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INFLUENCE OF GRAVITY AND INITIAL STRESS ON RAYLEIGH WAVE PROPAGATION IN MAGNETO-THERMOELASTIC MEDIUM

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Abstract. The main aim of present study is to show the effect of gravity, initial stress and magnetic field on the Rayleigh waves propagation in homogeneous orthotropic magneto-thermoelastic medium in the context of Three Phase Lag (TPL) model at two temperature. The governing equations of thermoelasticity have been solved by normal mode technique to deduce the frequency equation for Rayleigh wave with relevant boundary conditions. Special cases have been derived for isothermal and thermally insulated surfaces. Computer simulation is used for numerical discussion to show the effects of various parameters on phase velocity of Rayleigh waves. The variation in phase velocity corresponding to wave number has been demonstrated graphically in the presence of gravity, initial stress and magnetic field.

Keywords: thermoelasticity; Rayleigh waves; normal mode analysis; three phase lag; two temperature.

2010 AMS Subject Classification: 74F05, 37N15.

1. Introduction

It is well known fact that classical coupled theory of thermoelasticity had major drawback regarding thermal wave showing infinite speed with parabolic heat equation. To overcome this inconsistency, various theories had been proposed in last five decades. Lord and Shulman [1] proposed a theory in which one relaxation parameter was used in modified fourier law and the

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heat conduction equation transformed into hyperbolic form. Green and Lindsay [2] formulated a new theory of generalized thermoelasticity that involves heat conduction equation with two relaxation times. Green and Nagdhi [3-5] formulated three generalized thermoelastic theories are known as GN-I, II, III for homogeneous isotropic materials that involves propagation of thermal waves with finite speed. GN-II & III theory involves the heat conduction equation without and with energy dissipation respectively. Tzou [6] developed the Dual Phase Lag (DPL) theory of thermoelasticity which consider interaction of phonon-electron at the microscopic level. In Dual Phase Lag (DPL) theory, τ_q (heat flux gradient) and τ_T (temperature gradient) are used in modified form of fourier law of heat conduction. Choudhri [7] formulated Three Phase Lag (TPL) thermoelasticity that involves modified Fourier law using three phase lag in τ_q (heat flux gradient), τ_T (temperature gradient) and τ_v (displacement gradient) in heat conduction equation. Three Phase Lag (TPL) model have been used in catalytic reactions, phonon-electron interactions and nuclear boiling problem.

During the last one-decade, extensive work has been done in analyzing the effect of various parameters on the propagation of surface waves in thermoelastic medium. Abo-Dahab [8] investigated the impact of various parameter like rotation, initial stress and effects of voids on P-waves. Abo-Dahab [9] and Ivanov and Savona [10] reviewed orthotropic half space for propagation of surface waves in thermoelastic medium. Othman and Said [11] studied the influence of diffusion and internal heat source on the thermoelastic medium at two temperature in the context of Three Phase Lag (TPL) model. Singh and Verma [12] investigated the Rayleigh wave propagation in thermoelastic medium in reference to various theories of generalised thermoelasticity. Rossikin and Shitikova [13] studied the polarized thermoelastic medium for the propagation of surface waves in the presence of heat conduction and thermal relaxation times. Sharma and Kaur [14] studied the impact of voids in the rotating thermoelastic medium on the propagation of Rayleigh waves. Sharma et al. [15] discussed Rayleigh waves propagation in isotropic solids under the effect of different parameters like relaxation times, micropolarity and microstretch. Shaw and Mukhopadhyay [16] explained thermo-microstretch isotropic half space in the presence of electromagnetic effects for the propagation of Rayleigh waves.

Abd-Alla et al. [17] investigated the magneto-thermoelastic half space for Rayleigh wave propagation under the influence of gravity, initial stress and rotation. Kumar and Kansal [18] studied the propagation of Rayleigh waves in thermoelastic medium with thermally insulated boundary within the framework of generalized thermoelasticity. Stroh [19] used the mathematical methods to study the basic equations of elasticity in anisotropic medium. Vinh and Seriani [20] analyzed the basic equations for the gravitational effect on Rayleigh wave propagation in a non-homogeneous orthotropic thermoelastic medium. Kumar and Gupta [21] studied the influence of phase lag on propagation of Rayleigh wave in thermoelastic medium with mass diffusion. Kumar et al. [22] discussed the effect of viscosity on phase velocity of Rayleigh wave propagation in anisotropic medium in the frame of Three Phase Lag (TPL) model. Biswas et al. [23] examined the Rayleigh wave propagation in the context of Three Phase Lag (TPL) model of thermoelasticity in orthotropic solid.

Chen et al. [24-26] formulated the theory of heat conduction that based upon two temperatures Θ and T where Θ represent conductive temperature, T represent thermodynamic temperature and a^* represent material parameter. If $a^* \to 0$, this imply that $\Theta \to T$, hence two temperature theory coincide with classical theory. Warren and Chen [27] examined the effect of two temperature on propagation of wave in thermoelastic medium. Youssef [28] formulated a theory of generalized thermoelasticity by considering the hypothesis that heat supply in elastic bodies depends upon conductive and thermodynamic temperature and derived the frequency equation. Puri and Jordan [29] examined plane waves propagation in thermoelastic medium with two temperature theory.

In this present study, orthotropic magneto-thermoelastic half space has been examined for propagation of Rayleigh waves in context of Three Phase Lag (TPL) model with two temperature. The frequency equation of Rayleigh wave has been derived using normal mode technique at two temperature with relevant boundary conditions. The variation of phase velocity of Rayleigh waves corresponding to wave number are represented graphically to demonstrate the effect of gravity, initial stress and magnetic field.

2. Formulation of the Problem and Governing Equations

Consider an orthotropic thermoelastic solid in the presence of initial stress P_1 and plane strain is parallel to $x_1 - x_3$ plane whereas the boundary x_3=0 is considered to be stress free. The Rayleigh surface waves propagates along the direction of x_1 axis. Body forces in the presence of acceleration of gravity are X_1=0; X_3=-g if initial stress field is hydrostatic

$$\sigma_{11} = \sigma_{33} = \Upsilon, \tau_{13} = 0 \tag{1}$$

where the function of depth represented by Υ. The initial stress in equilibrium form is given by:

$$\Upsilon_{,1} = 0; \Upsilon_{,3} - \rho g = 0$$

Consider Maxwell equation for perfectly electric conductor in the absence of displacement current by assuming medium perfectly conductor (Mukhopadhaya and Roy [30])

$$\nabla \times \vec{b} = \vec{J} \tag{2}$$

$$\nabla \times \vec{E} = -\mu_e \frac{\partial \vec{b}}{\partial t} \tag{3}$$

$$\nabla . \vec{b} = 0 \tag{4}$$

Here $\vec{b}$ is perturbed magnetic field $\vec{b} = \nabla \times (\vec{u} \times \vec{B})$; $\vec{B} = \vec{B_0} + \vec{b}$; $\vec{u} = (u_1, 0, u_3)$; $\vec{B} = (0, B_0, 0)$ g represent gravity, $\vec{E}$ represent the electric density, B_0 represent primary magnetic field, μ_e represent magnetic permeability.

$$\sigma_{11,1} + \tau_{13,3} - P_1 \omega_{13,3} + \mu_e B_0^2 (u_{1,11} + u_{3,13}) - \rho g u_{3,1} = \rho \ddot{u_1} \tag{5}$$

$$\sigma_{33,3} + \tau_{13,1} - P_1 \omega_{31,3} + \mu_e B_0^2 (u_{3,33} + u_{1,13}) - \rho g u_{1,1} = \rho \ddot{u_3} \tag{6}$$

$$\begin{aligned} K_3 \left[1 + \tau_T \frac{\partial}{\partial t}\right] \frac{\partial}{\partial t} T_{,33} + K_1 \left[1 + \tau_T \frac{\partial}{\partial t}\right] \frac{\partial}{\partial t} T_{,11} + K_3^* \left[1 + \tau_v \frac{\partial}{\partial t}\right] T_{,33} + K_1^* \left[1 + \tau_v \frac{\partial}{\partial t}\right] T_{,11} \\ = \left[1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2} \frac{\partial^2}{\partial t^2}\right] \frac{\partial^2}{\partial t^2} \left[\rho c_e \Theta + T_0(\beta_1 u_{1,1} + \beta_3 u_{3,3})\right] \end{aligned} \tag{7}$$

Here τ_T represents temperature gradient, τ_q represents heat flux gradient, τ_v represents thermal displacement gradient, K_{ij} represents the components of thermal conductivity, K_{ij}^* represents material constant characteristics, β_{ij}represents thermoelastic tensor, ρ represents mass density. The two temperature relation is given by

$$\Theta = T - a^*[T_{,11} + T_{,33}] \tag{8}$$

In two temperature relation Θ represents conductive temperature, T represents absolute temperature, a^* represents two temperature parameter.
The stress-displacement relation for incremental are given by

$$\sigma_{11} = (C_{11} + P_1)u_{1,1} + (C_{13} + P_1)u_{3,3} - \beta_1\Theta \tag{9}$$

$$\sigma_{33} = C_{13}u_{1,1} + C_{33}u_{3,3} - \beta_3\Theta \tag{10}$$

$$\tau_{13} = \frac{1}{2}(C_{11} - C_{13})(u_{1,3} + u_{3,1}) \tag{11}$$

$$\omega_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{12}$$

Substituting the equations (9)-(12) in (5)-(6), hence we obtained as:

$$\begin{aligned} (C_{11} + P_1 + \mu_e B_0^2)u_{1,11} + \frac{1}{2}(C_{11} - C_{13} - P_1)u_{1,33} + \left[\frac{C_{11} + C_{13}}{2} + \frac{3P_1}{2} + \mu_e B_0^2\right] u_{3,13} \\ - \beta_1\Theta_{,1} - \rho g u_{3,1} = \rho \ddot{u}_1 \end{aligned} \tag{13}$$

$$\begin{aligned} \left[\frac{C_{11} + C_{13}}{2} + \frac{P_1}{2} + \mu_e B_0^2\right] u_{1,13} + \frac{1}{2}\left[C_{11} - C_{13} + P_1\right] u_{3,11} - \beta_3\Theta_{,3} \\ + (C_{33} + \mu_e B_0^2)u_{3,33} + \rho g u_{1,1} = \rho \ddot{u}_3 \end{aligned} \tag{14}$$

3. Solution of the Problem

The relationship between displacement potentials $\phi(x_1, x_3, t)$, $\psi(x_1, x_3, t)$ and displacement components u_1, u_3 are assumed as follows:

$$\begin{cases} u_1 = \phi_{,1} - \psi_{,3} \\ u_3 = \phi_{,3} + \psi_{,1} \end{cases} \tag{15}$$

Substituting the value of expression (15) in (13)-(14) and (7), it is observed Φ and Ψ satisfied the equations:

$$(C_{11}+P_1+\mu_e B_0^2)(\phi_{,11}+\phi_{,33})-\rho g\phi_{,3}-\rho g\psi_{,1}-\beta_1\Theta=\rho\ddot{\phi} \tag{16}$$

$$\left[\frac{C_{33}-C_{13}-C_{11}-P_1}{2}\right]\psi_{,33}+\frac{1}{2}(C_{11}-C_{13}+P_1)\psi_{,11}+\rho g\phi_{,1}=\rho\ddot{\psi} \tag{17}$$

$$\begin{aligned}K_3\left[1+\tau_T\frac{\partial}{\partial t}\right]\frac{\partial}{\partial t}T_{,33}+K_1\left[1+\tau_T\frac{\partial}{\partial t}\right]\frac{\partial}{\partial t}T_{,11}+K_3^*\left[1+\tau_v\frac{\partial}{\partial t}\right]T_{,33}+K_1^*\left[1+\tau_v\frac{\partial}{\partial t}\right]T_{,11}\\=\left[1+\tau_q\frac{\partial}{\partial t}+\frac{\tau^2}{2}\frac{\partial^2}{\partial t^2}\right]\frac{\partial^2}{\partial t^2}[\rho c_e\Theta+T_0[\beta_1(\phi_{,11}-\psi_{,13}]+\beta_3(\phi_{,33}+\psi_{,31})]\end{aligned} \tag{18}$$

Equations (16) and (17) are in equivalence to (13) and (14) respectively in the context of boundary conditions. Here equations (16), (17) and (18) are considered as the solutions.
The following are mechanical and thermal boundary conditions assumed in case of thermally stress free surface:

1 Normal stress component vanished $\sigma_{33}=0$
2 Tangential stress component vanished $\tau_{13}=0$
3 Thermal conditions $q_3+h\Theta=0$ and for thermally insulated surface $h\to 0$ and for isothermal Surface $h\to\infty$

4. NORMAL MODE ANALYSIS

Consider the harmonic waves propagates along x_1 axis and ϕ, ψ and T can be assumed in the following form using normal mode technique.

$$\begin{cases}\phi=U(x_3)e^{i\alpha(x_1-ct)}\\ \psi=V(x_3)e^{i\alpha(x_1-ct)}\\ T=W(x_3)e^{i\alpha(x_1-ct)}\end{cases} \tag{19}$$

Here phase velocity is represented by c and wave number is represented by α. Using the value of ϕ,ψ and T in the Equation (16), (17) and (18)

$$[(C_{11}+P_1+\mu_e B_0^2)(D^2-\alpha^2)-\rho gD+\rho\alpha^2c^2]U-\rho gi\alpha V-\beta_1(1+a^*\alpha^2-a^*D^2)W=0 \tag{20}$$

$$\rho g i\alpha U+\left[\left(\frac{2C_{33}-C_{13}-C_{11}-P_1}{2}\right)D^2-\alpha^2\left(\frac{C_{11}-C_{13}+P_1}{2}\right)+\rho\alpha^2c^2\right]V=0 \tag{21}$$

$$\begin{aligned}[(i\alpha^3cK_1m_1-\alpha^2m_2K_1^*+\rho c_e(1+a^*\alpha^2-a^*D^2)\alpha^2c^2)+(K_3^*m_2-iK_3\alpha cm_1)D^2]W\\+(T_0\beta_3D^2-T_0\beta_1\alpha^2)\alpha^2c^2U+(i\alpha T_0\beta_3-i\alpha T_0\beta_1)\alpha^2c^2)DV=0\end{aligned} \tag{22}$$

Where $D^2=\frac{d^2}{dx_3^2},\ m_1=\frac{m_3}{m_5},\ m_2=\frac{m_4}{m_5},\ m_3=1-i\alpha c\tau_T,\ m_4=1-i\alpha c\tau_v,$
$m_5=1-i\alpha c\tau_q-\frac{\alpha^2c^2\tau_q^2}{2}$

Eliminating U,V and W from equations (20),(21) and(22). Hence we get

$$(D^6-ID^4+JD^2-K)(U(x_3),V(x_3),W(x_3))=0$$

In other words it can be reduced to as follows :

$$(D^2-\Gamma_1^2)(D^2-\Gamma_2^2)(D^2-\Gamma_3^2)(U(x_3),V(x_3),W(x_3))=0 \tag{23}$$

Where Γ_1, Γ_2 and Γ_3 are positive solutions of following characteristic equation

$$\Gamma^6-I\Gamma^4+J\Gamma^2-K=0 \tag{24}$$

Equation (24) gives the positive roots as follows:

$$\Gamma_1=\sqrt{\frac{1}{3}[2dsin(e)-I]},\Gamma_2=\sqrt{\frac{1}{3}[-I-d(\sqrt{3}cose+sine)]},\Gamma_3=\sqrt{\frac{1}{3}[-I+d(\sqrt{3}cose+sine)]}$$

Where $d=\sqrt{I^2-3J},\ e=\frac{sin^{-1}f}{3}\ and\ f=\frac{2I^3-9IJ+27K}{2d^3}$

$$I=\frac{a_1a_2a_9+a_1a_2a_7a_{10}-a^*a_1a_8a_{10}+a_1a_8a_{11}-a^*a_2a_4a_{10}+a_2a_4a_{11}+a^*\beta_1a_2a_{13}-a^*\beta_1a_8a_{12}}{a_1a_2a_{11}-a^*a_1a_2a_{10}-a^*\beta_1a_2a_{12}}+\frac{\beta_1a_2a_7a_{12}+a^*a_2a_5a_{10}-a_2a_5a_{11}}{a_1a_2a_{11}-a^*a_1a_2a_{10}-a^*\beta_1a_2a_{12}}$$

$$J=\frac{a_1a_8a_9+a_1a_7a_8a_{11}+a_2a_4a_9+a_2a_4a_7a_{10}-a^*a_4a_8a_{10}+a_4a_8a_{11}-a^*a_6^2a_{10}+a_6^2a_{11}}{a_1a_2a_{11}-a^*a_1a_2a_{10}-a^*\beta_1a_2a_{12}}+\frac{a^*\beta_1a_8a_{13}-\beta_1a_2a_7a_{13}+\beta_1a_7a_8a_{12}+a^*a_5a_8a_{10}-a_5a_8a_{11}+a^*\beta_1a_6a_{14}}{a_1a_2a_{11}-a^*a_1a_2a_{10}-a^*\beta_1a_2a_{12}}$$

$$K=\frac{a_4a_8a_9+a_4a_7a_8a_{10}+a_6^2a_9+a_6^2a_7a_{10}-\beta_1a_7a_8a_{13}-a_5a_8a_9-a_5a_7a_8a_{10}-\beta_1a_6a_7a_{14}}{a_1a_2a_{11}-a^*a_1a_2a_{10}-a^*\beta_1a_2a_{12}}$$

$$a_1=C_{11}+P_1+\mu_eB_0^2,\ a_2=\left(\frac{2C_{33}-C_{13}-C_{11}-P_1}{2}\right),\ a_3=\left(\frac{C_{11}-C_{13}+P_1}{2}\right),\ a_4=-a_1\alpha^2+\rho\alpha^2c^2$$

$$a_5=-\rho g,\ a_6=\rho gi\alpha,\ a_7=1+a^*\alpha^2,\ a_8=-a_3\alpha^2+\rho\alpha^2c^2,\ a_9=i\alpha^3cK_1m_1-\alpha^2m_2K_1^*,$$

$$a_{10}=\rho c_e\alpha^2c^2,\ a_{11}=K_3^*m_2-iK_3\alpha cm_1,\ a_{12}=T_0\beta_3\alpha^2c^2,\ a_{13}=T_0\beta_3\alpha^4c^2,\ a_{14}=(i\alpha T_0\beta_3-i\alpha T_0\beta_1)\alpha^2c^2$$

when $x_3\to\infty$ the equation is bounded and in other words it can be drafted as follows:

$$\begin{cases}U(x_3)=\sum_{i=1}^{3}A_iexp[-\Gamma_ix_3]\\ V(x_3)=\sum_{i=1}^{3}B_iexp[-\Gamma_ix_3]\\ W(x_3)=\sum_{i=1}^{3}C_iexp[-\Gamma_ix_3]\end{cases}\tag{25}$$

Where A_i,B_i,C_i are constants for i=1,2,3

$$B_i=b_iA_i\ and\ C_i=d_iA_i$$

Where

$$d_i=\frac{-a_6(a_{12}\Gamma_i^2-a_{13})}{a_6[a_9+a_{10}(a_7-a^*\Gamma_i^2)+a_{11}\Gamma_i^2]+a_{14}\Gamma_i[\beta_1(a_7-a^*\Gamma_i^2)]}$$

$$b_i=\frac{(a_1\Gamma_i^2+a_4+a_5\Gamma_i)-\beta_1(a_7-a^*\Gamma_i^2)d_i}{a_6}$$

Hence the solutions of equations (16), (17) and (18) are given by

$$\begin{cases}\phi=\sum_{i=1}^{3}A_iexp[-\Gamma_ix_3+i\alpha(x_1-ct)]\\ \psi=\sum_{i=1}^{3}b_iA_iexp[-\Gamma_ix_3+i\alpha(x_1-ct)]\\ T=\sum_{i=1}^{3}d_iA_iexp[-\Gamma_ix_3+i\alpha(x_1-ct)]\end{cases}\tag{26}$$

5. FREQUENCY EQUATION

The stress components in context of thermoelastic potentials at two temperature is given by:

$$\sigma_{33} = c_{13}\phi_{,11} + c_{33}\phi_{,33} - (c_{13} - c_{33})\psi_{,31} - \beta_3\Theta = 0 \tag{27}$$

$$\tau_{13} = \left(\frac{C_{11} - C_{13}}{2}\right)[2\phi_{,13} - \psi_{,33} + \psi_{,31}] = 0 \tag{28}$$

Two temperature gradient is associated to each other q_3 (Normal component of heat flux vector) by the following relation:

$$q_3 = \left[\frac{-K_3(1+\tau_T D^{'})D^{'} - K_3^*(1+\tau_v D^{'})}{D'(1+\tau_q D' + (\frac{\tau_q^2}{2})D'^2)}\right]\frac{\partial\Theta}{\partial x_3} \tag{29}$$

Where $\dfrac{\partial}{\partial t} = D^{'}$, $\quad \Theta = T - a^*[T_{,11} + T_{,33}]$

Using the boundary conditions in equations (27), (28) and (29). The linear equations in terms of A_1, A_2 and A_3 are obtained as follows:

$$\sum_{i=1}^{3}(a_7 - a^*\Gamma_i^2)(\eta\Gamma_i + h)d_i A_i = 0 \tag{30}$$

$$\sum_{i=1}^{3}[c_{33}\Gamma_i^2 - c_{13}\alpha^2 - (c_{13} - c_{33})i\alpha\Gamma_i b_i - \beta_3(a_7 - a^*\Gamma_i^2)d_i]A_i = 0 \tag{31}$$

$$\sum_{i=1}^{3}[b_i\Gamma_i^2 + 2i\alpha\Gamma_i + \alpha^2 b_i]A_i = 0 \tag{32}$$

Here $\eta = \frac{-iK_3\alpha c\tau_3 + K_3^*\tau_4}{-i\alpha c\tau_5}$

The non trivial solutions of equations (30), (31) and (32) exist if

(33)

$$[c_{33}\Gamma_1^2 - c_{13}\alpha^2 - (c_{13}-c_{33})i\alpha\Gamma_1 b_1 - \beta_3(a_7 - a^*\Gamma_1^2)d_1] \times [(b_2\Gamma_2^2 + 2i\alpha\Gamma_2 + \alpha^2 b_2)(a_7 - a^*\Gamma_3^2)(\eta\Gamma_3 + h)d_3 - (b_3\Gamma_3^2 + 2i\alpha\Gamma_3 + \alpha^2 b_3)(a_7 - a^*\Gamma_2^2)(\eta\Gamma_2 + h)d_2][c_{33}\Gamma_2^2 - c_{13}\alpha^2 - (c_{13}-c_{33})i\alpha\Gamma_2 b_2 - \beta_3(a_7 - a^*\Gamma_2^2)d_2] \times [(b_3\Gamma_3^2 + 2i\alpha\Gamma_3 + \alpha^2 b_3)(a_7 - a^*\Gamma_1^2)(\eta\Gamma_1 + h)d_1 - (b_1\Gamma_1^2 + 2i\alpha\Gamma_1 + \alpha^2 b_1)(a_7 - a^*\Gamma_3^2)(\eta\Gamma_3 + h)d_3] [c_{33}\Gamma_3^2 - c_{13}\alpha^2 - (c_{13}-c_{33})i\alpha\Gamma_3 b_3 - \beta_3(a_7 - a^*\Gamma_3^2)d_3] \times [(b_1\Gamma_1^2 + 2i\alpha\Gamma_1 + \alpha^2 b_1)(a_7 - a^*\Gamma_2^2)(\eta\Gamma_2 + h)d_2 - (b_2\Gamma_2^2 + 2i\alpha\Gamma_2 + \alpha^2 b_2)(a_7 - a^*\Gamma_1^2)(\eta\Gamma_1 + h)d_1] = 0$$

Equation (33) represents the required frequency equation for Rayleigh wave propagation in orthotropic thermoelastic half space in the presence of gravity, initial stress and magnetic field.

6. Special Cases

Case 1: For thermally insulated surface

By applying the boundary condition $q_3 = 0$ at $x_3 = 0$ for thermally insulated surface, then equation (33) transform into

(34)

$$[c_{33}\Gamma_1^2 - c_{13}\alpha^2 - (c_{13}-c_{33})i\alpha\Gamma_1 b_1 - \beta_3(a_7 - a^*\Gamma_1^2)d_1] \times [(b_2\Gamma_2^2 + 2i\alpha\Gamma_2 + \alpha^2 b_2)(a_7 - a^*\Gamma_3^2)\Gamma_3 d_3 - (b_3\Gamma_3^2 + 2i\alpha\Gamma_3 + \alpha^2 b_3)(a_7 - a^*\Gamma_2^2)\Gamma_2 d_2][c_{33}\Gamma_2^2 - c_{13}\alpha^2 - (c_{13}-c_{33})i\alpha\Gamma_2 b_2 - \beta_3(a_7 - a^*\Gamma_2^2)d_2] \times [(b_3\Gamma_3^2 + 2i\alpha\Gamma_3 + \alpha^2 b_3)(a_7 - a^*\Gamma_1^2)\Gamma_1 d_1 - (b_1\Gamma_1^2 + 2i\alpha\Gamma_1 + \alpha^2 b_1)(a_7 - a^*\Gamma_3^2)\Gamma_3 d_3][c_{33}\Gamma_3^2 - c_{13}\alpha^2 - (c_{13}-c_{33})i\alpha\Gamma_3 b_3 - \beta_3(a_7 - a^*\Gamma_3^2)d_3] \times [(b_1\Gamma_1^2 + 2i\alpha\Gamma_1 + \alpha^2 b_1)(a_7 - a^*\Gamma_2^2)\Gamma_2 d_2 - (b_2\Gamma_2^2 + 2i\alpha\Gamma_2 + \alpha^2 b_2)(a_7 - a^*\Gamma_1^2)\Gamma_1 d_1] = 0$$

Case 2: For isothermal surface

By applying the boundary condition $T = 0$ at $x_3 = 0$ for isothermal surface, then equation

(33)transform into

(35)

$$[c_{33}\Gamma_1^2-c_{13}\alpha^2-(c_{13}-c_{33})i\alpha\Gamma_1 b_1-\beta_3(a_7-a^*\Gamma_1^2)d_1]\times[(b_2\Gamma_2^2+2i\alpha\Gamma_2+\alpha^2 b_2)(a_7-a^*\Gamma_3^2)d_3-(b_3\Gamma_3^2+2i\alpha\Gamma_3+\alpha^2 b_3)(a_7-a^*\Gamma_2^2)d_2][c_{33}\Gamma_2^2-c_{13}\alpha^2-(c_{13}-c_{33})i\alpha\Gamma_2 b_2-\beta_3(a_7-a^*\Gamma_2^2)d_2]\times[(b_3\Gamma_3^2+2i\alpha\Gamma_3+\alpha^2 b_3)(a_7-a^*\Gamma_1^2)d_1-(b_1\Gamma_1^2+2i\alpha\Gamma_1+\alpha^2 b_1)(a_7-a^*\Gamma_3^2)d_3][c_{33}\Gamma_3^2-c_{13}\alpha^2-(c_{13}-c_{33})i\alpha\Gamma_3 b_3-\beta_3(a_7-a^*\Gamma_3^2)d_3]\times[(b_1\Gamma_1^2+2i\alpha\Gamma_1+\alpha^2 b_1)(a_7-a^*\Gamma_2^2)d_2-(b_2\Gamma_2^2+2i\alpha\Gamma_2+\alpha^2 b_2)(a_7-a^*\Gamma_1^2)d_1]=0$$

7. Numerical Results and Discussion

Relevant parameters for Cobalt material (Sharma et. al [31]) are assumed for numerical discussion. The results are represented graphically for the impact of various parameters like initial stress, gravity and magnetic field on phase velocity (c) of Rayleigh wave propagating in orthotropic thermoelastic medium.

$c_{11}=3.07\times10^{11}Nm^{-2},\;\; c_{33}=3.581\times10^{11}Nm^{-2},\;\; c_{13}=1.650\times10^{11}Nm^{-2}$

$c_{44}=1.51\times10^{11}Nm^{-2},\;\; c_e=4.270\times10^{2}J/KgK,\;\; T_0=298K$

$\beta_3=6.93\times10^{6}N/m^2K,\;\; \rho=8.837\times10^{3}kg/m^3,\;\; \beta_1=7.04\times10^{6}N/m^2K$

$K_1=6.89\times10^{2}W/mk,\;\; K_1^*=1.313\times10^{2}W/s,\;\; K_3=7.011\times10^{2}W/mk$

$K_3^*=1.54\times10^{2}W/s,\;\; \tau_q=2.0\times10^{-7}s,\;\; \tau_T=1.5\times10^{-7}s\,,\;\; \tau_v=1.01\times10^{-8}s$

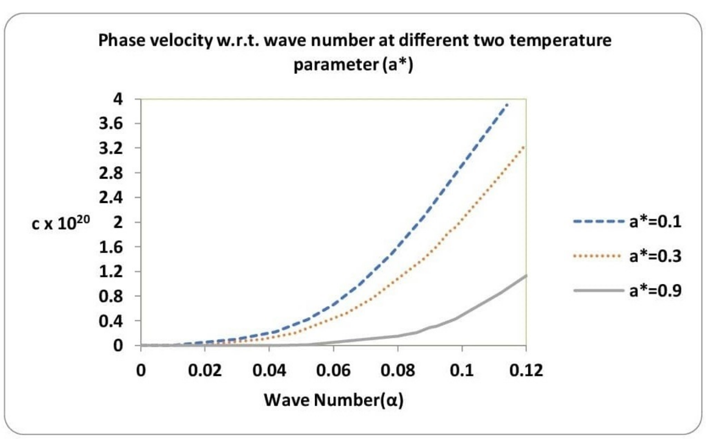

FIGURE 1. c (Phase velocity) corresponding to α (wave number) at distinct value of a^* in the presence of gravity (g =9.8)

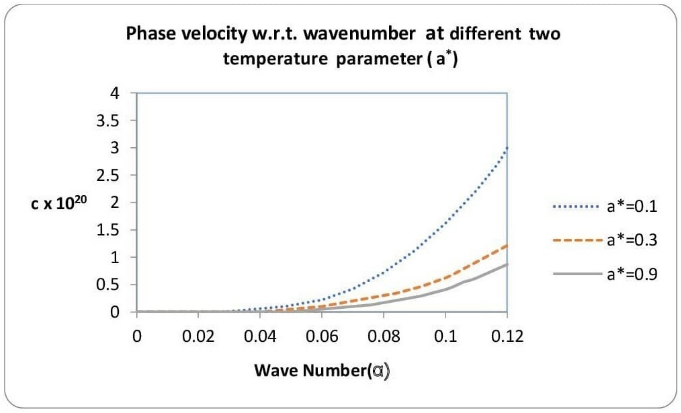

FIGURE 2. c (Phase velocity) corresponding to α (wave number) at distinct values of a^* in the presence of initial stress ($P_1 = 10^9$)

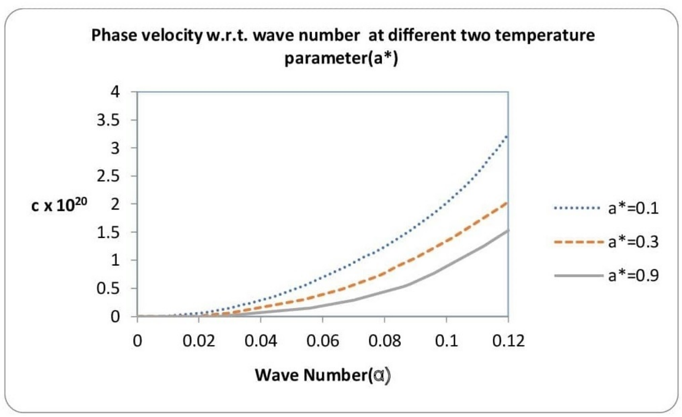

FIGURE 3. c (Phase velocity) corresponding to α (wave number) at distinct values of a^* in the presence of magnetic field ($B_0 = 1$)

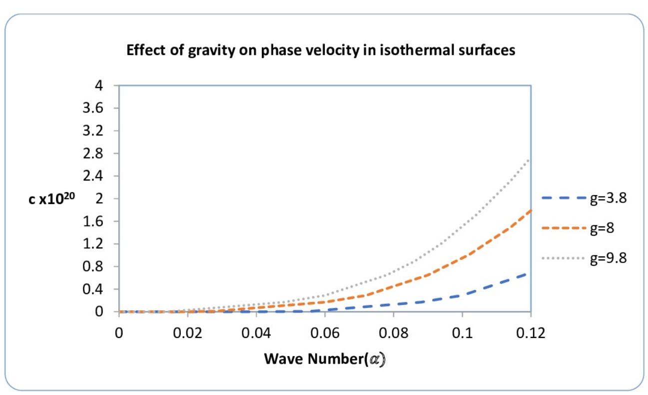

FIGURE 4. c (Phase velocity) corresponding to α (wave number) in isothermal surface at distinct values of g, $P_1 = 10^9$, B_0=1

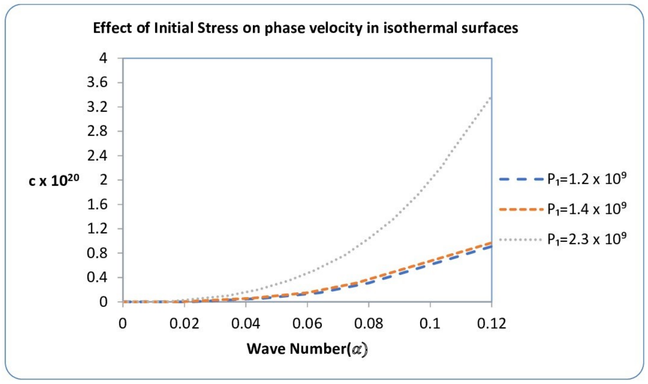

FIGURE 5. c (Phase velocity) corresponding to α (wave number) in isothermal surface at different values of P_1, g=9.8, $B_0 = 1$

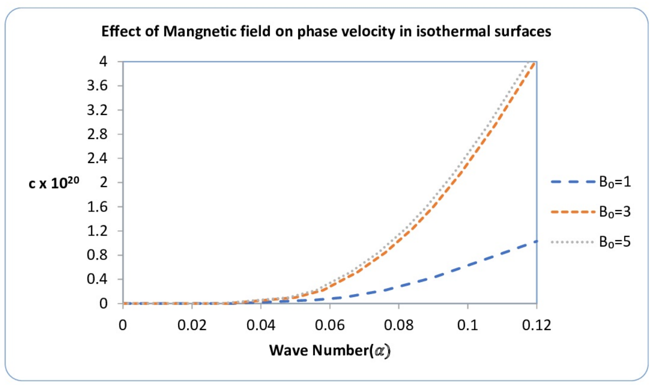

FIGURE 6. c (Phase velocity) corresponding to α (wave number) in isothermal surface at distinct values of B_0, g=9.8, $P_1 = 10^9$

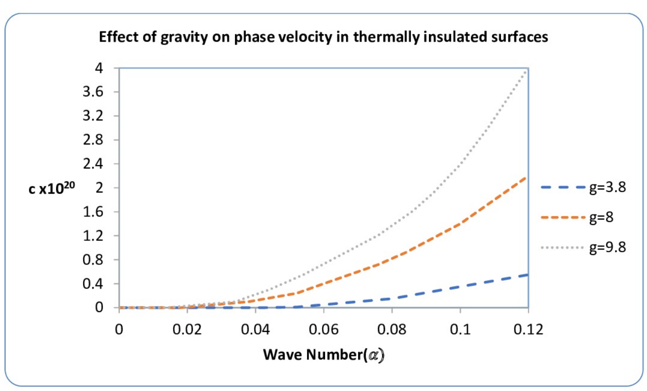

FIGURE 7. c (Phase velocity) corresponding to α (wave number) in thermally insulated surface at distinct values of g, $B_0 = 1, P_1 = 10^9$

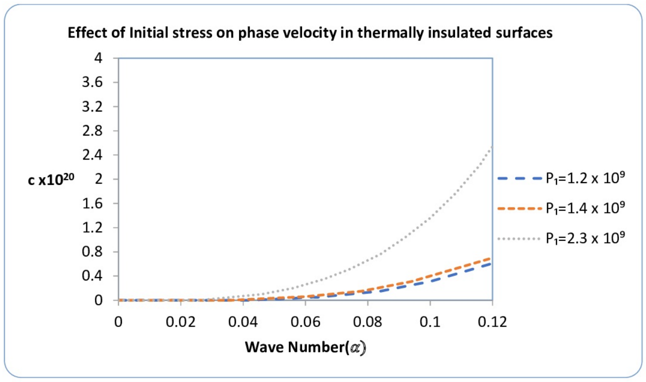

FIGURE 8. c (Phase velocity) corresponding to α (wave number) in thermally insulated surface at distinct values of P_1, g=9.8, $B_0 = 1$

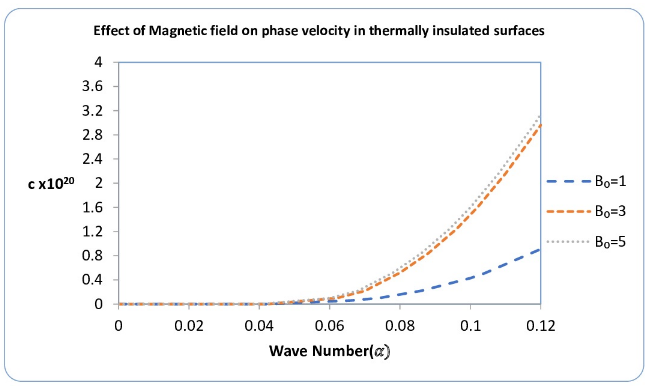

FIGURE 9. c (Phase velocity) corresponding to α (wave number) in thermally insulated surface at distinct values of B_0, g=9.8, $P_1 = 10^9$

Figures 1-3 represents the variation of c (phase velocity) with respect to α (wave number) in the presence of gravity (g=9.8), initial stress ($P_1 = 10^9$) and magnetic field ($B_0 = 1$) at distinct value of two temperature parameter (a^*). It has been observed that the phase velocity increases with the increase of wave number for the value of $a^* = 0.9$. The phase velocity increases relatively fast for $a^* = 0.3$ as compared to $a^* = 0.9$, but this increment of phase velocity is even more pronounced for the value of $a^* = 0.1$.

Figures 4-6 represents the variation of c (phase velocity) corresponding to α (wave number) at distinct values of gravity, initial stress and magnetic field in isothermal surface at the two temperature parameter ($a^* = 0.1$). It has been noticed that at low values of gravity, initial stress and magnetic field, the variation in phase velocity with increasing wave number is more evident than at higher values of these parameters, but this enhancement of phase velocity is almost identical in the case of initial stress ($P_1 = 1.2 \times 10^9$), ($P_1 = 1.4 \times 10^9$) and similar behaviour in variation of phase velocity has been observed at magnetic field ($B_0 = 3$) and ($B_0 = 5$).

Figures 7-9 represents the variation of c (phase velocity) corresponding to α (wave number) for distinct values of gravity, initial stress and magnetic field in thermally insulated surface at the two temperature parameter ($a^* = 0.1$). It has been noticed that phase velocity shows identical trends in thermally insulated surfaces for all the parameters gravity, initial stress and magnetic field like isothermal surface. It may be attributed to the fact that thermoelastic dissipation of energy may not be dominant in isothermal surfaces because the Rayleigh wave propagates with high phase velocity at low value of uniform absolute temperature. It shows that thermoelastic dissipation is negligible with high phase velocity in isothermal surfaces, but insulated boundary system retains the energy and has limited impact on Rayleigh wave's phase velocity.

8. Conclusion

The Rayleigh waves propagation in magneto-thermoelastic medium in the context of Three Phase Lag (TPL) model at two temperature has been studied for homogeneous orthotropic half space. Normal mode analysis technique has been employed to derive frequency equations for isothermal surfaces and thermally insulated surfaces with relevant boundary conditions. The impact of various parameters like initial stress, gravity and magnetic field on phase velocity of Rayleigh waves corresponding to wave number has been analysed. Based upon numerical and analytical observation, it can be concluded that:

(1) The phase velocity of Rayleigh waves in orthotropic solids increases with the increase of wave number and this trend of variation in phase velocity with respect to wave number is almost identical in the presence of initial stress, gravity and magnetic field.

(2) The variation of phase velocity with respect to wave number is not so much pronounced with the increase of two temperature parameter (a^*) in the presence of initial stress, gravity and magnetic field.

(3) The variation of phase velocity in reference to increase of wave number is more evident at the low value of gravity, initial stress and magnetic field than at higher values of these parameters.

(4) Even though this problem is considered as theoretical one, but all these observations can provide useful information for analysing the vital parameters in the field seismology, mine engineering and geophysics.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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