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SOME NEW OPERATORS ON $\mu_I g$-CLOSED SETS IN GITS

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Abstract. By making use of $\mu_I g$-closed sets, we have made out $\mu_I g$-Exterior, $\mu_I g$-border, $\mu_I g$-frontier and their properties are listed out. Also $q\mu_I g$-separated sets are introduced and their characters are contemplated. Some new forms of $\mu_I g$-closed sets are to be introduced. Also we introduce pre$^*\mu_I$-closed sets and their attributes are to be discussed.

Keywords: $q\mu_I g$-separated set; pre$^*\mu_I$–closed set; semi$^*\mu_I$–closed set; $\alpha^*\mu_I$–closed set; $\beta^*\mu_I$–closed set; regular $^*\mu_I$ –closed set; pre$^*\mu_I$-closure; pre$^*\mu_I$-interior.

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1. INTRODUCTION

Coker introduced intuitionistic set based on membership and non-membership degrees which gives flexible approaches to represent the mathematical objects that plays a great role with classical set logic . Later on using these concepts we made $\mu_I g$–closed set in GITS. Here we are yet to study about few operators in $\mu_I g$–closed sets and their natures are described.

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2. Preliminaries

In this section we list some definitions and basic results of generalized intuitionistic topological space.

Definition 2.1. *[1] Let X be a non-empty set. An intuitionistic set A is an object having the form $A =< X, A_1, A_2 >$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A while A_2 is called the set of non- members of A.*

Result 2.1. *Let X be a non-empty set and let A, B be an intuitionistic sets in the form $A =< X, A_1, A_2 >$ and $B =< X, B_1, B_2 >$ respectively. Then*

1) $A \subseteq B$ *if and only if* $A_1 \subseteq B_1$ *and* $B_2 \subseteq A_2$.
2) $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$.
3) $\overline{A} =< X, A_2, A_1 >$, *(in intuitionistic,* $\overline{A} = A^c$*)*
4) $A \cup B =< X, A_1 \cup B_1, A_2 \cap B_2 >$.
5) $A \cap B =< X, A_1 \cap B_1, A_2 \cup B_2 >$.
6) $A - B = A \cap \overline{B}$.
7) $\phi_\sim =< X, \phi, X >$; $X_\sim =< X, X, \phi >$.

Definition 2.2. *[1] An intuitionistic topology on a non-empty set X is a family τ of intuitionistic sets in X containing $\phi_\sim$, $X_\sim$ and closed under finite union and arbitrary intersection. The pair (X, τ) is called an intuitionistic topological space. Any intuitionistic set in τ is known as an intuitionistic open set (IOS) in X and the complement of IOS is called an intuitionistic closed set (ICS).*

Definition 2.3. *[7] Let X be a non-empty set and μ_I be the collection of intuitionistic subset of X. Then μ_I is called generalized intuitionistic topology on X if $\phi \in \mu_I$ and μ_I is closed under arbitrary unions. The elements of μ_I are called μ_I-open sets and their complements are called μ_I-closed sets.*

Definition 2.4. *[7] The μ_I-closure of A is the intersection of all μ_I-closed sets containing A, and the μ_I-interior of A (its denoted by $i_{\mu_I}(A)$) is the union of all μ_I-open sets contained in A.*

Definition 2.5. *[12] In (X,μ_I), an intuitionistic set A of X is said to be an intuitionistic generalized closed sets in generalized intuitionistic topological space (GITS) if $c_{\mu_I}(A) \subseteq U$ whenever $A \subseteq U$ and U is μ_I–open set and it is denoted by $\mu_I g$–closed. The complement of $\mu_I g$-closed set is $\mu_I g$–open set.*

Definition 2.6. *[12] The $\mu_I g$–closure of A, denoted by $c^*_{\mu_I}(A)$, is the intersection of all $\mu_I g$–closed supersets of A.*

Definition 2.7. *[12] For any $A \subseteq X$; the union of all $\mu_I g$-open sets contained in A is defined as the $\mu_I g$-interior of A and is denoted by $i^*_{\mu_I}(A)$.*

Result 2.2. *[12] Let (X,μ_I) be a GITS and $A,B \subseteq X$.*

1) $c^*_{\mu_I}(\phi_\sim) \neq \phi_\sim$; $c^*_{\mu_I}(X_\sim) = X_\sim$.
2) $i_{\mu_I}(X_\sim) \neq X_\sim$; $i_{\mu_I}(\phi_\sim) = \phi_\sim$.
3) *Monotonicity:*
   a) If $A \subseteq B$ then $c^*_{\mu_I}(A) \subseteq c^*_{\mu_I}(B)$.
   b) If $A \subseteq B$ then $i^*_{\mu_I}(A) \subseteq i^*_{\mu_I}(B)$.
4) *Idempotent property:* $c^*_{\mu_I}[c^*_{\mu_I}(A)] = c^*_{\mu_I}(A)$.
5) If A is $\mu_I g$–closed ($\mu_I g$–open) then $c^*_{\mu_I}(A) = A (i^*_{\mu_I}(A) \subseteq A)$.
6) $c^*_{\mu_I}(A) \cup c^*_{\mu_I}(B) \subseteq c^*_{\mu_I}(A \cup B)$.
7) $c^*_{\mu_I}(A \cap B) \subseteq c^*_{\mu_I}(A) \cap c^*_{\mu_I}(B)$.
8) $A \subseteq c^*_{\mu_I}(A) \subseteq c_{\mu_I}(A)$.
9) $i^*_{\mu_I}(A) \cup i^*_{\mu_I}(B) \subseteq i^*_{\mu_I}(A \cup B)$.
10) $i^*_{\mu_I}(A \cap B) \subseteq i^*_{\mu_I}(A) \cap i^*_{\mu_I}(B)$.
11) $i_{\mu_I}(A) \subseteq i^*_{\mu_I}(A) \subseteq A$.
12) a) $c^*_{\mu_I}(\overline{A}) = \overline{(i^*_{\mu_I}(A))}$
   b) $\overline{(c^*_{\mu_I}(A))} = i^*_{\mu_I}(\overline{A})$
   c) $\overline{(c^*_{\mu_I}(\overline{A}))} = i^*_{\mu_I}(A)$
   d) $c^*_{\mu_I}(A) = \overline{(i^*_{\mu_I}(\overline{A}))}$
13) *Every μ_I– closed set is a $\mu_I g$–closed set.*

Definition 2.8. *[6] Consider (X_1, τ_1) be an ITS, then the intuitionistic subset M of X_1 is said to be an*

i) Intuitionistic prefrontier ($IpFr$ shortly) if $IpFr(M) = Ipcl(M) - Ipint(M)$.

ii) Intuitionistic preborder ($Ipbr$ shortly) if $Ipbr(M) = M - Ipint(M)$.

Definition 2.9. *[6] For an intuitionistic subset N of X in ITS, intuitionistic α-exterior of N is defined as $I\alpha ext(N) = I\alpha int(X_\sim - N)$.*

Definition 2.10. *[6] For an intuitionistic subset N of X in ITS, intuitionistic pre-exterior of N is defined as $Ipext(N) = Ipint(X_\sim - N)$.*

Definition 2.11. *[6] Let (X, ψ) be an intuitionistic topological space. Two non-empty ISs M and N of X are said to be intuitionistic q-separated if $M \cap Icl(N) = \phi_\sim$ and $Icl(M) \cap N = \phi_\sim$. These both conditions are similar to the single condition $(M \cap Icl(N)) \cup (Icl(M) \cap N) = \phi_\sim$.*

Definition 2.12. *[7] Let (X, τ) be an ITS. Then intuitionistic set A of X is said to be*

i) μ_I α-closed set if $c_{\mu_I}(i_{\mu_I}(c_{\mu_I}(A))) \subseteq A$.

ii) μ_I semi-closed set if $i_{\mu_I}(c_{\mu_I}(A)) \subseteq A$.

iii) μ_I pre-closed set if $c_{\mu_I}(i_{\mu_I}(A)) \subseteq A$.

iv) μ_I β-closed set if $i_{\mu_I}(c_{\mu_I}(i_{\mu_I}(A))) \subseteq A$.

Definition 2.13. *[13] Let (X, μ_I) be a GTS and $A \subseteq X$. Then the μ-pre*-closure of A, denoted by $pre^*c_\mu(A)$, is the intersection of all μ- pre*closed sets containing A.*

3. $\mu_I g$- EXTERIOR OF GITS

Definition 3.1. *An intuitionistic subset A of X in GITS is said to be $\mu_I g$-Exterior (denoted by $E^*_{\mu_I}(A)$) if $E^*_{\mu_I}(A) = i^*_{\mu_I}(\overline{A})$.*

Theorem 3.1. *For intuitionistic subsets A and B of X in GITS, the following are hold.*

i) If $A \subseteq B$ then $E^*_{\mu_I}(B) \subseteq E^*_{\mu_I}(A)$.

ii) $E_{\mu_I}(A) \subseteq E^*_{\mu_I}(A)$ where $E_{\mu_I}(A)$ is the μ_I-Exterior of A.

iii) $E^*_{\mu_I}(A \cup B) \subseteq E^*_{\mu_I}(A) \cup E^*_{\mu_I}(B)$.

iv) $E^*_{\mu_I}(A) \cap E^*_{\mu_I}(B) \subseteq E^*_{\mu_I}(A \cap B)$.

Proof. (i) Suppose $A \subseteq B$, then $\overline{B} \subset \overline{A}$ which implies $i^*_{\mu_I}(\overline{B}) \subseteq i^*_{\mu_I}(\overline{A})$. Hence $E^*_{\mu_I}(B)$ $\subseteq E^*_{\mu_I}(A)$.

(ii) Suppose $x \in E_{\mu_I}(A)$, then $x \in i_{\mu_I}(\overline{A})$, which gives $x \in \overline{c_{\mu_I}(A)}$ and so $x \notin c_{\mu_I}(A)$. By the definition of $c_{\mu_I}(A)$, $x \notin \cap F$, F is μ_I-closed superset of A. Since every μ_I-closed set is a $\mu_I g$-closed set, $x \notin \cap F$, F is $\mu_I g$-closed superset of A. Hence we have $x \notin c^*_{\mu_I}(A)$. Then $x \in \overline{c^*_{\mu_I}(A)}$ = $i^*_{\mu_I}(\overline{A}) = E^*_{\mu_I}(A)$. Therefore $E_{\mu_I}(A) \subseteq E^*_{\mu_I}(A)$.

(iii) We know that $A \subseteq A \cup B$ and also $B \subseteq A \cup B$. Then $\overline{A \cup B} \subseteq \overline{A}$ and $\overline{A \cup B} \subseteq \overline{B}$. Hence $i^*_{\mu_I}(\overline{A \cup B}) \subseteq i^*_{\mu_I}(\overline{A})$ and $i^*_{\mu_I}(\overline{A \cup B}) \subseteq i^*_{\mu_I}(\overline{B})$. Therefore $E^*_{\mu_I}(A \cup B) \subseteq E^*_{\mu_I}(A) \cup E^*_{\mu_I}(B)$.

(iv) We know that $A \cap B \subseteq A$ and also $A \cap B \subseteq B$. Then we have $\overline{A} \subseteq \overline{A \cap B}$ and $\overline{B} \subseteq \overline{A \cap B}$. Hence $i^*_{\mu_I}(\overline{A}) \subseteq i^*_{\mu_I}(\overline{A \cap B})$ and $i^*_{\mu_I}(\overline{B}) \subseteq i^*_{\mu_I}(\overline{A \cap B})$. Therefore $E^*_{\mu_I}(A) \cap E^*_{\mu_I}(B) \subseteq E^*_{\mu_I}(A \cap B)$.

□

Theorem 3.2. $i^*_{\mu_I}(E^*_{\mu_I}(A)) = E^*_{\mu_I}(A)$.

Proof. $i^*_{\mu_I}(E^*_{\mu_I}(A)) = i^*_{\mu_I}(i^*_{\mu_I}(\overline{A})) = i^*_{\mu_I}(\overline{(c^*_{\mu_I}(A))}) = \overline{(c^*_{\mu_I}(c^*_{\mu_I}(A)))} = \overline{(c^*_{\mu_I}(A))} = i^*_{\mu_I}(\overline{A}) = E^*_{\mu_I}(A)$.

□

Result 3.1. *i)* $E^*_{\mu_I}(\phi_\sim) = i^*_{\mu_I}(X_\sim)$ *ii)* $E^*_{\mu_I}(X_\sim) = i^*_{\mu_I}(\phi_\sim)$
iii) $E^*_{\mu_I}(A)$ *is the largest* $\mu_1 g$*-open subset of* $\overline{A}$.

Proof. *i)* $E^*_{\mu_I}(\phi_\sim) = i^*_{\mu_I}(\overline{\phi_\sim}) = i^*_{\mu_I}(X_\sim)$.
ii) $E^*_{\mu_I}(X_\sim) = i^*_{\mu_I}(\overline{X_\sim}) = i^*_{\mu_I}(\phi_\sim)$.
iii) Since $i^*_{\mu_I}(A)$ is the largest $\mu_1 g$-open subset of A, $E^*_{\mu_I}(A)$ is the largest $\mu_1 g$-open subset of $\overline{A}$. □

Theorem 3.3. *i)* $E^*_{\mu_I}(A) \subseteq \overline{A}$ *ii)* $E^*_{\mu_I}(\overline{A}) \subseteq A$

Proof. *i)* $E^*_{\mu_I}(A) = i^*_{\mu_I}(\overline{A}) = \overline{(c^*_{\mu_I}(A))} \subseteq \overline{A}$
ii) $E^*_{\mu_I}(\overline{A}) = i^*_{\mu_I}(A) \subseteq A$ □

Theorem 3.4. *Let A be an intuitionistic subset of a GITS* (X, μ_I) *. Then*

i) $E^*_{\mu_I}(A) = X - c^*_{\mu_I}(A)$.
ii) $i^*_{\mu_I}(c^*_{\mu_I}(A)) \subseteq E^*_{\mu_I}(E^*_{\mu_I}(A))$.

iii) $i^*_{\mu_I}(A) \subseteq E^*_{\mu_I}(E^*_{\mu_I}(A))$.

Proof. i) $E^*_{\mu_I}(A) = i^*_{\mu_I}(\overline{A}) = \overline{c^*_{\mu_I}(A)} = X - c^*_{\mu_I}(A)$.
ii) Let $x \notin E^*_{\mu_I}(E^*_{\mu_I}(A)) = E^*_{\mu_I}(\overline{c^*_{\mu_I}(A)})$. Take $B = \overline{c^*_{\mu_I}(A)}$ Then $x \notin i^*_{\mu_I}(\overline{B}) = i^*_{\mu_I}(c^*_{\mu_I}(A))$ and hence $i^*_{\mu_I}(c^*_{\mu_I}(A)) \subseteq E^*_{\mu_I}(E^*_{\mu_I}(A))$.
iii). We know that $A \subseteq c^*_{\mu_I}(A)$. Then $i^*_{\mu_I}(A) \subseteq i^*_{\mu_I}(c^*_{\mu_I}(A)) = i^*_{\mu_I}\overline{(i^*_{\mu_I}(\overline{A})} = i^*_{\mu_I}\overline{(E^*_{\mu_I}(A))}$
$= E^*_{\mu_I}(E^*_{\mu_I}(A))$. Therefore $i^*_{\mu_I}(A) \subseteq E^*_{\mu_I}(E^*_{\mu_I}(A))$. □

Note 3.1. *From all the above discussions, we conclude that some properties such as enhancing, monotonicity and idempotency does not hold in $\mu_I g$-Exterior of GITS. $\mu_I g$-Exterior need not be $\mu_I g$-open since the union of $\mu_I g$-closed sets need not be $\mu_I g$-closed sets. Hence $E^*_{\mu_I}(A)$ need not be $\mu_I g$-open whenever $i^*_{\mu_I}(A) = A$.*

4. $\mu_I g$-Border of GITS

Definition 4.1. *The $\mu_I g$-border of A (denoted by $b^*_{\mu_I}(A)$) is defined as $b^*_{\mu_I}(A) = A - i^*_{\mu_I}(A)$.*

Theorem 4.1. *Let A be an intuitionistic subset of a GITS (X, μ_I). Then subsequent results are hold.*

i) $b^*_{\mu_I}(A) = A \cap c^*_{\mu_I}(X - A)$.
ii) $b^*_{\mu_I}(\phi_\sim) = \phi_\sim$.
iii) $b^*_{\mu_I}(A) \subseteq \overline{i^*_{\mu_I}(A)}$.
iv) $b^*_{\mu_I}(A) \subseteq A \subseteq c^*_{\mu_I}(A)$.

Proof. i) $b^*_{\mu_I}(A) = A - i^*_{\mu_I}(A) = A \cap \overline{i^*_{\mu_I}(A)} = A \cap c^*_{\mu_I}(\overline{A}) = A \cap c^*_{\mu_I}(X - A)$.
ii) $b^*_{\mu_I}(\phi_\sim) = \phi_\sim \cap \overline{i^*_{\mu_I}(\phi_\sim)} = \phi_\sim \cap \overline{\phi_\sim} = \phi_\sim$.
iii) $b^*_{\mu_I}(A) = A - i^*_{\mu_I}(A) = A \cap \overline{i^*_{\mu_I}(A)} \subseteq \overline{i^*_{\mu_I}(A)}$.
iv) By the definition of $\mu_I g$-border of A, $b^*_{\mu_I}(A) \subseteq A$. We know that $A \subseteq c^*_{\mu_I}(A)$. Therefore $b^*_{\mu_I}(A) \subseteq A \subseteq c^*_{\mu_I}(A)$. □

Theorem 4.2. *Let A be an intuitionistic subset of a GITS (X, μ_I) . Then*

i) $i^*_{\mu_I}(b^*_{\mu_I}(A)) \subseteq A$.
ii) $b^*_{\mu_I}(i^*_{\mu_I}(A)) \subseteq A$.

iii) $b^*_{\mu_I}(A) \subseteq b_{\mu_I}(A)$, *where* $b_{\mu_I}(A)$ *is the* μ_I*-border of* A.

Proof. $i)$ $i^*_{\mu_I}(b^*_{\mu_I}(A)) \subseteq b^*_{\mu_I}(A) \subseteq A$.
$ii)$ $b^*_{\mu_I}(i^*_{\mu_I}(A)) \subseteq i^*_{\mu_I}(A) \subseteq A$.
$iii)$ Suppose $x \notin b_{\mu_I}(A) = A \cap c_{\mu_I}(X-A)$, then $x \notin A$ and $x \notin c_{\mu_I}(X-A)$, which implies $x \notin A$ and $x \notin \cap F, F$ is μ_I-closed set and $(X-A) \subseteq F$. Then $x \notin A$ and $x \notin \cap F, F$ is $\mu_I g$-closed set and $(X-A) \subseteq F$ and hence $x \notin b^*_{\mu_I}(A)$.Therefore$b^*_{\mu_I}(A) \subseteq b_{\mu_I}(A)$. □

Theorem 4.3. *Let A and B be two intuitionistic subset of a GITS* (X, μ_I)*. Then*

i) $b^*_{\mu_I}(A \cup B) \subseteq b^*_{\mu_I}(A) \cup b^*_{\mu_I}(B)$.
ii) $b^*_{\mu_I}(A) \cap b^*_{\mu_I}(B) \subseteq b^*_{\mu_I}(A \cap B)$.

Proof. $i)$. $b^*_{\mu_I}(A \cup B) = (A \cup B) - i^*_{\mu_I}(A \cup B) = (A \cup B) \cap \overline{i^*_{\mu_I}(A \cup B)} = (A \cup B) \cap c^*_{\mu_I}\overline{(A \cup B)}$
$= (A \cup B) \cap c^*_{\mu_I}(\overline{A} \cap \overline{B}) \subseteq (A \cup B) \cap [c^*_{\mu_I}(\overline{A}) \cap c^*_{\mu_I}(\overline{B})] \subseteq (A \cap c^*_{\mu_I}(A)) \cup (B \cap c^*_{\mu_I}(B)) = b^*_{\mu_I}(A) \cup b^*_{\mu_I}(B)$.
$ii)$. The proof is similar to (i). □

Example 1. *The inclusion may be strict or equal, now we explain with an example.*
i). Let $X = \{i, j, k\}$*. Then* $\mu_I g$*-closed set =* $\{X_\sim, < X, \phi, \{i\} >, < X, \phi, \{i, j\} >, < X, \{j\}, \{i\} >, < X, \{k\}, \phi >, < X, \{k\}, \{i\} >, < X, \{k\}, \{j\} >, < X, \{k\}, \{i, j\} >, < X, \{j,k\}, \phi >, < X, \{j,k\}, \{i\} >, < X, \{k,i\}, \phi >, < X, \{k,i\}, \{j\} >$.
Let $A = < X, \{j,k\}, \phi >, B = < X, \{i,k\}, \phi >$.$A \cup B = < X, X, \phi > \Rightarrow b^*_{\mu_I}(A \cup B) = < X, \phi, \{i, j\} >$.$b^*_{\mu_I}(A) \cup b^*_{\mu_I}(B) = < X, \{k\}, \phi >$*. Therefore* $b^*_{\mu_I}(A \cup B) \subset b^*_{\mu_I}(A) \cup b^*_{\mu_I}(B)$*. Let* $A = < X, \{k\}, \{j\} >, B = < X, \{k\}, \{i, j\} >$.$(A \cup B) = < X, \{k\}, \{j\} > \Rightarrow b^*_{\mu_I}(A \cup B) = < X, \{k\}, \{j\} >$*. Then* $b^*_{\mu_I}(A) \cup b^*_{\mu_I}(B) = < X, \{k\}, \{j\} >$*. Therefore* $b^*_{\mu_I}(A \cup B) = b^*_{\mu_I}(A) \cup b^*_{\mu_I}(B)$.
ii). Let $X = \{s,t\}$*. Then* $\mu_I g$*-closed set =* $\{X_\sim, < X, \phi, \{s\} >, < X, \phi, \phi >, < X, \{s\}, \phi >, < X, \{t\}, \phi >, < X, \{t\}, \{s\} >\}$*. Let* $A = < X, \{s\}, \phi >, B = < X, \phi, \{t\} >$*. Then* $A \cap B = < X, \phi, \{t\} > \Rightarrow b^*_{\mu_I}(A \cap B) = < X, \phi, \{t\} >$*. Then* $b^*_{\mu_I}(A) \cap b^*_{\mu_I}(B) = < X, \phi, \{s,t\} >$*. Therefore* $b^*_{\mu_I}(A) \cap b^*_{\mu_I}(B) \subset b^*_{\mu_I}(A \cap B)$*. Let* $A = < X, \{t\}, \{s\} >, B = < X, \phi, \{s\} >$*. Then* $(A \cap B) = < X, \phi, \{s\} > \Rightarrow b^*_{\mu_I}(A \cap B) = < X, \phi, \{s\} >$ *and* $b^*_{\mu_I}(A) \cap b^*_{\mu_I}(B) = < X, \phi, \{s\} >$*. Therefore* $b^*_{\mu_I}(A) \cap b^*_{\mu_I}(B) = b^*_{\mu_I}(A \cap B)$.

Remark 4.1. *For any intuitionistic subset A in ITS, the following statements are valid.*

i) $b^*_{\mu_I}(A) \cup i^*_{\mu_I}(A) = A$.

ii) $b^*_{\mu_I}(A) \cap i^*_{\mu_I}(A) = \phi_\sim$.

But in GITS these are not valid. Now we explain with an example.

Let $X = \{0,1,2\}$. *Then* $\mu_I g$*-closed set* = $\{X_\sim, <X,\phi,\{0\}>, <X,\phi,\{0,1\}>, <X,\{1\},\{0\}>, <X,\{2\},\phi>, <X,\{2\},\{0\}>, <X,\{2\},\{1\}>, <X,\{2\},\{0,1\}>, <X,\{1,2\},\phi>, <X,\{1,2\},\{0\}>, <X,\{2,0\},\phi>, <X,\{2,0\},\{1\}>\}$.

Now take $A = <X,\{2,0\},\phi>$. *Then* $b^*_{\mu_I}(A) = <X,\phi,\{0\}>$ *and* $i^*_{\mu_I}(A) = <X,\{0\},\phi>$. *Therefore* $b^*_{\mu_I}(A) \cup i^*_{\mu_I}(A) = <X,\{0\},\phi> \neq A$. *Also* $b^*_{\mu_I}(A) \cap i^*_{\mu_I}(A) = <X,\phi,\{0\}>$ *which is not equal to* $<X,\phi,X> = \phi_\sim$.

Note 4.1. *For* $\mu_I g$*-border of GITS, the properties such as monotonicity, enhancing and idempotency does not hold.*

5. $\mu_I g$- FRONTIER OF GITS

Definition 5.1. *If A is an intuitionistic subset of a GITS* (X,μ_I)*, then* $\mu_I g$*-Frontier of A (denoted by* $Fr^*_{\mu_I}(A)$*) is defined as* $Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A) - i^*_{\mu_I}(A)$.

Theorem 5.1. *Let A be an intuitionistic subset of a GITS* (X,μ_I)*. Then the subsequent results are valid.*

i) $Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A) \cap c^*_{\mu_I}(\overline{A})$.

ii) $Fr^*_{\mu_I}(\overline{A}) = Fr^*_{\mu_I}(A)$.

iii) $\overline{Fr^*_{\mu_I}(A)} = i^*_{\mu_I}(\overline{A}) \cup i^*_{\mu_I}(A)$.

iv) $Fr^*_{\mu_I}(A) \subseteq Fr_{\mu_I}(A)$*, where* $Fr_{\mu_I}(A)$ *is the* μ_I*-Frontier of A.*

v) $b^*_{\mu_I}(A) \subseteq Fr^*_{\mu_I}(A)$.

Proof. i) $Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A) - i^*_{\mu_I}(A) = c^*_{\mu_I}(A) \cap \overline{i^*_{\mu_I}(A)} = c^*_{\mu_I}(A) \cap c^*_{\mu_I}(\overline{A})$.

ii) $Fr^*_{\mu_I}(\overline{A}) = c^*_{\mu_I}(\overline{A}) \cap c^*_{\mu_I}(A) = Fr^*_{\mu_I}(A)$.

iii) $\overline{Fr^*_{\mu_I}(A)} = \overline{c^*_{\mu_I}(A) \cap c^*_{\mu_I}(\overline{A})} = \overline{c^*_{\mu_I}(A)} \cup \overline{c^*_{\mu_I}(\overline{A})} = i^*_{\mu_I}(\overline{A}) \cup i^*_{\mu_I}(A)$.

(iv) $Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A) - i^*_{\mu_I}(A) \subseteq c_{\mu_I}(A) - i_{\mu_I}(A) = Fr_{\mu_I}(A)$.

(v) $b^*_{\mu_I}(A) = A \cap c^*_{\mu_I}(X - A) = A \cap \overline{i^*_{\mu_I}(A)} \subseteq c^*_{\mu_I}(A) \cap \overline{i^*_{\mu_I}(A)} = Fr^*_{\mu_I}(A)$. □

Theorem 5.2. *If an intuitionistic subset A is $\mu_I g$- closed in GITS (X,μ_I), then $A - Fr^*_{\mu_I}(A) \subseteq A$.*

Proof. We know that $A - Fr^*_{\mu_I}(A) = A \cap \overline{Fr^*_{\mu_I}(A)}$. Now $Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A) \cap (\overline{A}) \Rightarrow \overline{Fr^*_{\mu_I}(A)} = \overline{c^*_{\mu_I}(A)} \cup A \Rightarrow A \cap \overline{Fr^*_{\mu_I}(A)} = A \cap \overline{c^*_{\mu_I}(A)} \cup (A \cap A) \subseteq (A \cap (\overline{A})) \cup A = A$. Therefore $A - Fr^*_{\mu_I}(A) \subseteq A$. □

Remark 5.1. *The inclusion may be strict or equal. Now let us seen the following example. Let $X = \{x,y,z\}$. Then $\mu_I g$-closed set $= \{X_\sim, <X,\phi,\{x\}>, <X,\phi,\{x,y\}>, <X,\{z\},\{x\}>, <X,\{z\},\phi>, <X,\{y\},\{x\}>, <X,\{z\},\{y\}>, <X,\{z\},\{x,y\}>, <X,\{y,z\},\{x\}>, <X,\{z,y\},\phi>, <X,\{x,z\},\{y\}>, <X,\{x,z\},\phi>\}$. Take $A = <X,\phi,\{y\}>$.*
Then $A - Fr^*_{\mu_I}(A) = <X,\phi,\{y,z\}> \subset A$. Also we take $J = <X,\{y\},\phi>$. Then $J - Fr^*_{\mu_I}(J) = J$.

Theorem 5.3. *If an intuitionistic subset A is $\mu_I g$- closed in GITS (X,μ_I), then $Fr^*_{\mu_I}(A) \subseteq A$.*

Proof. $Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A) - i^*_{\mu_I}(A)$. Since A is $\mu_I g$- closed, $Fr^*_{\mu_I}(A) = A - i^*_{\mu_I}(A) = b^*_{\mu_I}(A) \subseteq A$. □

Note 5.1. *If an intuitionistic subset A is $\mu_I g$-closed in GITS (X,μ_I), then its border and frontier are equal.*

Theorem 5.4. *If an intuitionistic subset A is $\mu_I g$-open in GITS, then $Fr^*_{\mu_I}(A) \subseteq \overline{A}$.*

Proof. $Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A) \cap c{\mu_I}^*(\overline{A}) = c^*_{\mu_I}(A) \cap \overline{A} \subseteq \overline{A}$. □

Theorem 5.5. *Let A be an intuitionistic subset of a GITS (X,μ_I), then $A \cup Fr^*_{\mu_I}(A) \subseteq c^*_{\mu_I}(A)$.*

Proof. Now $A \cup Fr^*_{\mu_I}(A) = A \cup [c^*_{\mu_I}(A) \cap c^*_{\mu_I}(\overline{A})] = [A \cup c^*_{\mu_I}(A)] \cap [A \cup c^*_{\mu_I}(\overline{A})] = c^*_{\mu_I}(A) \cap [A \cup c^*_{\mu_I}(\overline{A})] \subseteq c^*_{\mu_I}(A)$.
The inclusion may be strict or equal,we discuss in the following example.
Let $X = \{x,y,z\}$. Then $\mu_I g$-closed set = $\{X_\sim, <X,\phi,\{x\}>, <X,\phi,\{x,y\}>, <X,\{z\},\{x\}>, <X,\{z\},\phi>, <X,\{y\},\{x\}>, <X,\{z\},\{y\}>, <X,\{z\},\{x,y\}>, <X, \{y,z\},\{x\}>, <X,\{z,y\},\phi>, <X,\{x,z\},\{y\}>, <X,\{x,z\},\phi>\}$. Take $A = <X,\{x\}, \phi>$. Then $Fr^*_{\mu_I}(A) = <X,\phi,\{x\}>$ and $c^*_{\mu_I}(A) = <X,\{z,x\},\phi>$. Therefore $A \cup Fr^*_{\mu_I}(A) \subset c^*_{\mu_I}(A)$. Take $A = <X,\{x\},\{z\}>$. Then $Fr^*_{\mu_I}(A) = <X,\{z\},\{x\}>$ and $c^*_{\mu_I}(A) = <X,\{z,x\},\phi>$. Therefore $A \cup Fr^*_{\mu_I}(A) = c^*_{\mu_I}(A)$. □

Theorem 5.6. *Let A be an intuitionistic subset of a GITS* (X,μ_I)*. Then* $Fr^*_{\mu_I}[c^*_{\mu_I}(A)] \subseteq Fr^*_{\mu_I}(A)$.

Proof. Let A be an intuitionistic subset of X. Now $Fr^*_{\mu_I}[c^*_{\mu_I}(A)] = c^*_{\mu_I}[c^*_{\mu_I}(A)] \cap c^*_{\mu_I}[\overline{c^*_{\mu_I}(A)}] = c^*_{\mu_I}(A) \cap c^*_{\mu_I}[\overline{c^*_{\mu_I}(A)}] \subseteq c^*_{\mu_I}(A) \cap c^*_{\mu_I}(\overline{A}) = Fr^*_{\mu_I}(A)$. Hence $Fr^*_{\mu_I}[c^*_{\mu_I}(A)] \subseteq Fr^*_{\mu_I}(A)$. □

Theorem 5.7. *Let A be an intuitionistic subset of a GITS* (X,μ_I)*. Then* $Fr^*_{\mu_I}[i^*_{\mu_I}(A)] \subseteq Fr^*_{\mu_I}(A)$.

Proof. Let A be an intuitionistic subset of X. Now $Fr^*_{\mu_I}[i^*_{\mu_I}(A)] = c^*_{\mu_I}[i^*_{\mu_I}(A)] \cap c^*_{\mu_I}[\overline{i^*_{\mu_I}(A)}] \subseteq c^*_{\mu_I}(A) \cap c^*_{\mu_I}(\overline{A}) = Fr^*_{\mu_I}(A)$. Hence $Fr^*_{\mu_I}[i^*_{\mu_I}(A)] \subseteq Fr^*_{\mu_I}(A)$. □

Remark 5.2. *In GITS we give some examples to show that the following statements are not valid.*

i) $c^*_{\mu_I}(A) = Fr^*_{\mu_I}(A) \cup i^*_{\mu_I}(A)$.

ii) $<X,\phi,X> = Fr^*_{\mu_I}(A) \cap i^*_{\mu_I}(A)$.

Let $X = \{u,v,w\}$*. Then* $\mu_I g$*-closed set =* $\{X_\sim, <X,\phi,\{u\}>, <X,\phi,\{u,v\}>, <X,\phi,\{v\}>, <X,\phi,X>, <X,\phi,\{v,w\}>, <X,\phi,\{w,u\}>, <X,\{v\},\{u\}>, <X,\{v\},\{w,u\}>, <X,\{w\},\phi>, <X,\{w\},\{u\}>, <X,\{w\},\{v\}>, <X,\{w\},\{u,v\}>, <X,\{v,w\},\phi>, <X,\{v,w\},\{u\}>, <X,\{w,u\},\phi>, <X,\{w,u\},\{v\}>\}$*. Take* $A = <X,\{u\},\phi>$*. Then* $i^*_{\mu_I}(A) = <X,\{u\},\phi>$ *and* $c^*_{\mu_I}(A) = <X,\{u,w\},\phi>$*. Also* $Fr^*_{\mu_I}(A) = <X,\phi,\{u\}>$*. Therefore* $c^*_{\mu_I}(A) \neq Fr^*_{\mu_I}(A) \cup i^*_{\mu_I}(A)$.

Let $X = \{u,v,w\}$*. Then* $\mu_I g$*-closed set =* $\{X_\sim, <X,\phi,\phi>, <X,\phi,\{v\}>, <X,\phi,\{w\}>, <X,\phi,\{v,w\}>, <X,\{u\},\{v\}>, <X,\{u\},\{w\}>, <X,\{u\},\phi>, <X,\{u\},\{v,w\}>, <X,\{w\},\{v\}>, <X,\{v,u\},\phi>, <X,\{v,u\},\{w\}>, <X,\{w\},\phi>, <X,\{w,u\},\phi>, <X,\{w,u\},\{v\}>, <X,\{v\},\{w\}>, <X,\{v,w\},\phi>\}$*. Take* $A = <X,\{u\},\phi>$*. Then* $i^*_{\mu_I}(A) = <X,\phi,\phi>$ *and* $Fr^*_{\mu_I}(A) = <X,\phi,\phi>$*. Therefore* $<X,\phi,X> \neq Fr^*_{\mu_I}(A) \cap i^*_{\mu_I}(A)$.

In $\mu_I g$*-Frontier the properties such as enhancing, monotonicity and idempotency fails. Also* $Fr^*_{\mu_I}(A \cap B)$ *and* $Fr^*_{\mu_I}(A) \cap Fr^*_{\mu_I}(B)$ *do not depends on each other. Hence there is no relation* *between $Fr^*_{\mu_I}(A\cap B)$ and $Fr^*_{\mu_I}(A)\cap Fr^*_{\mu_I}(B)$. Therefore both are independent. $\mu_I g$-Frontier need not be $\mu_I g$-closed, since the intersection of $\mu_I g$-closed sets need not be $\mu_I g$-closed sets. Hence $Fr^*_{\mu_I}(A)$ need not be $\mu_I g$-closed whenever $c^*_{\mu_I}(A) = A$.*

6. $q\mu_I g$- SEPARATED IN GITS

Definition 6.1. *Two non-empty intuitionistic subsets A and B of a GITS (X,μ_I) are said to be intuitionistic $q\mu_I g$-separated if $A\cap c^*_{\mu_I}(B) = \phi_\sim$ and $c^*_{\mu_I}(A)\cap B = \phi_\sim$. These both conditions are similar to the single condition $(A\cap c^*_{\mu_I}(B))\cup(c^*_{\mu_I}(A)\cap B) = \phi_\sim$.*

Note that any two intuitionistic $q\mu_I g$-separated sets are intuitionistic disjoint. But two intuitionistic disjoint sets are not necessarily intuitionistic $q\mu_I g$-separated. This condition can be seen in the following example.

Example 2. *Let $X = \{1,2,3\}$. Then $\mu_I g$-closed set = $\{X_\sim, <X,\phi,\{1\}>, <X,\phi,\{3\}>, <X,\phi,\{3,1\}>, <X,\{2\},\{1\}>, <X,\{2\},\phi>, <X,\{2\},\{3\}>, <X,\{2\},\{1,3\}>, <X,\{1,2\},\phi>, <X,\{2,3\},\{1\}>, <X,\{2,3\},\phi>\}$. Let $A = <X,\{1\},\{2,3\}>$, $B = <X,\{2,3\},\{1\}>$, $c^*_{\mu_I}(A) = <X,\{2,1\},\{3\}>$ and $c^*_{\mu_I}(B) = <X,\{2,3\},\{1\}>$. Then $A\cap c^*_{\mu_I}(B) = \phi_\sim$ but $c^*_{\mu_I}(A)\cap B \neq \phi_\sim$. Here A and B are intuitionistic disjoint sets but not intuitionistic $q\mu_I g$–separated.*

Theorem 6.1. *If A and B are intuitionistic $q\mu_I g$–separated sets of GITS (X,μ_I) and $M\subset A$ and $N\subset B$, then M and N are also intuitionistic $q\mu_I g$–separated.*

Proof. Given $M\subset A \Rightarrow c^*_{\mu_I}(M)\subset c^*_{\mu_I}(A)$ and $N\subset B\Rightarrow c^*_{\mu_I}(N)\subset c^*_{\mu_I}(B)$. Since A and B are intuitionistic $q\mu_I g$–separated sets, it gives $A\cap c^*_{\mu_I}(B) = \phi_\sim$ and $c^*_{\mu_I}(A)\cap B = \phi_\sim$. Hence $c^*_{\mu_I}(M)\cap N = \phi_\sim$ and $M\cap c^*_{\mu_I}(N) = \phi_\sim$. Therefore M and N are intuitionistic $q\mu_I g$–separated. □

7. SOME NEW CLOSED SETS IN GITS

The intersection of all $\mu_I g$-closed superset of A is called $\mu_I g$-closure of A and it is denoted $c^*_{\mu_I}(A)$. By using this operator $c^*_{\mu_I}$,we define the following.

Definition 7.1. *An intuitionistic subset A of X in GITS is said to be*

i) $\alpha^*\mu_I$–closed set if $c^*_{\mu_I}(i_{\mu_I}(c_{\mu_I}(A)))\subseteq A$.

ii) *$\alpha^*\mu_I$–open set if $A \subseteq i^*_{\mu_I}(c_{\mu_I}(i_{\mu_I}(A)))$.*

iii) *semi$^*\mu_I$-closed set if $i^*_{\mu_I}(c_{\mu_I}(A)) \subseteq A$.*

iv) *semi $^*\mu_I$-open set if $A \subseteq c^*_{\mu_I}(i_{\mu_I}(A))$.*

v) *pre$^*\mu_I$-closed set if $c^*_{\mu_I}(i_{\mu_I}(A)) \subseteq A$.*

vi) *pre$^*\mu_I$-open set if $A \subseteq i^*_{\mu_I}(c_{\mu_I}(A))$.*

vii) *$\beta^*\mu_I$–closed set if $i^*_{\mu_I}(c_{\mu_I}(i_{\mu_I}(A))) \subseteq A$.*

viii) *$\beta^*\mu_I$–open set if $A \subseteq c^*_{\mu_I}(i_{\mu_I}(c_{\mu_I}(A)))$.*

Theorem 7.1. *Every semi$^*\mu_I$-closed set is $\beta^*\mu_I$-closed set but the converse is not true.*

Proof. Suppose A is a semi$^*\mu_I$-closed set then $i^*_{\mu_I}(c_{\mu_I}(A)) \subseteq A$ which implies $i^*_{\mu_I}(c_{\mu_I}$ $(i_{\mu_I}(A))) \subseteq i^*_{\mu_I}(c_{\mu_I}(A)) \subseteq A$ and hence A is a $\beta^*\mu_I$-closed set. □

Example 3. *The converse of the above theorem need not be true. Let $X = \{s,t\}$. Then $\mu_I g$-closed set $= \{X_\sim, < X, \phi, \{s\} >, < X, \phi, \phi >, < X, \{s\}, \phi >, < X, \{t\}, \phi >, < X, \{t\}, \{s\} >\}$. Here $< X, \{t\}, \phi >$ is a $\beta^*\mu_I$-closed set but not a semi$^*\mu_I$-closed set.*

In GITS, we obtain that there is no relation between $\mu_I g$-closed sets and semi $^*\mu_I$-closed set, $\alpha^*\mu_I$-closed set, $\beta^*\mu_I$-closed set. So each one is independent to each other. But there is a relation between $\mu_I g$-closed set and pre$^*\mu_I$-closed set. Now we discuss about the characterization of pre$^*\mu_I$-closed set.

8. PRE$^*\mu_I$-CLOSED SET

Theorem 8.1. *Every $\mu_I g$-closed set is a pre$^*\mu_I$-closed set but the converse is not true.*

Proof. Suppose A is a $\mu_I g$–closed set, then $c^*_{\mu_I}(A) = A$. Also we know that $i_{\mu_I}(A) \subseteq A$ it gives $c^*_{\mu_I}(i_{\mu_I}(A)) \subseteq c^*_{\mu_I}(A) = A$. Therefore A is a pre$^*\mu_I$-closed set. □

Example 4. *The converse of the above theorem need not be true. Now we can see in the following illustration. Let $X = \{a,b,c\}$. Then $\mu_I g$–closed set $= \{< X, X, \phi >, < X, \phi, \{a\} >, < X, \phi, \{c\} >, < X, \phi, \{c,a\} >, < X, \{b\}, \{a\} >, < X, \{b\}, \phi >, < X, \{a,b\}, \phi >, < X, \{b\}, \{a,c\} >, < X, \{a,b\}, \{c\} >, < X, \{b,c\}, \phi >, < X, \{b,c\}, \{a\} >, < X, \{b\}, \{c\} >\}$ and pre $^*\mu_I$-closed set $= \{< X, X, \phi >, < X, \phi, \{a\} >, < X, \phi, \{c\} >, < X, \{c\}, \{a\} >$*

$, < X, \phi, \{c,a\} >, < X, \{b\}, \{a\} >, < X, \{b\}, \phi >, < X, \{a,b\}, \phi >, < X, \{ b\}, \{a,c\} >, < X, \{a,b\}, \{c\} >, < X, \{b,c\}, \phi >, < X, \{b,c\}, \{a\} >, < X, \{b\}, \{c\} >\}$ *In this example,* $< X, \{c\}, \{a\} >$ *is a pre*$^*\mu_I$*-closed set but not a* $\mu_I g$*-closed set.*

Theorem 8.2. *Every* $\alpha^*\mu_I$*-closed set is a pre*$^*\mu_I$*-closed set.*

Proof. Suppose A is a $\alpha^*\mu_I$-closed set, then $c^*_{\mu_I}(i_{\mu_I}(c_{\mu_I}(A))) \subseteq A$. Now $c^*_{\mu_I}(i_{\mu_I}(A)) \subseteq c^*_{\mu_I}(A)$ and hence A is a pre$^*\mu_I$-closed set. □

Example 5. *The converse of the above theorem need not be true.*
Let $X = \{1,2,3\}$. *Then pre*$^*\mu_I$*-closed set* $= \{X_\sim, < X, \phi, \{1\} >, < X, \phi, \{3\} >, < X, \phi, \{3,1\} >, < X, \{2\}, \{3\} >, < X, \{3\}, \{1\} >, < X, \{2,3\}, \phi >, < X, \{2\}, \phi >, < X, \{ 2\}, \{1\} >, < X, \{2\}, \{1,3\} >, < X, \{1,2\}, \{3\} < X, \{1,2\}, \phi >, < X, \{2,3\}, \{1\} >\}$. α^* μ_I*-closed set* $= \{< X, \{2\}, \{1\} >, < X, \{2\}, \{1,3\} >, < X, \{3,2\}, \{1\} >, < X, \{2\}, \phi >, X_\sim, \phi_\sim, < X, \phi, \{1\} >, < X, \phi, \{3\} >, < X, \phi, \{3,1\} >, < X, \{2\}, \{3\} >, < X, \{3\}, \{1\} >\}$. *Here* $< X, \{1,2\}, \phi >, < X, \{1,2\}, \{3\} >, < X, \{3,2\}, \phi >$ *are pre*$^*\mu_I$*-closed sets but not a* $\alpha * \mu_I$*-closed sets.*

Remark 8.1. *Union of two pre*$^*\mu_I$*-closed sets need not be pre*$^*\mu_I$*-closed set. Now we can see the successive illustration. Let* (X, μ_I) *be a GITS where* $X = \{a,b,c\}$. *Then pre*$^*\mu_I$*-closed set* $= \{< X, \phi, \{a\} >, < X, X, \phi >, < X, \phi, \{c\} >, < X, \phi, \{c,a\} >, < X, \{b\}, \{a\} >, < X, \{c\}, \{a\} >< X, \{b\}, \phi >, < X, \{a,b\}, \phi >, < X, \{b\}, \{a,c\} >, < X, \{a,b\}, \{c\} >, < X, \{b,c\}, \phi >, < X, \{b,c\}, \{a\} >, < X, \{b\}, \{c\} >\}$. *Let* $A =< X, \phi, \{a\} >$ *and* $B =< X, \phi, \{c\} >$ *be pre*$^*\mu_I$*-closed sets. Then* $A \cup B =< X, \phi, \phi >$ *which is not a pre*$*\mu_I$*-closed set.*

Theorem 8.3. *Arbitrary intersection of pre*$^*\mu_I$*-closed sets are pre*$^*\mu_I$*-closed set.*

Proof. Let $\{F_\alpha\}$ be the collection of pre$^*\mu_I$-closed sets. Then $c^*_{\mu_I}(i_{\mu_I}(F_\alpha)) \subseteq F_\alpha$, for each α. Now $c^*_{\mu_I}(i_{\mu_I}(\cap F_\alpha)) \subseteq c^*_{\mu_I}(\cap i_{\mu_I}(F_\alpha)) \subseteq \cap c^*_{\mu_I}(i_{\mu_I}(F_\alpha)) \subseteq \cap F_\alpha$. Therefore $\cap F_\alpha$ is a pre$^*\mu_I$-closed set. □

9. PRE$^*\mu_I$-CLOSURE IN GITS

Definition 9.1. *Let* (X, μ_I) *be a GITS and* $A \subseteq X$. *Then the pre*$^*\mu_I$*-closure of A, denoted by* $c^*_{p\mu_I}(A)$, *is the intersection of all pre*$^*\mu_I$*-closed sets containing A.*

Theorem 9.1. *Let (X,μ_I) be a GITS. Then $A \subseteq X$ is a pre$^*\mu_I$-closed set iff $c^*_{p\mu_I}(A) = A$.*

Proof. Assume that $A \subseteq X$ is a pre$^*\mu_I$-closed set. By the definition:9.1, we have $c^*_{p\mu_I}(A) = A$. Conversely assume $c^*_{p\mu_I}(A) = A$. Using theorem:8.3, we have $A \subseteq X$ is a pre$^*\mu_I$-closed set. □

Note 9.1. *i)* $c^*_{p\mu_I}(\phi_\sim) \neq \phi_\sim$.
ii) $c^*_{p\mu_I}(X_\sim) = X_\sim$.

Theorem 9.2. *(Enhancing Property) $A \subseteq c^*_{p\mu_I}(A)$.*

Proof. Since $c^*_{p\mu_I}(A)$ is the intersection of all pre$^*\mu_I$-closed sets containing A, $A \subseteq c^*_{p\mu_I}(A)$. □

Theorem 9.3. *(Monotonicity Property) If $A \subseteq B$ then $c^*_{p\mu_I}(A) \subseteq c^*_{p\mu_I}(B)$.*

Proof. Suppose $x \notin c^*_{p\mu_I}(B)$,then $x \notin \cap F$, F is pre$^*\mu_I$-closed set and $B \subseteq F$. This impiles $x \notin F$, for some pre$^*\mu_I$-closed superset F of B. Since $A \subseteq B$, $A \subseteq F$. Hence $x \notin F$, for some pre$^*\mu_I$-closed superset of A. So $x \notin c^*_{p\mu_I}(A)$. Therefore $c^*_{p\mu_I}(A) \subseteq c^*_{p\mu_I}(B)$. □

Theorem 9.4. *(Idempotency Property) $c^*_{p\mu_I}[c^*_{p\mu_I}(A)] = c^*_{p\mu_I}(A)$.*

Proof. From theorem:9.2 and 9.3, we have $c^*_{p\mu_I}(A) \subseteq c^*_{p\mu_I}[c^*_{p\mu_I}(A)]$. Let $x \notin c^*_{p\mu_I}(A)$. Then $x \notin F$, for some pre$^*\mu_I$-closed set F such that $A \subseteq F \Rightarrow c^*_{p\mu_I}(A) \subseteq c^*_{p\mu_I}(F) = F$ and hence $x \notin c^*_{p\mu_I}[c^*_{p\mu_I}(A)]$. Then we get $c^*_{p\mu_I}[c^*_{p\mu_I}(A)] \subseteq c^*_{p\mu_I}(A)$. Therefore $c^*_{p\mu_I}[c^*_{p\mu_I}(A)] = c^*_{p\mu_I}(A)$. □

Theorem 9.5. $A \subseteq c^*_{p\mu_I}(A) \subseteq c^*_{\mu_I}(A) \subseteq c_{\mu_I}(A)$.

Proof. Suppose $x \notin c_{\mu_I}(A)$,then $x \notin \cap F$, where F is a μ_I-closed superset of A and so $x \notin \cap F, F$ is a $\mu_I g$-closed superset of A. That is $x \notin c^*_{\mu_I}(A)$ which implies $x \notin \cap F, F$ is a pre$^*\mu_I$-closed superset of A. Then $x \notin F$ for some pre$^*\mu_I$-closed superset of A. Therefore $x \notin A$ and hence we have $A \subseteq c^*_{p\mu_I}(A) \subseteq c^*_{\mu_I}(A) \subseteq c_{\mu_I}(A)$. □

Theorem 9.6. $c^*_{p\mu_I}(A \cap B) \subseteq c^*_{p\mu_I}(A) \cap c^*_{p\mu_I}(B)$.

Proof. We know that $A \cap B \subseteq A$ and $A \cap B \subseteq B$. Then $c^*_{p\mu_I}(A \cap B) \subseteq c^*_{p\mu_I}(A)$ and $c^*_{p\mu_I}(A \cap B) \subseteq c^*_{p\mu_I}(B)$. Therefore $c^*_{p\mu_I}(A \cap B) \subseteq c^*_{p\mu_I}(A) \cap c^*_{p\mu_I}(B)$. □

Example 6. *The inclusion may be strict or equal, we can see the ensuing illustration.*

Let $X=\{a,b,c\}$. *Then pre*$^*\mu_I$*-closed set* $=\{<X,\phi,\{a\}>,<X,X,\phi>,<X,\phi,\{c\}>,<X,\phi,\{c,a\}>,<X,\{b\},\{a\}>,<X,\{c\},\{a\}>,<X,\{b\},\phi>,<X,\{a,b\},\phi>,<X,\{b\},\{a,c\}>,<X,\{a,b\},\{c\}>,<X,\{b,c\},\phi>,<X,\{b,c\},\{a\}>,<X,\{b\},\{c\}>\}$. *Let* $A=<X,\{c\},\phi>$ *and* $B=<X,\{b\},\{a,c\}>$. *Then* $c^*_{p\mu_I}(A)=<X,\{b,c\},\phi>, c^*_{p\mu_I}(B)=<X,\{b\},\{a,c\}>$ *which implies* $c^*_{p\mu_I}(A)\cap c^*_{p\mu_I}(B)=<X,\{b\},\{a,c\}>$. *Now,* $A\cap B=<X,\phi,\{a,c\}>$.*Then*$c^*_{p\mu_I}(A\cap B)=<X,\phi,\{a,c\}>$.*Hence* $c^*_{p\mu_I}(A\cap B)\subset c^*_{p\mu_I}(A)\cap c^*_{p\mu_I}(B)$.*Take* $A=<X,\phi,\phi>, B=<X,\phi,\{a\}>$. *Then* $A\cap B=<X,\phi,\{a\}>$ *which gives* $c^*_{p\mu_I}(A\cap B)=<X,\phi,\{a\}>$. $c^*_{p\mu_I}(A)=<X,\{b\},\phi>, c^*_{p\mu_I}(B)=<X,\phi,\{a\}>$. *Hence* $c^*_{p\mu_I}(A\cap B)=c^*_{p\mu_I}(A)\cap c^*_{p\mu_I}(B)$.

Theorem 9.7. $c^*_{p\mu_I}(A)\cup c^*_{p\mu_I}(B)\subseteq c^*_{p\mu_I}(A\cup B)$.

Proof. We know that $A\subseteq A\cup B$ and $B\subseteq A\cup B$. Then $c^*_{p\mu_I}(A)\subseteq c^*_{p\mu_I}(A\cup B)$ and $c^*_{p\mu_I}(B)\subseteq c^*_{p\mu_I}(A\cup B)$. Therefore $c^*_{p\mu_I}(A)\cup c^*_{p\mu_I}(B)\subseteq c^*_{p\mu_I}(A\cup B)$. □

Example 7. *The inclusion may be strict or equal, we can see the ensuing illustration.*

Let $X=\{p,q,r\}$ *be a GITS* (X,μ_I). *Then pre*$^*\mu_I$*-closed set* $=\{<X,\phi,\{p\}>,<X,X,\phi>,<X,\phi,\{r\}>,<X,\phi,\{r,p\}>,<X,\{q\},\{p\}>,<X,\{r\},\{p\}>,<X,\{q\},\phi>,<X,\{p,q\},\phi>,<X,\{q\},\{p,r\}>,<X,\{p,q\},\{r\}>,<X,\{q,r\},\phi>,<X,\{q,r\},\{p\}>,<X,\{q\},\{r\}>\}$. *Let* $A=<X,\phi,\{p\}>$ *and* $B=<X,\phi,\{r\}>$. *Then* $c^*_{p\mu_I}(A)=<X,\phi,\{p\}>, c^*_{p\mu_I}(B)=<X,\phi,\{r\}>$*which implies* $c^*_{p\mu_I}(A)\cup c^*_{p\mu_I}(B)=<X,\phi,\phi>$. *Now,* $A\cup B=<X,\phi,\phi>$. *Then* $c^*_{p\mu_I}(A\cup B)=<X,\{q\},\phi>$. *Hence* $c^*_{p\mu_I}(A)\cup c^*_{p\mu_I}(B)\subset c^*_{p\mu_I}(A\cup B)$. *Take* $A=<X,\phi,\phi>, B=<X,\phi,\{p\}>$. *Then* $A\cup B=<X,\phi,\phi>$ *which gives* $c^*_{p\mu_I}(A\cup B)=<X,\{q\},\phi>$. $c^*_{p\mu_I}(A)=<X,\{q\},\phi>, c^*_{p\mu_I}(B)=<X,\phi,\{p\}>$. *Hence* $c^*_{p\mu_I}(A\cup B)=c^*_{p\mu_I}(A)\cup c^*_{p\mu_I}(B)$.

10. PRE$^*\mu_I$-OPEN IN GITS

Definition 10.1. *Let* (X,μ_I) *be a GITS. Then* $A\subseteq X$ *is called pre*$^*\mu_I$*-open (denoted by* $i^*_{p\mu_I}(A)$*) if the complement of A is a pre*$^*\mu_I$*-closed set.*

Theorem 10.1. *Every* $\mu_I g$*-open set is a pre*$^*\mu_I$*-open set but the converse is not true.*

Proof. Suppose A is a $\mu_I g$-open set then $i^*_{\mu_I}(A) = A$. Also we know that $A \subseteq c_{\mu_I}(A)$ which gives $i^*_{\mu_I}(c_{\mu_I}(A)) \supseteq i^*_{\mu_I}(A) = A$. Therefore A is a pre$^*\mu_I$-open set. □

Example 8. *The converse of the above theorem need not be true. Now we can see the following illustration.*

Let $X = \{a,b,c\}$. Then $\mu_I g$-open set $= \{<X,\phi,X>, <X,\{a\},\phi>, <X,X,\phi>, <X,\{b\},\phi>, <X,\{a,b\},\phi>, <X,\{a\},\{b\}>, <X,\{a,c\},\{b\}>, <X,\phi,\{c\}>, <X,\{a\},\{c\}>, <X,\{b\},\{c\}>, <X,\{a,b\},\{c\}>, <X,\phi,\{b,c\}>, <X,\{a\},\{b,c\}>, <X,\phi,\{c,a\}>, <X,\{c,a\},\phi>, <X,\{b\},\{c,a\}>, <X,\{b,c\},\phi>\}$ *and pre$^*\mu_I$-open set* $= \{<X,\phi,X>, <X,\{a\},\phi>, <X,X,\phi>, <X,\{b\},\phi>, <X,\{a,b\},\phi>, <X,\{a\},\{b\}>, <X,\{a,c\},\{b\}>, <X,\phi,\{c\}>, <X,\{a\},\{c\}>, <X,\{b\},\{c\}>, <X,\{a,b\},\{c\}>, <X,\phi,\{b,c\}>, <X,\{a\},\{b,c\}>, <X,\phi,\{c,a\}>, <X,\{c,a\},\phi>, <X,\{b\},\{c,a\}>, <X,\{b,c\},\phi>, <X,\{c\},\phi>, <X,\{c\},\{a\}>, <X,\{b,c\},\{a\}>, <X,\{c\},\{b\}>, <X,\{c\},\{a,b\}>, <X,\phi,\phi>\}$. *In this example,* $<X,\{c\},\{a\}>, <X,\{c\},\phi>, <X,\{b,c\},\{a\}>, <X,\{c\},\{b\}>, <X,\{c\},\{a,b\}>$ *and* $<X,\phi,\phi>$ *are pre$^*\mu_I$-open sets but not a $\mu_I g$-open sets.*

Theorem 10.2. *Arbitrary union of pre$^*\mu_I$-open sets are pre$^*\mu_I$-open set.*

Proof. Let $\{U_\alpha\}$ be a collection of pre$^*\mu_I$-open sets. Then $\{X - \{U_\alpha\}\}$ is a collection of pre$^*\mu_I$-closed sets. By theorem:8.3, $\cap\{X - \{U_\alpha\}\}$ is a pre$^*\mu_I$-closed sets. Therefore $\cup\{U_\alpha\}$ is a pre$^*\mu_I$-open set. □

Remark 10.1. *Intersection of any two pre$^*\mu_I$-open sets need not be pre$^*\mu_I$-open set. Now we can see the following example. Let $X = \{a,b,c\}$ be a GITS (X,μ_I).*
Then pre$^*\mu_I$-open set $= \{<X,\{a\},\phi>, <X,\phi,X>, <X,\{c\},\phi>, <X,\{c,a\},\phi>, <X,\{a\},\{b\}>, <X,\{a\},\{c\}>, <X,\phi,\{b\}>, <X,\phi,\{a,b\}>, <X,\{a,c\},\{b\}>, <X,\{c\},\{a,b\}>, <X,\phi,\{b,c\}>, <X,\{a\},\{b,c\}>, <X,\{c\},\{b\}>\}$. *Let $A = <X,\{a\},\phi>$ and $B = <X,\{c\},\phi>$ be pre$^*\mu_I$-open sets. Then $A \cap B = <X,\phi,\phi>$ which is not a pre$^*\mu_I$-open set.*

11. PRE$^*\mu_I$-INTERIOR IN GITS

Definition 11.1. *Let (X,μ_I) be a GITS and $A \subseteq X$. Then the pre$^*\mu_I$-interior of A, denoted by $i^*_{p\mu_I}(A)$, is the union of all pre$^*\mu_I$-open sets contained in A.*

Theorem 11.1. *Let (X,μ_I) be a GITS. Then $A \subseteq X$ is a pre$^*\mu_I$-open set iff $i^*_{p\mu_I}(A) = A$.*

Proof. Suppose $A \subseteq X$ is a pre$^*\mu_I$-open set, by the definition we get $i^*_{p\mu_I}(A) = A$. Conversely suppose $i^*_{p\mu_I}(A) = A$. By theorem:10.2, we get A is a pre$^*\mu_I$-open set. □

Note 11.1. (i) $i^*_{p\mu_I}(\phi_\sim) = \phi_\sim$.
(ii) $i^*_{p\mu_I}(X_\sim) \neq X_\sim$.

Theorem 11.2. *(Enhancing Property) $i^*_{p\mu_I}(A) \subseteq A$.*

Proof. Since $i^*_{p\mu_I}(A)$ is the union of all pre$^*\mu_I$-open sets contained in A, $i^*_{p\mu_I}(A) \subseteq A$. □

Theorem 11.3. *(Monotonicity Property) If $A \subseteq B$ then $i^*_{p\mu_I}(A) \subseteq i^*_{p\mu_I}(B)$.*

Proof. Given that $A \subseteq B$,then $x \in i^*_{p\mu_I}(A)$. Then $x \in \cup G$, G is a pre$^*\mu_I$-open set and $G \subseteq A$. This impiles $x \in G$, for all pre$^*\mu_I$-open set G contained in B. Hence $x \in \cup G, G$ is a pre$^*\mu_I$-open set contained in B. So $x \in i^*_{p\mu_I}(B)$. Therefore $i^*_{p\mu_I}(A) \subseteq i^*_{p\mu_I}(B)$. □

Theorem 11.4. *(Idempotency Property) $i^*_{p\mu_I}[i^*_{p\mu_I}(A)] = i^*_{p\mu_I}(A)$.*

Proof. From theorem:11.2 and 11.3, we have $i^*_{p\mu_I}[i^*_{p\mu_I}(A)] \subseteq i^*_{p\mu_I}(A)$. Let $x \in i^*_{p\mu_I}(A)$. Then $x \in G$, for some pre$^*\mu_I$-open set G such that $G \subseteq A \Rightarrow G = i^*_{p\mu_I}(G) \subseteq i^*_{p\mu_I}(A)$ and hence $x \in i^*_{p\mu_I}[i^*_{p\mu_I}(A)]$. Then we get $i^*_{p\mu_I}(A) \subseteq i^*_{p\mu_I}[i^*_{p\mu_I}(A)]$. Therefore $i^*_{p\mu_I}[i^*_{p\mu_I}(A)] = i^*_{p\mu_I}(A)$. □

Theorem 11.5. $i_{\mu_I}(A) \subseteq i^*_{\mu_I}(A) \subseteq i^*_{p\mu_I}(A) \subseteq A$.

Proof. Suppose $x \in i_{\mu_I}(A)$. Then $x \in \cup G$, where G is a μ_I- open set contained in A. It gives $x \in \cup G$, where G is a $\mu_I g-$ open set contained in A. That is $x \in i^*_{\mu_I}(A)$ which implies $x \in \cup G$, where G is a pre$^*\mu_I$-open set contained in A. Then $x \in i^*_{p\mu_I}(A)$ and by theorem:11.2, we have $x \in A$. Therefore $i_{\mu_I}(A) \subseteq i^*_{\mu_I}(A) \subseteq i^*_{p\mu_I}(A) \subseteq A$. □

Theorem 11.6. $i^*_{p\mu_I}(A \cap B) \subseteq i^*_{p\mu_I}(A) \cap i^*_{p\mu_I}(B)$.

Proof. We know that $A \cap B \subseteq A$ and $A \cap B \subseteq B$. Then $i^*_{p\mu_I}(A \cap B) \subseteq i^*_{p\mu_I}(A)$ and $i^*_{p\mu_I}(A \cap B) \subseteq i^*_{p\mu_I}(B)$. Therefore $i^*_{p\mu_I}(A \cap B) \subseteq i^*_{p\mu_I}(A) \cap i^*_{p\mu_I}(B)$. □

Example 9. *The inclusion may be strict or equal, we can see the ensuing illustration, Let* $X=\{a,b,c\}$. *Then pre*$^*\mu_I$*-closed set* $=\{<X,\phi,X>,<X,\phi,\{a\}>,<X,X,\phi>,<X,\phi\{b\},<X,\phi,\{a,b\}>,<X,\{b\},\{a\}>,<X,\{a,c\},\{b\}>,<X,\phi,\{c\}>,<X,\{c\},\phi>,<X,\{b\},\{c\}>,<X,\{a,b\},\{c\}>,<X,\phi,\{b,c\}>,<X,\{a\},\{b,c\}>,<X,\phi,\{c,a\}>,<X,\{c,a\},\phi>,<X,\{b\},\{c,a\}>,<X,\{b,c\},\phi>,<X,\{c\},\{a\}>,<X,\{b,c\},\{a\}>,<X,\{c\},\{b\}>,<X,\{c\},\{a,b\}>,<X,\phi,\phi>\}$. *Let* $A=<X,\{a\},\phi>$ *and* $B=<X,\{c\},\{a\}>$. *Then* $i^*_{p\mu_I}(A)=<X,\{a\},\phi>, i^*_{p\mu_I}(B)=<X,\{c\},\{a\}>$ *which implies* $i^*_{p\mu_I}(A)\cap i^*_{p\mu_I}(B)=<X,\phi,\{a\}>$. *Now,* $A\cap B=<X,\phi,\{a\}>$. *Then* $i^*_{p\mu_I}(A\cap B)=<X,\phi,\{a,c\}>$. *Hence* $i^*_{p\mu_I}(A\cap B)\subset i^*_{p\mu_I}(A)\cap i^*_{p\mu_I}(B)$. *Take* $A=<X,\phi,\phi>, B=<X,\phi,\{a\}>$. *Then* $A\cap B=<X,\phi,\{a\}>$ *which gives* $i^*_{p\mu_I}(A\cap B)=<X,\phi,\{c,a\}>$. $i^*_{p\mu_I}(A)=<X,\phi,\phi>, i^*_{p\mu_I}(B)=<X,\phi,\{c,a\}>$. *Hence* $i^*_{p\mu_I}(A\cap B)=i^*_{p\mu_I}(A)\cap i^*_{p\mu_I}(B)$.

Theorem 11.7. $i^*_{p\mu_I}(A)\cup i^*_{p\mu_I}(B)\subseteq i^*_{p\mu_I}(A\cup B)$.

Proof. We know that $A\subseteq A\cup B$ and $B\subseteq A\cup B$. Then $i^*_{p\mu_I}(A)\subseteq i^*_{p\mu_I}(A\cup B)$ and $i^*_{p\mu_I}(B)\subseteq i^*_{p\mu_I}(A\cup B)$. Therefore $i^*_{p\mu_I}(A)\cup i^*_{p\mu_I}(B)\subseteq i^*_{p\mu_I}(A\cup B)$. □

Example 10. *The inclusion may be strict or equal, we can see the following illustration, Let* $X=\{u,v,w\}$ *be a GITS* (X,μ_I). *Then pre*$^*\mu_I$*-closed set* $=\{<X,\phi,\{v\},<X,X,\phi>,<X,\phi,\{w\}>,<X,\phi,\{v,w\}>,<X,\{u\},\{v\}>,<X,\{u\},\{w\}>,<X,\{u\},\phi>,<X,\{w\},\phi>,<X,\{u\},\{v,w\}>,<X,\{w\},\{v\}>,<X,\{u,v\},\phi>,<X,\{u,v\},\{w\}>,<X,\{u,w\},\{v\}>,<X,\phi,\phi>,<X,\{u,w\},\phi>\}$. *Let* $A=<X,\{v,w\},\{u\}>$ *and* $B=<X,\{w,u\},\{v\}>$. *Then* $i^*_{p\mu_I}(A)=<X,\{v,w\},\{u\}>, i^*_{p\mu_I}(B)=<X,\{w\},\{u,v\}>$ *which implies* $i^*_{p\mu_I}(A)\cup i^*_{p\mu_I}(B)=<X,\{v,w\},\{u\}>$. *Now,* $A\cup B=<X,X,\phi>$. *Then* $i^*_{p\mu_I}(A\cup B)=<X,\{v,w\},\phi>$. *Hence* $i^*_{p\mu_I}(A)\cup i^*_{p\mu_I}(B)\subset i^*_{p\mu_I}(A\cup B)$. *Take* $A=<X,\phi,\phi>, B=<X,\phi,\{v\}>$. *Then* $A\cup B=<X,\phi,\phi>$ *which gives* $i^*_{p\mu_I}(A\cup B)=<X,\phi,\phi>$. $i^*_{p\mu_I}(A)=<X,\phi,\phi>, i^*_{p\mu_I}(B)=<X,\phi,\{u,v\}>$. *Hence* $i^*_{p\mu_I}(A\cup B)=i^*_{p\mu_I}(A)\cup i^*_{p\mu_I}(B)$.

Relation between Pre$^*\mu_I$-Closure and Pre$^*\mu_I$-Interior in GITS.

Property 11.1. *Let* (X,μ_I) *be a GITS and A be a subset of X. Afterwards the subsequent statements are hold.*

i) $c^*_{p\mu_I}(\overline{A}) = \overline{i^*_{p\mu_I}(A)}$

ii) $\overline{c^*_{p\mu_I}(A)} = i^*_{p\mu_I}(\overline{A})$

iii) $\overline{c^*_{p\mu_I}(\overline{A})} = i^*_{p\mu_I}(A)$

iv) $c^*_{p\mu_I}(A) = \overline{i^*_{p\mu_I}(\overline{A})}$.

Proof. i) Let $x \in c^*_{p\mu_I}(\overline{A})$. Then $x \in \cap F, F$ is a pre$^*\mu_I$-closed set and $\overline{A} \subseteq F$, which implies $x \in F$, for all pre$^*\mu_I$-closed set F such that $\overline{A} \subseteq F$. Therefore $x \notin X - F$, for all pre$*\mu_I$-open set $X - F$ such that $X - F \subseteq A$. Then $x \notin i^*_{p\mu_I}(A)$ and hence $x \in \overline{i^*_{p\mu_I}(A)}$ which implies $c^*_{p\mu_I}(\overline{A}) \subseteq \overline{i^*_{p\mu_I}(A)}$. Suppose $x \notin c^*_{p\mu_I}(\overline{A})$, then $x \notin \cap F, F$ is pre$*\mu_I$-closed set and $\overline{A} \subseteq F$, which implies $x \notin F$, for some pre$*\mu_I$-closed set contains $\overline{A}$. Therefore $x \in X - F$, for some pre$*\mu_I$-open set $X - F$ such that $X - F \subseteq A$ and consequently $x \in i^*_{p\mu_I}(A)$ which implies $x \notin \overline{i^*_{p\mu_I}(A)}$. Then $\overline{i^*_{p\mu_I}(A)} \subseteq c^*_{p\mu_I}(\overline{A})$ and we get a result.

ii) Proof is similar to i).

iii) Following by taking complements in i).

iv) Replacing A by $(\overline{A})$ in i). □

12. Conclusion

In this article, we dealt with $\mu_I g$-Exterior, $\mu_I g$-border and $\mu_I g$-Frontier,pre$^*\mu_I$-closed and pre$^*\mu_I$-open set. In future we wish to do our research in $\mu_I g$-dence, $\mu_I g$-connected, $\mu_I g$-compact and $\mu_I g$-continuous and so on.

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Conflict of Interests

The author(s) declare that there is no conflict of interests.

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