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STUDY OF DOUBLE-DIFFUSIVE CONVECTION IN A ROTATING COUPLE-STRESS FERROMAGNETIC FLUID IN THE PRESENCE OF VARYING GRAVITATIONAL FIELD AND HORIZONTAL MAGNETIC FIELD SATURATING IN A POROUS MEDIUM

PULKIT KUMAR NADIAN*, RIMPLE PUNDIR, SUDHIR KUMAR PUNDIR

Department of Mathematics, S.D. (P.G.) College, Muzaffarnagar, U.P.-251001, India

Abstract. The results of double-diffusive convection on the liquid coatings of ferromagnetic liquids is studied by researchers by various theoretical and hydrodynamics concepts of hydrodynamics. However this study is done in presence of varying gravitational field. The study of such couple-stress ferromagnetic fluids in presence of fluctuating gravitational field is still not explored. As we know that gravitational field varies on different points on the earth; for this reason the study of couple-stress ferromagnetic liquids in presence of variable gravity fields is highly required. In this paper we studied the impact of medium permeability, stable solute gradient, couple-stress, rotation, magnetic field, and magnetization of a couple-stress ferromagnetic fluid in presence of variable gravitational field. For this study a coating of couple-stress ferromagnetic fluid saturating in a porous medium when fluid layer is heated and soluted. We examined the results of the magnetic field on couple-stress ferromagnetic fluids, its thermosolutal instability during rotation. A linearized concept and normal mode technique are used to acquire the dispersion relation. For the case of stationary convection, medium permeability, couple-stress, and magnetic field have both stabilizing and destabilizing results under certain conditions. A stable solute gradient has a stabilizing result on the system. Also, the rotation has a stabilizing result for $\lambda > 0$ and destabilizing result for $\lambda < 0$ the system. It is likewise observed that in the absence of a stable solute gradient, magnetization has a stabilizing

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*Corresponding author
E-mail address: pknadian2204@gmail.com
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result on the system for both $\lambda > 0$ and $\lambda < 0$. The critical Rayleigh number for the onset of instability is decided numerically and graphically also. The principle of exchange of stabilities is observed to keep genuine in the absence of rotation, magnetic field, and stable solute gradient under certain conditions.

Keywords: double-diffusive convection; couple-stress fluid; ferromagnetic fluid; magnetization; magnetic field intensity.

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1. INTRODUCTION

Chandrasekhar [1] noted the results of the onset of thermal instability on the liquid coating through various theoretical and hydrodynamics concepts of hydrodynamics. The concept of fluid-stress fluid is provided by Stokes [2]. The idea of a couple-stress comes from the way a mechanical connection is followed by a liquid. In this view, the rotating field is defined by the velocity vector itself, and the rotating vector is equal to half the velocity vector curl. Pressure compression is no longer equal here. An introduction to this concept is found in the monograph "Theories of Liquids with Microstructure-An Introduction" written by Stokes [3] himself. Goel et al. [4] discussed the hydromagnetic stability of a combination of binary liquids with non-compressed fluctuations in the exchange of fixed gradients of heat and solute concentration. Sunil et al. [5] investigated the stability of high-pressure couples' pressure liquids in a toxic environment on hydromagnetics. The result of fixed particles on the formation of a coating of liquid at warm pressure and dissolved from below in the framed area observed by Sunil et al. [6]. Thermosolutal convection in a compressible fluid-stress fluid where there are rotation and the magnetic force is mentioned with the help of using Singh et al. [7]. Kumar et al. [8] read about the effect of a horizontal magnetic field and a horizontal rotation in a dusty liquid that is heated and soluted from below by a porous medium: the Brinkman Model. Banyal et al. [9] have discussed the magneto-thermosolutal convection in Rivlin-Ericksen viscoelastic fluid in a porous medium. Also, onset of thermosolutal convection in couple-stress fluid in a porous media in the presence of magnetic field has been discussed by Banyal and Singh [10]. Some instability problems related to viscoelastic fluids has been studied by Aggarwal and Dixit [11, 12, 13].

Rosensweig [14] has produced a direct introduction to the magnetic field in his monograph. Finlayson [15] observed the thermal conductivity of ferromagnetic fluid within the presence of the magnetic field. Bénard's transfer to ferromagnetic liquids has been discussed by many authors (Siddheswar [16, 17], Venkatasubramaniam, and Kaloni [18]). Thermosolutal convection in ferromagnetic fluid filling porous medium has been suggested with the help of Sunil et al. [19]. And Sunil et al. [20] have studied the result of Magnetic-Field-Dependent viscosity on thermosolutal convection of the ferromagnetic fluid cycle that fills the porous medium. Magneto-rotational convection of ferromagnetic fluid within the pressure finding and heat source with a porous medium has been discussed with the help of the use of Sharma et al. [21]. Aggarwal and Makhija [22] have studied the effect of hall current on thermosolutal convection of ferromagnetic fluids in porous medium. Recently, Nadian et al. [23] discussed the effect of rotation on thermal convection of couple-stress ferromagnetic fluid within the presence of varying gravity field. Thermal convection in a layer of couple-stress ferromagnetic fluid in the presence of rotation, magnetic field and varying gravitational field has been discussed by Nadian et al. [24]. Also, the effect of rotation on double-diffusive convection of couple-stress ferromagnetic fluid layer in varying gravitational field has been discussed by Nadian et al. [25]. The present study is designed to examine the result of rotation and magnetic field on the thermosolutal instability of ferromagnetic fluid that emphasizes the porous medium in front of the flexible magnetic field. It can be considered an extension of a research paper entitled "Thermosolutal instability of couple-stress rotating fluid in the presence of magnetic field" studied by Kumar et al. [26].

2. FORMULATION OF THE PROBLEM

Consider an infinite, horizontal, electrically non-conducting incompressible thin coating of couple-stress ferromagnetic fluid of thickness d heated and soluted from below. This coating is anticipated to detain between two infinite horizontal planes located at $z = 0$ and $z = d$ (see Fig. 1), that is acted upon via way of means of a uniform rotation $\mathbf{\Omega}(0,0,\Omega)$ and variable gravity field

$\boldsymbol{g}(0,0,-g)$, where $g=\lambda g_0$. A uniform magnetic field $\boldsymbol{H}(0,0,H)$ works along with the $z-$direction. The coating of couple-stress ferromagnetic fluid is heated and soluted from below leading to an inauspicious temperature gradient β and $\beta^{'}$. This fluid coating is believed to be flowing through an isotropic and homogeneous porous medium of porosity ε^* and medium permeability k_1.

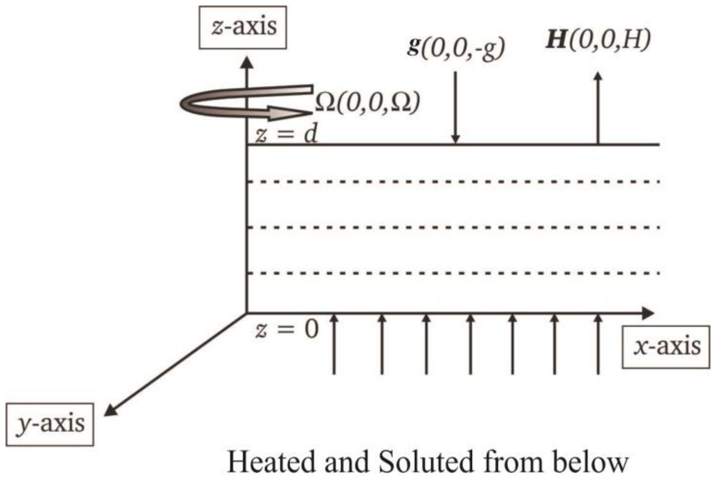

Fig.1. Geometrical Configuration

Let $p,\rho,T^*,C^*,\alpha,\alpha^{'},\nu,\mu^{'},\kappa_T,\kappa_S$ and $\boldsymbol{q}^*(u_1^*,u_2^*,u_3^*)$ denotes pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, kinematic viscosity, couple-stress viscosity, thermal diffusivity, solute diffusivity and velocity of the fluid respectively. Then the mathematical equations of motion and continuity of couple-stress ferromagnetic fluid saturating a porous medium for the above model are as follows:

(1)
$$\left\{\begin{aligned}&\frac{1}{\varepsilon^*}\left[\frac{\partial \boldsymbol{q}^*}{\partial t}+\frac{1}{\varepsilon^*}\left(\boldsymbol{q}^*.\nabla\right)\boldsymbol{q}^*\right]=-\frac{1}{\rho_0}\nabla p+\boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\nu\boldsymbol{q}^*+\frac{1}{\rho_0}M\nabla H+\left(\nu-\frac{\mu^{'}}{\rho_0}\nabla^2\right)\nabla^2\boldsymbol{q}^*\\&+\frac{2}{\varepsilon^*}\left(\boldsymbol{q}^*\times\boldsymbol{\Omega}\right)+\frac{\mu_e}{4\pi\rho_0}\left[\left(\nabla\times\boldsymbol{H}\right)\times\boldsymbol{H}\right],\end{aligned}\right.$$

(2)
$$\nabla.\boldsymbol{q}^*=0.$$

The Maxwell's equations [27] of electromagnetism are given by,

(3)
$$\varepsilon^*\frac{\partial \boldsymbol{H}}{\partial t}=\left(\boldsymbol{H}.\nabla\right)\boldsymbol{q}^*+\varepsilon^*\eta\nabla^2\boldsymbol{H},$$

(4)
$$\nabla.\boldsymbol{H}=0.$$

The equation of state is,

(5) $$\rho = \rho_0\left[1-\alpha\left(T^* - T_0^*\right)+\alpha'\left(C^* - C_0^*\right)\right],$$

Where ∇H is the magnetic field gradient. Also, $H = |\boldsymbol{H}|, M = |\boldsymbol{M}|$.

Let c_v, ρ_s, c_s denote the heat capacity of the fluid at constant volume, the density of the solid (porous) material, and heat capacity of the solid (porous) material respectively. Then, the equation of heat conduction gives,

(6) $$E\frac{\partial T^*}{\partial t}+\left(\boldsymbol{q}^*.\nabla\right)T^* = \kappa_T \nabla^2 T^*,$$

and

(7) $$E'\frac{\partial C^*}{\partial t}+\left(\boldsymbol{q}^*.\nabla\right)C^* = \kappa_S \nabla^2 C^*,$$

Where $E = \varepsilon^* + \left(1-\varepsilon^*\right)\frac{\rho_s c_s}{\rho_0 c_v}$ is a constant, E' is a constant analogous to E and the kinematic viscosity ν , couple-stress viscosity μ' , thermal diffusivity κ_T , solute diffusivity κ_S , coefficient of thermal expansion α , and an analogous coefficient of expansion α' are all assumed to be constant.

Now, we consider magnetization to be independent of magnetic field intensity so that $M = M\left(T^*, C^*\right)$. As a first approximation, we consider that

(8) $$M = M_0\left[1-K_1\left(T^* - T_0^*\right)+K_2\left(C^* - C_0^*\right)\right],$$

Where $K_1 = \frac{1}{M_0}\left(\frac{\partial M}{\partial T^*}\right)_H, K_2 = \frac{1}{M_0}\left(\frac{\partial M}{\partial C^*}\right)_H$ and M_0 is the magnetization at $T^* = T_0^*$ with T_0^* being the reference temperature.

3. BASIC STATE, PERTURBATION EQUATIONS AND NORMAL MODE ANALYSIS

The basic state of which we wish to examine the stability is characterized by,

(9) $$\begin{cases} \boldsymbol{q}^* = (0,0,0), p = p(z), \rho = \rho(z) = \rho_0\left(1-\alpha\beta z+\alpha'\beta' z\right), T^* = T_0^* - \beta z, C^* = C_0^* - \beta' z, \\ \boldsymbol{\Omega} = (0,0,\Omega), \boldsymbol{H} = (0,0,H),\ M = M_0\left(1-K_1\beta z+K_2\beta' z\right), M = M(z). \end{cases}$$

Here, β and $\beta^{'}$ may be either positive or negative.

Assuming small perturbations around the basic state and let $\boldsymbol{q}^*\left(u_1{}^*, u_2{}^*, u_3{}^*\right), \boldsymbol{h}\left(h_x, h_y, h_z\right), \theta^*, \gamma^*$, $\delta\rho, \delta p$ and δM denote respectively the perturbations in fluid velocity, magnetic field, temperature, concentration, density, pressure, and magnetization. Hence, the perturbed flow may be represented as,

$$\begin{cases} \boldsymbol{q}^* = (0,0,0) + \left(u_1{}^*, u_2{}^*, u_3{}^*\right), \boldsymbol{h} = (0,0,H) + \left(h_x, h_y, h_z\right), T^* = T^*(z) + \theta^*, \\ C^* = C^*(z) + \gamma^*, \rho = \rho(z) + \delta\rho,\ p = p(z) + \delta p, M = M(z) + \delta M. \end{cases} \tag{10}$$

Linearizing the equation in perturbation and analyzing the perturbation into normal modes, we assume that the perturbation quantity is of the form,

$$\left(u_3{}^*, \theta^*, \gamma^*, \zeta, \xi, h_z\right) = \left[W(z), \Theta(z), \Gamma(z), \mathrm{Z}(z), X(z), K(z)\right].\mathrm{e}^{\left(ik_x x + ik_y y + nt\right)}, \tag{11}$$

Where, $k = \left(k_x{}^2 + k_y{}^2\right)^{\frac{1}{2}}$ is the resultant wave number of disturbances and n is the frequency of any arbitrary disturbance.

We eliminate the physical quantities using the non-dimensional parameters $a = kd, \sigma = \frac{nd^2}{\nu}, p_1 = \frac{\nu}{\kappa_T}, q_1 = \frac{\nu}{\kappa_S}, p_2 = \frac{\nu}{\eta}, F = \frac{\mu^{'}}{\rho_0 d^2 \nu}, P_1 = \frac{k_1}{d^2}, D^* = dD.$ Also, dropping (*) for convenience, we obtain,

$$\begin{cases} \left(D^2 - a^2\right)\left[\left(\frac{\sigma}{\varepsilon^*} + \frac{1}{P_1}\right) + F\left(D^2 - a^2\right)^2 - \left(D^2 - a^2\right)\right]W + \frac{\lambda\alpha a^2 d^2}{\nu}\left(g_0 - \frac{K_1 M_0 \nabla H}{\rho_0 \alpha\lambda}\right)\Theta \\ -\frac{\lambda\alpha^{'} a^2 d^2}{\nu}\left(g_0 - \frac{K_2 M_0 \nabla H}{\rho_0 \alpha^{'} \lambda}\right)\Gamma + \frac{2\Omega d^3}{\varepsilon^* \nu} DZ - \frac{\mu_e H d}{4\pi\rho_0 \nu}\left(D^2 - a^2\right)DK = 0, \end{cases} \tag{12}$$

$$\left[\left(\frac{\sigma}{\varepsilon^*} + \frac{1}{P_1}\right) + F\left(D^2 - a^2\right)^2 - \left(D^2 - a^2\right)\right]Z = \frac{2\Omega d}{\varepsilon^* \nu} DW + \frac{\mu_e H d}{4\pi\rho_0 \nu} DX, \tag{13}$$

$$\left(D^2 - a^2 - \sigma p_2\right)X = -\frac{Hd}{\varepsilon^* \eta} DZ, \tag{14}$$

$$\left(D^2-a^2-\sigma p_2\right)K=-\frac{Hd}{\varepsilon^*\eta}DW, \tag{15}$$

$$\left(D^2-a^2-\sigma Ep_1\right)\Theta=-\frac{\beta d^2}{\kappa_T}W, \tag{16}$$

$$\left(D^2-a^2-E'q_1\sigma\right)\Gamma=-\frac{\beta' d^2}{\kappa_S}W, \tag{17}$$

Now, eliminating X,Θ,Γ,K and Z among Eqs. (12) – (17), we obtain the stability governing equation,

$$\left\{\begin{aligned}&\left(D^2-a^2\right)\left[\left(\frac{\sigma}{\varepsilon^*}+\frac{1}{P_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W-\lambda R_f a^2\left[\frac{1}{\left(D^2-a^2-\sigma Ep_1\right)}\right]W+\lambda a^2 S_f\left[\frac{1}{\left(D^2-a^2-\sigma E'q_1\right)}\right]W\\&+\frac{T_A}{\varepsilon^{*2}}\left\{\frac{\left(D^2-a^2-\sigma p_2\right)}{\left\{\left[\left(\frac{\sigma}{\varepsilon^*}+\frac{1}{P_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2-\sigma p_2\right)+\frac{Q}{\varepsilon^*}D^2\right\}}\right\}D^2W+\frac{Q}{\varepsilon^*}\frac{\left(D^2-a^2\right)}{\left(D^2-a^2-\sigma p_2\right)}D^2W=0,\end{aligned}\right. \tag{18}$$

Where, $\frac{\alpha\beta d^4}{\nu\kappa_T}\left(g_0-\frac{K_1M_0\nabla H}{\rho_0\alpha\lambda}\right)=R_f$ is the Rayleigh number for ferromagnetic liquids, $\frac{\alpha'\beta' d^4}{\nu\kappa_S}\left(g_0-\frac{K_2M_0\nabla H}{\rho_0\alpha'\lambda}\right)=S_f$ is the solute Rayleigh number for ferromagnetic fluids, $\left(\frac{2\Omega d^2}{\nu}\right)^2=T_A$ is modified Taylor number and $\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}=Q$ is Chandrasekhar number.

Now, the appropriate boundary condition (when we take the case of two free boundaries) are,

$$W=0, D^2W=0, D^4W=0, \Theta=0, \Gamma=0, Z=0, X=0, DZ=0, DX=0, DK=0 \text{ at } z=0 \text{ and } z=1. \tag{19}$$

The proper solution to Equation (18) characterizing the lowest mode is,

$$W=W_0\sin\pi z, (W_0 = \text{constant}) \tag{20}$$

Now, using Equation (20) in Equation (18), we get,

$$\begin{cases} R_1 = \dfrac{(1+x_w)}{\lambda x_w}\left[\left(\dfrac{i\sigma_1}{\varepsilon^*}+\dfrac{1}{P}\right)+F_1(1+x_w)^2+(1+x_w)\right](1+x_w+i\sigma_1 Ep_1)+S_1\dfrac{(1+x_w+i\sigma_1 Ep_1)}{(1+x_w+i\sigma_1 E' q_1)} \\ +\dfrac{T_{A_1}}{\lambda\varepsilon^{*2}x_w}\left\{\dfrac{(1+x_w+i\sigma_1 p_2)(1+x_w+i\sigma_1 Ep_1)}{\left\{\left[\left(\dfrac{i\sigma_1}{\varepsilon^*}+\dfrac{1}{P_1}\right)+F_1(1+x_w)^2+(1+x_w)\right](1+x_w+i\sigma_1 p_2)+\dfrac{Q_1}{\varepsilon^*}\right\}}\right\}+\dfrac{Q_1}{\varepsilon^*}\dfrac{(1+x_w)}{\lambda x_w}\dfrac{(1+x_w+i\sigma_1 Ep_1)}{(1+x_w+i\sigma_1 p_2)}, \end{cases} \tag{21}$$

Where, $x_w=\dfrac{a^2}{\pi^2}, i\sigma_1=\dfrac{\sigma}{\pi^2}, F_1=\pi^2 F, R_1=\dfrac{R_f}{\pi^4}, S_1=\dfrac{S_f}{\pi^4}, T_{A_1}=\dfrac{T_A}{\pi^4}, Q_1=\dfrac{Q}{\pi^2}, P=\pi^2 P_1.$

4. ANALYTICAL DISCUSSION

- **Stationary Convection**

When stability sets in at stationary convection, the marginal state will be characterized by $\sigma_1=0.$ So, put $\sigma_1=0$ in Equation (21), we get,

$$R_1=\frac{(1+x_w)^2}{\lambda x_w}\left[\frac{1}{P}+F_1(1+x_w)^2+(1+x_w)\right]+S_1+\frac{T_{A_1}}{\varepsilon^{*2}\lambda x_w}\left\{\frac{(1+x_w)^2}{\left[\dfrac{(1+x_w)}{P}+F_1(1+x_w)^3+(1+x_w)^2+\dfrac{Q_1}{\varepsilon^*}\right]}\right\}+\frac{Q_1}{\varepsilon^*\lambda}\frac{(1+x_w)}{x_w}. \tag{22}$$

Now, to study the effect of medium permeability, solute gradient, couple-stress, rotation, and magnetic field, we examine the behavior of $\dfrac{dR_1}{dP},\dfrac{dR_1}{dS_1},\dfrac{dR_1}{dF_1},\dfrac{dR_1}{dT_{A_1}}$ and $\dfrac{dR_1}{dQ_1}$.

(a) Effect of medium permeability

$$\frac{dR_1}{dP}=\frac{(1+x_w)^2}{\lambda x_w P^2}\left\{\frac{T_{A_1}(1+x_w)}{\varepsilon^{*2}\left[\dfrac{(1+x_w)}{P}+F_1(1+x_w)^3+(1+x_w)^2+\dfrac{Q_1}{\varepsilon^*}\right]^2}-1\right\},$$

clearly, medium permeability has a stabilizing result on the system under condition,

$$\lambda>0, T_{A_1}(1+x_w)>\varepsilon^{*2}\left[\frac{(1+x_w)}{P}+F_1(1+x_w)^3+(1+x_w)^2+\frac{Q_1}{\varepsilon^*}\right]^2$$

and $\lambda < 0, T_{A_1}(1+x_w) < \varepsilon^{*2}\left[\frac{(1+x_w)}{P} + F_1(1+x_w)^3 + (1+x_w)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$.

Also, medium permeability has a destabilizing result on the system under condition,

$$\lambda > 0, T_{A_1}(1+x_w) < \varepsilon^{*2}\left[\frac{(1+x_w)}{P} + F_1(1+x_w)^3 + (1+x_w)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$$

and $\lambda < 0, T_{A_1}(1+x_w) > \varepsilon^{*2}\left[\frac{(1+x_w)}{P} + F_1(1+x_w)^3 + (1+x_w)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$.

In the absence of rotation and magnetic field,

$$\frac{dR_1}{dP} = -\frac{(1+x_w)^2}{\lambda x_w P^2},$$

clearly, medium permeability has a stabilizing result on the system if $\lambda < 0$ and destabilizing result if $\lambda > 0$.

(b) Effect of stable solute gradient

$$\frac{dR_1}{dS_1} = 1$$

clearly, solute gradient has a stabilizing result on the system.

(c) Effect of couple-stress

$$\frac{dR_1}{dF_1} = \frac{(1+x_w)^4}{\lambda x_w}\left\{1 - \frac{T_{A_1}(1+x_w)}{\varepsilon^{*2}\left[\frac{(1+x_w)}{P} + F_1(1+x_w)^3 + (1+x_w)^2 + \frac{Q_1}{\varepsilon^*}\right]^2}\right\},$$

clearly, couple-stress has a stabilizing result on the system under condition,

$$\lambda > 0, T_{A_1}(1+x_w) < \varepsilon^{*2}\left[\frac{(1+x_w)}{P} + F_1(1+x_w)^3 + (1+x_w)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$$

and $\lambda < 0, T_{A_1}(1+x_w) > \varepsilon^{*2}\left[\frac{(1+x_w)}{P} + F_1(1+x_w)^3 + (1+x_w)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$.

Also, couple-stress has a destabilizing result on the system under condition,

$$\lambda > 0, T_{A_1}\left(1+x_w\right) > \varepsilon^{*2}\left[\frac{\left(1+x_w\right)}{P} + F_1\left(1+x_w\right)^3 + \left(1+x_w\right)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$$

and $\lambda < 0, T_{A_1}\left(1+x_w\right) < \varepsilon^{*2}\left[\frac{\left(1+x_w\right)}{P} + F_1\left(1+x_w\right)^3 + \left(1+x_w\right)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$.

In the absence of rotation and magnetic field,

$$\frac{dR_1}{dF_1} = \frac{\left(1+x_w\right)^4}{\lambda x_w},$$

clearly, couple-stress has a stabilizing result on the system if $\lambda > 0$ and destabilizing result if $\lambda < 0$.

(d) Effect of rotation

$$\frac{dR_1}{dT_{A_1}} = \frac{1}{\varepsilon^{*2}\lambda x_w}\left\{\frac{\left(1+x_w\right)^2}{\left[\frac{\left(1+x_w\right)}{P} + F_1\left(1+x_w\right)^3 + \left(1+x_w\right)^2 + \frac{Q_1}{\varepsilon^*}\right]}\right\},$$

clearly, rotation has a stabilizing result on the system if $\lambda > 0$ and destabilizing result if $\lambda < 0$.

(e) Effect of magnetic field

$$\frac{dR_1}{dQ_1} = \frac{\left(1+x_w\right)}{\lambda\varepsilon^* x_w}\left\{1 - \frac{T_{A_1}\left(1+x_w\right)}{\varepsilon^{*2}\left[\frac{\left(1+x_w\right)}{P} + F_1\left(1+x_w\right)^3 + \left(1+x_w\right)^2 + \frac{Q_1}{\varepsilon^*}\right]^2}\right\},$$

clearly, magnetic field has a stabilizing result on the system under condition,

$$\lambda > 0, T_{A_1}\left(1+x_w\right) < \varepsilon^{*2}\left[\frac{\left(1+x_w\right)}{P} + F_1\left(1+x_w\right)^3 + \left(1+x_w\right)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$$

and $\lambda < 0, T_{A_1}\left(1+x_w\right) > \varepsilon^{*2}\left[\frac{\left(1+x_w\right)}{P} + F_1\left(1+x_w\right)^3 + \left(1+x_w\right)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$.

Also, magnetic field has a destabilizing result on the system under condition,

$$\lambda > 0, T_{A_1}\left(1+x_w\right) > \varepsilon^{*2}\left[\frac{\left(1+x_w\right)}{P} + F_1\left(1+x_w\right)^3 + \left(1+x_w\right)^2 + \frac{Q_1}{\varepsilon^*}\right]^2$$

and $\lambda<0, T_{A_1}\left(1+x_w\right)<\varepsilon^{*2}\left[\frac{\left(1+x_w\right)}{P}+F_1\left(1+x_w\right)^3+\left(1+x_w\right)^2+\frac{Q_1}{\varepsilon^*}\right]^2$.

In the absence of rotation,

$$\frac{dR_1}{dQ_1}=\frac{\left(1+x_w\right)}{\lambda\varepsilon^* x_w},$$

clearly, magnetic field has a stabilizing result on the system if $\lambda>0$ and destabilizing result if $\lambda<0$.

(f) To see the result of magnetization, we study the result of $\frac{dR}{dM_0}$ analytically.

$$\frac{dR}{dM_0}=\left\{\frac{\pi^4\left(1+x_w\right)^2}{\lambda x_w}\left[\frac{1}{P}+F_1\left(1+x_w\right)^2+\left(1+x_w\right)\right]+S_1\pi^4+\frac{\pi^4 T_{A_1}}{\varepsilon^{*2}\lambda x_w}\left[\frac{\left(1+x_w\right)^2}{\left(\frac{\left(1+x_w\right)}{P}+F_1\left(1+x_w\right)^3+\left(1+x_w\right)^2+\frac{Q_1}{\varepsilon^*}\right)}\right]+\frac{\pi^4 Q_1}{\varepsilon^*\lambda}\frac{\left(1+x_w\right)}{x_w}\right\}$$

(23)
$$.\left(1-\frac{\gamma' M_0\nabla H}{\rho_0\alpha g_0\lambda}\right)^{-2}\left(\frac{\gamma'\nabla H}{\rho_0\alpha g_0\lambda}\right),$$

clearly, in the absence of a stable solute gradient, magnetization has a stabilizing result on the system for both $\lambda>0$ and $\lambda<0$.

- **The case of Oscillatory Modes**

Multiplying Equation (12) by W^* (conjugate of W) and integrate over the range of z and making use of Eqs. (13) – (17) together with the boundary conditions (19), we get,

$$\left\{\begin{array}{l}\left(\frac{\sigma}{\varepsilon^*}+\frac{1}{P_1}\right)I_1+I_2+FI_3+d^2\left[\left(\frac{\sigma^*}{\varepsilon^*}+\frac{1}{P_1}\right)I_4+I_5+FI_6+\frac{\mu_e\varepsilon^*\eta}{4\pi\rho_0\nu}\left(I_7+p_2\sigma I_8\right)\right]+\frac{\mu_e\varepsilon^*\eta}{4\pi\rho_0\nu}\left(I_9+p_2\sigma^* I_{10}\right)\\ -\frac{\lambda\alpha a^2\kappa_T}{\beta\nu}\left(g_0-\frac{K_1M_0\nabla H}{\rho_0\alpha\lambda}\right)\left(I_{11}+\sigma^* Ep_1 I_{12}\right)+\frac{\lambda\alpha' a^2\kappa_S}{\beta'\nu}\left(g_0-\frac{K_2M_0\nabla H}{\rho_0\alpha'\lambda}\right)\left(I_{13}+E' q_1\sigma^* I_{14}\right)=0,\end{array}\right.$$

(24)

Where, $I_1=\int\left(\left|DW\right|^2+a^2\left|W\right|^2\right)dz, I_2=\int\left(\left|D^2W\right|^2+2a^2\left|DW\right|^2+a^4\left|W\right|^2\right)dz,$

$$I_3=\int\left(\left|D^3W\right|^2+3a^2\left|D^2W\right|^2+3a^4\left|DW\right|^2+a^6\left|W\right|^2\right)dz, I_4=\int\left(\left|Z\right|^2\right)dz,$$

$$I_5=\int\left(\left|DZ\right|^2+a^2\left|Z\right|^2\right)dz, I_6=\int\left(\left|D^2Z\right|^2+2a^2\left|DZ\right|^2+a^4\left|Z\right|^2\right)dz,$$

$$I_7 = \int\left(|DX|^2 + a^2|X|^2\right)dz, I_8 = \int|X|^2 dz,$$

$$I_9 = \int\left(\left|D^2K\right|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz, I_{10} = \int\left(|DK|^2 + a^2|K|^2\right)dz,$$

$$I_{11} = \int\left(|D\Theta|^2 + a^2|\Theta|^2\right)dz, I_{12} = \int\left(|\Theta|^2\right)dz.$$

$$I_{13} = \int\left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz, I_{14} = \int\left(|\Gamma|^2\right)dz.$$

All these integrals from I_1 to I_{14} are positive definite.

Now, putting $\sigma = i\sigma_i$ (where σ_i is real) in Equation (24) and equating the imaginary part, we get,

$$\begin{cases} \sigma_i\left\{\frac{I_1}{\varepsilon^*} - \frac{d^2I_4}{\varepsilon^*} + \frac{\mu_e\varepsilon^*\eta d^2}{4\pi\rho_0\nu}p_2I_8 - \frac{\mu_e\varepsilon^*\eta}{4\pi\rho_0\nu}p_2I_{10} + Ep_1\left[\frac{\lambda\alpha a^2\kappa_T}{\beta\nu}\left(g_0 - \frac{K_1M_0\nabla H}{\rho_0\alpha\lambda}\right)\right]I_{12}\right. \\ \left. \qquad - E'q_1\left[\frac{\lambda\alpha' a^2\kappa_S}{\beta'\nu}\left(g_0 - \frac{K_2M_0\nabla H}{\rho_0\alpha'\lambda}\right)\right]I_{14}\right\} = 0. \end{cases} \tag{25}$$

In the absence of rotation and magnetic field, Equation (25) turns into,

$$\sigma_i\left\{\frac{I_1}{\varepsilon^*} + \frac{\mu_e\varepsilon^*\eta d^2}{4\pi\rho_0\nu}p_2I_8 + Ep_1\left[\frac{\lambda\alpha a^2\kappa_T}{\beta\nu}\left(g_0 - \frac{K_1M_0\nabla H}{\rho_0\alpha\lambda}\right)\right]I_{12} - E'q_1\left[\frac{\lambda\alpha' a^2\kappa_S}{\beta'\nu}\left(g_0 - \frac{K_2M_0\nabla H}{\rho_0\alpha'\lambda}\right)\right]I_{14}\right\} = 0 \tag{26}$$

If $g_0 \geq \frac{K_1M_0\nabla H}{\rho_0\alpha\lambda}$ and $g_0 \leq \frac{K_2M_0\nabla H}{\rho_0\alpha'\lambda}$, then the terms with inside the bracket are positive definite, which suggests that $\sigma_i = 0$. Hence, oscillatory modes aren't allowed and the principle of exchange of stabilities is satisfied if $g_0 \geq \frac{K_1M_0\nabla H}{\rho_0\alpha\lambda}$ and $g_0 \leq \frac{K_2M_0\nabla H}{\rho_0\alpha'\lambda}$.So, we will say that inside the system, oscillatory modes are delivered because of the presence of rotation and magnetic field, while in their absence, the principle of exchange of stabilities is satisfied.

5. NUMERICAL COMPUTATIONS

The dispersion relation (22) is analyzed numerically also. The numerical value of the thermal Rayleigh number R_1 is determined for various values of medium permeability P, solute gradient

S_1, couple-stress F_1, rotation T_{A_1}, magnetic field Q_1, and magnetization M_0. Also, graphs have been plotted between R_1 and these parameters as demonstrated in figures (2) – (18).

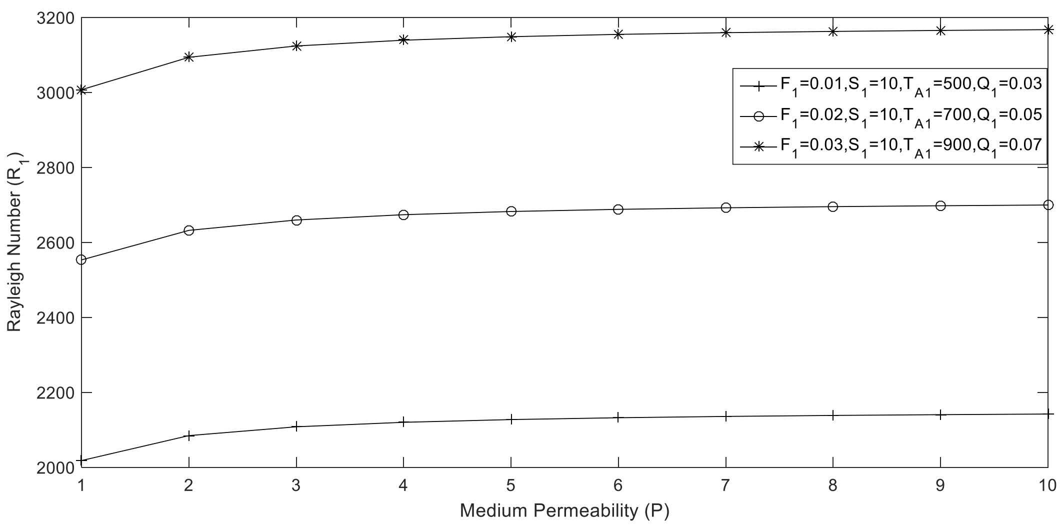

Fig. 2. R_1 vs. P for $\lambda > 0(\lambda = 1)$.

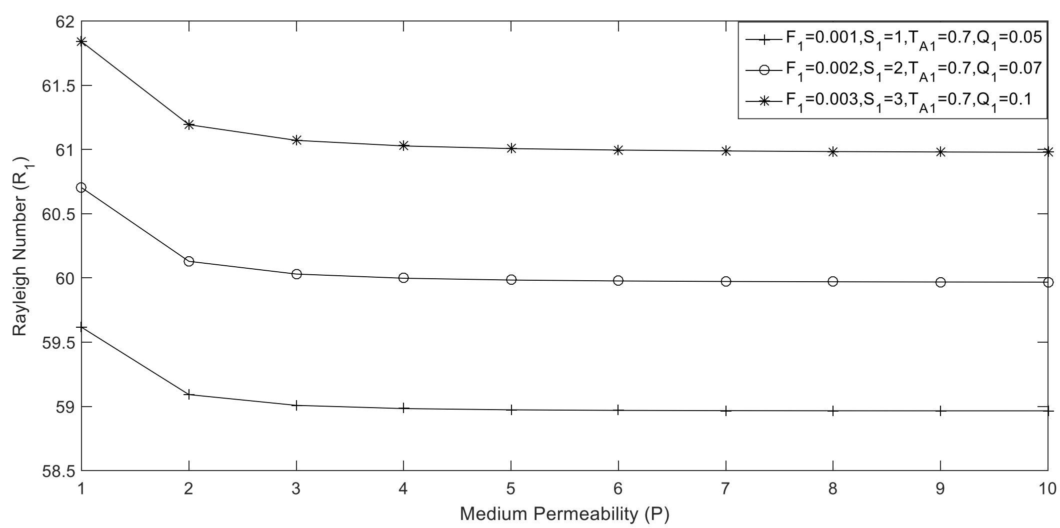

Fig. 3. R_1 vs. P for $\lambda > 0(\lambda = 0.5)$.

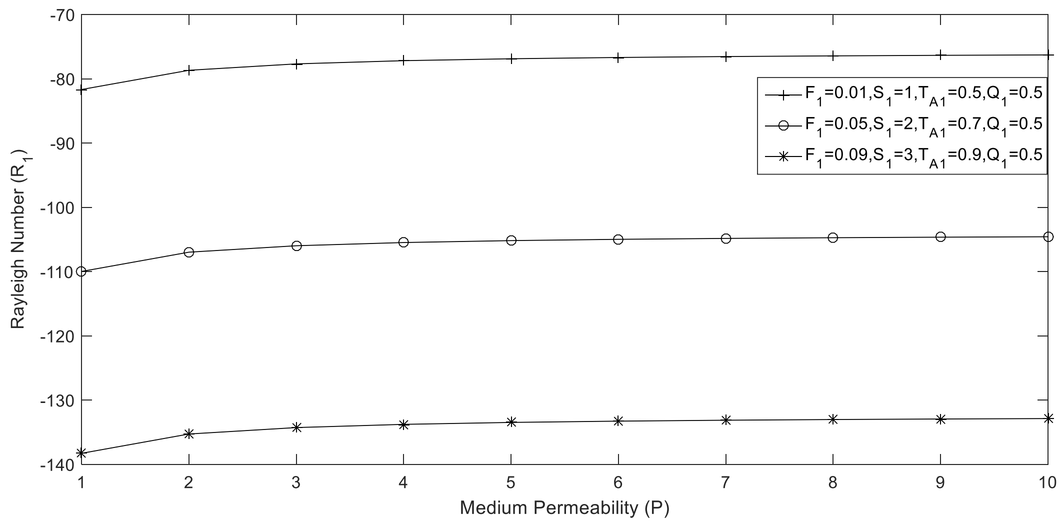

Fig. 4. R_1 vs. P for $\lambda < 0 (\lambda = -2)$.

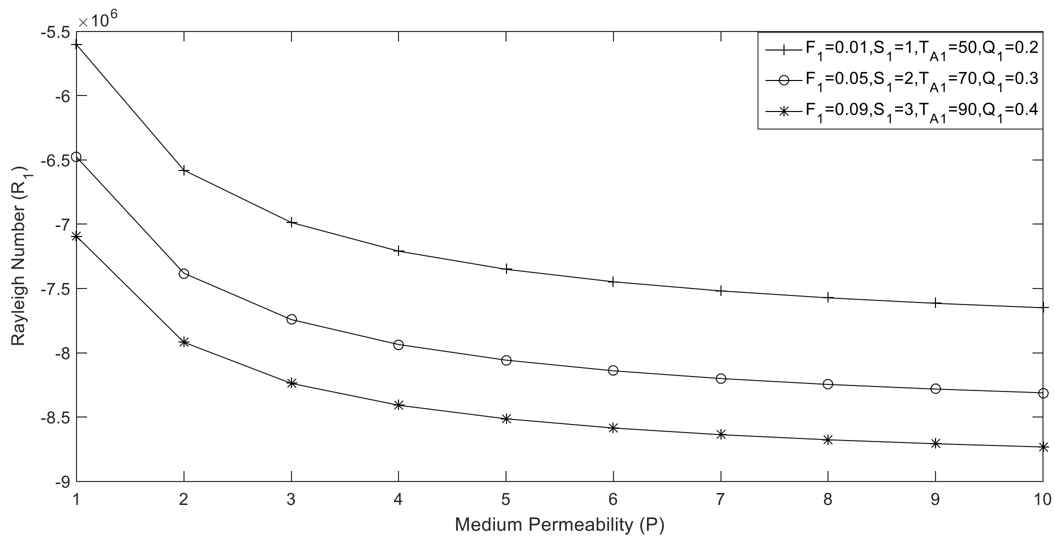

Fig. 5. R_1 vs. P for $\lambda < 0 (\lambda = -2)$.

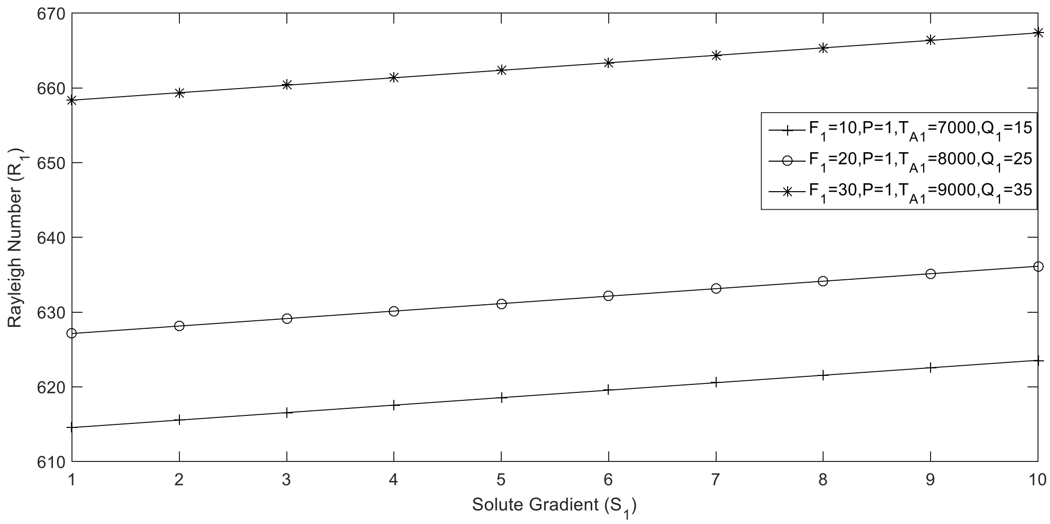

Fig. 6. R_1 vs. S_1 .

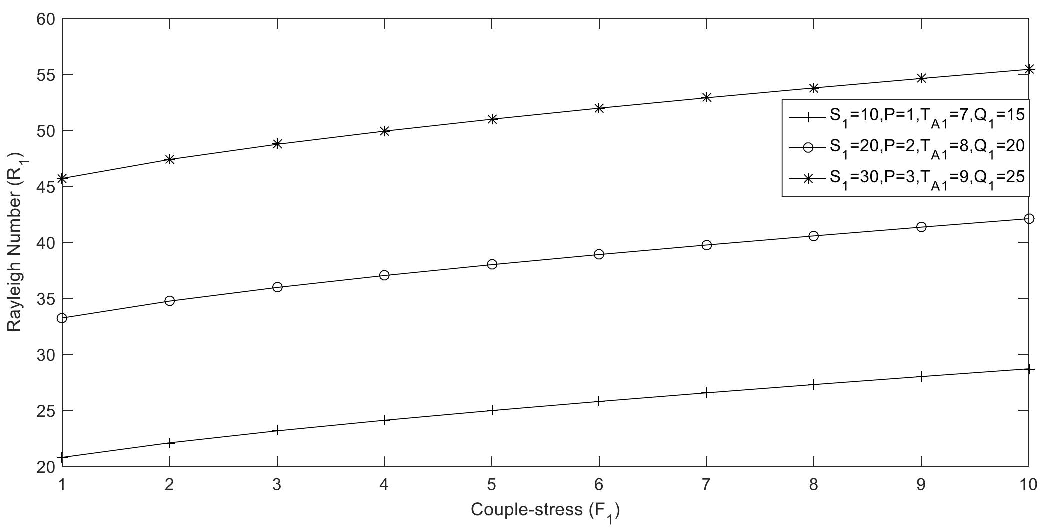

Fig. 7. R_1 vs. F_1 for $\lambda > 0(\lambda = 20)$.

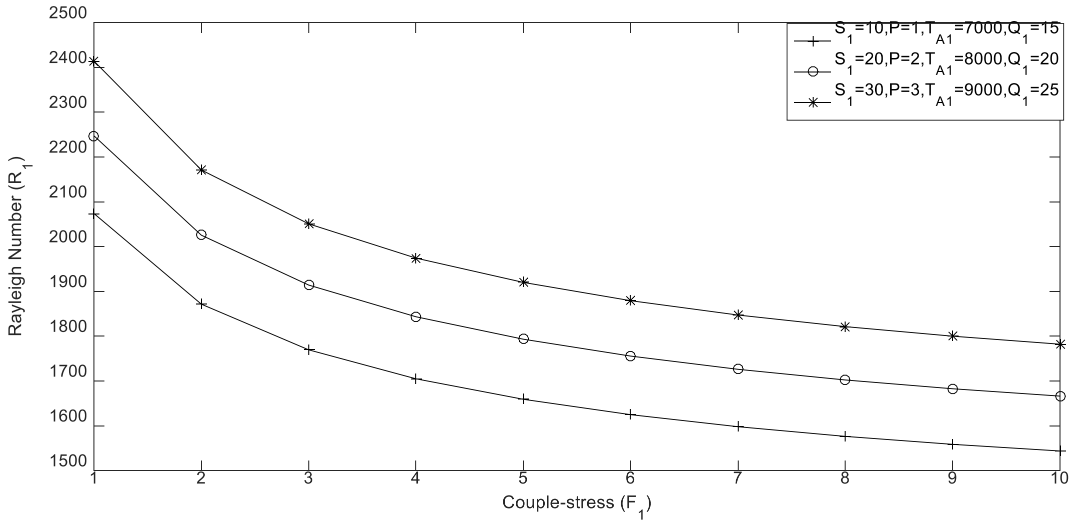

Fig. 8. R_1 vs. F_1 for $\lambda > 0(\lambda = 2)$.

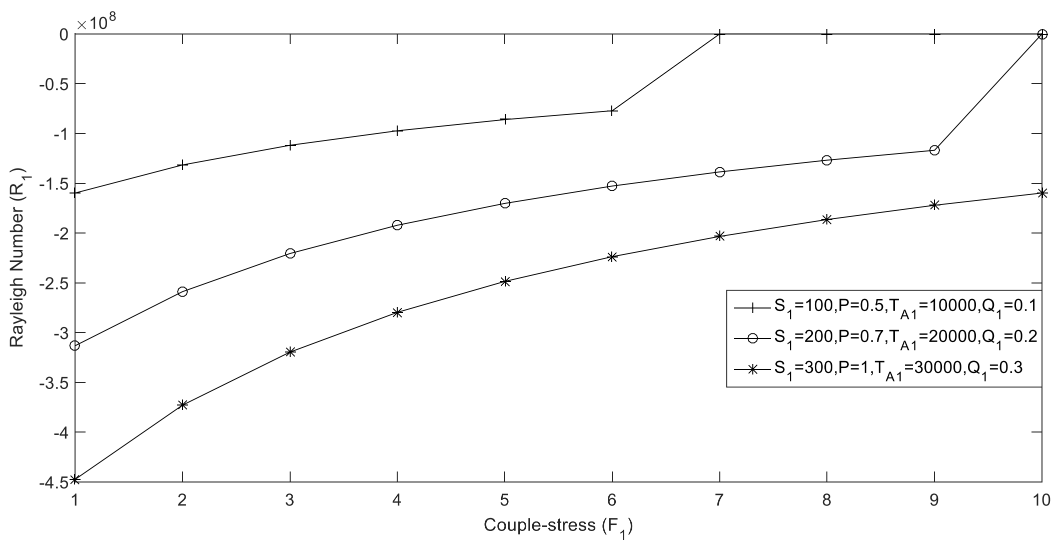

Fig. 9. R_1 vs. F_1 for $\lambda < 0(\lambda = -10)$.

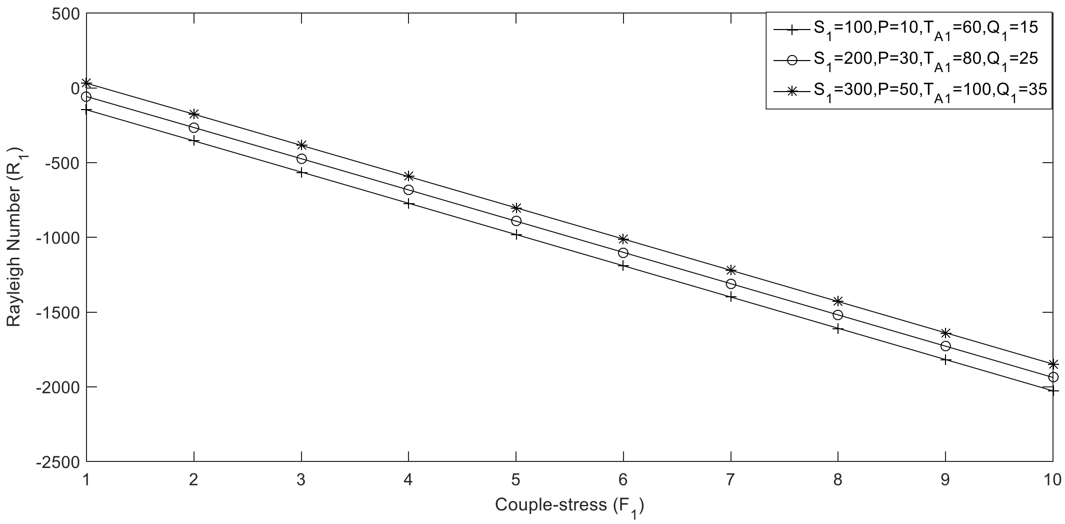

Fig. 10. R_1 vs. F_1 for $\lambda < 0(\lambda = -7)$.

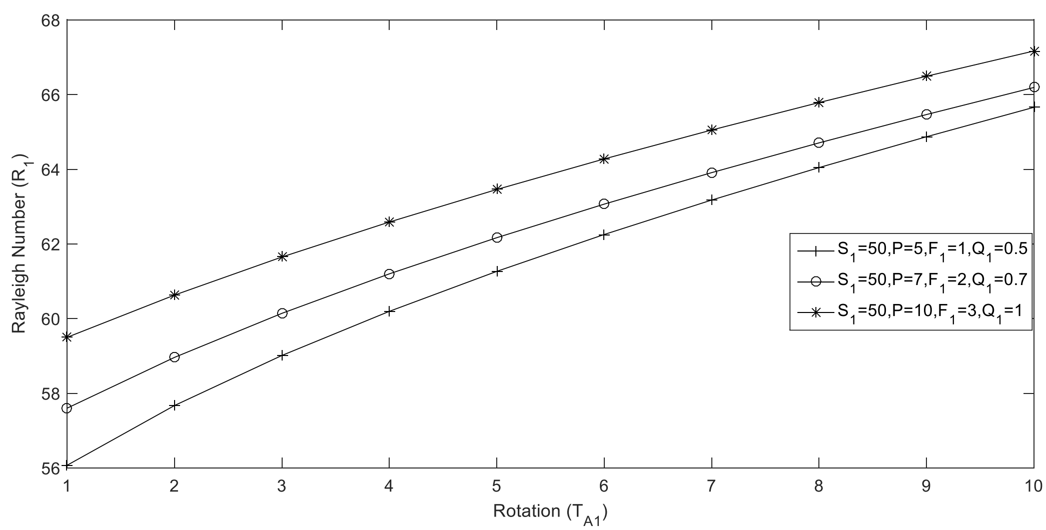

Fig. 11. R_1 vs. T_{A_1} for $\lambda > 0(\lambda = 7)$.

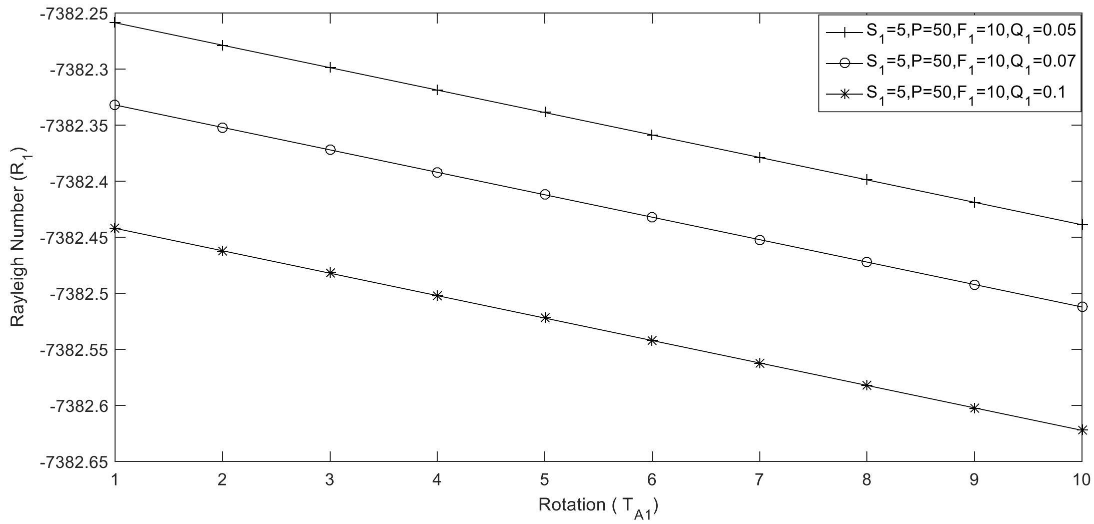

Fig. 12. R_1 vs. T_{A_1} for $\lambda < 0\left(\lambda = -2\right)$.

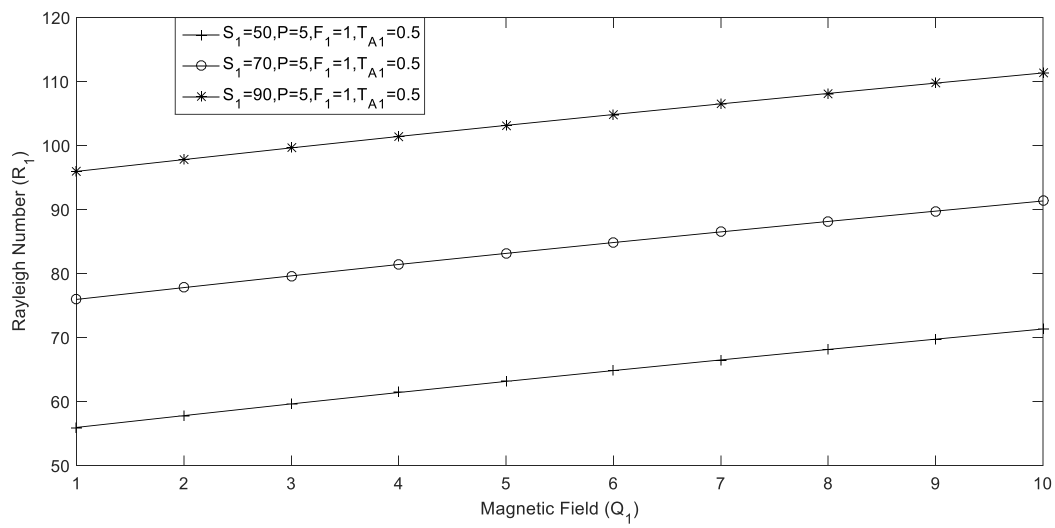

Fig. 13. R_1 vs. Q_1 for $\lambda > 0\left(\lambda = 7\right)$.

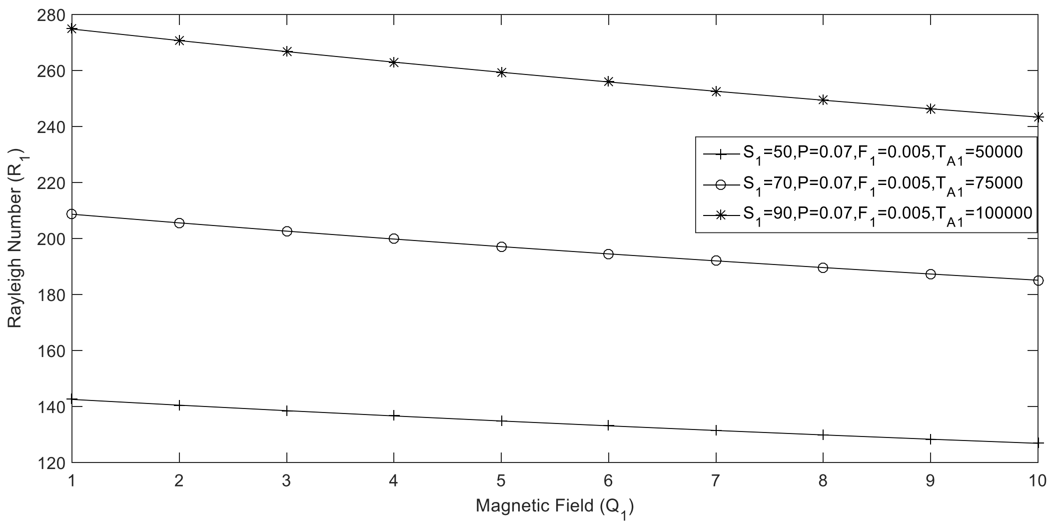

Fig. 14. R_1 vs. Q_1 for $\lambda > 0(\lambda = 1000)$.

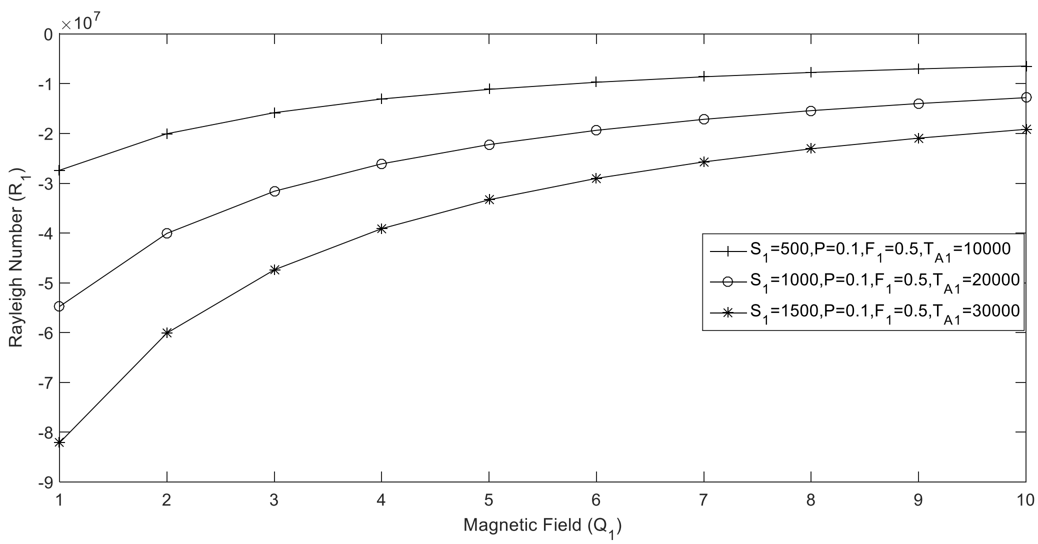

Fig. 15. R_1 vs. Q_1 for $\lambda < 0(\lambda = -15)$.

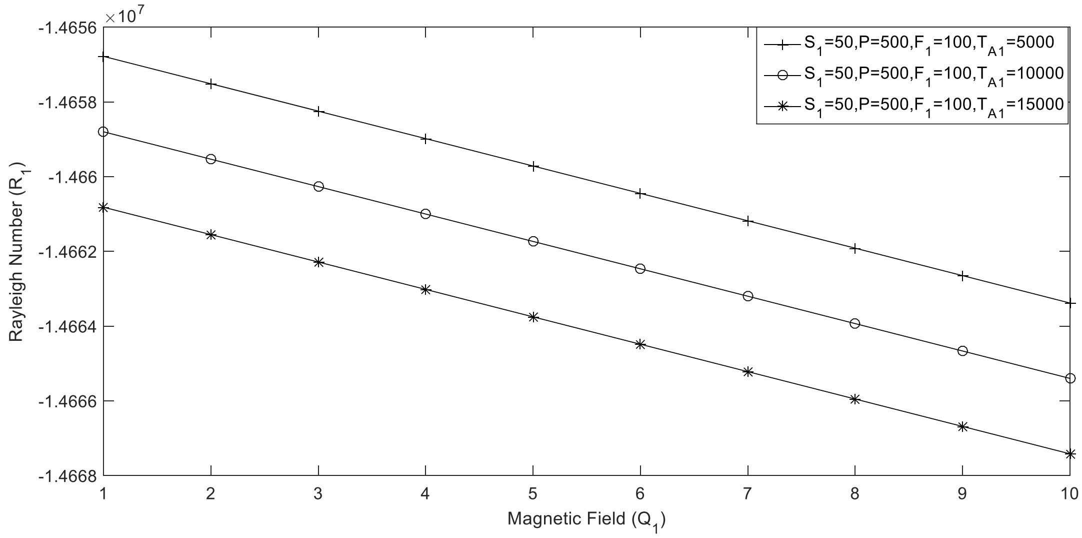

Fig. 16. R_1 vs. Q_1 for $\lambda < 0(\lambda = -0.01)$.

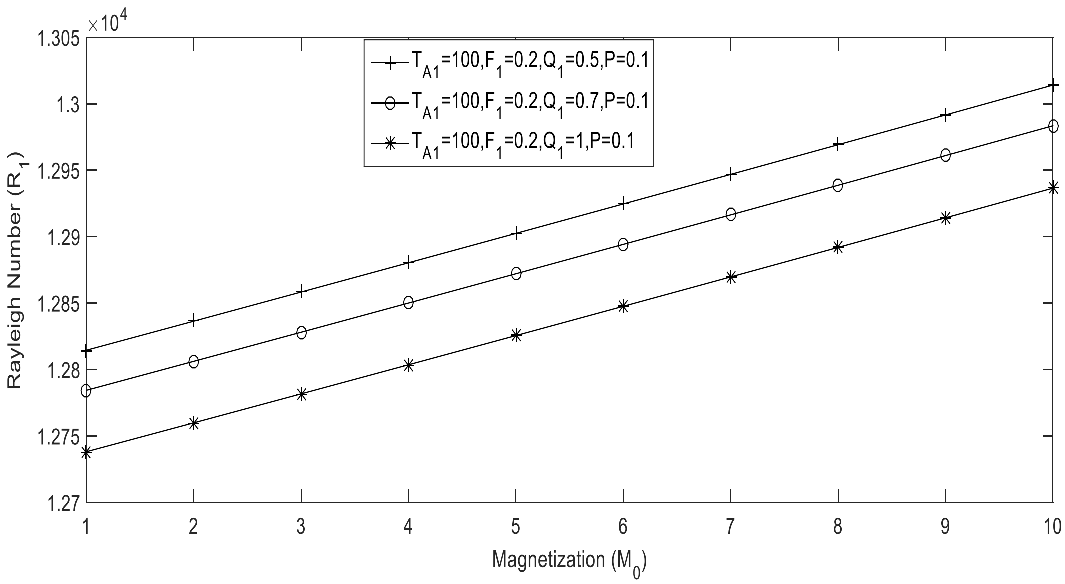

Fig. 17. R_1 vs. M_0 for $\lambda > 0(\lambda = 3)$.

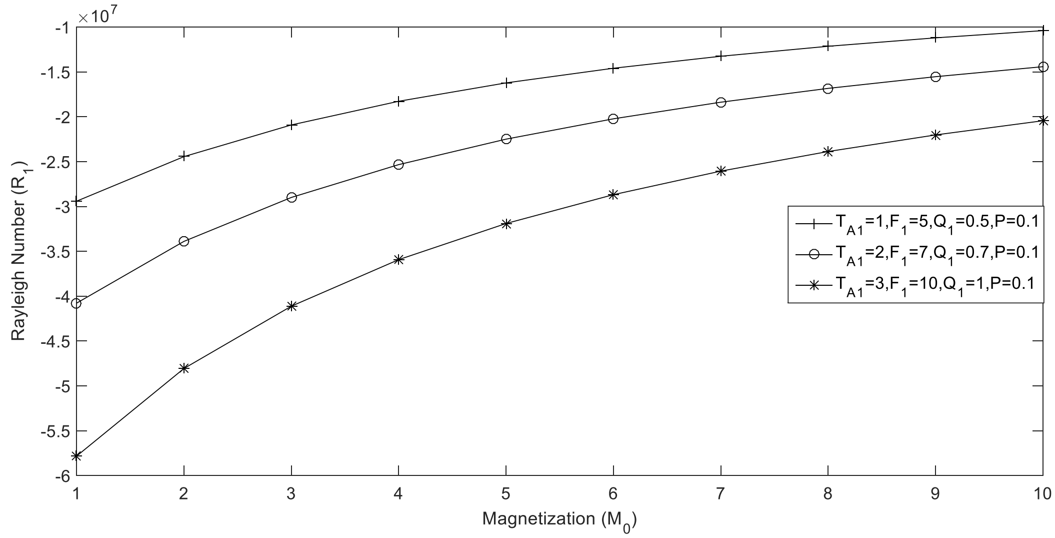

Fig. 18. R_1 vs. M_0 for $\lambda < 0 (\lambda = -0.02)$.

These are some graphs showing stabilizing and destabilizing effect between R_1 and these parameters (medium permeability, solute gradient, couple-stress, rotation, magnetic field and magnetization).

In Fig. 2, the critical Rayleigh number R_1 will increase with growth in medium permeability parameter P. So, medium permeability has a stabilizing result on the system. In Fig. 3, the critical Rayleigh number R_1 decreases with growth in medium permeability parameter P. So, medium permeability has a destabilizing result on the system. In Fig. 4, the critical Rayleigh number R_1 will increase with growth in the medium permeability parameter P. So, medium permeability has a stabilizing result on the system. In Fig. 5, the critical Rayleigh number R_1 decreases with growth in the medium permeability parameter P. So, medium permeability has a destabilizing result on the system. In Fig. 6, the critical Rayleigh number R_1 will increase with growth in the solute gradient parameter S_1. So, the solute gradient has a stabilizing result on the system. In Fig. 7, the critical Rayleigh number R_1 will increase with growth in the couple-stress parameter F_1. So, the couple-stress has a stabilizing result on the system. In Fig. 8, the critical Rayleigh number R_1 decreases with growth in the couple-stress parameter F_1. So, the couple-stress has a destabilizing result on the system. In Fig. 9, the critical Rayleigh number R_1 will increase with growth in the couple-stress parameter F_1. So, the couple-stress has a stabilizing result on the system. In Fig. 10, the critical Rayleigh number R_1 decreases with growth in the couple-stress parameter F_1. So, the couple-stress has a destabilizing result on the system. In Fig. 11, the critical Rayleigh number R_1 will increase with growth in the rotation parameter T_{A_1}. So, the rotation has a stabilizing result on the system. In Fig. 12, the critical Rayleigh number R_1 decreases with growth in rotation parameter T_{A_1}. So, the rotation has a destabilizing result on the system. In Fig. 13, the critical Rayleigh number R_1 will increase with growth in the magnetic field parameter Q_1. So, the magnetic field has a stabilizing result on the system. In Fig. 14, the critical Rayleigh number R_1 decreases with growth in the magnetic field parameter Q_1. So, the magnetic field has a destabilizing result on the system. In Fig. 15, the critical Rayleigh number R_1 will increase with growth in the magnetic field parameter Q_1. So, the magnetic field has a stabilizing result on the system. In Fig. 16, the critical Rayleigh number R_1 decreases with growth in the magnetic field parameter Q_1. So, the magnetic field has a destabilizing result on the system. In Fig. 17, the critical Rayleigh number R_1 will increase with growth in the magnetization parameter M_0. So, magnetization has a stabilizing result on the system. In Fig. 18, the critical Rayleigh number R_1 will increase with growth in the magnetization parameter M_0. So, magnetization has a stabilizing result on the system.

6. CONCLUSIONS

In the present paper, we have discussed the combined result of rotation and magnetic field on couple-stress ferromagnetic fluid heated and soluted from below in the presence of a variable gravity field saturating in a porous medium. To obtain the dispersion relation, we obtain used the linearized theory and normal mode technique. As we have discussed before that the present paper is an extension of the research paper entitled "Thermosolutal instability of couple-stress rotating fluid in the presence of magnetic field" studied by Kumar et al.[26], we have done the comparison between the results of these two which is given as below:

Case I: In the paper entitled "Thermosolutal instability of couple-stress rotating fluid in the presence of magnetic field", the author has discussed the case of $\lambda > 0$. The results obtained by him and we are given as below:

i. Kumar et al. [26] have not discussed the result of medium permeability on the system, but according to us, medium permeability has both stabilizing and destabilizing result on the system $\lambda > 0$ under certain conditions. And in the absence of rotation and magnetic field, medium permeability has a destabilizing result $\lambda > 0$.

ii. A stable solute gradient has a stabilizing result on the system.

iii. Couple-stress and magnetic fields have both stabilizing and destabilizing results on the system $\lambda > 0$ under certain conditions. Furthermore, in the absence of rotation and magnetic field, the couple-stress has a stabilizing result on the system $\lambda > 0$. And in the absence of rotation, the magnetic field has a stabilizing result on the system $\lambda > 0$.

iv. The rotation has a stabilizing result on the system $\lambda > 0$.

v. In the absence of a stable solute gradient, magnetization has a stabilizing result on the system $\lambda > 0$. This case was not discussed by Kumar et al. [26].

Case II: We also discussed the case when $\lambda < 0$ (This case had not been discussed in the paper entitled "Thermosolutal instability of couple-stress rotating fluid in the presence of magnetic field" studied by Kumar et al. [26]). The important results obtained are as given below:

i. Medium permeability has both stabilizing and destabilizing results on the system under certain conditions, while in the absence of rotation and magnetic field, medium permeability has a stabilizing result on the system $\lambda < 0$.

ii. Couple-stress and magnetic field have both stabilizing and destabilizing result on the system under certain conditions, while in the absence of rotation and magnetic field, couple-stress has a destabilizing result on the system for $\lambda < 0$. And in the absence of rotation, the magnetic field has a destabilizing result on the system $\lambda < 0$.

iii. The rotation has a destabilizing result on the system $\lambda < 0$.

iv. In the absence of a stable solute gradient, magnetization has a stabilizing result on the system $\lambda < 0$.

Also, the principle of exchange of stabilities is not valid for the present problem under consideration, whereas in the absence of rotation and magnetic field, the principle of exchange of stabilities (PES) is valid for the present problem if $g_0 \geq \frac{K_1 M_0 \nabla H}{\rho_0 \alpha \lambda}$ and $g_0 \leq \frac{K_2 M_0 \nabla H}{\rho_0 \alpha' \lambda}$.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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