistant to these insecticides and secondly the World Health Organization[11], recommends a few number of these chemicals because of their harmful effects to human. As a better replacement to this insecticide control method, we present in this research work, the Sterile Insect Technique which does not cause any harm to human and his surrounding environment. It is a birth control that involves the mass rearing of the male Aedes Mosquitoes species in a factory, sterilized and treated with radiation such as X-rays and Gamma rays and then released into the field, the endemic area by air where they compete and mate with the wild female mosquitoes which results into laying of eggs without hatching thereby reducing the population of the Aedes Mosquitoes in the next generation, the release of the sterile male species of the Aedes Mosquitoes into the population of the wild mosquitoes will greatly eliminate the wild Aedes Mosquitoes population that cause Zika virus infection and as such eliminates the Zika

Virus disease from a population. The main aim of this work is to obtain an approximate solution to zika virus vector population control model using the Sterile Insect Technology,this is achievable using the Differential Transformation Method.

This Method was first introduced by Zhou[12] for solving non-linear and linear initial value problems in Electrical circuit analysis.The method was also applied to solve algebraic and differential equations by [13].Schrodinger equations were also solved by [14] using this technique.Solution of fractional differential equations is similarly presented by [15],while[16] presented the solution of free vibration analysis of rotating beams,usteady rolling motion of spheres equation inclined tubes using the Differential Transformation Method.In this present work, we present the solution of a vector control population Sterile Insect Technology (SIT) model presented by [17] using the Differential Transformation Method.

2. Mathematical Model

The mathematical equations that represent the Sterile Insect Technology(SIT) model for the control of zika virus vector is given by:

$$
\text{(1)} \qquad \begin{cases}
\dfrac{dA}{dt} = \Lambda_1 F_{NM} - \gamma A - \mu_A A - \mu_\rho A^2 \\
\dfrac{dF_M}{dt} = \phi\gamma A - (\beta_1\rho_\omega + \beta_2\rho_s)F_M - \mu_F F_M \\
\dfrac{dM_M}{dt} = (1-\phi)\gamma A - \mu_M M_M \\
\dfrac{dF_{NM}}{dt} = \beta_1\rho_\omega F_M - \mu_F F_{NM} \\
\dfrac{dF_{SM}}{dt} = \beta_2\rho_s F_M - \mu_F F_{SM} \\
\dfrac{dM_S}{dt} = pq\Lambda_2 - \mu_S M_S
\end{cases}
$$

2.1. Model variables.

Descriptions of the model variables used in this work is presented in table (1) below:

TABLE 1. Model variables and descriptions.

S/N	Variables	Descriptions
1	A	Aquatic mosquito class
2	M_M	Male mosquitoes(wild)
3	F_M	Female Mosquitoes not yet laying eggs
4	F_{NM}	Female non-sterile mosquitoes(can lay and hatch eggs)
5	F_{SM}	Female sterile mosquitoes(can lay but do not hatch)
6	M_S	Sterile male mosquitoes

2.2. Model parameters.

Descriptions of the model parameters used is presented in table (2) below:

TABLE 2. Model parameters and descriptions.

Parameters	Descriptions
Λ_1	Oviposition rate of fertilized female mosquitoes
ϕ	proportion of emerging female mosquitoes
$(1-\phi)$	Male Mosquitoes emerging population
β_i	Mating rate, where $i = 1, 2$
γ	Maturity rate of mosquitoes
μ_M	Natural death rate of wild male mosquitoes
μ_S	Natural death rate of sterile mosquitoes
μ_ρ	Density dependent death rate of aquatic mosquito class
μ_A	Natural death rate of aquatic mosquito class
μ_F	Natural death rate of female mosquitoes
ρ_ω	Female mosquito probability to mate with wild male mosquito
ρ_s	Female mosquito probability to mate with wild sterile male mosquito
p	Fraction of the released male that can join the wild mosquitoes
q	Mean mating competitiveness of the sterile

3. APPROXIMATE SOLUTION OF THE MODEL USING THE DIFFERENTIAL TRANSFORMATION METHOD(DTM)

Let $f(x)$ be an arbitrary function, then the function $f(x)$ can be expanded using Taylor series about a point $x = 0$ as:

$$f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}[\frac{d^k f}{dx^k}]_{x=0}$$

The differential transformation of the function $f(x)$ is given as:

$$F(x) = \frac{1}{k!}[\frac{d^k f}{dx^k}]_{x=0}$$

With the inverse differential transform given as:

$$f(x) = \sum_{k=0}^{\infty} x^k F(k)$$

In this section, we present in table (3), the fundamental mathematical operations performed by [16], while table (4) below shows, the value of model variables and parameters used for the implementation of the Differential Transformation Method (DTM).

Using also the transformed function of the original function presented in table (3), we now have a recurrence relation of the model (1) as presented in equation (2).

TABLE 3. The fundamental operations of Differential Transformation Method(DTM).

Original Function	**Transformed Function**
$y(x) = g(x)+h(x)$	$Y = G(K)+H(K)$
$y(x) = \alpha g(x)$	$Y = \alpha G(K)$
$y(x) = \frac{dg(x)}{dx}$	$Y(K) = (K+1)G(K+1)$
$y(x) = \frac{d^2g(x)}{dx^2}$	$Y(K) = (K+1)(K+2)G(K+2)$
$y(x) = \frac{d^mg(x)}{dx^m}$	$Y(K) = (K+1)(K+2)...(K+m)G(K+m)$
$y(x) = 1$	$Y(K) = \delta(K)$
$y(x) = x$	$Y(K) = \delta(K-1)$
$y(x) = x^m$	$Y(K) = \delta(K-m) = 1, if k = m$
$y(x) = x^m$	$Y(K) = \delta(K-m) = 0, if k \neq m$
$y(x) = g(x)h(x)$	$Y(K) = \sum_{m=0}^{k=0} H(m)g(k-m)$
$y(x) = \ell^{\lambda x}$	$Y(k) = \frac{\lambda^k}{k!}$
$y(x) = (Hx)^m$	$Y(k) = \frac{m(m-1)...(m-k+1)}{k!}$

TABLE 4. Initial Values of Variables and Parameters used.

Variables	**Numerical Value**	**Source**
$A(0)$	2500	Assumed
$M_M(0)$	150	Assumed
$F_M(0)$	650	Assumed
$F_{NM}(0)$	250	Assumed
$F_{SM}(0)$	100	Assumed
$M_M(0)$	200	Assumed
μ_ρ	0.00002	[1]
Λ_1	70	[5]
Λ_2	0.7	Assumed
ϕ	0.6	[5]
γ	0.00006	[18]
β_1	0.7	Assumed
ρ_ω	0.6	Assumed
ρ_s	0.4	Assumed
β_2	0.3	Assumed
μ_F	0.000057	Assumed
μ_A	0.00003	[19]
μ_M	0.000025	[20]
μ_S	0.00003	[21]
p	0.6	Assumed
q	0.4	Assumed

$$
(2) \quad \begin{cases}
A(K+1) = \frac{1}{k+1}[\Lambda_1 F_{NM}(k) - \gamma A(k) - \mu_A A(k) - \mu_\rho \sum_{m=0}^{k} A(m)A(k-m)] \\
F_M(K+1) = \frac{1}{k+1}[\phi\gamma A(k) - [\beta_1\rho_\omega + \beta_2\rho_s]F_M(k) - \mu_F F_M(k)] \\
M_M(K+1) = \frac{1}{k+1}[(1-\phi)\gamma A(k) - \mu_M M_M(k)] \\
F_{NM}(K+1) = \frac{1}{k+1}[\beta_1\rho_\omega F_M(k) - \mu_F F_{NM}(k)] \\
F_{SM}(K+1) = \frac{1}{k+1}[\beta_2\rho_s F_M(k) - \mu_F F_{SM}(k)] \\
M_S(K+1) = \frac{1}{k+1}[pq\Lambda_2 - \mu_S M_S(k)]
\end{cases}
$$

When $k = 0, m = 0$

Using the initial mosquitoes populations and parameter values tabulated in table(4), we have that:

$$
(3) \quad \begin{cases}
A(1) = [\Lambda_1 F_{NM}(0) - \gamma A(0) - \mu_A A(0) - \mu_\rho A(0)A(0)] \\
\qquad = [70(250) - 0.00006(2500) - 0.00003(2500) - 0.00002(2500)(2500)] = 17375 \\
F_M(1) = [\phi\gamma A(0) - [\beta_1\rho_\omega + \beta_2\rho_s]F_M(0) - \mu_F F_M(0)] \\
\qquad = [0.6(0.00006)(2500) - ((0.7)(0.6) + (0.3)(0.4))650 - 0.000025(650)] = -351 \\
M_M(1) = [(1-\phi)\gamma A(0) - \mu_M M_M(0)] \\
\qquad = [(1-0.6)(0.00006)(2500) - (0.000025)(150)] = 0.05625 \\
F_{NM}(1) = [\beta_1\rho_\omega F_M(0) - \mu_F F_{NM}(0)] \\
\qquad = [0.7(0.6)(650) - 0.000057(250)] = 273 \\
F_{SM}(1) = [\beta_2\rho_s F_M(0) - \mu_F F_{SM}(0)] \\
\qquad = [0.3(0.4)(650) - 0.000057(100)] = 78 \\
M_S(1) = [pq\Lambda_2 - \mu_S M_S(0)] \\
\qquad [(0.6)(0.4)(0.7) - 0.00003(200)] = 0.162
\end{cases}
$$

Therefore:

$$(4)\qquad \begin{cases} A(1) = 17375, F_M(1) = -351, M_M(1) = 0.05625, F_{NM}(1) = 273, \\ F_{SM}(1) = 78, M_S(1) = 0.162. \end{cases}$$

When $k = 1, m = 0, 1$:

(5)

$$\begin{cases} A(2) = \frac{1}{2}[\Lambda_1 F_{NM}(1) - \gamma A(1) - \mu_A A(0) - \mu_\rho (A(0)A(1) + A(1)A(0)] \\ \quad = \frac{1}{2}[70(273) - 0.00006(17375) - 0.00003(17375) - 0.00002[2500(17375) + 17375(2500)]] \\ \quad = 8685 \\ F_M(2) = \frac{1}{2}[\phi\gamma A(1) - [\beta_1\rho_\omega + \beta_2\rho_s]F_M(1) - \mu_F F_M(1)] \\ \quad = \frac{1}{2}[0.6(0.00006)(17375) - ((0.7)(0.6) + (0.3)(0.4))(-351) - 0.000025(-351)] = 95 \\ M_M(2) = \frac{1}{2}[(1-\phi)\gamma A(1) - \mu_M M_M(1)] \\ \quad = \frac{1}{2}[(1-0.6)(0.00006)(17375) - (0.000025)(0.05625)] = 0.208 \\ F_{NM}(2) = \frac{1}{2}[\beta_1\rho_\omega F_M(1) - \mu_F F_{NM}(1)] \\ \quad = \frac{1}{2}[0.7(0.6)(-351) - 0.000057(273)] = -74 \\ F_{SM}(2) = \frac{1}{2}[\beta_2\rho_s F_M(1) - \mu_F F_{SM}(1)] \\ \quad = \frac{1}{2}[0.3(0.4)(-351) - 0.000057(78)] = -21 \\ M_S(2) = \frac{1}{2}[pq\Lambda_2 - \mu_S M_S(1)] \\ \quad \frac{1}{2}[(0.6)(0.4)(0.7) - 0.00003(0.162)] = 0.0839 \end{cases}$$

Therefore:

$$(6)\qquad \begin{cases} A(2) = 8685, F_M(2) = 95, M_M(2) = 0.208, F_{NM}(2) = -74, \\ F_{SM}(2) = -21, M_S(2) = 0.0839 \end{cases}$$

When $k = 1, m = 0, 1, 2$:

(7)
$$\begin{cases} A(3) = \frac{1}{3}[\Lambda_1 F_{NM}(2) - \gamma A(2) - \mu_A A(2) - \mu_\rho [A(0)A(2) + A(2)A(1) + A(2)A(0)]] \\ \quad = \frac{1}{3}[70(-74) - 0.00006(8685) - 0.00003(8685) - 0.00002[2500(8685) + 8685(17375) \\ \quad + (8685)(2500)]] = 3022 \\ F_M(3) = \frac{1}{3}[\phi\gamma A(2) - [\beta_1\rho_\omega + \beta_2\rho_s]F_M(2) - \mu_F F_M(2)] \\ \quad = \frac{1}{3}[0.6(0.00006)(8685) - ((0.7)(0.6) + (0.3)(0.4))(95) - 0.000025(95)] = -17 \\ M_M(3) = \frac{1}{3}[(1-\phi)\gamma A(2) - \mu_M M_M(2)] \\ \quad = \frac{1}{3}[(1-0.6)(0.00006)(8685) - (0.000025)(0.208)] = 0.07 \\ F_{NM}(3) = \frac{1}{3}[\beta_1\rho_\omega F_M(1) - \mu_F F_{NM}(2)] \\ \quad = \frac{1}{3}[0.7(0.6)(95) - 0.000057(-74)] = 13.3 \\ F_{SM}(3) = \frac{1}{3}[\beta_2\rho_s F_M(2) - \mu_F F_{SM}(2)] \\ \quad = \frac{1}{3}[0.3(0.4)(95) - 0.000057(-21)] = 3.8 \\ M_S(3) = \frac{1}{3}[pq\Lambda_2 - \mu_S M_S(2)] \\ \quad \frac{1}{3}[(0.6)(0.4)(0.7) - 0.00003(0.0839)] = 0.0559 \end{cases}$$

Therefore:

(8)
$$\begin{cases} A(3) = 3022, F_M(3) = -17, M_M(3) = 0.07, F_{NM}(3) = 13.3, \\ F_{SM}(3) = 3.8, M_S(3) = 0.0559 \end{cases}$$

Then the closed form of the solution is presented in equation (9) below:

(9)
$$\begin{cases} A(t) = \sum_{m=0}^{k} A(t)t^k, F_M(t) = \sum_{m=0}^{k} F_M(t)t^k, M_M(t) = \sum_{m=0}^{k} M_M(t)t^k, \\ F_{NM}(t) = \sum_{m=0}^{k} F_{NM}(t)t^k, F_{SM}(t) = \sum_{m=0}^{k} F_{SM}(t)t^k, M_S(t) = \sum_{m=0}^{k} M_S(t)t^k. \end{cases}$$

Therefore, the closed form of the solution expressed in equation (9) above can be expanded and presented in equation (10), this represents the Model Approximate Solution using the Differential Transformation Method(DTM).

$$(10)\qquad \begin{cases} A(t) = 2500 + 17375t + 8685t^2 + 3022t^3 \\ F_M(t) = 650 - 351t + 95t^2 - 17t^3 \\ M_M(t) = 150 + 0.05625t + 0.208t^2 + 0.07t^3 \\ F_{NM}(t) = 250 + 273t - 74t^2 + 13.3t^3 \\ F_{SM}(t) = 100 + 78t - 21t^2 + 3.8t^3 \\ M_S(t) = 200 + 0.162t + 0.0839t^2 + 0.0559t^3 \end{cases}$$

4. CONCLUSION

In this work,Differential Transformation Method (DTM) has been used to determine the Approximate Solution of the zika virus vector population control model using the Sterile Insect Technology(SIT) with initial conditions.The method does not require discretization, linearization or perturbation as it is been applied directly and as such can be considered an easy alternative mathematical tool to solutions of linear and non linear problems that arise in Sciences and Technology.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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