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NONMINIMALLY SUPPORTED DESIGN FOR THREE PARAMETERS GENERALIZED EXPONENTIAL MODEL

TATIK WIDIHARIH*, MUSTAFID, ALAN PRAHUTAMA, SUDARNO

Department of Statistics, Diponegoro University, Semarang, Indonesia

Abstract: The exponential models are widely applied in several fields as a growth curve. The D-optimal design is a minimally supported design, the number of supported designs is the same as the number of parameters with uniform weight. Nonminimally supported design is a design with the number of supported designs is greater than the number of parameters. In this paper, we investigated nonminimally supported design that is built using four supported points with uniform weight and determination of the supported designs by maximizing the determinant of information matrix. Determination of the supported designs use two ways, first by deriving the objective function formula, which is determinant of the information matrix then maximized it, second by adding one supported point to the minimally supported design. Based on the numerical simulation of two methods, nonminimally supported design with the maximum determinant value is the best nonminimally supported design.

Keywords: minimally supported design; nonminimally supported design; information matrix; generalized exponential model.

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*Corresponding author
E-mail address: widiharih@gmail.com
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1. INTRODUCTION

The Exponential models are widely used to describe a growth functions. Researcher usually use this model with the curve always rising. The curve of exponential model consists of two types that is increasing/decreasing monotone and an unimodal. Hathout [1] use exponential model to describe the world population. Archontoulis and Miguez [2] use the exponential model with monotone go up and unimodal forms for modeling in agricultural field. Ricker and von Rosen [3] use a generalization of the exponential model. Al-Eideh and Al-Omar [4] use exponential model to estimate population. Ma [5], use the exponential model to estimate an epidemic.

D-optimal design is a design with the selection of supported designs by maximizing the determinant of the information matrix. The maximized determinant of the information matrix that gives the variance of the estimator of parameter is small, so the hypothesis that parameter equal zero will be rejected. The D-optimal design is a minimally supported design (the number of supported is the same as the number of parameters) with uniform weight ([6],[7]). Information matrix of nonlinear model contain the parameter which unknown value. The formula determinant of the information matrix becomes complicated. Supported designs can be obtained if we have the information about the value of parameters. Maximizing the determinant of the information matrix is done numerically. D-optimal design for exponential model have been done, including ([8], [9], [10], [11], [12]).

The design with the number of supported points equal to k + 1 (*k* is number of parameter in the model), is the simplest of nonminimally supported design. Initially, the research on this case was conducted by Khinkis et al [13], they use the Hill model. Other researchers who have conducted this research, Su and Raghavarao [14], used the sigmoid model (cuving from the starting point then rises, reaches at maximum point then is relatively constant) including the Logistics, Probit and Gompertz models. They construct nonminimally supported design by adding one supported design to D-optimal design. One supported design is chose one of the supported design in D-optimal design or the average of the supported design in D-optimal design, than calculate the determinant information matrix for all alternative design and chose the design that have the highest value of determinant information matrix.

Generalized Exponential distribution was introduced by Gupta and Kundu [15]. The distribution is viewed as an alternative of Gamma distribution or Weibull distribution. The formula of the Generalized Exponential distribution as follows:

(1) $$f(t) = \alpha\lambda e^{-\lambda t}\left(1-e^{-\lambda t}\right)^{\alpha-1}, t \ge 0, \alpha \ge 1, \lambda > 0$$

Based on equation (1) in this paper, we use the three parameter Generalized Exponential model as follows:

(2) $$y = \theta_3 e^{-\theta_1 t}\left(1-e^{-\theta_1 t}\right)^{\theta_2} + \varepsilon, t \ge 0, \theta_1, \theta_2, \theta_3 > 0$$

The model in equation (2) is a nonlinear model. To build this model, it is necessary to select some points of t to be observed, then the points are called supported points. The model obtained is expected to be significant, that is the variable factor influences the response, in other words, the test whose parameters equal zero is rejected.

Nonminimally supported design will be built by adding one supported design from the minimally supported design, so we use four supported design with uniform proportion. We use two approaches, first by deriving the formula of determinant of the information matrix, then determining the points that maximize this determinant, second by adding one supported design to the D-optimal design which randomly selected (in this case we chose 133 point) from the design region. Based on the two methods, all of the alternative of nonminimally supported design are calculated the information matrix and their determinant. Based on the value of the determinant, the best design is design that have the highest value of determinant of information matrix.

2. PRELIMINARIES

The nonlinear model is denote by:

(3) $$y = \eta(t, \theta) + \varepsilon$$

with independent $\varepsilon \sim N(0, \sigma^2)$

(4) $$E(Y \mid t) = \eta(t, \theta)$$

Designs ξ of *4* supported points ($t_i, i = 1,2,3,4$) and their proportions ($w_i = \frac{1}{4}, i = 1,2,3,4$) is denoted by:

(5) $$\xi = \begin{pmatrix} t_1 & t_2 & t_3 & t_4 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

The information matrix of design ξ for model (5) is:

(6) $$M(\xi,\theta) = \frac{1}{4}\sum_{i=1}^{4} h(t_i,\theta)h^T(t_i,\theta)$$

where: $\boldsymbol{h}(t,\theta) = \frac{\partial\eta(t,\theta)}{\partial\theta} = \big(h_1(t,\theta), h_2(t,\theta), h_3(t,\theta), h_4(t,\theta)\big)^{\boldsymbol{T}}$ is the vector of partial derivatives of the conditional expectation $E(Y|t)$ with respect to the parameters θ. $\boldsymbol{M}(\xi,\theta)$ is 3×3 symetric matrix. $\boldsymbol{M}(\xi,\theta)$ contains parameters with unknown values so initial information about the parameter values is required

Nonminimally supported designs are built in two ways as follows:

A. Design I

Based on the model in equation (2) and the design ξ equation (5), determination of nonminimally supported designs as follows:

i. Calculate the element of information matrix.

ii. Construct the information matrix

iii. Determine the formula determinant of the information matrix.

iv. Enter the value of parameters based on initial information about the parameter values in (iii) and determine the design region.

v. Determine the supported designs by maximizing the determinant of the information matrix (iv).

B. Design II

Nominimally supported design is obtained by adding one supported point from the D-optimal design. These points are selected randomly in the given design region. Based on all of the alternative, design that has the highest value of determinant of information matrix is chosen as the best nonminimally supported design. The complete algorithm is as follows:

a. Determine the minimally supported design (t_1, t_1, t_3) obtained from D-optimal design for model (2).
b. Adding one supported point (t_4) selected randomly in the given design region.
c. Given uniform weight to t_1, t_2, t_3, t_4
d. Determine the determinant of information matrix for all alternative nonminimally supported design.
e. Chose the design that has the highest value of determinant of information matrix as the best nonminimally supported design.

3. MAIN RESULTS

3.1. Nonminimally Supported Designs For Three Parameters Generalized Exponential Model Design I

Consider model (2) :

$$y = \theta_3 e^{-\theta_1 t}\left(1 - e^{-\theta_1 t}\right)^{\theta_2} + \varepsilon, \quad x \geq 0, \theta_1, \theta_2, \theta_3 > 0.$$

The curve of model (2) for $\theta_3 = 1.65$; $\theta_1 = 0.75$ at several of θ_2 and for $\theta_3 = 1.65$; $\theta_2 = 2.25$ at several of θ_1 are presented in Figure 1 and Figure 2.

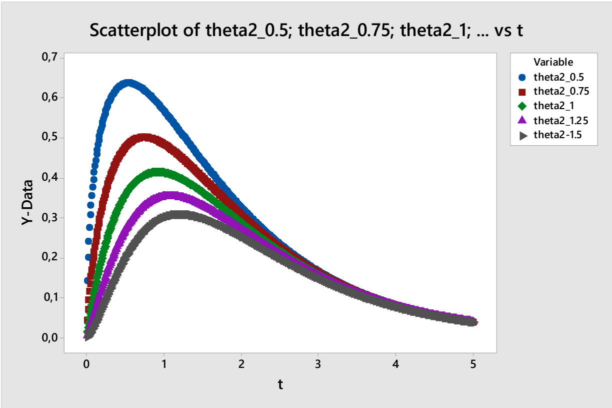

Figure 1. The curve of model (2) for $\theta_3 = 1.65$; $\theta_1 = 0.75$ at several of θ_2

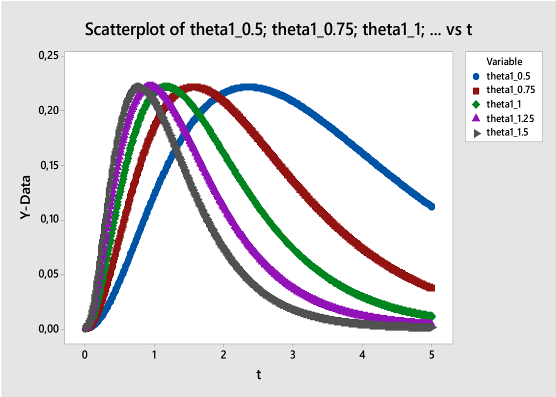

Figure 2. The curve of model (2) for $\theta_3 = 1.65$; $\theta_2 = 2.25$ at several of θ_1

Based on Figure 1, if θ_3 and θ_1 fixed with varying θ_2 the smaller value of θ_2 then *t* also the smaller but the maximum value is greater. Based on Figure 2, if θ_3 and θ_2 fixed and several of θ_1 each curve has maximum value are relatively the same but the smaller of value θ_1 then the greater of *t* value.

Based on equation (2) and (4) so that:

$$\eta(t,\theta) = \theta_3 e^{-\theta_1 t}\left(1 - e^{-\theta_2 t}\right)^{\theta_2}$$

(7)
$$h(t,\theta) = \begin{pmatrix} \theta_3 t e^{-\theta_1 t}\left(1-e^{-\theta_1 t}\right)^{\theta_2 - 1}\left((1+\theta_2)e^{-\theta_1 t}-1\right) \\ \theta_3 e^{-\theta_1 t}\left(1-e^{-\theta_1 t}\right)^{\theta_2}\ln\left(1-e^{-\theta_1 t}\right) \\ e^{-\theta_1 t}\left(1-e^{-\theta_1 t}\right)^{\theta_2} \end{pmatrix}$$

Designs ξ of *4* supported points

$$\xi = \begin{pmatrix} t_1 & t_2 & t_3 & t_4 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

The information matrix based on equation (6) and (7) as follows:

(8)
$$M(\xi,\theta) = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix}$$

where the element of information matrix as follows:

$$m_{11} = \frac{1}{4}\sum_{i=1}^{4} {\theta_3}^2 {t_i}^2 e^{-2\theta_1 t_i}\left(1 - e^{-\theta_1 t_i}\right)^{2(\theta_2 - 1)}\left((1+\theta_2)e^{-\theta_1 t_i} - 1\right)^2$$

$$m_{22} = \frac{1}{4}\sum_{i=1}^{4} {\theta_3}^2 e^{-2\theta_1 t_i}\ \left(1 - e^{-\theta_1 t_i}\right)^{2\theta_2} ln^2\left(1 - e^{-\theta_1 t_i}\right)$$

$$m_{33} = \frac{1}{4}\sum_{i=1}^{4} e^{-2\theta_1 t_i}\left(1 - e^{-\theta_2 t_i}\right)^{2\theta_2}$$

$$m_{12} = \frac{1}{4}\sum_{i=1}^{4} {\theta_3}^2 t_i e^{-2\theta_1 t_i}\left(1 - e^{-\theta_1 t_i}\right)^{2\theta_2 - 1}\left((1+\theta_2)e^{-\theta_1 t_i} - 1\right) ln\left(1 - e^{-\theta_1 t_i}\right)$$

$$m_{13} = \frac{1}{4}\sum_{i=1}^{4} \theta_3 t_i e^{-2\theta_1 t_i}\left(1 - e^{-\theta_1 t_i}\right)^{2\theta_2 - 1}\ \left((1+\theta_2)e^{-\theta_1 t_i} - 1\right)$$

$m_{23} = \frac{1}{4}\sum_{i=1}^{4} \theta_3 e^{-2\theta_1 t_i}\left(1 - e^{-\theta_2 t_i}\right)^{2\theta_2} \ln\left(1 - e^{-\theta_2 t_i}\right)$

The formula of $|\boldsymbol{M}(\xi, \theta)|$ is very complicated, we use the notation:

$x_1 = 1 - e^{-\theta_1 t_1}$

$y_1 = 1 - e^{-\theta_1 t_2}$

$z_1 = 1 - e^{-\theta_1 t_3}$

$k_1 = 1 - e^{-\theta_1 t_4}$

$x_2 = (1 + \theta_2)e^{-\theta_1 t_1} - 1$

$y_2 = (1 + \theta_2)e^{-\theta_1 t_2} - 1$

$z_2 = (1 + \theta_2)e^{-\theta_1 t_3} - 1$

$k_2 = (1 + \theta_2)e^{-\theta_1 t_4} - 1$

Based on this notation can be find the formula of $|\boldsymbol{M}(\xi, \theta)|$ as follows:

(9) $$|M(\xi, \theta)| \propto A + B + C + D$$

where:

$A_1 = e^{-2\theta_1(t_1+t_2+t_3)}$

$A_{11} = {x_1}^{2\theta_2} ln^2(x_1)\, {y_1}^{2\theta_2-2} {z_1}^{2\theta_2-2}(t_3.y_1z_2 - t_2.z_1y_2)^2$

$A_{12} = {y_1}^{2\theta_2} ln^2(y_1)\, {x_1}^{2\theta_2-2} {z_1}^{2\theta_2-2}(t_3.x_1z_2 - t_1.z_1x_2)^2$

$A_{13} = {z_1}^{2\theta_2} ln^2(z_1)\, {y_1}^{2\theta_2-2} {x_1}^{2\theta_2-2}(t_2.x_1y_2 - t_1.y_1x_2)^2$

$A_{14} = {x_1}^{2\theta_2-1} ln(x_1)\, {y_1}^{2\theta_2-1} ln(y_1) {z_1}^{2\theta_2-1}[t_1.x_2(t_3.z_2y_1 - t_2.y_2z_1) + t_2.y_2(t_3.z_2x_1 - t_1.x_2z_1)]$

$A_{15} = {x_1}^{2\theta_2-1} ln(x_1)\, {z_1}^{2\theta_2-1} ln(z_1) {y_1}^{2\theta_2-1}[t_1.x_2(t_2.y_2z_1 - t_3.z_2y_1) + t_3.z_2(t_2.y_2x_1 - t_1.x_2y_1)]$

$A_{16} = {y_1}^{2\theta_2-1} ln(y_1)\, {z_1}^{2\theta_2-1} ln(z_1) {x_1}^{2\theta_2-1}[t_2.y_2(t_1.x_2z_1 - t_3.z_2x_1) + t_3.z_2(t_1.x_2y_1 - t_2.y_2x_1)]$

$A_{17} = {x_1}^{2\theta_2-1} ln(x_1)\, {z_1}^{2\theta_2-1} ln(z_1) {y_1}^{2\theta_2-2}\, t_2.y_2[t_3.y_1x_1z_2 + t_1.y_1z_1x_2 - 2.t_2.y_2x_1z_1]$

$A_{18} = {x_1}^{2\theta_2-1} ln(x_1)\, {y_1}^{2\theta_2-1} ln(y_1) {z_1}^{2\theta_2-2} t_3.z_2[t_2.z_1x_1y_2 + t_1.y_1z_1x_2 - 2.t_3.z_2x_1y_1]$

$A_{19} = {y_1}^{2\theta_2-1} ln(y_1)\, {z_1}^{2\theta_2-1} ln(z_1) {x_1}^{2\theta_2-2} t_1.x_2[t_2.z_1x_1y_2 + t_1.y_1x_1z_2 - 2.t_1.x_2z_1y_1]$

$A = A_{11} + A_{12} + A_{13} + A_{14} + A_{15} + A_{16} + A_{17} + A_{18} + A_{19}$

$B_1 = e^{-2\theta_1(t_1+t_2+t_4)}$

$B_{11} = {x_1}^{2\theta_2} ln^2(x_1)\, {y_1}^{2\theta_2-2} {k_1}^{2\theta_2-2}(t_4. y_1 k_2 - t_2. k_1 y_2)^2$

$B_{12} = {y_1}^{2\theta_2} ln^2(y_1)\, {x_1}^{2\theta_2-2} {k_1}^{2\theta_2-2}(t_4. x_1 k_2 - t_1. k_1 x_2)^2$

$B_{13} = {k_1}^{2\theta_2} ln^2(k_1)\, {y_1}^{2\theta_2-2} {x_1}^{2\theta_2-2}(t_2. x_1 y_2 - t_1. y_1 x_2)^2$

$B_{14} = {x_1}^{2\theta_2-1} ln(x_1)\, {y_1}^{2\theta_2-1} ln(y_1) {k_1}^{2\theta_2-1}[t_1. x_2(t_4. k_2 y_1 - t_2. y_2 k_1) + t_2. y_2(t_4. k_2 x_1 - t_1. x_2 k_1)]$

$B_{15} = {x_1}^{2\theta_2-1} ln(x_1)\, {k_1}^{2\theta_2-1} ln(k_1) {y_1}^{2\theta_2-1}[t_1. x_2(t_2. y_2 k_1 - t_4. k_2 y_1) + t_4. k_2(t_2. y_2 x_1 - t_1. x_2 y_1)]$

$B_{16} = {y_1}^{2\theta_2-1} ln(y_1)\, {k_1}^{2\theta_2-1} ln(k_1) {x_1}^{2\theta_2-1}[t_2. y_2(t_1. x_2 k_1 - t_4. k_2 x_1) + t_4. k_2(t_1. x_2 y_1 - t_2. y_2 x_1)]$

$B_{17} = {x_1}^{2\theta_2-1} ln(x_1)\, {k_1}^{2\theta_2-1} ln(k_1) {y_1}^{2\theta_2-2}\, t_2. y_2[t_4. y_1 x_1 k_2 + t_1. y_1 k_1 x_2 - 2. t_2. y_2 x_1 k_1]$

$B_{18} = {x_1}^{2\theta_2-1} ln(x_1)\, {y_1}^{2\theta_2-1} ln(y_1) {k_1}^{2\theta_2-2} t_4. k_2[t_2. k_1 x_1 y_2 + t_1. y_1 k_1 x_2 - 2. t_4. k_2 x_1 y_1]$

$B_{19} = {y_1}^{2\theta_2-1} ln(y_1)\, {k_1}^{2\theta_2-1} ln(k_1) {x_1}^{2\theta_2-2} t_1. x_2[t_2. k_1 x_1 y_2 + t_1. y_1 x_1 k_2 - 2. t_1. x_2 k_1 y_1]$

$B = B_{11} + B_{12} + B_{13} + B_{14} + B_{15} + B_{16} + B_{17} + B_{18} + B_{19}$

$C_1 = e^{-2\theta_1(t_1+t_4+t_3)}$

$C_{11} = {x_1}^{2\theta_2} ln^2(x_1)\, {k_1}^{2\theta_2-2} {z_1}^{2\theta_2-2}(t_3. k_1 z_2 - t_4. z_1 k_2)^2$

$C_{12} = {k_1}^{2\theta_2} ln^2(k_1)\, {x_1}^{2\theta_2-2} {z_1}^{2\theta_2-2}(t_3. x_1 z_2 - t_1. z_1 x_2)^2$

$C_{13} = {z_1}^{2\theta_2} ln^2(z_1)\, {k_1}^{2\theta_2-2} {x_1}^{2\theta_2-2}(t_4. x_1 k_2 - t_1. k_1 x_2)^2$

$C_{14} = {x_1}^{2\theta_2-1} ln(x_1)\, {k_1}^{2\theta_2-1} ln(k_1) {z_1}^{2\theta_2-1}[t_1. x_2(t_3. z_2 k_1 - t_4. k_2 z_1) + t_4. k_2(t_3. z_2 x_1 - t_1. x_2 z_1)]$

$C_{15} = {x_1}^{2\theta_2-1} ln(x_1)\, {z_1}^{2\theta_2-1} ln(z_1) {k_1}^{2\theta_2-1}[t_1. x_2(t_4. k_2 z_1 - t_3. z_2 k_1) + t_3. z_2(t_4. k_2 x_1 - t_1. x_2 k_1)]$

$C_{16} = {k_1}^{2\theta_2-1} ln(k_1)\, {z_1}^{2\theta_2-1} ln(z_1) {x_1}^{2\theta_2-1}[t_4. k_2(t_1. x_2 z_1 - t_3. z_2 x_1) + t_3. z_2(t_1. x_2 k_1 - t_4. k_2 x_1)]$

$C_{17} = {x_1}^{2\theta_2-1} ln(x_1)\, {z_1}^{2\theta_2-1} ln(z_1) {k_1}^{2\theta_2-2}\, t_4. k_2[t_3. k_1 x_1 z_2 + t_1. k_1 z_1 x_2 - 2. t_4. k_2 x_1 z_1]$

$C_{18} = {x_1}^{2\theta_2-1} ln(x_1)\, {k_1}^{2\theta_2-1} ln(k_1) {z_1}^{2\theta_2-2} t_3. z_2[t_4. z_1 x_1 k_2 + t_1. k_1 z_1 x_2 - 2. t_3. z_2 x_1 k_1]$

$C_{19} = {k_1}^{2\theta_2-1} ln(k_1)\, {z_1}^{2\theta_2-1} ln(z_1) {x_1}^{2\theta_2-2} t_1. x_2[t_4. z_1 x_1 k_2 + t_1. k_1 x_1 z_2 - 2. t_1. x_2 z_1 k_1]$

$C = C_{11} + C_{12} + C_{13} + C_{14} + C_{15} + C_{16} + C_{17} + C_{18} + C_{19}$

$D_1 = e^{-2\theta_1(t_4+t_2+t_3)}$

$D_{11} = {k_1}^{2\theta_2} ln^2(k_1)\, {y_1}^{2\theta_2-2} {z_1}^{2\theta_2-2}(t_3. y_1 z_2 - t_2. z_1 y_2)^2$

$D_{12} = {y_1}^{2\theta_2} ln^2(y_1)\, {k_1}^{2\theta_2-2} {z_1}^{2\theta_2-2}(t_3. k_1 z_2 - t_4. z_1 k_2)^2$

$D_{13} = {z_1}^{2\theta_2} ln^2(z_1)\, {y_1}^{2\theta_2-2} {k_1}^{2\theta_2-2}(t_2. k_1 y_2 - t_4. y_1 k_2)^2$

$D_{14} = {k_1}^{2\theta_2-1} ln(k_1)\, {y_1}^{2\theta_2-1} ln(y_1) {z_1}^{2\theta_2-1} [t_4.k_2(t_3.z_2y_1 - t_2.y_2z_1) + t_2.y_2(t_3.z_2k_1 - t_4.k_2z_1)]$

$D_{15} = {k_1}^{2\theta_2-1} ln(k_1)\, {z_1}^{2\theta_2-1} ln(z_1) {y_1}^{2\theta_2-1} [t_4.k_2(t_2.y_2z_1 - t_3.z_2y_1) + t_3.z_2(t_2.y_2k_1 - t_4.k_2y_1)]$

$D_{16} = {y_1}^{2\theta_2-1} ln(y_1)\, {z_1}^{2\theta_2-1} ln(z_1) {k_1}^{2\theta_2-1} [t_2.y_2(t_4.k_2z_1 - t_3.z_2k_1) + t_3.z_2(t_4.k_2y_1 - t_2.y_2k_1)]$

$D_{17} = {k_1}^{2\theta_2-1} ln(k_1)\, {z_1}^{2\theta_2-1} ln(z_1) {y_1}^{2\theta_2-2}\, t_2.y_2[t_3.y_1k_1z_2 + t_4.y_1z_1k_2 - 2.t_2.y_2k_1z_1]$

$D_{18} = {k_1}^{2\theta_2-1} ln(k_1)\, {y_1}^{2\theta_2-1} ln(y_1) {z_1}^{2\theta_2-2} t_3.z_2[t_2.z_1k_1y_2 + t_4.y_1z_1k_2 - 2.t_3.z_2k_1y_1]$

$D_{19} = {y_1}^{2\theta_2-1} ln(y_1)\, {z_1}^{2\theta_2-1} ln(z_1) {k_1}^{2\theta_2-2} t_4.k_2[t_2.z_1k_1y_2 + t_3.y_1k_1z_2 - 2.t_4.k_2z_1y_1]$

$D = D_{11} + D_{12} + D_{13} + D_{14} + D_{15} + D_{16} + D_{17} + D_{18} + D_{19}$

Supported points $t_i, i = 1,2,3,4$ is obtained by maximizing $|\boldsymbol{M}(\xi,\theta)|$, but in the $|\boldsymbol{M}(\xi,\theta)|$ contains the parameter $\theta_i, i = 1,2,3$, so we need theirs predetermined values of $\theta_i, i = 1,2,3$. Numerical simulation nonminimally supported designs for model (2) by maximizing equation (11) for some value of $\theta_i, i = 1,2,3$ and design region (0 , 5) is presented in Table (1).

Table 1. Nonminimally Supported Design Model (2) For Some Value Of $\theta_i, i = 1,2,3$ with Design Region (0, 5)

			Supported design				$\lvert\boldsymbol{M}(\xi,\theta)\rvert$
θ_3	θ_2	θ_1	t_1	t_2	t_3	t_4	
1.65	0.50	0.75	0.041277	0.523139	2.124812	2.124813	0.000492164159
	0.75		0.105879	0.719584	2.385058	2.385058	0.000069652143
	1.00		0.180906	0.888596	2.597196	2.597196	0.000015289309
	1.25		0.259130	1.037199	2.776853	2.776857	0.000004385068
	1.50		0.336589	1.169961	2.933091	2.932960	0.000001510158
	1.75		0.412425	1.290033	3.071344	3.071335	0.000000594740
	2.00		0.485752	1.399686	3.19560	3.195588	0.000000259647
	2.25		0.556318	1.500664	3.308648	3.308435	0.000000123026
1.65	2.25	0.50	0.834507	0.834419	2.250930	4.962697	0.000000276808
		0.75	0.556318	1.500664	3.308648	3.308435	0.000000123026
		1.00	0.417237	1.125487	2.481425	2.481341	0.000000069202
		1.25	0.333788	0.900372	1.984997	1.985158	0.000000044289
		1.50	0.278161	0.750369	1.654388	1.654299	0.000000030756
		1.75	0.238421	0.643130	1.417891	1.417959	0.000000022597
		2.00	0.562779	0.208622	1.240480	1.241032	0.000000017301
		2.25	0.500209	0.185438	1.102815	1.102833	0.000000013670

Based on Table (1) shows that from all of the cases value of $\theta_i, i = 1,2,3$ four supported designs two of them are equal. In order word nonminimally supported design have three supported design with the weight are 0.25 , 0.25 and 0.5.

3.2 Nonminimally Supported Designs For Three Parameters Generalized Exponential Model Design II

D-optimal design for model (2) has been investigated [11].

Supported design (t_1, t_1, t_3) is obtained by maximizing:

(10) $$|M(\xi,\theta)| \propto e^{-2\theta_1 \sum_{i=1}^{3} t_i} \prod_{i=1}^{3} \left(1-e^{-\theta_1 t}\right)^{2(\theta_2-1)} \left[\sum_{i=1}^{3} R_i + \sum_{i=1}^{3} S_i\right]$$

where:

$$R_i = t_i^2\left(1-e^{-\theta_1 t_j}\right)^2\left(1-e^{-\theta_1 t_k}\right)^2\left((1+\theta_2)e^{-\theta_1 t_i} - 1\right)^2 A$$

$$A = \left[ln\left(1-e^{-\theta_1 t_j}\right) - ln\left(1-e^{-\theta_1 t_k}\right)\right]^2$$

$$S_i = t_i t_j\left(1-e^{-\theta_1 t_i}\right)\left(1-e^{-\theta_1 t_j}\right)\left(1-e^{-\theta_1 t_k}\right)^2\left((1+\theta_2)e^{-\theta_1 t_i} - 1\right) C$$

$$C = \left((1+\theta_2)e^{-\theta_1 t_i} - 1\right) ln\frac{1-e^{-\theta_1 t_k}}{1-e^{-\theta_1 t_i}}\ ln\frac{1-e^{-\theta_1 t_j}}{1-e^{-\theta_1 t_k}}$$

$$i \neq j \neq k, j = 2,3, k = 1,2,3$$

Supported designs $t_i, i = 1,2,3,$ is obtained by maximizing $|\boldsymbol{M}(\xi,\theta)|$, but the determinant $|\boldsymbol{M}(\xi,\theta)|$ contains three parameters $\theta_i, i = 1,2,3,$ so we need theirs predetermined values of $\theta_i, i = 1,2,3$. Numerical simulation D-optimal desing with model (2) by maximixing equation (12) for some value of $\theta_i, i = 1,2,3$ and design region (0 , 5) is presented in Table (2).

Table 2. D-optimal Design Model (1) for Some Value Of $\theta_i, i = 1,2,3$ with Design Region (0,5)

			Supported design		
θ_3	θ_2	θ_1	t_1	t_2	t_3
1.65	0.5	0.75	0.041277	0.523139	2.124813
	0.75		0.105879	0.719584	2.385058
	1		0.180906	0.888596	2.597197
	1.25		0.258913	1.037199	2.776856
	1.5		0.336589	1.169960	2.933019
	1.75		0.412425	1.290030	3.071334
	2		0.485752	1.399684	3.195590
	2.25		0.556314	1.500625	3.308469
1.65	2.25	0.5	0.834449	2.250904	4.962401
		0.75	0.556314	1.500625	3.308468
		1	0.417235	1.125469	2.481318
		1.25	0.333788	0.900374	1.985072
		1.5	0.278157	0.750307	1.654231
		1.75	0.238417	0.643104	1.417466
		2	0.562734	0.208618	1.240676
		2.25	0.500081	0.185381	1.102537

Table (2) shows that from many value of $\theta_i, i = 1,2,3,$ these three points supported designs are equal to the supported design given in table (1). Adding one supported point (t_4) selected randomly in (0 , 5) is a very important to decide the best nonminimally supported design based on the value of determinant information matrix. In the cases t_1, t_2, t_3, t_4 have the same weight i.e 0.25, so the information matrix as in equation (8). We select 133 point randomly in (0, 5) for all value of $\theta_i, i = 1,2,3$. Based on the simulation, for all value of $\theta_i, i = 1,2,3$, the determinan of information matrix have same patern for all value of t_4. As an illustration the scatterplot of determinan of information matrix and t_4 for $\theta_3 = 1.65$; $\theta_2 = 0.75$, $\theta_1 = 0.5$ is presented in Figure 3.

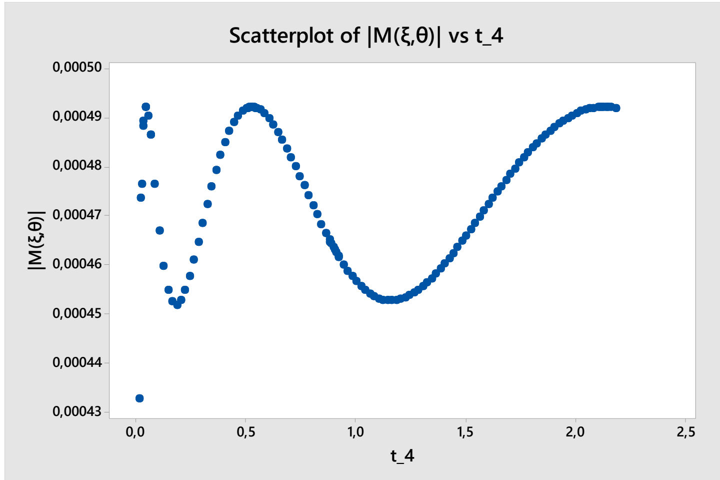

Figure 3. Scatterplot $|\boldsymbol{M}(\xi,\theta)|$ vs t_4 for $\theta_3 = 1.65$; $\theta_2 = 0.75$, $\theta_1 = 0.5$

Based on the Figure (3) shows that the highest value of determinan of information matrix happens at three time of t_4 that are 0.0412771728; 0.523138991; 2.124812716. The value of determinan of information matrix is 0.000492 . We can show that t_4 in this cases is one of t_1, t_2, t_3, in table (2). In other word Design I and Desin II have the same result.

4. CONCLUSION

The results of this research indicate that the contracting nonminimally supported design with uniform proportion and based on the values determinant of information matrix can be carry out in two ways. First, by constructing the formula of determinan of information matrix than maximize it. Second, by adding one supported design to the D-optimal design, it selected randomly in design area, each of supported have the same proportion, calculat their determinan of information matrix and the best nonminimally supported design is the design with maximum value of determinant information matrix. Design I and Design II have the same result.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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