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IMPLEMENTATION OF FUZZY OPTIMIZATION APPROACH TO FACULTATIVE WASTEWATER STABILIZATION PONDS PROBLEM CONSIDERING FUZZY PARAMETERS

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Abstract. This article proposed a decision-making tool in the form of a fuzzy programming approach to be used in determining the optimal decision on facultative stabilization ponds in a wastewater treatment facility. The specific problem was approaching the uncertain inflow rate of the wastewater to the pond as a fuzzy variable and this required the formulation of a mathematical optimization model containing the variables which was solved using fuzzy programming to achieve the optimal load of the wastewater and corresponding storing time as the optimal decision. Moreover, a numerical experiment was conducted using secondary and data generated from observing the Sewon wastewater treatment facility located in Yogyakarta, Indonesia containing four facultative ponds. The results produced the optimal decision with the wastewater load for each pond found to be 1199.5 kg while the corresponding storing time was 0.23, 0.23, 0.79, and 0.71 day for the pond I, II, III, and IV respectively.

Keywords: biological oxygen demand; decision-making; fuzzy optimization; wastewater management; wastewater treatment.

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1. INTRODUCTION

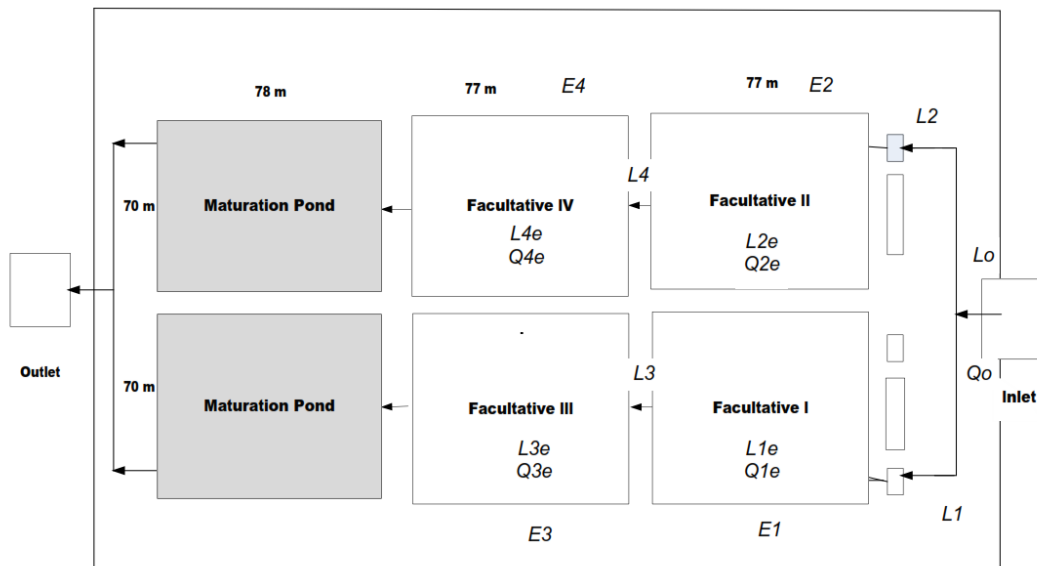


FIGURE 1. Sewon wastewater treatment facility [3]

One of the human efforts to minimize the negative effects of wastewater on nature is by building a treatment facility. This involves the reduction of the pollutant concentration in wastewater by employing bacteria, algae, and zooplankton through a natural process [1] which is conducted in several steps based on the facility's design. This article focused on Sewon wastewater treatment plant located in Bantul, Yogyakarta, Indonesia and, as illustrated in Figure (1), the wastewater enters the treatment facility via inlet containing a filter for the physical matters to be removed, stored in the facultative ponds for the time decided by the decision-maker, and moved to the maturation ponds to be ready for disposal to nature or reused [3]. In order to optimize the process in the facultative pond, some models have been developed by researchers to determine the optimal decision for treatment such as the storing duration. These models were formulated based on the condition of facilities and approaches employed. For example, a quantitative approach was used for pollutant degradation analysis [2], linear programming to optimize wastewater treatment in a facultative pond in [3], effectiveness analysis for biological oxygen demand concentration in a wastewater treatment plant [4], a model to optimize the construction cost of a wastewater treatment facility [5], a natural adsorbent approach to analyse the treatment process [6], and several others. Apart from the efforts to optimize, studies were also

conducted for other purposes such as sewage treatment [7], nitrogen removal analysis [8], maturation pond analysis [9], energy and mitigation analysis [10], reuse of effluents and sludge [11], removal of microplastics in wastewater [12], and others. Meanwhile, some literature showed the utilization of wastewater residual to make bricks [13], re-used [14], and produce bioenergy [15, 16]. Moreover, some specific methods have also been used to maintain the wastewater treatment process such as statistical analysis [17], distillation column optimization model [18], phosphorus concentration analysis [19], a hydraulic-physio-chemical & microbiological analysis [20], oxygen electrode [21], mathematical optimization for organism & organic matter [22], and a quadratic mathematical optimization [23].

It is possible for the wastewater treatment analysis in more complicated conditions to contain uncertain parameters with the value unknown. For example, a model with a probabilistic parameter was developed for wastewater optimization problem [24], but it requires historical/observation data to formulate the probability distribution function for the corresponding uncertain parameter. The model previously mentioned was developed to analyse the performance of a wastewater treatment plant in a probabilistic environment through the use of a probabilistic programming approach. This method is considered not to be appropriate as long as there is no historical/observation data and a possible approach in such situation is applying fuzzy theory which involves the membership function formulated by the decision-maker to represent the uncertain parameter. This method was employed in this article to optimize wastewater treatment process involving fuzzy parameters. Several problems solved using this approach have shown its usefulness as observed in its application in fibre reinforced polymer [25], energy hub optimization [26], and radiotherapy optimization [27], selecting process on project risk response [28], and others.

A fuzzy programming approach was used in this article as the decision-making tool to obtain an optimal decision for a facultative wastewater stabilization pond. The decision variable was made under an uncertain condition such that the wastewater inflow rate into the pond was unknown and represented as a fuzzy parameter. The total load volume of the wastewater and biological oxygen demand (BOD) degradation efficiency index were maximized by the model

while some constraints were formulated to handle the conditions required to be held. Moreover, the computational experiment was used to describe how the model was evaluated and the optimal decision made.

2. MATERIAL AND METHOD

The wastewater processing in Sewon wastewater treatment facility was described in Figure (1) with the assumption that the focus is only on the facultative ponds. Some of the assumptions made in the model formulation are as follows:

- (1) The inflow rate of the wastewater through the inlet valve was uncertain and approached as a fuzzy parameter,
- (2) The BOD degradation efficiency index value was formulated as the percentage of the BOD degradation rate multiply by the storing time and divided by BOD degradation rate multiply by the storing time plus one,
- (3) Before the disposal of the wastewater to nature, it is required to satisfy the minimum required quality standard, especially with the BOD concentration. The value was assumed to be in line with the local government decree,
- (4) The BOD concentration was measured in some finite sample points in the facultative pond and assumed to be uniformly distributed at all points with the average value obtained,
- (5) The formulated model only included the treatment process in the facultative pond while others such as those implemented in the maturation pond were decided separately.

The problem was solved using the following process. First, the membership function for the fuzzy parameters was formulated by the decision-maker based on intuition or observation after which the objective function to be maximized in order to obtain the best performance in the facultative pond was developed. Furthermore, a reference tracking term was added for BOD degradation efficiency index value into the objective function to bring the value to some point decided by the decision-maker. After that, the constraints were formulated to handle the problem including the quality standard requirement, load volume limitation, and efficiency index formulation. This was followed by writing the fuzzy optimization model formulated into

the optimization tool, LINGO 18.0 software, which was interfaced on Microsoft Excel in the input-output form to the user. Finally, data, the computation was initiated after the data has been inputted and the optimal decision obtained at the end.

3. MATHEMATICAL MODEL

The mathematical notations used in the model are as follow:

decision variables:

L_i^e : wastewater load entering the facultative pond i (kg/day),

t : Storing time (day)

fuzzy parameters:

L_i^e : wastewater load entering the facultative pond i (kg/day)

deterministic parameters:

L_0 : wastewater load entering the facility via inlet valve (kg/day);

L_i : wastewater load before it entering the facultative pond i (kg/day);

C_i : BOD concentration in facultative pond i (mg/L);

L : total wastewater load in all facultative ponds (kg/day);

E_i : BOD degradation efficiency index of pond i (in percentage);

BM : wastewater quality standard requirement.

The intention was to maximize the wastewater load entering all facultative ponds to ensure the BOD degradation is as close as possible to the desired value or trajectory while other constraints were held constant. The first term of the objective function was formulated as the total amount of wastewater load in all facultative ponds to be maximized and denoted by Z_1 , while the second term was the trajectory reference term for the BOD degradation efficiency index value in the form of quadratic form to be minimized and denoted by Z_2 , therefore

$$(1) \quad \begin{cases} \max Z_1 = \sum_{i=1}^4 L_i^e, \\ \min Z_2 = \sum_{i=1}^4 (E_i - E_i^r)^2. \end{cases}$$

Due to the possibility of converting the first one into minimizing the problem, the optimization problem above is then can be rewritten as

$$(2) \quad \min Z = - \sum_{i=1}^4 L_i^e + \sum_{i=1}^4 (E_i - E_i^r)^2.$$

The decision needs to hold certain conditions while some constraints were formulated based on the conditions of the problem. The constraint functions employed in the model are as follow:

$$(3) \quad E_i C_i \leq BM, i = 1, 2, 3, 4;$$

$$(4) \quad L_i^e \leq L_i, i = 1, 2, 3, 4;$$

$$(5) \quad L_i^e = \frac{(\mathcal{Q}_i^e \cdot C_i)}{1000}, i = 1, 2, 3, 4;$$

$$(6) \quad E_i = \frac{k.t}{1 + k.t}, i = 1, 2, 3, 4;$$

where,

- The BOD concentration in each pond should not be more than the standard quality
- The wastewater load for pond i should not be more than the value for the corresponding pond
- The wastewater load entering the pond is the flow rate of the wastewater with the organic matter
- The efficiency index value formula E_i with k denotes the BOD degradation rate in one day while t indicates the processing duration (in day).

It is important to note that \mathcal{Q}_i^e is a fuzzy number and its membership function was determined by the decision-maker and it could be in the form of triangular, trapezoidal, or other functions. Meanwhile, a piecewise fuzzy membership function was applied in the numerical experiment and due to the presence of a fuzzy variable in the problem, a fuzzy optimization method was needed. Therefore, fuzzy expected-based programming was employed in this study with the general form of the optimization problem with fuzzy parameters expressed as

$$(7) \quad \begin{cases} \min f(x, \xi) \\ \text{s.t. } g_i(x, \xi) \geq 0, i = 1, 2, \dots, p, \end{cases}$$

where $f(x, \xi)$ is the objective function containing fuzzy variables, $g_i(x, \xi)$ are the constraint functions containing fuzzy variables, x is the decision variable, and ξ is a vector using a fuzzy

variable as its element. Due to the fact that a crisp feasible set was not produced, the method to solve the problem directly was not well-defined and of the proposed method to obtain optimal decision was fuzzy expected approach proposed by Liu and Liu [29] due to its ability to minimize the fuzzy expected value of the objective function subject to those for the constraint formulated as

$$(8) \quad \begin{cases} \min \mathcal{E} [f(x, \xi)] \\ \text{s.t. } \mathcal{E} [g_i(x, \xi)] \geq 0, i = 1, 2, \dots, p, \end{cases}$$

where $\mathcal{E} [\cdot]$ denotes the fuzzy expected value defined as

$$\mathcal{E} [\xi] = \int_0^\infty Cr \{ \xi \geq r \} dr - \int_{-\infty}^0 Cr \{ \xi \leq r \} dr$$

and $Cr [\cdot]$ indicates the credibility value. A further result showed the two independent fuzzy variables ξ and ζ with finite expected values, and arbitrary real numbers a and b as

$$\mathcal{E} [a\xi + b\zeta] = a\mathcal{E} [\xi] + \mathcal{E} [\zeta].$$

The application of this concept to solve Eq. (2) and the simplification of the constraint functions led to the formulation of the optimization Eq. (2) as fuzzy expected optimization problem as follows

$$(9) \quad \min Z = \mathcal{E} \left[- \sum_{i=1}^4 L_i^e + \sum_{i=1}^4 (E_i - E_i^r)^2 \right]$$

subject to:

$$(10) \quad \mathcal{E} [E_i \cdot 1000 \cdot L_i - BM \cdot \mathcal{Q}_i^e] \leq 0, i = 1, 2, 3, 4;$$

$$(11) \quad E_i = \frac{k.t}{1 + k.t}, i = 1, 2, 3, 4;$$

$$(12) \quad L_1^e \leq L_1;$$

$$(13) \quad L_3^e \leq L_1 - L_1^e;$$

$$(14) \quad L_2^e \leq L_2;$$

$$(15) \quad L_4^e \leq L_2 - L_2^e.$$

TABLE 1. Membership value data for fuzzy variable $Q_i^e(j)$

j	1	2	3	4	5	6	7	8	9	10
$\mu_{Q_1^e(j)}$	0.10	0.20	0.40	0.80	1.00	0.90	0.80	0.50	0.20	0.01
$w_{Q_1^e(j)}$	0.05	0.05	0.10	0.20	0.15	0.05	0.15	0.15	0.09	0.01
$Q_1^e(j)$	10450	10500	10650	10800	11000	11200	11250	11350	11400	11500
$\mu_{Q_2^e(j)}$	0.20	0.25	0.30	0.50	0.80	1.00	0.70	0.30	0.20	0.10
$w_{Q_2^e(j)}$	0.10	0.025	0.025	0.10	0.15	0.25	0.20	0.05	0.05	0.05
$Q_2^e(j)$	10800	10900	10950	11000	11100	11150	11250	11300	11350	11400
$\mu_{Q_3^e(j)}$	0.20	0.25	0.35	0.60	0.70	1.00	0.90	0.40	0.25	0.20
$w_{Q_3^e(j)}$	0.10	0.025	0.05	0.125	0.05	0.20	0.25	0.075	0.025	0.10
$Q_3^e(j)$	10500	10600	10650	10850	11000	11200	11300	11450	11500	11550
$\mu_{Q_4^e(j)}$	0.40	0.50	0.80	1.00	0.80	0.90	0.70	0.60	0.40	0.20
$w_{Q_4^e(j)}$	0.20	0.05	0.15	0.15	0.00	0.10	0.05	0.10	0.10	0.10
$Q_4^e(j)$	10200	10250	10350	10400	10450	10500	10550	10650	10850	11000

It is, however, possible to express the constraints as

$$E_i \cdot 1000 \cdot L_i - BM \cdot \mathcal{E}[\mathcal{Q}_i^e] \leq 0, i = 1, 2, 3, 4.$$

Let the membership function of \mathcal{Q}_i^e be formulated as a piecewise function which is defined by

$$(16) \quad \mu_{\mathcal{Q}_i^e} = \begin{cases} \mu_{\mathcal{Q}_i^e(j)} & \text{if } \mathcal{Q}_i^e = \mathcal{Q}_i^e(j), j = 1, 2, \dots, J, \\ 0, & \text{others.} \end{cases}$$

Then the fuzzy expected value of \mathcal{Q}_i^e is

$$(17) \quad \mathcal{E}[\mathcal{Q}_i^e] = \sum_{j=1}^J w_{\mathcal{Q}_i^e(j)}(\mathcal{Q}_i^e(j))$$

where $w_{\mathcal{Q}_i^e(j)}$ is derived from

$$w_{\mathcal{Q}_i^e(j)} = \frac{1}{2} \left(\max_{1 \leq j < i} \mu_{\mathcal{Q}_i^e(j)} - \max_{1 \leq j < i} \mu_{\mathcal{Q}_i^e(j)} + \max_{i \leq j \leq J} \mu_{\mathcal{Q}_i^e(j)} - \max_{1 < j \leq J} \mu_{\mathcal{Q}_i^e(j)} \right).$$

4. RESULTS & DISCUSSION

Let the decision-maker decides the membership function of $W_i^e(j)$ following a discrete membership function defined as Eq. (2) which yields the fuzzy expected value of Eq. (17) with the values of $\pi_{Q_i^e(j)}$ and $w_{Q_i^e(j)}$ provided in Table 1.

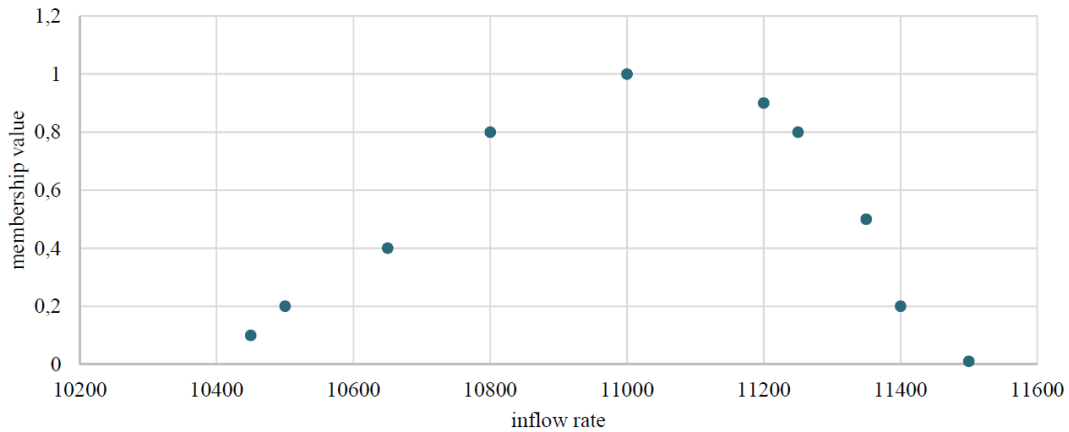


FIGURE 2. Graphical illustration of the membership function of the fuzzy variable $\mu_{Q_i^e}$

The membership function of the fuzzy variable $\mu_{Q_i^e}$ was comprehensively explained using the graphical illustration of $\mu_{Q_i^e}$ in Figure (2). This function was formulated by the decision-maker based on intuition or observation since the data was assumed not to be available. The membership value was interpreted as the decision maker’s belief level of the expected corresponding inflow rate value and was reported to be set at 11000 kg and membership value 1 shows it occurs often. The remaining parameters were obtained from the authors’ previous articles as shown in Table 2 with the total wastewater load at inflow $L_0 = 4.799,6$ kg/day while the values for ponds I and II were half of L_0 , and pond III and IV were half of I and II. Moreover, the BOD degradation coefficient was $k = 1.1\%$ [3] while the minimum standard quality of wastewater was 50 mg/L [30]. The optimization problem Eq. (9) was solved using LINGO 18.0 optimization tool by employing the Generalized Reduced Gradient (GRG) algorithm and the results are shown in Table 2.

The $\mathcal{E}[Q_i^e(j)]$ column in Table 2 shows the fuzzy value expected for the fuzzy variable $Q_i^e(j)$ which was derived from Eq. (17) while the E_i^f column shows the reference value for the efficiency index decided by the decision-maker. Moreover, $E_1^f = 0.2$ means the decision-maker

TABLE 2. Optimization result

Pond (<i>i</i>)	$\mathcal{E}[Q_i^e(j)]$ (m)	L_i^e (kg)	E_i (%)	E_i^r (%)	S_i (day)
1	11013.00	1199.5	0.20	0.20	0.23
2	11131.25	1199.5	0.20	0.20	0.23
3	11120.00	1199.5	0.46	0.50	0.79
4	10492.50	1199.5	0.44	0.50	0.71

wanted to make the efficiency index value of the BOD degradation in the pond I as close as possible to 0.2. The optimization results, therefore, presented the optimal decision with the first being the optimal load of wastewater required to be in each pond which was is 1199.5 kg and the storing time shown in column S_i to be 0.23 day at the pond I and II, 0.79 at III, and 0.71 at IV. This decision is expected to provide an efficiency index of 0.2% at ponds I and II, 0.46% at III, and 0.44% at IV.

The optimization derived, therefore, suggested the following managerial aspects:

- (1) The pollutant concentration is expected to be more reduced following the storing time of the degradation process. A smaller BOD value was observed to have the possibility of producing longer storing time.
- (2) Due to the uncertainty of the inflow rate value, the optimal decision achieved from the proposed mathematical optimization is just an estimation and this means it is possible to obtain different results in the field.
- (3) The wastewater load and storing time produced were just a recommendation from the proposed mathematical optimization model and this means it is possible for the decision-maker to change the decision based on observation.

The BOD degradation rate used in this study was 1.1 measured in a sunny season and this means there is a possibility of different value following the weather condition during the wastewater processing. Therefore, the decision-maker may re-observe the value and re-run the optimization to obtain new optimal decision.

5. CONCLUDING REMARKS

This article proposed a decision-making tool via fuzzy optimization model to make a decision for facultative stabilization pond in a wastewater treatment facility where the inflow rate variable was approached as a fuzzy variable. The experiment showed the model was able to produce an optimal decision as observed in the value of the wastewater load to be processed in the pond and the corresponding storing time.

Future research direction is required to develop an optimization model with the ability to handle a multi-period case by generating optimal decision for multi-period of storing time in one time calculating process. Moreover, another interesting suggestion is to integrate the model to ensure the wastewater processing involves the maturation pond.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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