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RANDIC TYPE SDI ENERGY OF GRAPH

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Abstract. Let G be a simple graph of order n and m edges, we define Randic type SDI matrix as follows

$$RSDI_{ij} = \begin{cases} {d_u}^2{d_v}^2 & \text{if } v_i \sim v_j, \\ 0 & otherwise. \end{cases}$$

We establish the bounds for Randic type SDI energy. We generalize this energy for few classes of graphs.

Keywords: randic type SDI connectivity matrix; randic type SDI energy; randic type SDI characteristic polynomial.

2010 AMS Subject Classification: 05C50.

1. INTRODUCTION

Let $\{v_1, v_2, ..., v_n\}$ be the vertices of a graph G, then if two vertices v_i and v_j are adjacent, then we use the notation $v_i \sim v_j$. We use the notation d_i, to represent the degree of the vertex v_i. In 1978, Ivan Gutman introduced the concept of energy of graph, he defined it as the sum of the absolute values of eigenvalues of adjacency matrix with respect that graph.[2]. For additional information on energy of graph, refer [2]and [3]. Different sort of energy of graphs exist in the literature. In the recent years, many researchers attracted towards the topological indices,

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due to the applications in mathematical chemistry. We are taking such an important molecular descriptor into the study, that is Randic type SDI index. [8]

$$RSDI(G) = {d_u}^2 {d_v}^2$$

. The work on Randic energy influenced us to introduce the Randic type SDI matrix $RSDI(G)$. The Randic type SDI matrix $RSDI(G) = (RSDI_{ij})_{n\times n}$ is defined as

$$RSDI_{ij} = \begin{cases} {d_u}^2 {d_v}^2 & \text{if } v_i \sim v_j, \\ 0 & otherwise \end{cases}$$

2. The Randic Type SDI Energy of Graph

The characteristic polynomial of the Randic type SDI matrix $RSDI(G)$ is denoted by $\phi_{SL}(G,\lambda) = det(\lambda I - RSDI(G))$. This matrix is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1 > \lambda_2 > \cdots \lambda_n$. The Randic type SDI energy is given by

$$RSDIE(G) = \sum_{i=1}^{n} |\lambda_i| \tag{1}$$

3. Randic Type SDI Energy of Few Graph Structures

Theorem 3.1. *The Randic type SDI energy of a complete graph K_n is $RSDIE(K_n) = 2(n-1)^5$.*

Proof. Let K_n be the complete graph with vertex set $V = \{v_1, v_2 ... v_n\}$. The Randic type SDI matrix is

$$RSDI(K_n) = \left(\ (n-1)^4 \, (J-I) \ \right).$$

Characteristic equation is

$$\left(\lambda + (n-1)^4\right)^{n-1} \left(\lambda - (n-1)^5\right) = 0$$

and the spectrum is $Spec_{RSDI}(K_n) = \begin{pmatrix} (n-1)^4 & (n-1)^5 \\ n-1 & 1 \end{pmatrix}$.

Therefore, $RSDIE(K_n) = 2(n-1)^5$. □

Theorem 3.2. *The Randic type SDI energy of Crown graph S_n^0 is*

$$RSDIE(S_n^0) = 4(n-1)^5.$$

Proof. Let S_n^0 be a crown graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The Randic type SDI matrix is

$$RSDIE(S_n^0) = (n-1)^4 \begin{pmatrix} 0_{n\times n} & (J-I)_{n\times n} \\ (J-I)_{n\times n} & 0_{n\times n} \end{pmatrix}.$$

Characteristic equation is

$$(\lambda-(n-1)^4)^{n-1}(\lambda+(n-1)^4)^{n-1}(\lambda+(n-1)^5)(\lambda+(n-1)^5) = 0$$

spectrum is $Spec_{RSDI}(S_n^0)$
$= \begin{pmatrix} (n-1)^5 & -(n-1)^5 & (n-1)^4 & -(n-1)^4 \\ 1 & 1 & n-1 & n-1 \end{pmatrix}.$
Therefore,

$$RSDIE(S_n^0) = 4(n-1)^5.$$

□

Theorem 3.3. *For complete bipartite graph $K_{m,n}$. The Randic type SDI energy of $K_{m,n}$ is*

$$RSDIE(K_{m,n}) = 2(mn)^{\frac{5}{2}}.$$

Proof. $RSDI(K_{m,n}) = RSDI(K_{m,n}) = (mn)^2 \begin{pmatrix} 0_{m\times m} & J_{m\times n} \\ J_{n\times m} & 0_{n\times n} \end{pmatrix}.$

Characteristic equation is

$$\lambda^{m+n-2}(\lambda^2-(mn)^5) = 0$$

Hence, spectrum is

$Spec_{RSDI}(K_{m,n})$

$$= \begin{pmatrix} 0 & (mn)^{\frac{5}{2}} & -(mn)^{\frac{5}{2}} \\ m+n-2 & 1 & 1 \end{pmatrix}.$$

Therefore, $RSDIE(K_{m,n}) = 2(mn)^{\frac{5}{2}}$. □

Theorem 3.4. *The energy of the cocktail party graph $K_{n\times 2}$ is*

$$SDDE(K_{n\times 2}) = 16(n-1)^3.$$

Proof. Let $K_{n\times 2}$ be the cocktail party graph of order $2n$ having vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The degree sum square matrix is

$$\begin{bmatrix} 0 & 0 & 4(n-1)^2 & 4(n-1)^2 & \dots & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 \\ 0 & 0 & 4(n-1)^2 & 4(n-1)^2 & \dots & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 \\ 4(n-1)^2 & 4(n-1)^2 & 0 & 0 & \dots & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 \\ 4(n-1)^2 & 4(n-1)^2 & 0 & 0 & \dots & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & \dots & 0 & 0 & 4(n-1)^2 & 4(n-1)^2 \\ 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & \dots & 0 & 0 & 4(n-1)^2 & 4(n-1)^2 \\ 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & \dots & 4(n-1)^2 & 4(n-1)^2 & 0 & 0 \\ 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & 4(n-1)^2 & \dots & 4(n-1)^2 & 4(n-1)^2 & 0 & 0 \end{bmatrix}.$$

In that case, the characteristic equation is

$$\lambda^n(\lambda+4)^{n-1}(\lambda-(4n-4)) = 0$$

and hence the spectrum becomes

$$Spec_{SDD}(K_{n\times 2}) = \begin{pmatrix} -8(n-1)^3 & 0 & -8(n-1)^2 \\ 1 & n & n-1 \end{pmatrix}.$$

Therefore we arrive at the required result:

$$SDDE(K_{n\times 2}) = 16(n-1)^3.$$

□

Definition 3.5. [?] *Let G be a graph and $P_k = \{V_1, V_2, ..., V_k\}$ be a partition of its vertex set V. Then the k-complement of G is obtained as follows: For all V_i and V_j in P_k, $i \neq j$ remove the edges between V_i and V_j and add the edges between the vertices of V_i and V_j which are not in G and is denoted by $\overline{(G)_k}$.*

Theorem 3.6. *The Randic type SDI energy of the complement $\overline{K_n}$ of the complete graph K_n is*

$$RSDIE(\overline{K_n}) = 0.$$

Proof. Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$. The complement of complete graph is edge less disconnected graph. Thus the matrix is

$$RSDI(\overline{K_n}) = \begin{pmatrix} 0_{n\times n} \end{pmatrix}.$$

All the eigenvalues are null.
Thus, $RSDIE(\overline{K_n}) = 0$. □

Theorem 3.7. *The Randic type SDI energy of the complement $\overline{K_{n\times 2}}$ of the cocktail party graph $K_{n\times 2}$ of order $2n$ is*

$$RSDIE\overline{(K_{n\times 2})} = 2n.$$

Proof. Let $\overline{(K_{n\times 2})}$ be the cocktail party graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The Randic type SDI matrix is

$$RSDI(\overline{K_{n\times 2}}) = \begin{pmatrix} 0_{n\times n} & 1(I)_{n\times n} \\ 1(I)_{n\times n} & 0_{n\times n} \end{pmatrix}.$$

Characteristic equation is

$$(\lambda + 1)^n (\lambda - 1)^n = 0$$

Hence, spectrum is $Spec_{RSDI}(K_{n\times 2})$
$= \begin{pmatrix} 1 & -1 \\ n & n \end{pmatrix}.$
Therefore, $RSDIE(\overline{K_{n\times 2}}) = 2n$. □

4. Some Properties of the Randic type SDI Energy of a Graph

Let us consider the number

$$RSDIE(G) = \sum_{i=1}^{n} [d_u{}^2 d_v{}^2]^2 \tag{2}$$

Proposition 4.1. *The first three coefficients of the polynomial $\phi_{RSDI}(G,\lambda)$ are 1, 0 and $-\sum_{i=1}^{n}[d_u{}^2 d_v{}^2]^2$ respectively.*

Proof. **(i)** From the definition of the characteristic polynomial we get $a_0 = 1$ after easy calculations.

(ii) The sum of the determinants of all 1×1 principal submatrices is equal to the trace.

$$a_1 = (-1)^1 \cdot \text{trace of } [RSDI(G)] = 0.$$

(iii) Similarly we have

$$\begin{aligned}
(-1)^2 a_2 &= \sum_{1\le i<j\le n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix} \\
&= \sum_{1\le i<j\le n} a_{ii}a_{jj} - a_{ji}a_{ij} \\
&= \sum_{1\le i<j\le n} a_{ii}a_{jj} - \sum_{1\le i<j\le n} a_{ji}a_{ij} \\
&= -\sum_{i=1}^{n} [d_u{}^2 d_v{}^2]^2.
\end{aligned}$$

□

The following results can be easily proven.

Proposition 4.2. *If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the Randic type SDI eigenvalues of $RSDI(G)$, then*

$$\sum_{i=1}^{n} \lambda_i{}^2 = 2\sum_{i=1}^{n} [d_u{}^2 d_v{}^2]^2.$$

The next results gives the upper bound and lower bound for the Randic type SDI energy of a graph G.

Theorem 4.3. *Let G be a graph with n vertices. Then*

$$RSDIE(G) \leq \sqrt{2n\sum_{i=1}^{n}[{d_u}^2{d_v}^2]^2}$$

.

Theorem 4.4. *Let G be a graph with n vertices.*

$$RSDIE(G) \geq \sqrt{2\sum_{i=1}^{n}[{d_u}^2{d_v}^2]^2 + n(n-1)[Det(RSDI(G))]^{\frac{2}{n}}}.$$

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] S. Barnard, J.M. Child, Higher Algebra, Macmillan, New York, 1959.

[2] I. Gutman, The energy of a graph, Ber. Math. Stat. Sekt. Forschungsz. Graz, 103 (1978), 1-22.

[3] I. Gutman, The Energy of a Graph: Old and New Results, in: A. Betten, A. Kohnert, R. Laue, A. Wassermann (Eds.), Algebraic Combinatorics and Applications, Springer Berlin Heidelberg, Berlin, Heidelberg, 2001: pp. 196–211.

[4] K.N. Prakasha, P.S.K. Reddy, I.N. Cangul, Partition Laplacian energy of a graph, Adv. Stud. Contemp. Math. 27(4) (2017), 477-494.

[5] K.N Prakasha, K. Kiran, S. Rakshith, Randic type SDI index of graph. TWMS J. Appl. Eng. Math. 9 (2019), 894-900.

[6] R. Todeschini, V. Consonni, Handbook of Molecular Descriptors. Wiley–VCH, Weinheim, 2000

[7] R. Todeschini, V. Consonni, Molecular descriptors for chemoinformatics. Wiley-VCH, New York, 2009.

[8] D. Vuki čević, M. Ga š perov, Bond Additive Modeling 1. Adriatic Indices, Croat. Chem. Acta, 83 (2010), 243–260.