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STEADY STATE ANALYSIS OF QUEUEING SYSTEM WITH RANDOM VACATION SUBJECT TO SUPPLEMENTARY VARIABLE METHOD

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Abstract: This article, considered about a M/G/1 retrial system with multi option customer single working vacation. The administration is state subordinate wherein server gives first fundamental help to all showing up clients and second discretionary to just not many of them who requests for them. After the administration finishing of a positive client on the off chance that the circle gets unfilled, at occasion server goes for a working excursion. Excursion interference happens when the server is exposed to breakdown because of the appearance of negative clients. Further, the server goes for a fix at whatever point there is a breakdown. There is additionally a plausibility of postponement before its fix could be begun. Strengthening variable strategy is applied to frame the arrangement of administering conditions. We have likewise determined peripheral likelihood disseminations, which are additionally used to process other helpful performance measures.

Keywords: steady state; retrial system; random vacation; supplementary variable method.

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1. INTRODUCTION

Queueing hypothesis is the numerical search of investment up lines or line. The suggestion empowers numerical assessment of a only some of related forms, including poignant base at the line, holding up in the line and being served by the server at the front of the line. The assumption allows the induction and computation of a few presentation measures including the normal property up time in the line or the framework, the ordinary number of clients and the possibility of experiencing the framework in certain states, for example, vacant, full, having an available server or trusting that a specific time will be served. In this situation the circumstance where a server is inaccessible for essential clients in periodic interims of time is known as repair. The excursions may speak to server's taking a shot at some advantageous employments, performing server upkeep review and fixes, that intrude on the client service. Allowing servers to take repair makes the line models increasingly practical and adaptable in considering genuine circumstances. In the event that it isn't in task individuals regularly go for some other stimulation. In such cases, the landing rate will be low. This reasonable circumstance precisely fits into the present model.

Gupta and Sikdar (2004) [6] have analysed a single server finite-buffer bulk-service queueing model with particular vacation in which the interarrival and service times are considered to be exponentially and arbitrarily distributed, respectively. Madan and Al-Rawwash (2005) [8] have analysed a single server queue with batch arrivals and general service time distribution. Baba (2012) [2] used a batch arrival M/G/1 queue with multiple working vacation and obtained probability generating function and stochastic rotting structure of the system and some performance indices, mean system length and the mean waiting time. Vijaya Laxmi et al (2013) [11] Analysed a finite barrier renewal input single working vacation queue with state dependent vacations. They also presented an efficient computation algorithm and computed the stationary queue length for the above model along with different performance measures.

Sree Parimala and Palaniammal (2014) [12] studied bulk service queueing model with server’s single and delayed vacation. For this model, the steady state solutions and the system characteristics are derived and analysed. Ibe (2015) [7] has considered a single server vacation queueing system with server time out and derived terminology for the mean waiting time and studied N-policy scheme. Balamani (2014) [3] has studied a two stage batch arrival queue with compulsory server vacation and second optional repair and has derived the steady state solutions also computed the mean queue length and the mean waiting time. Ebenesar Anna Bagyam et al (2015) [5] have considered immensity arrival multi-stage retrial queue with Bernoulli vacation, the steady state distributions of the server state and the queue size have been obtained along with important measures. Maragathasundari and Karthikeyan (2015) [9] have studied about a Non Markovian queueing model with extended repair. Balasubramanian et al. (2016) [4], A Markovian queue with blocking and catastrophes. Arivukkarasu V, Bharathidass S, Ganesan V. (2016) [1] Customer impatience in bulk arrival queue with optional service. Moganraj D. et al. (2018) [10] derived, A study on transient solution of single server queueing system with repair process by using generating functions of the system under the system down.

Using the above ideas, the consistent state line size dispersion at a subjective time is acquired. Execution estimates like the normal line duration, expected length of occupied and inactive periods are inferred. The holding up point in time in the line is additionally got. A cost model is additionally inferred.

2. MODEL DESCRIPTION

We are used to develop the queue size distribution for following notation. Let X denote the customers arrival in random, Customer arrival when the server is busy is on Poisson arrival rate λ_b and λ_b be the when the server is on repair Poisson arrival rate, P_m be the probability that m customers arrive in a batch and G_(z) be its probability generating function. Let $S_s(x)$ and $S_r(x)$ be the cumulative distributions of the service time and repair time, respectively. Let $S_s(x)$ and $S_r(x)$ be the probability density functions of service time and repair time, respectively. $S_s^*(t)$ denotes the remaining service time of a batch at an arbitrary time t and $S_r^*(x)$ denotes the remaining vacation time of a server at an arbitrary time t. Let $\tilde{S}_s(t)$ and $\tilde{S}_r(t)$ denote the Laplace- Stieltjes transforms of S and R, respectively. $T_s(t)$ and $T_w(t)$ are the total customers under service and total customers in the waiting, respectively.

$$X(t) = \begin{cases} 0, & \text{if the server is massiveness service} \\ 1, & \text{if the server is on patch up.} \\ 2, & \text{if the server is working vacation} \end{cases}$$

3. STEADY STATE EQUATION

$$-\frac{d}{dx}p_{i0}(x)$$
$$= -\lambda p_{i0}(x) + \sum_{m=a}^{b} p_{i0}(0)s(x) + \sum_{i=1}^{M} Q_{1i}(0)s(x) + \lambda \sum_{n=0}^{a-1} T_i g_{i-n}\, s(x), \quad a \le i \le b \qquad (1)$$

$$-\frac{d}{dx}p_{ij}(x) = -\lambda p_{ij}(x) + \lambda \sum_{n=0}^{j} p_{ij-k}(x)\, k_m \qquad a \le i \le b-1, j \le 1 \tag{2}$$

$$-\frac{d}{dx}p_{bj}(x)$$

$$= -\lambda p_{bj}(x) + \textstyle\sum_{m=a}^{b} p_{m\,b+j}\,(0)s(x) + \sum_{i=i}^{m} q_{1b+j}\,(0)\mathrm{s(x)} + \sum_{n=0}^{a-1} \mathrm{T_n}\, \lambda \mathrm{g_{b+j-n}} \mathrm{s(x)} \tag{3}$$

$$0 = -\lambda_0 T_0 + q_{m0}(0) \tag{4}$$

$$0 = -\lambda_0 T_0 + q_{mn}(0) + \sum_{k=1}^{b} T_{n-k}\ \lambda_0 g_{k,} \quad 1 \le n \le a-1 \tag{5}$$

$$-\frac{d}{dx}q_{10}(x) = -\lambda_0 q_{10}(x) + \sum_{m=a}^{b} p_{m0}(0)\, v(x) \tag{6}$$

$$-\frac{d}{dx}q_{1n}(x) = -\lambda_0 q_{1n}(x) + \sum_{m=a}^{b} p_{mn}(0)\, v(x) + \sum_{k=1}^{n} q_{1n-k}\,(x)\lambda_0 k_{m,}\ \ 1 \le n \ge a-1 \tag{7}$$

$$-\frac{d}{dx}q_{1n}(x) = -\lambda_0 q_{1n}(x) + \sum_{k=1}^{n} q_{1n-k}\,(x)\lambda_0 k_m, \qquad n \ge a \tag{8}$$

$$-\frac{d}{dx}q_{jn}(x) = -\lambda_0 q_{jn}(x) + q_{jn-1}(0)r(x) + \sum_{k=1}^{n} q_{jn-k}\,(x)\lambda_0 k_m, \qquad n \ge a \tag{9}$$

Where $1 = n = a-1, 2 = j == m$

$$-\frac{d}{dx}q_{jn}(x) = -\lambda_0 q_{jn}(x) + \sum_{k=1}^{n} q_{jn-k}\,(x)\lambda_0 g_{k,}\ n \ge a,\ \ 2 \le j \le\le m \tag{10}$$

Taking Laplace transform equation from (1) to equation (10), then we get

$$\theta\, \tilde{p}_{i0}(\theta) - p_{i0}(0)$$

$$= \lambda \tilde{p}_{i0}(\theta) - \left[\sum_{m=a}^{b} p_{mi}\,(0) + \sum_{i=1}^{M} q_{1i}\,(0) + \lambda \sum_{n=0}^{a-1} T_n g_{i-n}\right] \tilde{s}(\theta) \tag{11}$$

$$\theta\, \tilde{p}_{ij}(\theta) - p_{ij}(0)$$

$$= \lambda \tilde{p}_{ij}(\theta) - \lambda \sum_{k=1}^{j} p_{ij-k}(\theta) k_m \tag{12}$$

$$\theta\ \tilde{q}_{10}(\theta) - q_{10}(0)$$
$$= \lambda_0 q_{10}(\theta) - \sum_{m=a}^{b} p_{m0}\,(0)\tilde{r}(\theta) \tag{13}$$
$$\theta\ \tilde{q}_{1n}(\theta) - q_{1n}(0)$$
$$= \lambda_0 q_{1n}(\theta) - \sum_{m=a}^{b} p_{mn}\,(0)\tilde{r}(\theta) - \lambda_0 \sum_{k=1}^{n} \tilde{q}_{1n-k}(\theta)\, k_m \tag{14}$$
$$\theta\ \tilde{q}_{1n}(\theta) - q_{1n}(0) = \lambda_0 q_{1n}(\theta) - \lambda_0 \sum_{k=1}^{j} \tilde{q}_{1n-k}(\theta)\, k_m \quad ; n \geq a \tag{15}$$
$$\theta\ \tilde{q}_{j0}(\theta) - q_{j0}(0) = \lambda_0 q_{j0}(\theta) - \tilde{q}_{j-10}(0)\tilde{r}\ (\theta),\ \ 2 \leq j \leq k \tag{16}$$
$$\theta\ \tilde{q}_{jn}(\theta) - q_{jn}(0) = \lambda_0 \tilde{q}_{jn}(\theta) - \tilde{q}_{j-1n}(\theta) - \tilde{q}_{j-1n}(0)\lambda_0 \sum_{k=1}^{n} \tilde{q}_{1n-k}(\theta)\ \ k_m \tag{17}$$
$$\theta\ \tilde{q}_{jn}(\theta) - q_{jn}(0) = \ \lambda_0 \tilde{q}_{jn}(\theta) - \lambda_0 \sum_{k=1}^{n} \tilde{q}_{jn-k}(0)\ \ k_m,\quad 2 \leq j \leq K, n \geq a \tag{18}$$

4. QUEUE TIME DISTRIBUTION

To obtain the Probability generating function of queue time distribution at an arbitrary time, then we defined as

$$\tilde{p}_i(z,\theta) = \sum_{n=0}^{\infty} \tilde{p}_{in}(\theta)\, z^n;\ p_i(z,0) = \sum_{n=0}^{\infty} p_{in}(0)\, z^n; \quad a \leq i \leq b$$
$$\tilde{q}_j(z,\theta) = \sum_{n=0}^{\infty} \tilde{q}_{jn}(\theta)\, z^n;\ q_j(z,0) = \sum_{n=0}^{\infty} q_{jn}(0)\, z^n; \quad 1 \leq j \leq M$$
$$C(z) = \sum_{n=0}^{a-1} C_n z^n \tag{21}$$

By using equation (21) we get,

$$\big(\theta - \lambda_0 + \lambda_0\, X(z)\big)\ \tilde{q}_1(z,\theta) = \tilde{q}_1(z,0) - \tilde{v}(\theta)\ \sum_{n=0}^{a-1} \sum_{m=a}^{b} p_{mn}\,(0) z^n \tag{22}$$

By using equation (22) we get,

$$\big(\theta - \lambda_0 + \lambda_0\, X(z)\big)\tilde{q}_j(z,\theta) = \tilde{q}_j(z,0) = \tilde{v}(\theta) \sum_{n=0}^{a-1} q_{j-1n}\,(0) z^n \qquad 2 \leq \mathrm{K} \leq jm \tag{23}$$

$$\left(\theta-\lambda+\lambda X(z)\right)\tilde{p}_i(z,\theta) = p_i(z,0)-\tilde{s}(\theta)\left[\sum_{m=a}^{b} p_{mi}(0)+\sum_{i=1}^{m} q_{1i}(0)+\lambda\sum_{n=0}^{a-1} C_n g_{i-n}\right] \quad (24)$$

Where $a=i=b-1$

$$z^b\left(\theta-\lambda+\lambda X(z)\right)\tilde{p}_b(z,\theta)$$
$$= z^b p_b(z,0) - \tilde{s}(\theta)\left\{\left[\sum_{m=a}^{b}\left(p_m(z,0)-\sum_{j=0}^{b-1} p_{mj}(0)z^j\right)\right]+\left[\sum_{i=1}^{m}\left(q_1(z,0)-\sum_{j=0}^{b-m-1} g_i z^j\right)\right] + \lambda\left[C(z)X(z)-\sum_{m=0}^{a-1} C_m z^m \sum_{j=1}^{b-m-1}\sum_{j=0}^{b-m-1} g_i z^j\right]\right\} \quad (25)$$

$$q_1(z,0) = \tilde{R}\left(\lambda_0-\lambda_0 X(z)\right)\sum_{n=0}^{a-1}\sum_{m=a}^{b} p_{mn}(0)z^n \quad (26)$$

$$q_j(z,0) = \tilde{R}\left(\lambda_0-\lambda_0 X(z)\right)\sum_{n=0}^{a-1} q_{j-1n}(0)z^n \qquad 2\le j\le M \quad (27)$$

From the equation 24 and 25 we get

$$p_i(z,0)=\tilde{S}\left(\lambda-\lambda X(z)\right)\left[\sum_{m=a}^{b} p_{mi}(0)+\sum_{i=1}^{M} q_{1i}(0)+\lambda\sum_{n=0}^{a-1} C_n g_{i-n}\right] \quad a\le \mathrm{i}\le \mathrm{b}-1 \quad (28)$$

$$\left(z^b-\tilde{S}\left(\lambda-\lambda X(z)\right)\right) p_b(z,0)=\tilde{S}\left(\lambda-\lambda X(z)\right)\left\{\left[\left(\textstyle\sum_{m=a}^{b-1} p_m(z,0)-\sum_{m=a}^{b}\sum_{j=0}^{b-1} p_{mj}(0)z^j\right)\right]+\left[\textstyle\sum_{i=1}^{m}\left(q_1(z,0)-?_{j=0}^{b-1} q_{1j}(0)z^j\right)\right]+\lambda\left[C(z)X(z)-\textstyle\sum_{m=0}^{a-1} C_m z^m \sum_{j=1}^{b-m-1} K_{jz^j}\right]\right\} \quad (29)$$

From the above equation, we obtain

$$p_b(z,0)=\frac{\tilde{S}\left(\lambda-\lambda X(z)\right) f(z)}{\left(z^b-\tilde{S}\left(\lambda-\lambda X(z)\right)\right)} \quad (30)$$

$$f(z)=\sum_{m=a}^{b-1} p_m(z,0)-\sum_{m=a}^{b}\sum_{j=0}^{b-1} p_{mj}(0)\, z^j+\sum_{i=1}^{M}\left(q_1(z,0)-\sum_{j=0}^{b-1} q_{ij}(0)z^j\right) + \lambda\left[C(z)X(z)-\sum_{m=0}^{a-1} C_m z^m \sum_{j=1}^{b-m-1} k_j z^j\right] \quad (31)$$

From the above equation, we get

$$\tilde{q}_j(z,\theta) = \frac{\left[\tilde{R}\left(\lambda_0 - \lambda_0\, X(z)\right) - \tilde{R}(\theta)\right] \sum_{n=0}^{a-1} \sum_{j=1}^{M} q_{j-n}(0) z^n}{\left(\theta - \lambda_0 - \lambda_0\, X(z)\right)} \qquad 2 \le j \le M \tag{32}$$

$$p_i(z,\theta) = \frac{\left[\tilde{S}\left(\lambda - \lambda X(z)\right) - \tilde{S}(\theta)\right]\left(\sum_{m=a}^{b} p_{mi}\,(0) + \sum_{j=1}^{M} q_{ij}(0) + \lambda \sum_{n=0}^{M} C_n k_{i-n}\right)}{\left(\theta - \lambda - \lambda X(z)\right)} \tag{33}$$

We consider p(z) the queue time of PGF at an arbitrary epoch. Then

$$p(z) = \sum_{m=a}^{b-1} \tilde{p}_m\,(z,0) + \tilde{p}_b(z,0) + \sum_{m=a}^{b-1} \tilde{p}_m\,(z,0) + C(z) \tag{36}$$

$$p(z) = \frac{\left[\tilde{S}\left(\lambda - \lambda X(z)\right) - 1\right] \sum_{i=a}^{b}\left(\sum_{m=a}^{b} p_{mi}\,(0) + \sum_{j=1}^{M} q_{ji}\,(0) + \sum_{n=0}^{a-1} C_n k_{i-n}\right)}{\left(-\lambda - \lambda X(z)\right)}$$

$$+\ \frac{\left[\tilde{S}\left(\lambda - \lambda X(z)\right) - 1\right] f(x)}{\left(z^b - \tilde{S}\left(\lambda - \lambda X(z)\right)\right)\left(-\lambda - \lambda X(z)\right)} + \frac{\left(\tilde{R}\ (\lambda_0 + \lambda_0 X(z)\right) - 1 \sum_{n=0}^{a-1} \sum_{m=a}^{b} p_{mn}(0) z^n}{\left(-\lambda_0 - \lambda_0 X(z)\right)}$$

$$+ \frac{\left(\tilde{R}\ (\lambda_0 + \lambda_0 X(z)\right) - 1) \sum_{n=0}^{a-1} \sum_{j=1}^{n} q_{jn} z^n}{\left(-\lambda_0 - \lambda_0 X(z)\right)} \tag{37}$$

$$P_i = \sum_{m=a}^{b} p_{mi,}(0) \qquad Q_i = \sum_{i=1}^{M} q_{ji}(0) \qquad D_i = P_i + Q_i \tag{38}$$

Modify equation (37) and (38), we have

$$\left[\tilde{S}\left(\lambda - \lambda X(z)\right) - 1\right]\left(-\lambda_0 - \lambda_0 X(z)\right) \sum_{m=a}^{b-1} (z^b - z^m)\ D^m$$

$$+\left(\tilde{R}\ (\lambda_0 + \lambda_0 X(z) - 1\right)\langle \tilde{S}\left(\lambda - \lambda X(z)\right) - 1\rangle\left(-\lambda_0 - \lambda_0 X(z)\right)$$

$$+\left(z^b - \tilde{S}\left(\lambda - \lambda X(z)\right)\left(-\lambda - \lambda X(z)\right)\right) \sum_{n=0}^{a-1} D_n Z_n$$

$$+\ ?\, C(z)\left(-?_0 - ?_0\, X(z)\right)(X(z) - 1)\ (z^b - 1) + \tilde{S}\left(? - ?\, X(z)\right) - 1$$

$$p(z) = \frac{\left(-\lambda_0 - \lambda_0 X(z)\right)\lambda \sum_{i=a}^{b-1}\left((z^b - z^i\right) \sum_{m=0}^{a-1} C_m k_{i-m}}{\left(-\lambda + \lambda X(z)\right)\left(\left(-\lambda_0 - \lambda_0 X(z)\right)\right)\left(z^b - \tilde{S}\left(\lambda - \lambda X(z)\right)\right)} \tag{39}$$

5. PERFORMANCE MEASURE

At the point when the quantity of repair become $? = 8$ at that point, $\mathrm{C(z)}$ will become zero, henceforth using equation (39), then obtain the form in the given below,

$$p(z) = \frac{\begin{array}{c}[\tilde{S}(\lambda - \lambda X(z)) - 1](-\lambda_0 - \lambda_0 X(z)) \sum_{m=a}^{b-1} (z^b - z^m)\ D^m \\ +(\tilde{R}\ (\lambda_0 + \lambda_0 X(z) - 1)\langle \tilde{S}(\lambda - \lambda X(z)) - 1\rangle(-\lambda_0 - \lambda_0 X(z)) \\ + \left(z^b - \tilde{S}(\lambda - \lambda X(z))(-\lambda - \lambda X(z))\right) \sum_{n=0}^{a-1} D_n Z_n\end{array}}{(-\lambda + \lambda X(z))\left((-\lambda_0 - \lambda_0 X(z))\right)\left(z^b - \tilde{S}(\lambda - \lambda X(z))\right)}$$

The above P(z) gives the PGF of line length dissemination of an Queueing framework with state subordinate landings and repairs. The outcome precisely agrees with the queue length circulation of of Ebenesar Anna bakyam [9].

Expected Waiting Time in customer

The mean waiting time of the customers in the queue E (W)(or) W_q can be easily obtained using Little's formula

$$E(W) = {E(q)}/{\lambda E(X)}$$

Ideal period and Busy Period

The time period from the repair commencement epoch to the busy period beginning epoch is called the idle time period. Let I be the random number for idle period.$\eta_j = 0,1,2, \dots a - 1$ number of customers visit in the system during the idle period.

$I_j =$ 1, j customers visit to the system in the idle period

$I_j =$ 0, otherwise

Using the above condition on the size of queue at the time of service finish epoch, we have

$$\eta_j = \tau_j + \sum_{k=0}^{a-1} \tau_k\, p(I_{j=k} = 1) \quad , j = 1,2,3 \dots\dots a - 1 \tag{40}$$

In estimated length of busy period is inferred which is helpful to locate the general expense of the framework. Utilizing a contingent desire idea, the normal length of occupied period is determined as pursues

$$E(BP) = {E(S)}/{p(j = 0)} \tag{41}$$

Total Cost Analysis

The all-out normal expense of the Queueing framework is inferred with the accompanying suppositions:

C_b : begin-up expense per cycle

C_h : Handling expense per client per unit time

C_m : Maintaining expense per unit time

C_r : Reward because of repair per unit time.

The cycle duration is the aggregate of the infert period and occupied period. Presently, the normal length of cycle, $E(T_C)$ is obtained.

$$E(T_C) = E(IP) + E(BP)$$

$$E(T_C) = \frac{I}{\lambda_0}\sum_{j=0}^{a-1}\eta_j + E(R)\sum_{j=1}^{M} j\left(I - \sum_{n=0}^{a-1}\sum_{i=0}^{n}\tau_i\,(p_{n-1} + p_{n+1})\right) + E(BP) \qquad (42)$$

Table.1 Arrival rate and Total Average cost

μ	λ	E(Q)	E(W)	E(BP)	E(IP)	TAC
2.5	2	0.685	0.173	2.305	0.567	6.156
	2.4	0.689	0.188	3.228	0.485	7.1134
	2.7	1.098	0.230	4.966	0.433	7.9381
	3.0	1.487	0.239	5.328	0.376	8.5331
3.0	2	0.547	0.143	1.532	0.607	5.9171
	2.4	0.821	1.672	1.546	0.525	6.6310
	2.7	0.965	1.921	1.823	0.428	7.1316
	3.0	1.773	2.654	2.431	0.356	7.9882

Table.2 Arrival rate with system state

λ	$\mu = 2$		$\mu = 2.5$		$\mu = 3$	
	P(BP)	P(R)	P(BP)	P(R)	P(BP)	P(R)
1.0	0.044	0.035	0.032	0.040	0.025	0.049
1.2	0.072	0.031	0.050	0.039	0.037	0.048
1.4	0.212	0.025	0.072	0.039	0.053	0.047
1.6	0.195	0.026	0.103	0.038	0.073	0.047
1.8	0.257	0.020	0.145	0.035	0.098	0.046
2.0	0.461	0.015	0.204	0.031	0.129	0.045

CONCLUSION

We determined some significant framework attributes through the key likelihood producing capacity and also derived the expected waiting time of the customers and the function of the line length distribution. The model is critical since progressively broad circumstances in reasonable applications are considered in the model.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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