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USING ADOMIAN DECOMPOSITION METHOD FOR SOLVING SYSTEMS OF SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we introduce application of Adomian Decomposition Method (ADM) for solving systems of Ordinary Differential Equations (ODEs). This method is illustrated by four examples of (ODEs) and solutions are obtained. One of the most important advantages of this method is its simplicity in using.

Keywords: Adomian method; initial conditions; systems of second order equations.

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1. INTRODUCTION

The literature on the Adomian decomposition method (ADM) and its modifications [1-7] tells us that this method is proven to be efficient to solve linear and nonlinear ODEs, DAEs, PDEs, SDEs, integral equations and integrointegral equations. More importantly, such method has been applied to a wide class of problems in physics, biology and chemical reaction. The reason of such spread and application of the method lies in the fact that the ADM provides the solution in a rapid convergent series with computable terms. In this manuscript, we aim at introducing a new reliable modification of ADM. For this reason, a new differential operator for

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solving high-order and system of differential equations. In order to illustrate the application of the modified form of the ADM, we would provide a set of examples to show the advantages of using the proposed method to solve the initial value problems.

2. Analysis of the ADM

We consider the following system of ordinary differential equations of second order

$$u'' + p_1(x)u' + f_1(x,u,v,w,...) = g_1(x),$$

$$v'' + p_2(x)v' + f_2(x,u,v,w,...) = g_2(x),$$

$$w'' + p_3(x)w' + f_3(x,u,v,w,...) = g_3(x). \tag{1}$$

$$\vdots$$

With the following initial conditions

$$u(0) = a_1,\ u'(0) = a_2,$$

$$v(0) = b_1,\ v'(0) = b_2,$$

$$w(0) = d_1,\ w'(0) = d_2,$$

$$\vdots$$

where $f_1, f_2, ...f_i$ are nonlinear functions, $p_i(x)$,and $g_i(x)$ are given functions.
According to the ADM we rewrite the system of equations(1) in terms of operator from as

$$Lu = g_1(x) - f_1(x,u,v,w,...),$$

$$Lv = g_2(x) - f_2(x,u,v,w,...),$$

$$Lw = g_3(x) - f_3(x,u,v,w,...) \tag{2}$$

$$\vdots$$

where L_i are differential operators given by

$$L_i = e^{-\int p_i(x)dx}\frac{d}{dx}e^{\int p_i(x)dx}\frac{d}{dx},\ \ i = 1,2,...$$

and their inverse integral operators are defined as

$$L_i^{-1}(.) = \int_0^x e^{-\int p_i(x)dx} \int_0^x e^{\int p_i(x)dx}(.)dxdx. \tag{3}$$

Applying L_i^{-1} on (2) we get

$$u = \gamma_1(x) + L_1^{-1} g_1(x) - L_1^{-1} f_1(x,u,v,w,...),$$

$$v = \gamma_2(x) + L_2^{-1} g_2(x) - L_2^{-1} f_2(x,u,v,w,...),$$

$$w = \gamma_3(x) + L_3^{-1} g_3(x) - L_3^{-1} f_3(x,u,v,w,...). \tag{4}$$

$$\vdots$$

such that

$$L\gamma_i(x) = 0, \;\; i = 1,2,3,...$$

We decompose $u(x), v(x), ...w(x)$ and $f_i(x,u,v,...w)$ see in as

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \quad f_1(x,u,v,w,...) = \sum_{n=0}^{\infty} A_{1n},$$

$$v(x) = \sum_{n=0}^{\infty} v_n(x), \quad f_2(x,u,v,w,...) = \sum_{n=0}^{\infty} A_{2n},$$

$$w(x) = \sum_{n=0}^{\infty} w_n(x), \quad f_3(x,u,v,w,...) = \sum_{n=0}^{\infty} A_{3n},$$

$$\vdots$$

$$r(x) = \sum_{n=0}^{\infty} r_n(x), \quad f_i(x,u,v,w,...) = \sum_{n=0}^{\infty} A_{in}, \tag{5}$$

where A_{in} are the Adomian polynomials [8] are given

$$A_{in} = \frac{1}{n!}\frac{d^n}{d\lambda^n}[f_i(x, \sum_{j=0}^{\infty} u_j \lambda^j, \sum_{j=0}^{\infty} v_j \lambda^j, \sum_{j=0}^{\infty} w_j \lambda^j, ...)]_{\lambda=0}, \;\; i = 1,2,... \tag{6}$$

From (4)and (5) we have

$$\sum_{n=0}^{\infty} u_n(x) = \gamma_1(x) + L_1^{-1} g_1(x) - L_1^{-1}[\sum_{n=0}^{\infty} A_{1n}]$$

$$\sum_{n=0}^{\infty} v_n(x) = \gamma_2(x) + L_2^{-1} g_2(x) - L_2^{-1}[\sum_{n=0}^{\infty} A_{2n}]$$

$$\sum_{n=0}^{\infty} w_n(x) = \gamma_3(x) + L_1^{-1} g_3(x) - L_n^{-1}[\sum_{n=0}^{\infty} A_{3n}]$$

$$\vdots$$

(7) $$\sum_{n=0}^{\infty} r_n(x) = \gamma_n(x) + L_1^{-1} g_n(x) - L_n^{-1}[\sum_{n=0}^{\infty} A_{in}]$$

then we define:

$$u_0 = \gamma_1(x) + L^{-1} g_1(x), \quad u_{n+1} = -L_1^{-1} A_{1n},$$

$$v_0 = \gamma_2(x) + L^{-1} g_2(x), \quad v_{n+1} = -L_2^{-1} A_{2n},$$

(8) $$w_3 = \gamma_3(x) + L^{-1} g_3(x), \quad w_{n+1} = -L_i^{-1} A_{3n}, \ \ n \geq 0$$

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From (6) and (8), we can determine the components $u_n, v_n, w_n, \ldots$ can be immediately obtained.

3. Applications of the Method

In this section, we will provide four numerical examples that shows this method.

Example 1.

Consider the system of linear second order ordinary differential equations:

$$u'' + e^x u' + v = 3 + 2xe^x + x^3,$$

(9) $$v'' + e^{-x} v' + w = 1 + 6x + 3x^2 e^{-x} + x^4,$$

$$w'' - e^x w' + u = 1 + 13x^2 - 4x^3 e^x,$$

with initial conditions

$$u(0) = 1, u'(0) = 0, v(0) = 1, v'(0) = 0, w(0) = 1, w'(0) = 0,$$

The exact solution is

$u(x) = 1+x^2, v(x) = 1+x^3$, and $w(x) = 1+x^4$.

In an operator form eq.(9) became

$$Lu = 3+2xe^x+x^3-v,$$

$$Lv = 1+6x+3x^2e^{-x}+x^4-w, \tag{10}$$

$$Lw = 1+13x^2-4x^3e^x-u,$$

where

$$Lu = e^{e^x}\frac{d}{dx}e^{-e^x}\frac{d}{dx}(u),$$
$$Lv = e^{e^{-x}}\frac{d}{dx}e^{-e^{-x}}\frac{d}{dx}(v),$$
$$Lw = e^{-e^x}\frac{d}{dx}e^{e^x}\frac{d}{dx}(w),$$

and

$$L^{-1}(u) = \int_0^x e^{e^x}\int_0^x e^{-e^x}(u)dxdx,$$

$$L^{-1}(v) = \int_0^x e^{e^{-x}}\int_0^x e^{-e^{-x}}(v)dxdx,$$

$$L^{-1}(w) = \int_0^x e^{-e^x}\int_0^x e^{e^x}(w)dxdx.$$

Applying L^{-1} on both side of eq.(10) and using the initial conditions, we get

$$u(x) = 1+\frac{3x^2}{2}-\frac{x^3}{6}-\frac{x^4}{24}+\frac{7x^5}{120}-\frac{x^6}{720}-L^{-1}v,$$
$$v(x) = 1+\frac{x^2}{2}+\frac{7x^3}{6}+\frac{11x^4}{24}-\frac{5x^5}{24}+\frac{29x^6}{720}-L^{-1}w,$$
$$w(x) = 1+\frac{x^2}{2}+\frac{x^3}{6}+\frac{29x^4}{24}+\frac{11x^5}{120}+\frac{41x^6}{720}-L^{-1}u.$$

We use the following scheme

$$u_0 = 1 + \frac{3x^2}{2} - \frac{x^3}{6} - \frac{x^4}{24} + \frac{7x^5}{120} - \frac{x^6}{720}, \quad u_{n+1} = -L^{-1}v_n,$$

$$v_0 = 1 + \frac{x^2}{2} + \frac{7x^3}{6} + \frac{11x^4}{24} - \frac{5x^5}{24} + \frac{29x^6}{720}, \quad v_{n+1} = -L^{-1}w_n,$$

$$w_0 = 1 + \frac{x^2}{2} + \frac{x^3}{6} + \frac{29x^4}{24} + \frac{11x^5}{120} + \frac{41x^6}{720}, \quad w_{n+1} = -L^{-1}u_n.$$

Therefore

$$u_1 = \frac{-x^2}{2} + \frac{x^3}{6} - \frac{7x^5}{120} - \frac{x^6}{120},$$

$$v_1 = \frac{-x^2}{2} - \frac{x^3}{6} - \frac{x^5}{120} - \frac{2x^6}{45},$$

$$w_1 = \frac{-x^2}{2} - \frac{x^3}{6} - \frac{x^4}{4} - \frac{11x^5}{120} - \frac{11x^6}{180},$$

and

$$u_2 = \frac{x^4}{24} - \frac{x^6}{180},$$

$$v_2 = \frac{x^4}{24} + \frac{x^5}{60} + \frac{x^6}{180},$$

$$w_2 = \frac{x^4}{24} + \frac{x^6}{180}.$$

Approximations to the solution of the above system with three iterations of ADM, yields:

$$u(x) = 1 + x^2 - \frac{11x^6}{720},$$

$$v(x) = 1 + x^3 + \frac{5x^4}{12} - \frac{x^5}{6} + \frac{x^6}{720},$$

and

$$w(x) = 1 + x^4 + \frac{x^6}{720}.$$

In this example, we note the solution by ADM close to the exact solution.

Example 2.

We study the system of nonlinear equation of Emden-Fowler type

$$u'' + (\frac{1}{x})u' + u^2 v - (4x^2+5)u = 0,$$

$$v'' + (\frac{2}{x})v' + v^2 u - (4x^2-5)v = 0, \tag{11}$$

with initial conditions

$$u(0) = 1,\ u'(0) = 0$$

$$v(0) = 1,\ v'(0) = 0$$

with the exact solution see in[9]

$$(u(x), v(x)) = (e^{x^2}, e^{-x^2}),$$

where $p_1(x) = \frac{1}{x}$,$p_2(x) = \frac{2}{x}$ we find

$$L_1(.) = x^{-1}\frac{d}{dx}x\frac{d}{dx}(.),$$

$$L_2(.) = x^{-2}\frac{d}{dx}x^2\frac{d}{dx}(.),$$

the inverse operators L^{-1} are given by

$$L_1^{-1}(.) = \int_0^x x^{-1}\int_0^x x(.)dxdx,$$

$$L_2^{-1}(.) = \int_0^x x^{-2}\int_0^x x^2(.)dxdx,$$

applying the inverse operators L_1,L_2 on (11) and using the initial conditions we get

$$u = 1 + L^{-1}((4x^2+5)u) - L^{-1}(u^2 v)$$

$$v = 1 + L^{-1}((4x^2-5)v) - L^{-1}(v^2 u). \tag{12}$$

We use the following scheme

$$u_0 = 1,\ u_{n+1} = L^{-1}((4x^2+5)u_n) - L^{-1}A_{1n},\ n \geq 0,$$

$$v_0 = 1,\ v_{n+1} = L^{-1}((4x^2-5)v_n) - L^{-1}A_{2n},\ n \geq 0, \tag{13}$$

where A_{1n}, A_{2n} are Adomian polynomials that represent nonlinear term. Which are given by

$$A_{1n}(x) = u^2(x)v(x), A_{2n}(x) = v^2(x)u(x)$$

The comonents of the Adomian polynomials are given by

$$A_{10} = u_0^2 v_0,$$

$$A_{11} = u_0^2 v_1 + 2u_0 v_0 u_1,$$

$$A_{12} = u_0^2 v_2 + 2u_0 v_1 u_1 + 2u_0 v_0 u_2 + u_1^2 v_0$$

$$\dots$$

and the nonlinear term v^2 ,has the few Adomian polynomials A_{2n} are given by

$$A_{20} = v_0^2 u_0,$$

$$A_{21} = v_0^2 u_1 + 2v_0 v_1 u_0,$$

$$A_{22} = v_0^2 u_2 + 2v_0 u_1 v_1 + 2v_2 v_0 u_0 + v_1^2 u_0$$

$$\dots$$

leads to

$$u_0 = 1,$$

$$v_0 = 1,$$

$$u_1 = x^2 + \frac{x^4}{4},$$

$$v_1 = -x^2 + \frac{x^4}{5},$$

$$u_2 = \frac{x^4}{4} + \frac{91x^6}{720} + \frac{x^8}{64},$$

$$v_2 = \frac{3x^4}{10} - \frac{113x^6}{840} + \frac{x^8}{90},$$

so

$$u_3 = \frac{x^6}{24} + \frac{627x^8}{35840} + \frac{1091x^{10}}{288000} + \frac{x^{12}}{2304},$$

$$v_3 = \frac{-23x^6}{840} + \frac{12689x^8}{362880} - \frac{77767x^{10}}{11088000} + \frac{x^{12}}{3510}.$$

Approximations to the solutions are as follows:

$$u(x) = 1 + x^2 + 0.5x^4 + 0.168056x^6 + 0.0331194x^8 + 0.00378819x^{10} + \dots.$$

$$v(x) = 1 - x^2 + 0.5x^4 - 0.161905x^6 + 0.0460786x^8 - 0.00701362x^{10} + \dots.$$

From the previous example we note that, the solution by ADM converges to the exact solution.

Example 3.

We sutdy the system of nonlinear equations of Emden-Fowler type

$$u'' + \frac{2}{x}u' + v^2 - u^2 + 6v = 6 + 6x^2,$$

$$v'' + \frac{2}{x}v' + u^2 - v^2 - 6v = 6 - 6x^2, \tag{14}$$

with initial conditions

$$u(0) = 1, \;\; u'(0) = 0$$

$$v(0) = -1, \;\; v'(0) = 0$$

The exact solutions see in[9] are

$$(u(x), v(x)) = (x^2 + e^{x^2}, x^2 - e^{x^2}),$$

,where $p_1(x) = p_2(x) = \frac{2}{x}$.

System (14) we can write as

$$Lu = 6 + 6x^2 - 6v - v^2 + u^2,$$

$$Lv = 6 - 6x^2 + 6v + v^2 - u^2, \tag{15}$$

where $Lu, \;\; Lv$ define by:

$$Lu = x^{-2}\frac{d}{dx}x^2\frac{d}{dx}u,$$

$$Lv = x^{-2}\frac{d}{dx}x^2\frac{d}{dx}v. \tag{16}$$

And the inverse operators L^{-1} define by:

$$L^{-1}(.) = \int_0^x x^{-2} \int_0^x x^2(.),$$

$$L^{-1}(.) = \int_0^x x^{-2} \int_0^x x^2(.). \tag{17}$$

Applying L^{-1} on equation (15), and using the initial conditions, we get

$$u(x) = 1 + L^{-1}(6 + 6x^2 - 6v - v^2 + u^2),$$

$$v(x) = -1 + L^{-1}(6 - 6x^2 + 6v + v^2 - u^2), \tag{18}$$

by assuming that

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \;\; v(x) = \sum_{n=o}^{\infty} v_n(x). \tag{19}$$

By substituting equation (19) in (18) we have

$$\sum_{n=0}^{\infty} u_n(x) = 1 + L^{-1}(6 + 6x^2) - L^{-1}[6\sum_{n=o}^{\infty} v_n(x) + \sum_{n=o}^{\infty} A_{1n}(x) - \sum_{n=0}^{\infty} A_{2n}],$$

$$\sum_{n=0}^{\infty} v_n(x) = -1 + L^{-1}(6 - 6x^2) + L^{-1}[6\sum_{n=0}^{\infty} v_n + \sum_{n=o}^{\infty} A_{1n}(x) - \sum_{n=0}^{\infty} A_{2n}],$$

where

$$u_0 = 1 + L^{-1}(6 + 6x^2), \;\; u_{n+1} = -L^{-1}[6\sum_{n=0}^{\infty} v_n + \sum_{n=o}^{\infty} A_{1n} - \sum_{n=0}^{\infty} A_{2n}], n \geq 0,$$

$$v_0 = -1 + L^{-1}(6 - 6x^2), \;\; v_{n+1} = L^{-1}[6\sum_{n=0}^{\infty} v_n + \sum_{n=o}^{\infty} A_{1n}(x) - \sum_{n=0}^{\infty} A_{2n}], n \geq 0, \tag{20}$$

where A_{1n}, A_{2n} are Adomian polynomials define by

$$A_{1n} = v_n^2, \qquad A_{2n} = u_n^2,$$

$$A_{10} = v_0^2, \qquad A_{20} = u_0^2,$$

$$A_{11} = 2v_0v_1 \qquad A_{21} = 2u_0u_1.$$

Hence

$$u_0 = 1 + x^2 + \frac{3x^4}{10},$$

$$v_0 = -1 + x^2 - \frac{3x^4}{10},$$

as well as

$$u_1 = x^2 - \frac{x^4}{10} + \frac{3x^6}{70} + \frac{x^8}{60},$$
$$v_1 = -x^2 + \frac{x^4}{10} - \frac{3x^6}{70} - \frac{x^8}{60},$$

and

$$u_2 = \frac{3x^4}{10} + \frac{17x^6}{210} - \frac{x^8}{504} + \frac{19x^{10}}{7700},$$
$$v_2 = \frac{-3x^4}{10} - \frac{17x^6}{210} + \frac{x^8}{504} - \frac{19x^{10}}{7700}.$$

Therefore

$$u(x) = 1 + 2x^2 + \frac{x^4}{2} + \frac{13x^6}{105} + \frac{37x^8}{2520} + \frac{19x^{10}}{7700} + \dots$$
$$v(x) = -1 - \frac{x^4}{2} - \frac{13x^6}{105} - \frac{37x^8}{2520} - \frac{19x^{10}}{7700} + \dots$$

This gives the exact solution of Eq.(14) which is given as follows

$$(u(x), v(x)) = (x^2 + e^{x^2}, x^2 - e^{x^2})$$

Example 4.

Consider the system of non-liner equations:

$$u'' - u' + v^2 = 2e^{-x} + e^{2x},$$

$$v'' + v' + u^2 = 2e^{x} + 2e^{-2x}, \tag{21}$$

with initial conditions

$$u(0) = 1, u'(0) = -1,$$

$$v(0) = 1, v'(0) = 1.$$

The exact solutions are $(u(x), v(x)) = (e^{-x}, e^{x})$.

Re-written the system of non-liner eq.(21), as

$$Lu = 2e^{-x} + e^{2x} - v^2,$$

$$Lv = 2e^{x} + 2e^{-2x} - u^2, \tag{22}$$

where

$$L(.) = e^x \frac{d}{dx} e^{-x} \frac{d}{dx}(.),$$
$$L(.) = e^{-x} \frac{d}{dx} e^x \frac{d}{dx}(.).$$

The L^{-1}, are considered as two fold integral operator defined by

$$L^{-1}(.) = \int_0^x e^x \int_0^x e^{-x}(.)dxdx,$$

$$L^{-1}(.) = \int_0^x e^{-x} \int_0^x e^x(.)dxdx. \tag{23}$$

Applying L^{-1}(23) on (22), and using the initial conditions, we have

$$u(x) = 2 - e^x + L^{-1}(2e^{-x} + e^{2x} - v^2),$$

$$v(x) = 2 - e^{-x} + L^{-1}(2e^x + 2e^{-2x} - u^2). \tag{24}$$

Using Adomian decomposition for (u, v) as given in (24), we obtain

$$\sum_{n=0}^{\infty} u_n = 2 - e^x + L^{-1}(2e^{-x} + e^{2x}) - L^{-1} \sum_{n=0}^{\infty} A_{2n},$$

$$\sum_{n=0}^{\infty} v_n = 2 - e^{-x} + L^{-1}(2e^x + 2e^{-2x}) - L^{-1} \sum_{n=0}^{\infty} A_{1n}, \tag{25}$$

the components(u_n, v_n) can be recursively determined by using the relation

$$u_0 = 2 - e^x + L^{-1}(2e^{-x} + e^{2x}), u_{n+1} = -L^{-1}(A_{2n}), n \geq 0,$$

$$v_0 = 2 - e^{-x} + L^{-1}(2e^x + 2e^{-2x}), v_{n+1} = -L^{-1}(A_{1n}), n \geq 0, \tag{26}$$

where A_{1n},A_{2n} are Adomian polynomials of nonlinear (u^2, v^2), we are give by

$$A_{1n}(x) = u^2(x), A_{2n} = v^2,$$

we get

$$A_{10} = u_0^2,$$

$$A_{11} = 2u_0u_1,$$

$$A_{12} = 2u_0u_2 + u_1^2, \ldots$$

And

$$A_{20} = v_0^2,$$

$$A_{21} = 2v_0v_1,$$

$$A_{22} = 2v_0v_2 + v_1^2, \ldots$$

This in turn given

$$u_0 = 1 - x + x^2 + \frac{x^3}{3} + \frac{x^4}{3} + \frac{7x^5}{60} + \frac{2x^6}{45} + \frac{31x^7}{2520} + \frac{x^8}{315} + \frac{127x^9}{181440} + \frac{2x^{10}}{14175},$$

$$v_0 = 1 + x + \frac{3x^2}{2} - \frac{5x^3}{6} + \frac{5x^4}{8} - \frac{29x^5}{120} + \frac{7x^6}{80} - \frac{25x^7}{1008} + \frac{17x^8}{2688} - \frac{509x^9}{362880} + \frac{341x^{10}}{1209600},$$

$$u_1 = \frac{-x^2}{2} + \frac{x^3}{6} - \frac{5x^4}{24} + \frac{x^5}{40} - \frac{7x^6}{240} - \frac{7x^7}{720} - \frac{101x^8}{8064} - \frac{2477x^9}{362880} - \frac{12941x^{10}}{3628800},$$

$$v_1 = \frac{-x^2}{2} - \frac{x^3}{6} - \frac{7x^4}{24} - \frac{x^5}{120} - \frac{43x^6}{720} + \frac{251x^7}{5040} - \frac{1879x^8}{40320} + \frac{10151x^9}{362880} - \frac{10247x^{10}}{725760},$$

$$u_2 = \frac{-x^4}{12} + \frac{x^5}{20} - \frac{x^6}{20} + \frac{x^7}{252} - \frac{37x^8}{2880} - \frac{19x^9}{8640} - \frac{533x^{10}}{226800},$$

$$v_2 = \frac{-x^4}{12} - \frac{x^5}{20} - \frac{13x^6}{180} + \frac{x^7}{252} - \frac{499x^8}{20160} + \frac{113x^9}{12096} - \frac{3097x^{10}}{453600},$$

$$\ldots$$

This gives the exact solution of Eq.(21) which is given by

$$(u(x), v(x)) = (e^{-x}, e^x)$$

CONCLUSIONS

In this paper, the application of ADM is investigated to obtain approximations solutions of some linear and nonlinear system of (ODEs). This work emphasized our belief that the method is a reliable technique to handle linear and nonlinear system of (ODEs).

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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