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NUMERICAL APPROXIMATION OF 1D AND 2D REACTION DIFFUSION SYSTEM WITH MODIFIED CUBIC UAH TENSION B-SPLINE DQM

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Abstract: In this paper, a new numerical approach “Modified cubic UAH tension B-spline DQM” is projected to find the numerical approximation of 1D and 2D Reaction-Diffusion system. The modified cubic UAH tension B-spline is used in space to discretize the partial derivatives. The obtained system of ODE is dealt with SSP-RK43 scheme. To check the adaptability and efficiency of the proposed scheme, five numerical examples are discussed. The present method is easy to implement and economical as compared to the existing approaches available in literature for different types of linear and non-linear PDEs.

Keywords: 1D and 2D reaction-diffusion system; differential quadrature method; uniform algebraic hyperbolic (UAH) tension B-spline; SSP-RK43 scheme.

2010 AMS Subject Classification: 35Q68, 65M06.

1. INTRODUCTION

1D Reaction Diffusion System:

1D non-linear Reaction Diffusion system of equations is as follows:

$$u_t = a_1 u_{xx} + g_1(u, v) \quad (1)$$

$$v_t = a_2 v_{xx} + g_2(u, v) \quad (2)$$

with Dirichlet or Neumann Boundary conditions in the computational domain $[a, b]$. Where $u(x, t)$

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and $v(x, t)$ are the real valued functions, g_1 and g_2 are the arbitrary constants.

Model of reaction Diffusion systems are the mathematical models related to several physical processes. Reaction Diffusion systems have a wide number of applications in different areas alike, geology, biology, physics, ecology and many others. Reaction Diffusion systems can represent several models like Semi linear partial differential equations including Brusselator model [1], Gray Scott model [2], Isothermal model [3], Schnakenberg model [4] and many more. Many researchers have solved Reaction-Diffusion systems numerically. Sahin [5] implemented FE method for getting numerical approximation. By different methods [6-8] represents the numerical approximation of Reaction Diffusion systems.

2D Reaction Diffusion System:

2D Reaction Diffusion Brusselator system of the non-linear system of partial differential equations is as follows:

$$u_t = B_1 + u^2v + [A_1 + 1]u + \alpha[u_{xx} + u_{yy}] \quad (3)$$

$$v_t = A_1 u - u^2v + \alpha[v_{xx} + v_{yy}] \quad (4)$$

Where $x \in [0, 1]$ and $y \in [0, L]$. Where $u(x, y, t)$ and $v(x, y, t)$ are the provided in 2D region R^2 closed by the curve C along with

Initial conditions:

$$u(x, y, 0) = f_1(x, y) \text{ and } v(x, y, 0) = f_2(x, y) \quad (5)$$

and Neumann Boundary conditions on boundary ∂C are defined by the lines $x = 0, x = L, y = 0$ and $y = L$.

$$u_x(0, y, t) = u_x(L, y, t) = 0, \quad t \geq 0 \quad (6)$$

$$u_y(x, 0, t) = u_y(x, L, t) = 0, \quad t \geq 0 \quad (7)$$

$$v_x(0, y, t) = v_x(L, y, t) = 0, \quad t \geq 0 \quad (8)$$

$$v_y(x, 0, t) = v_y(x, L, t) = 0, \quad t \geq 0 \quad (9)$$

Where A_1, B_1, α are the given constants. $f_1(x, y), f_2(x, y)$ are the prescribed functions. The non-linear system represented by equations (3) and equation (4) validates an important model to study the processes in the chemical kinetics, like evolution of Brusselator system in the formation of ozone by oxygen by means of the triple collision. Also it is related to the processes of some chemical Reaction Diffusion processes like, enzyme reaction, in laser physics, in plasma and others. Because of such importance, these equations are very important from the numerical point of view. Such problems have been solved by eminent researchers. Dehghan et al. [9-14] gave the numerical

schemes for 1D heat equation and 1D Advection Diffusion equation, 3D Advection-Diffusion equation, 2D Transport equation and Coupled Burgers' equation. Different researchers have proposed the numerical approximation as well as the stability analysis for the Brusselator system given in equation (3) and equation (4). Adomian [15] and Wazwaz [16] gave the decomposition method. Twizell et al. [17] proposed 2D FD scheme for the solution of Brusselator RD system. Ang [18] gave the dual-Reciprocity Boundary element approximation for the solution of Brusselator system numerically.

B-spline is an important tool to develop some effective numerical regimes to solve the complex linear and non-linear partial differential equations. Different researchers have developed the numerical methods by using a series of B-splines. Bashan et al. [19] used quintic B-spline to solve KdVB equation. Bashan et al. [20] implemented quintic B-spline to get the solution of modified Burgers' equation. Mittal and Dahiya [21] employed the notion of modified cubic B-spline to solve Hyperbolic-Diffusion equation. Bashan et al. [22] used quintic B-spline for attaining the numerical solution of complex modified KdV equation. Singh et al. [23] implemented the notion of modified cubic B-spline for the approximation of 3D non-linear wave equation. Arora and Joshi [24] used the B-spline and Trigonometric B-spline to solve 1D Hyperbolic Telegraph equation. Mittal and Rohila [25] implemented the concept of the modified cubic B-spline to solve Reaction-Diffusion systems. Tamsir et al. [26] implemented exponential modified cubic B-spline to solve the non-linear Burgers' equation. Arora and Joshi [27] implemented the notion of modified trigonometric cubic B-spline for the solution of 1D and 2D Burgers' equation.

Differential Quadrature Method:

DQM has attained the noticeable attention over some previous decades. The initial knowledge was based upon the work of Bellman and Casti [28]. This regime owes its higher popularity due to its simplification and higher accuracy and efficiency. It is included in several applications of engineering and sciences. A comprehensive review of DQM was proposed by Bert and Malik [29]. DQM is actually a numerical discretization technique, in which several test functions can be used to get the weighting coefficients for the approximation of derivatives [30-32]. A lot of work has been reported in literature related to DQM. Korkmaz and Dag [33] implemented the notion of DQM to solve non-linear Schrodinger equation. Korkmaz [34] gave the numerical solution of KdV equation by using DQM. Shukla et al. [35] implemented exponential modified cubic B-spline DQM for solving 3D non-linear wave equation. Bashan and Esen [36] used DQM to solve the fourth order extended Fisher-Kolmogorov equation. Korkmaz and Dag [37] implemented Crank-

Nicolson-DQM for Kawahara equation. Korkmaz and Dag [38] gave the solitary wave solutions of the complex modified KdV equation using DQM.

After exploring the literature in detail, it is noticed that UAH tension B-spline has never been used to get the numerical approximation of 1D and 2D system of Reaction-Diffusion equations. As per author's knowledge, this scheme will open some new dimensions in the research of the numerical approximation of complex non-linear partial differential equations. This paper is organized into different sections. In Section 2, complete detail of the numerical scheme is provided. In Section 3, five test problems are provided. In Section 4, the crux of this research is given as conclusion.

2. NUMERICAL METHODOLOGY

[Modified Cubic UAH tension B-spline DQM]

Uniform algebraic hyperbolic tension B-spline of order 4 is defined as follows:

$UAHB_{i,4}(x) =$

$$\left\{ \begin{array}{ll} \frac{\delta_{i,3}\delta_{i,2}}{\tau\sinh(\tau h)} \left[(x_{i-2} - x) + \frac{\sinh[\tau(x - x_{i-2})]}{\tau} \right], & [x_{i-2}, x_{i-1}) \\ \delta_{i,3} \left[\frac{\delta_{i,2}}{\tau\sinh(\tau h)} \left\{ (x_{i-2} - x_{i-1}) + \frac{\sinh[\tau(x_{i-1} - x_{i-2})]}{\tau} \right\} + (x - x_{i-1}) \right. \\ \left. - \frac{\delta_{i,2}}{\tau\sinh(\tau h)} \left\{ (x_{i-1} - x) + \frac{1}{\tau} (\sinh(\tau(x - x_i)) + \sinh(\tau(x_i - x_{i-1}))) \right\} \right. \\ \left. - \frac{\delta_{i+1,2}}{\tau\sinh(\tau h)} \left\{ (x_{i-1} - x) + \frac{\sinh(\tau(x - x_{i-1}))}{\tau} \right\} \right] \\ \frac{\delta_{i+1,3}\delta_{i+1,2}}{\tau\sinh(\tau h)} \left\{ (x_{i-1} - x) + \frac{\sinh(\tau(x - x_{i-1}))}{\tau} \right\}, & [x_{i-1}, x_i) \\ 1 - \frac{\delta_{i,3}\delta_{i+1,2}}{\tau\sinh(\tau h)} \left\{ (x - x_{i+1}) - \frac{\sinh(\tau(x - x_{i+1}))}{\tau} \right\} \\ - \delta_{i+1,3} \left[\frac{\delta_{i+1,2}}{\tau\sinh(\tau h)} \left\{ (x_{i-1} - x_i) + \frac{\sinh(\tau(x_i - x_{i-1}))}{\tau} \right\} \right. \\ \left. + (x - x_i) - \frac{\delta_{i+1,2}}{\tau\sinh(\tau h)} \left\{ (x_i - x) + \frac{(\sinh(\tau(x - x_{i+1})) + \sinh(\tau(x_i - x_{i+1})))}{\tau} \right\} \right. \\ \left. - \frac{\delta_{i+2,2}}{\tau\sinh(\tau h)} \left\{ (x_i - x) + \frac{\sinh(\tau(x - x_i))}{\tau} \right\} \right], & [x_i, x_{i+1}) \\ \frac{\delta_{i+1,3}\delta_{i+2,2}}{\tau\sinh(\tau h)} \left[(x - x_{i+2}) - \frac{\sinh(\tau(x - x_{i+2}))}{\tau} \right], & [x_{i+1}, x_{i+2}) \\ 0, & \text{otherwise} \end{array} \right. \quad (10)$$

$$\vec{q}^{(1)}[i] = \begin{pmatrix} q_{i,1}^{(1)} \\ q_{i,2}^{(1)} \\ q_{i,3}^{(1)} \\ \vdots \\ \vdots \\ q_{i,N-1}^{(1)} \\ q_{i,N}^{(1)} \end{pmatrix} \text{ and } \vec{V}[1] = \begin{pmatrix} 2b_5 \\ b_4 - b_5 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix} \quad \vec{V}[2] = \begin{pmatrix} b_5 \\ 0 \\ b_4 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}, \dots, \vec{V}[n] = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ b_5 - b_4 \\ 2b_4 \end{pmatrix}$$

Second and higher order partial derivatives can be obtained by using the recurrence relation [40] given as follows,

$$a_{ij}^{(r)} = r \left[a_{ij}^{(1)} a_{ii}^{(r-1)} - \frac{a_{ij}^{(r-1)}}{x_i - x_j} \right] \text{ for } i \neq j \quad (13)$$

Where $i = 1, 2, 3, \dots, N$ and $r = 2, 3, 4, \dots, N - 1$

$$a_{ii}^{(r)} = - \sum_{j=1, j \neq i}^N a_{ij}^{(r)} \quad \text{for } i = j \quad (14)$$

Similarly the weighting coefficients $b_{ij}^{(r)}$ for second or higher order derivatives can be obtained by following formula [40],

$$b_{ij}^{(r)} = r \left[b_{ij}^{(1)} b_{ii}^{(r-1)} - \frac{b_{ij}^{(r-1)}}{y_i - y_j} \right] \text{ for } i \neq j \quad (15)$$

Where $i = 1, 2, 3, \dots, N$ and $r = 2, 3, 4, \dots, N - 1$

$$b_{ii}^{(r)} = - \sum_{j=1, j \neq i}^N b_{ij}^{(r)} \quad \text{for } i = j \quad (16)$$

After spatial discretization of the partial derivatives, the system of partial differential equations got transformed in to the system of ordinary differential equation, which is tackled by SSP-RK43 scheme [41].

3. NUMERICAL EXPERIMENTS AND DISCUSSION

In present section five numerical examples are elaborated to check the adaptability and efficiency of the proposed scheme. Among these five examples, first two examples are concerned to 1D Reaction-Diffusion system of equations and rest three examples are associated the notion of 2D Reaction-Diffusion system of equations.

In Figure 1, Numerical approximation of $U(x, t)$ is provided at the mentioned time levels for $N =$

21, $\Delta t = 0.00001$, $a_1 = 10^{-4}$, $a_2 = 10^{-4}$, $\eta_1 = 1$, $b_1 = 3.4$ and $\tau = 0.01$. In Figure 2, Numerical approximation of $V(x, t)$ is provided at the mentioned time levels for $N = 21$, $\Delta t = 0.00001$, $a_1 = 10^{-4}$, $a_2 = 10^{-4}$, $\eta_1 = 1$, $b_1 = 3.4$ and $\tau = 0.01$. In Table 2, Numerical $U(x, t)$ and $V(x, t)$ are given at $t = 0.1, 0.5$ and 0.8 for $N = 11$, $\Delta t = 0.001$, $\tau = 0.01$, $a_1 = 10^{-4}$, $a_2 = 10^{-4}$, $\eta_1 = 1$, $b_1 = 3.4$. In Figures 3 and 4 Numerical $U(x, t)$ and $V(x, t)$ are presented graphically for $N = 101$, $\Delta t = 0.0001$, $\tau = 0.01$ and $k = 9$. In Table 3, Numerical $U(x, t)$ and $V(x, t)$ are evaluated at $t = 10, 50$ and 70 for $N = 21$, $\Delta t = 0.0001$, $\tau = 0.01$ and $k = 0.9$. In Figures 5 and 6, Numerical approximations of $U(x, y, t)$ and $V(x, y, t)$ are provided for $N = 21$, $\Delta t = 0.0001$, $\tau = 1$, $A_1 = 1$, $B_1 = 3.4$ and $\alpha = 0.002$ at the mentioned time levels. In Table 4, Numerical approximation of $U(x, y, t)$ and $V(x, y, t)$ are given at time levels $t = 1.0$ and 3.0 for $N = 11$, $\Delta t = 0.0001$, $\tau = 1$, $A_1 = 1$, $B_1 = 3.4$ and $\alpha = 0.002$. In Figures 7 and 8 Numerical U and V are presented graphically at time levels $1.0, 2.0, 3.0$ and 4.0 respectively. In Table 5, Numerical U and V are given at $t = 0.1$ and 1.0 respectively for $A_1 = 0.5$, $B_1 = 1$ and $\alpha = 0.002$. In Figures 9 and 10, Numerical approximations of U and V are given graphically at $t = 1, 2, 3$ and 4 respectively for $A_1 = 1$, $B_1 = 2$ and $\alpha = 0.002$. In Table 6, Numerical U and V are evaluated at $t = 5$ and $t = 10$ respectively.

Example 1:

Brusselator model [42] was proposed by Brussels school of Prigogine. Present model represents the Hypo-theoretical tri-molecular natured reaction, having very important traits in the chemical science area.

Problem is defined as [43], [25],

$$u_t = a_1 u_{xx} - (b_1 + 1)u + u^2 v + \eta_1 \quad (17)$$

$$v_t = a_2 v_{xx} + b_1 u - u^2 v \quad (18)$$

Computational domain : $[0, 1]$

Initial conditions:

$$u(x, 0) = 0.5 \text{ and } v(x, 0) = 1 + 5x \quad (19)$$

Boundary conditions:

Natural boundary conditions are considered

$$u(0, t) = 0, \quad u(1, t) = 0 \quad (20)$$

$$v(0, t) = 0, \quad v(1, t) = 0 \quad (21)$$

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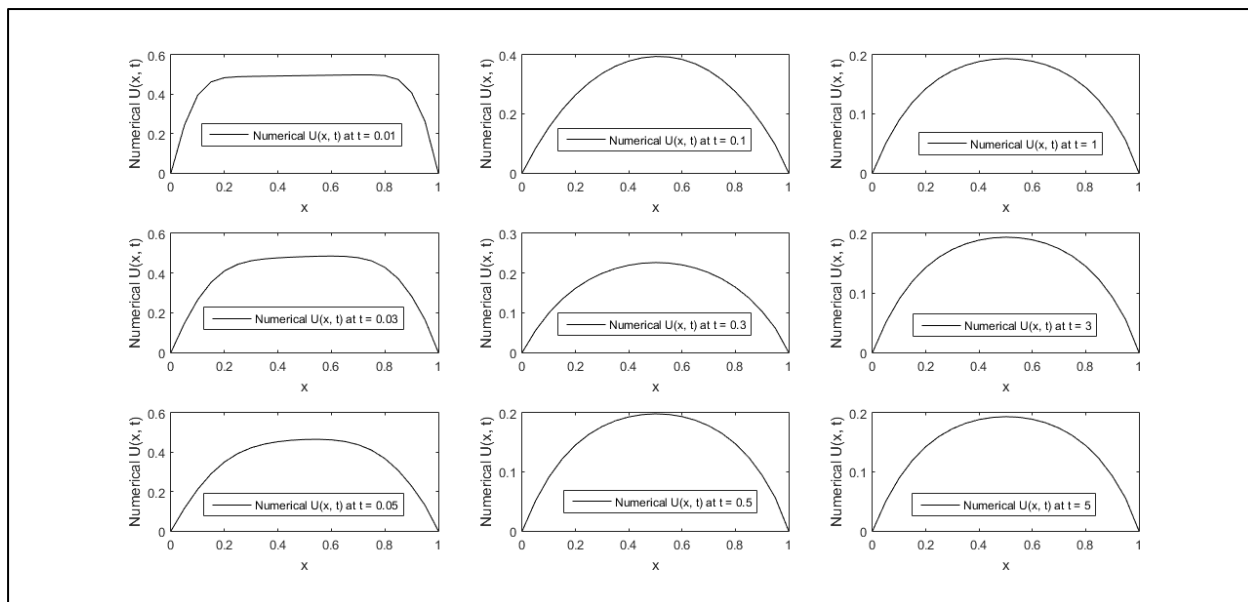


Figure 1: Numerical profiles of $U(x, t)$ at different time levels for $N = 21$, $\Delta t = 0.00001$, $a_1 = 10^{-4}$, $a_2 = 10^{-4}$, $\eta_1 = 1$, $b_1 = 3.4$ and $\tau = 0.01$

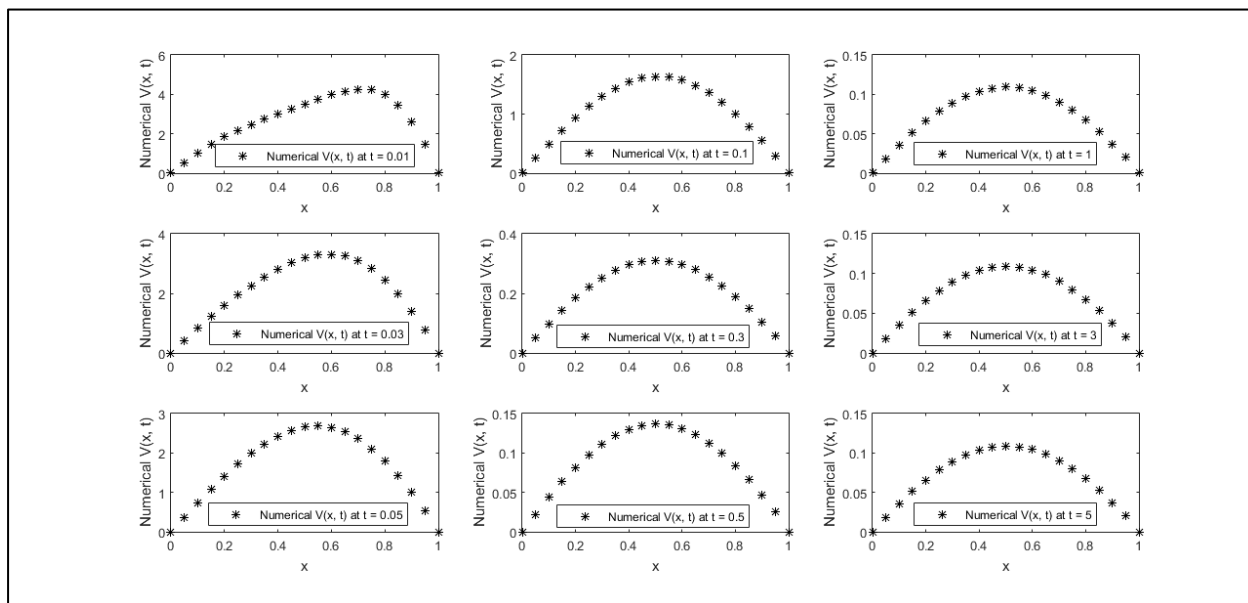


Figure 2: Numerical profiles of $V(x, t)$ at different time levels for $N = 21$, $\Delta t = 0.00001$, $a_1 = 10^{-4}$, $a_2 = 10^{-4}$, $\eta_1 = 1$, $b_1 = 3.4$ and $\tau = 0.01$

Table 2: Numerical solutions for $U(x, t)$ and $V(x, t)$ at $t = 0.1, 0.5$ and 0.8 for $N = 11, \Delta t = 0.001, \tau = 0.01, \alpha_1 = 10^{-4}, \alpha_2 = 10^{-4}, \eta_1 = 1$ and $b_1 = 3.4$

x	$t = 0.1$		$t = 0.5$		$t = 0.8$	
	Numerical U	Numerical V	Numerical U	Numerical V	Numerical U	Numerical V
0.1	0.153	0.4967	0.0898	0.0442	0.0884	0.0358
0.2	0.2629	0.9453	0.1448	0.083	0.1422	0.0669
0.3	0.3361	1.3074	0.1766	0.113	0.1729	0.0909
0.4	0.3784	1.5496	0.193	0.1321	0.1887	0.106
0.5	0.3943	1.6484	0.1984	0.139	0.1938	0.1115

Example 2:

Isothermal chemical system is given as [44], [25]:

$$u_t = u_{xx} - uv \tag{22}$$

$$v_t = v_{xx} - kv + uv \tag{23}$$

Computational Domain: $[0, 200]$

Initial conditions:

$$u(x, 0) = 1 \text{ and } v(x, 0) = \exp(-x^2) \tag{24}$$

Boundary conditions:

$$\frac{\partial u}{\partial x}(0, t) = 0, \quad u(200, t) = 1 \tag{25}$$

$$\frac{\partial v}{\partial x}(0, t) = 0, \quad v(200, t) = 0 \tag{26}$$

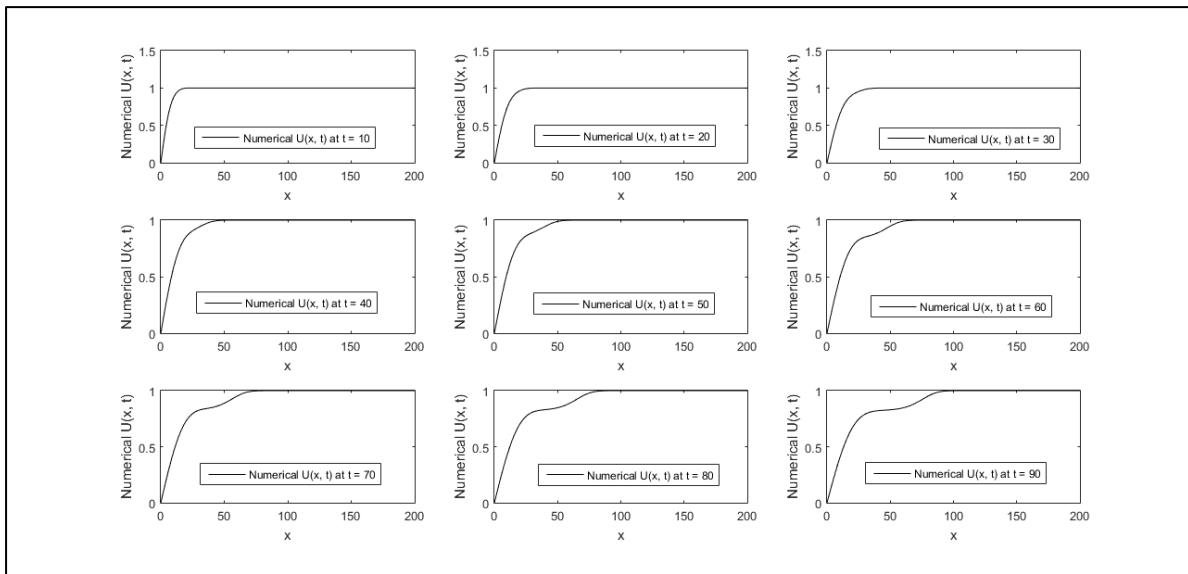


Figure 3: Numerical profile of $U(x, t)$ at the mentioned time levels for $N = 101, \Delta t = 0.0001, \tau = 0.01$ and $k = 0.9$

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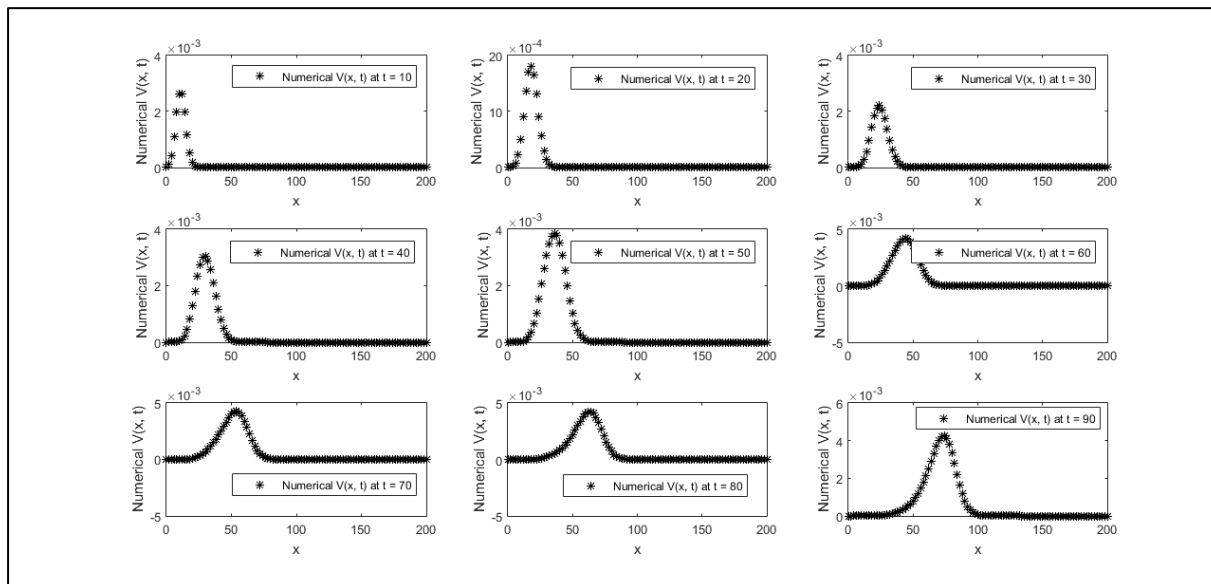


Figure 4: Numerical profile of $V(x, t)$ at the mentioned time levels for $N = 101, \Delta t = 0.0001, \tau = 0.01$ and $k = 0.9$

Table 3: Numerical Approximations of $U(x, t)$ and $V(x, t)$ for $N = 21, \Delta t = 0.0001, \tau = 0.01, k = 0.9$ at the time levels $t = 10, 50$ and 70 respectively

x	$t = 10$		$t = 50$		$t = 70$	
	Numerical U	Numerical V	Numerical U	Numerical V	Numerical U	Numerical V
10	0.7892	0.0027	0.518	0	0.4508	0
20	0.993	0.0006	0.8053	0.0004	0.7334	0.0001
30	1.0009	-0.0001	0.904	0.0026	0.839	0.001
40	0.9999	0	0.9669	0.0021	0.8781	0.0028
50	1	0	0.9997	0	0.9481	0.0028

Example 3:

In present example, Brusselator system (3) and (4) is considered along with the Neumann boundary conditions (6)-(9) and the following initial conditions [45, 46].

Initial Conditions:

$$u(x, y, 0) = 0.5 + y \text{ and } v(x, y, 0) = 1 + 5x \tag{27}$$

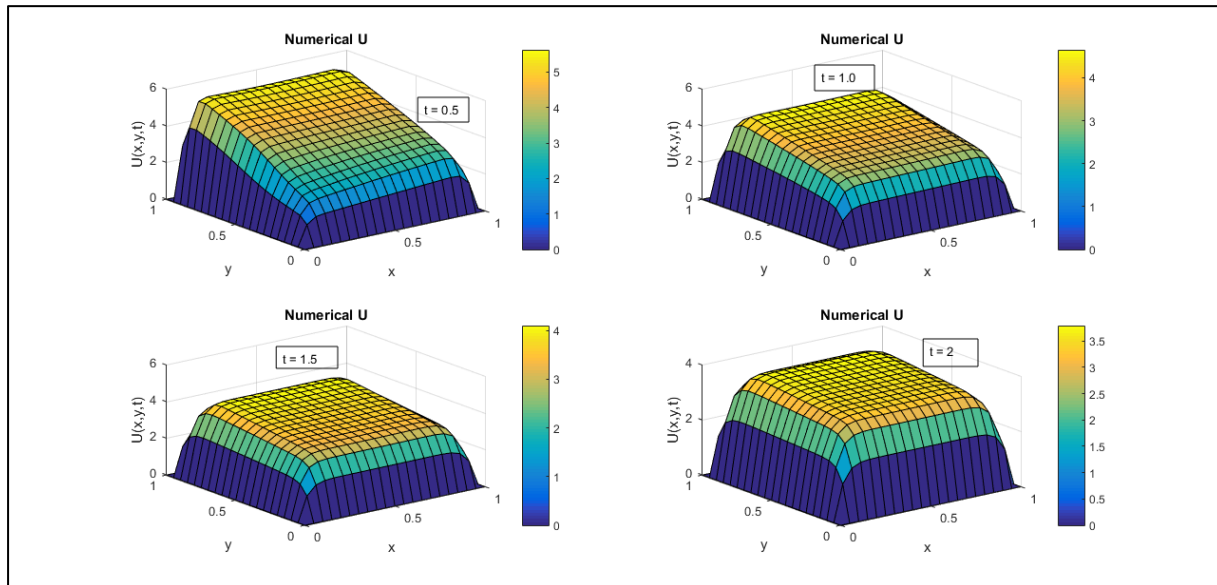


Figure 5: Numerical approximations of $U(x, y, t)$ at $t = 0.5, 1, 1.5$ and 2 respectively for $N = 21, \Delta t = 0.0001, \tau = 1, A_1 = 1, B_1 = 3.4$ and $\alpha = 0.002$

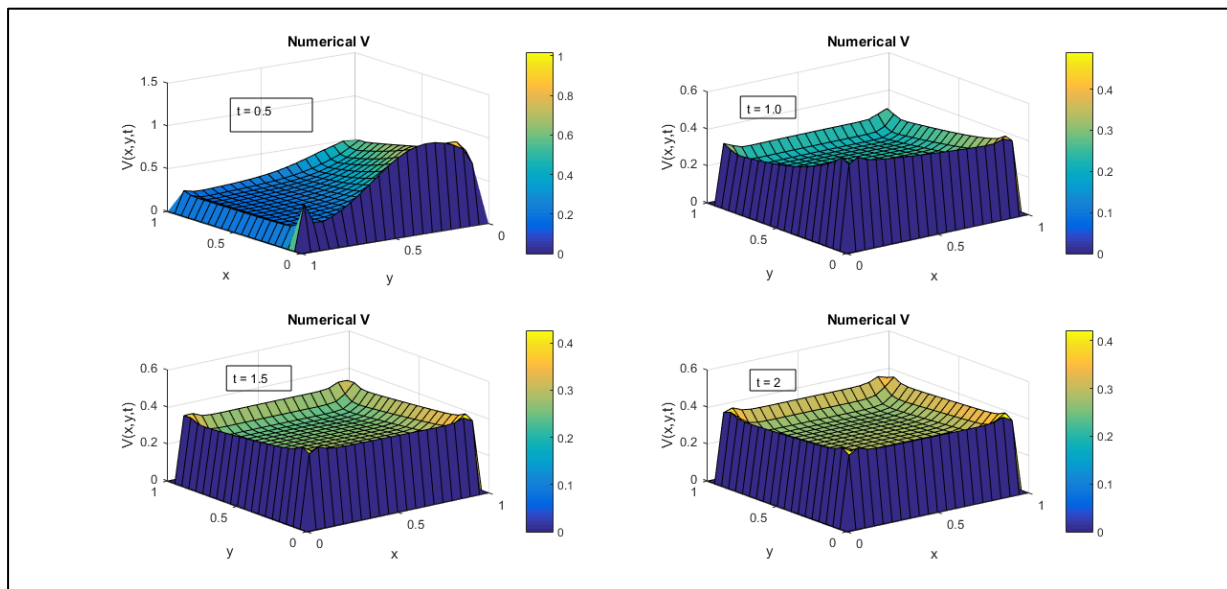


Figure 6: Numerical approximations of $V(x, y, t)$ at $t = 0.5, 1, 1.5$ and 2 respectively for $N = 21, \Delta t = 0.0001, \tau = 1, A_1 = 1, B_1 = 3.4$ and $\alpha = 0.002$

Table 4: Numerical $U(x, y, t)$ and Numerical $V(x, y, t)$ for $N = 11$, $\Delta t = 0.0001$, $\tau = 1$, $A_1 = 1$, $B_1 = 3.4$ and $\alpha = 0.0002$

(x, y)	Numerical U		Numerical V	
	$t = 1.0$	$t = 3.0$	$t = 1.0$	$t = 3.0$
$(0.1, 0.1)$	2.371421	2.357533	0.435414	0.388941
$(0.2, 0.3)$	3.364027	3.356909	0.302192	0.299404
$(0.3, 0.5)$	3.597468	3.419233	0.27674	0.292093
$(0.5, 0.5)$	4.000117	3.46825	0.247439	0.287738
$(0.7, 0.8)$	4.416162	3.460378	0.224095	0.289474

• **Example 4:**

In this example non-linear PDE related to the Brusselator system (3) and (4) is considered with the Neumann boundary conditions (6)-(9) along with the following initial conditions [46, 47].

Initial Conditions:

$$u(x, y, 0) = 0.5x^2 - \frac{1}{3}x^3 \text{ and } v(x, y, 0) = 0.5y^2 - \frac{1}{3}y^3 \quad (28)$$

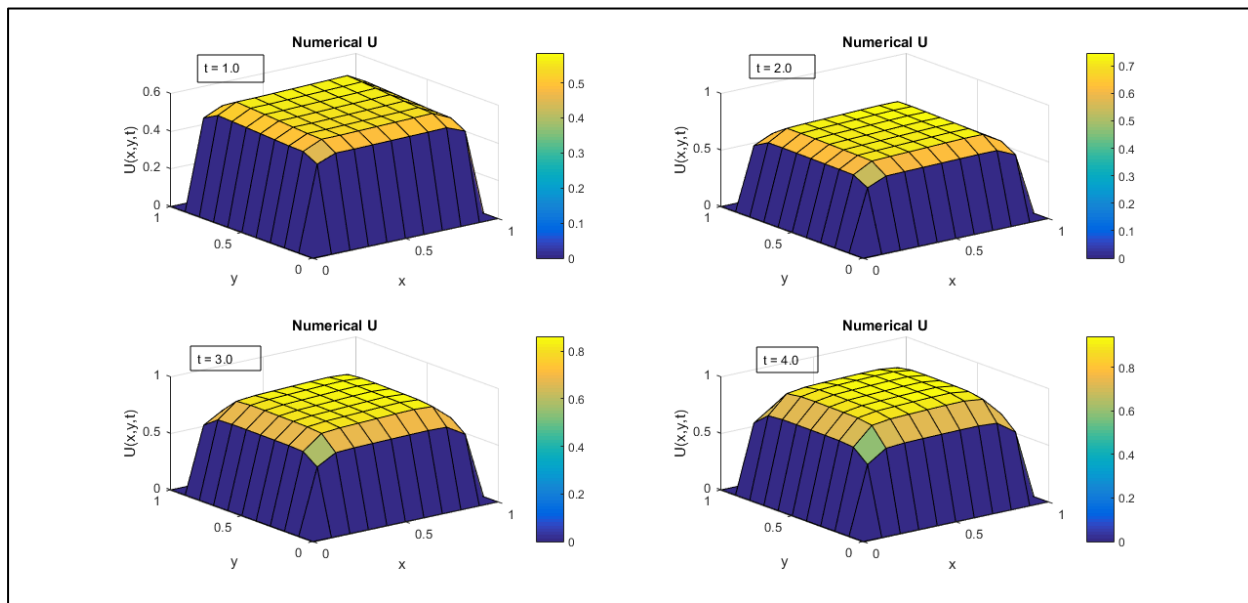


Figure 7: Numerical representation of $U(x, y, t)$ at time levels $t = 1, 2, 3$ and 4 respectively for $N = 11$, $\Delta t = 0.0001$, $\tau = 1$, $A_1 = 0.5$, $B_1 = 1$, $\alpha = 0.002$

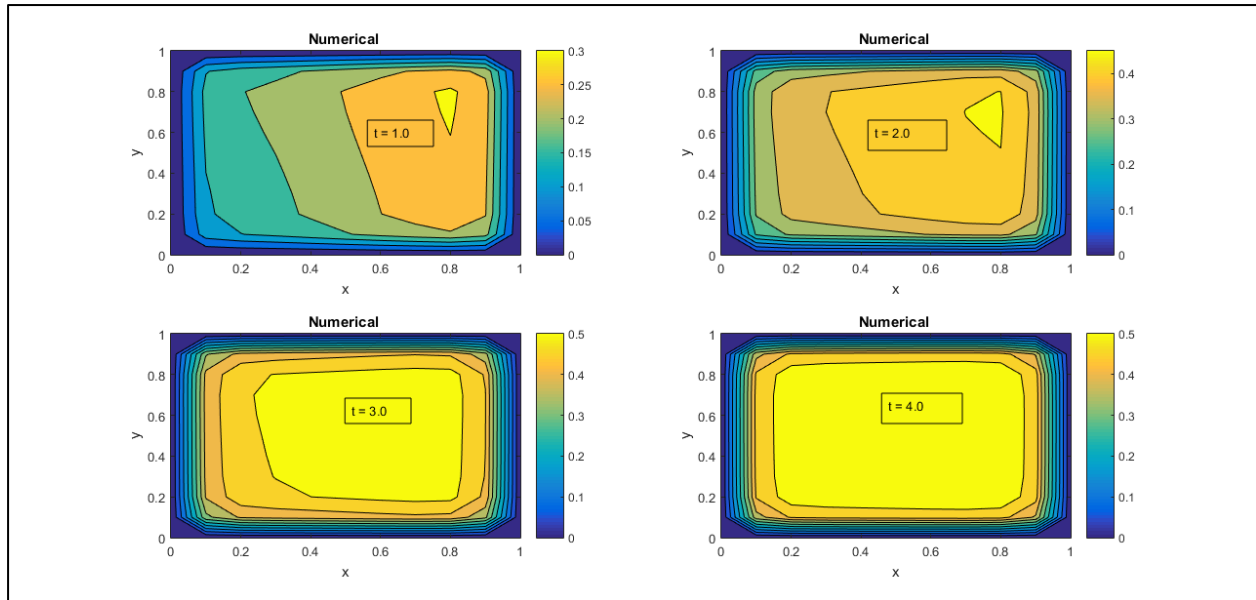


Figure 8: Numerical representation of $V(x, y, t)$ at time levels $t = 1, 2, 3$ and 4 respectively for $N = 11, \Delta t = 0.0001, \tau = 1, A_1 = 0.5, B_1 = 1, \alpha = 0.002$

Table 5: Numerical $U(x, y, t)$ and Numerical $V(x, y, t)$ at $t = 0.1$ and $t = 1.0$ respectively for $N = 11, \tau = 1, A_1 = 0.5, B_1 = 1$ and $\alpha = 0.002$

(x, y)	Numerical U		Numerical V	
	$t = 0.1$		$t = 1.0$	
$(0.1, 0.1)$	0.095021	0.007259	0.451174	0.12438
$(0.2, 0.3)$	0.108066	0.039338	0.535286	0.187069
$(0.3, 0.5)$	0.124037	0.087301	0.54291	0.231837
$(0.5, 0.7)$	0.164821	0.136566	0.559991	0.281197
$(0.7, 0.8)$	0.206027	0.157547	0.575987	0.304336

Example 5:

Considered Brusselator system (3) and (4) along with the Neumann boundary conditions (6)-(9) with the following initial conditions [46, 48].

Initial Conditions:

$$u(x, y, 0) = 2 + 0.25y \text{ and } v(x, y, 0) = 1 + 0.8y \quad (29)$$

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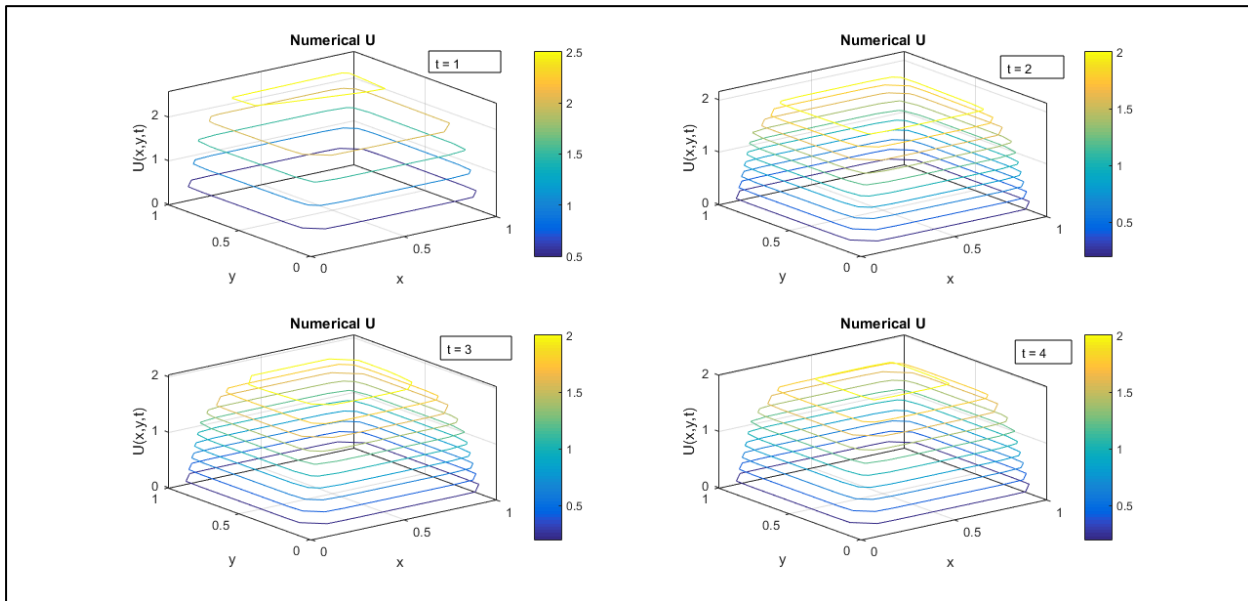


Figure 9: Numerical profiles of $U(x, y, t)$ at time levels $t = 1, 2, 3$ and respectively for $N = 11$, $\Delta t = 0.0001$, $\tau = 1$, $A_1 = 1$, $B_1 = 2$, $\alpha = 0.002$

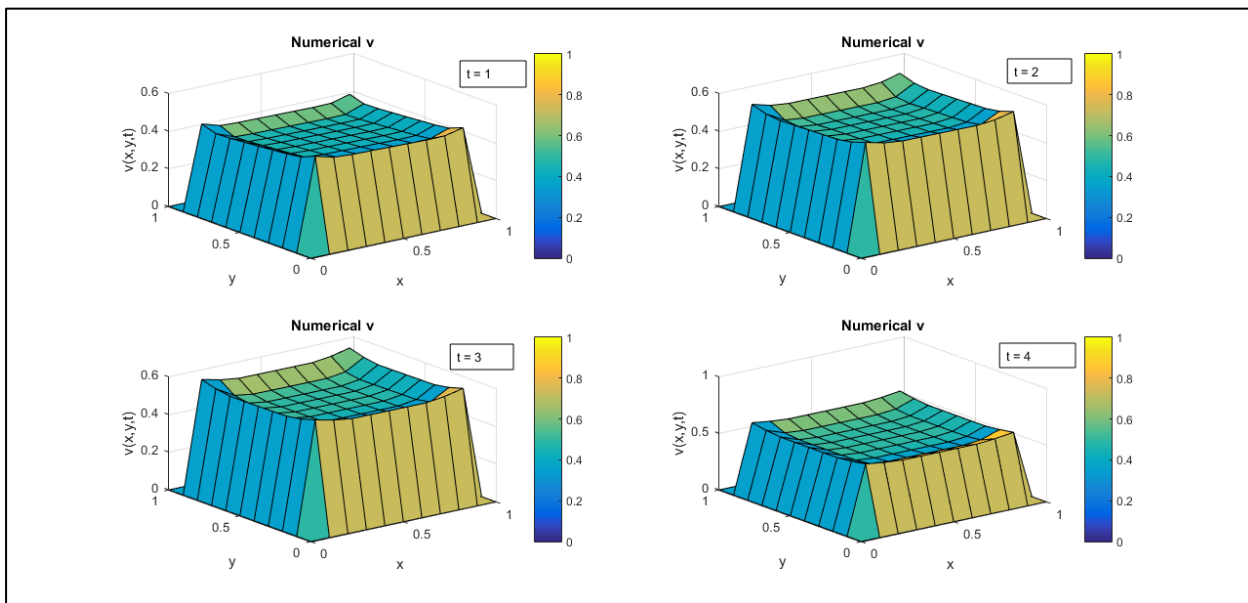


Figure 10: Numerical profiles of $U(x, y, t)$ at time levels $t = 1, 2, 3$ and respectively for $N = 11$, $\Delta t = 0.0001$, $\tau = 1$, $A_1 = 1$, $B_1 = 2$, $\alpha = 0.002$

Table 6: Numerical approximation of $U(x, y, t)$ and $V(x, y, t)$ for $N = 11$, $\Delta t = 0.0001$, $\tau = 1$, $A_1 = 1$, $B_1 = 2$, $\alpha = 0.002$

(x, y)	$t = 5$		$t = 10$	
	Numerical U	Numerical V	Numerical U	Numerical V
$(0.1, 0.1)$	1.279108	0.607737	1.279679	0.607713
$(0.1, 0.3)$	1.589723	0.572393	1.590001	0.572418
$(0.3, 0.5)$	1.999091	0.499876	1.99825	0.500253
$(0.5, 0.7)$	1.999316	0.499778	1.99825	0.500253
$(0.5, 0.8)$	1.970627	0.514103	1.970119	0.514418

4. CONCLUSION

In present paper, modified cubic UAH tension B-spline based DQM is developed to solve the linear and non-linear partial differential equations. Solving such complex non-linear partial differential equations analytically is not always possible. That is why, it is a major need of time to develop some efficient and accurate numerical regimes. The obtained ODE system is dealt by SSP-RK43 scheme. Five numerical examples are discussed in this paper. Numerical approximation of 1D and 2D Reaction-Diffusion system is obtained. This scheme will help researchers in their future work to solve some other complex partial differential equations numerically, mainly where analytical solution is not available.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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