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ON SOLVING INTERVAL DATA BASED ASSIGNMENT PROBLEM UNDER FUZZY ENVIRONMENT

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Abstract. This article introduces an algorithm for solving the interval data based assignment problem (IDAP) under fuzzy environment. The proposed algorithm for obtaining optimum solution of IDAP is purely based on the fuzzification and defuzzification of various fuzzy quantities of symmetric shapes. Moreover, this article will give the solution to select the suitable shape of fuzzy quantity for obtaining the optimum solution of IDAP through the numerical illustration.

Keywords: interval number; fuzzy assignment problem; fuzzy number; fuzzification; defuzzification.

2010 AMS Subject Classification: 94D05, 03E72, 97M40.

1. Introduction

Mostly the figures are collected in a vague manner on today's environment. But such type of figures cannot be observed as a fuzzy numbers whereas those are observed as interval numbers. Many researchers are interested in order to solve the assignment problems involving such type of interval numbers on nowadays. But, there is confusion in the minds of authors and

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researchers to select the suitable shape of fuzzy numbers for representing interval numbers in solving interval data based assignment problems using fuzzy set theory. Recently many works are being done in this regard with various shapes of fuzzy numbers.

In 2006 [22], a new heuristic method to find the feasible solution of the channel assignment problems was proposed by Won-Young Shin et al and they exhibited that their method is better than existing methods to give optimum solutions to Philadelphia benchmark instances. Yi Pan et. al [23] has solved NP-complete channel assignment problem by using an algorithm which combines heuristic method into genetic algorithm that consists of regular interval assignment, greedy assignment and the genetic algorithm assignment stages by in 2006. Also Philadelphia problem was solved by their proposed algorithm and compared the solutions with existing solutions. Linzhong Liu and Yinzhen in 2006 [10] formulated the fuzzy quadratic assignment problem with penalty as expected value model, chance-constrained programming and dependent-chance programming based on different decision criteria and proposed the initialization algorithm, crossover algorithm and mutation algorithm for FQAP. Also they have designed hybrid genetic algorithm to solve fuzzy programming models. In 2006, a chance-constrained goal programming model for two-objective fuzzy k-cardinality assignment problem was proposed by Yuan Feng and Lixing [25] and tabu search algorithm based on fuzzy simulation was designed to find the best solution. In 2010 [13], Nagarajan and Solairaju have found the solution of the fuzzy assignment problem using Hungarian method and used Robust's ranking method to rank the fuzzy numbers where the parameter cost is represented by triangular or trapezoidal fuzzy numbers. In 2012 [19], fuzzy goal programming method to solve multi-objective assignment problem has been developed by Surapati Pramanik and Pranab Biswas in which the elements of cost matrix, consumed time matrix and ineffectiveness level matrix are represented by generalized trapezoidal fuzzy numbers and also Euclidean distance function was used to find the appropriate priority structure of fuzzy goals. Srinivasan and Geetharamani solved the fuzzy assignment problem using Ones assignment method and Robust's ranking method where parameters are either triangular or trapezoidal fuzzy numbers in 2013 [18]. A fuzzy assignment problem was transformed into crisp assignment problem in LPP form using new ranking function which was proposed by Nagoor Gani and Mohamed in 2013 [15] and solved by

LINGO 9.0 where cost is represented by triangular or trapezoidal fuzzy numbers. Abhijet A. Chincholkar and Chaitali H. Thakare [1] have presented three-stage algorithm such as regular interval assignment stage, the greedy assignment stage and the genetic algorithm assignment stage which is used to find the optimum solution for channel assignment problem in a cellular mobile communication system in 2014. Ones assignment method was presented by Vimala and Krishna Prabha [21] for solving fuzzy assignment problems in which cost is considered as a triangular fuzzy numbers and they have used ranking technique to convert fuzzy assignment problem into crisp balanced and unbalanced assignment problems in 2016. Arokiamary and Nithya in 2016 [2, 3] have used ones assignment method in solving fuzzy assignment problem to compute maximum and minimum values of the objective function involving hexagonal fuzzy numbers and robust's ranking technique for ranking the fuzzy numbers to give optimal total cost of the problem. Moreover, revised ones assignment method was described to calculate the minimum and maximum total costs of fuzzy assignment problem. In 2016 [9], Krishna Prabha and Vimala explained the optimal solution of triangular fuzzy assignment problem using best candidate method and used method of magnitude to rank the fuzzy numbers. A fuzzy and intutionistic fuzzy assignment problems were solved by using ones assignment method with hexagonal fuzzy numbers and robust ranking technique was used for ranking the objective function by Narayanamoorthy et al in 2017 [14]. Jayamani, Ashwini and Srinivasan in 2017 [8] solved fuzzy assignment problem using ones assignment method and robust's ranking method. In 2018 [24], Xiaofeng Xu, Jun Hao, Lean Yu and Yirui Deng have presented a new cost-time-quality multi-objevtive programming of N-N task–resource assignment to find an optimal allocation model of fuzzy resources for multi-stage random logistics tasks involving six-point trapezoidal fuzzy numbers. Thorani and Ravi Shankar have proposed fuzzy multi-objective assignment model with three parameters fuzzy cost, fuzzy time and fuzzy quality involving generalized LR trapezoidal fuzzy numbers and used centroid of centroids from its' λ-cuts for linear and nonlinear reference functions for ranking the fuzzy numbers in 2017 [20]. In 2018 [4], Hungarian method has been used to solve fuzzy assignment problem in which cost is represented by symmetric hexagonal fuzzy numbers by Arumugam, Abdul Saleem and Arunvasan. In 2018, Mohamed Assarudeen, Sudhakar and Balakrishnan [12] described fuzzy assignment problem in which cost matrix is considered as neutrosophic elements using zero suffix method to find optimal solution. Genetic algorithm based method with fuzzy optimization presented by Hamdy et.al [6] is used to solve optimal components assignment problem (i.e.) optimal solution with maximum system reliability, minimum total assignment cost, and minimum total lead-time in 2019. Dhanasekar, Kanimozhi and Manivannan in 2019 [5] have proposed diagonal optimal method to solve assignment problem with hexagonal fuzzy numbers and used Yager's ranking technique to order the fuzzy numbers in their method. In 2020 [11], an ant colony optimization algorithm based on MAX-MIN ant system was developed and implemented it in C++ language by Marko Peras and Nikola Ivkovic to solve channel assignment problem in mobile cellular networks. Sanjivani M. Ingle and Kirtiwant P. Ghadle in 2020 [17] have derived two formulae: one is for odd number of fuzzy numbers and second is for even number of fuzzy numbers to find optimal solution from the existing solution to fuzzy assignment problem whose parameters are considered as pentagonal and hexagonal fuzzy numbers. In 2020, A new method has been proposed to rank neutrosophic numbers using its magnitude and developed an algorithm to find the solution of Neutrosophic Assignment problems in which parameters are pentagonal neutrosophic numbers by Radhika and Arun Prakash [14].

After making this sample survey, we came to know that the two things are placing a very important role in solving interval data based assignment (optimization) problems using fuzzy set theory which are fuzzfication and defuzzification techniques. Because, in the first phase for finding the optimum solution, the interval data based assignment (optimization) problems are converted as fuzzy assignment (optimization) problems using fuzzificaiton techniques and in the second phase, the fuzzy optimization problems are converted as crisp optimization problems using defuzzification techniques.

In this paper, we have made an attempt to analyse the nature of the various fuzzy quantities in order to obtain the optimum solution of interval data based assignment problems based on proposed fuzzication and defuzzification techniques. The main objective of this work is to introduce an algorithm for obtaining n fuzzy assignment problems from interval data based optimization problems using proposed fuzzification techniques based on the various shape of the fuzzy quantities and then the fuzzy problems are converted as crisp problems using proposed defuzzification techniques to find its optimum solutions. Finally we select the best optimum solutions from the n optimum solutions obtained from fuzzy assignment problems. Moreover, this work may give the solution to the confusions of the researchers in selecting the suitable shape of fuzzy quantity to find the better solution of interval data based assignment problems.

2. PRELIMINARIES

Definition 1 (Fuzzy Set)**.** If the set defined by the function $\mu_{\tilde{A}} : A \to [0,1]$ giving the grade for membership of the element x of the universal set Ω for its belongingness, then it is said to be a fuzzy set. It is defined and denoted as

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in \Omega\}$$

Definition 2 (Fuzzy Number)**.** If a fuzzy set $\tilde{A}$ defined on the universal set of all real numbers $\mathbb{R}$ with the membership function satisfying the following properties, then it is said to be a generalized or non-normal fuzzy number.

(i) $\mu_{\tilde{A}}(x)$ is piecewise continuous in its domain.

(ii) $\tilde{A}$ is normal, i.e., there is a $x_0 \in X$ such that $\mu_{\tilde{A}}(x_0) = \omega$.

(iii) $\tilde{A}$ is convex, i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, $\forall x_1, x_2$ in X.

It is said to be a normal fuzzy number when $\mu_{\tilde{A}}(x_0) = 1$.

Definition 3 (Triangular Fuzzy Number)**.** The fuzzy set of three elements defined on the real line R, is called the generalized triangular fuzzy number of the form $\tilde{A} = (\tau_1, \tau_2, \tau_3; w)$, where $\tau_1 < \tau_2 < \tau_3$, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq \tau_1 \\ w\left(\frac{x-\tau_1}{\tau_2-\tau_1}\right), & \tau_1 \leq x \leq \tau_2 \\ w\left(\frac{\tau_3-x}{\tau_3-\tau_2}\right), & \tau_2 \leq x \leq \tau_3 \\ 0, & x > \tau_3. \end{cases}$$

Definition 4 (Trapezoidal Fuzzy Number)**.** The fuzzy set of four elements defined on the real line $\mathbb{R}$, is called the generalized trapezoidal fuzzy number of the form $\tilde{A} = (\tau_1, \tau_2, \tau_3, \tau_4; w)$, where $\tau_1 < \tau_2 < \tau_3 < \tau_4$, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w\left(\frac{x-\tau_1}{\tau_2-\tau_1}\right), & \tau_1 \leq x \leq \tau_2 \\ w, & \tau_2 \leq x \leq \tau_3 \\ w\left(\frac{x-\tau_4}{\tau_3-\tau_4}\right), & \tau_3 \leq x \leq \tau_4 \\ 0, & \text{otherwise} \end{cases}$$

Definition 5 (Pentagonal Fuzzy Number)**.** The fuzzy set of five elements defined on the real line R, is called the generalized pentagonal fuzzy number of the form $\tilde{A} = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5; w)$, where $\tau_1 < \tau_2 < \tau_3 < \tau_4 < \tau_5$, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < \tau_1 \\ w\left(\frac{x-\tau_1}{\tau_2-\tau_1}\right), & \tau_1 \leq x \leq \tau_2 \\ w\left(\frac{x-\tau_2}{\tau_3-\tau_1}\right), & \tau_2 \leq x \leq \tau_3 \\ w, & x = \tau_3 \\ w\left(\frac{\tau_4-x}{\tau_4-\tau_3}\right), & \tau_3 \leq x \leq \tau_4 \\ w\left(\frac{\tau_5-x}{\tau_5-\tau_4}\right), & \tau_4 \leq x \leq \tau_5 \\ 0, & x > \tau_5 \end{cases}$$

Definition 6 (Hexagonal Fuzzy Number)**.** The fuzzy set of six elements defined on the real line R, is called the generalized hexagonal fuzzy number of the form $\tilde{A} = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6; w)$, where $\tau_1 < \tau_2 < \tau_3 < \tau_4 < \tau_5 < \tau_6$, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq \tau_1 \\ \frac{1}{2}\left(\frac{x-\tau_1}{\tau_2-\tau_1}\right), & \tau_1 \leq x \leq \tau_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-\tau_2}{\tau_3-\tau_2}\right), & \tau_2 \leq x \leq \tau_3 \\ 1, & \tau_3 \leq x \leq \tau_4 \\ 1 - \frac{1}{2}\left(\frac{x-\tau_4}{\tau_5-\tau_4}\right), & \tau_4 \leq x \leq \tau_5 \\ \frac{1}{2}\left(\frac{\tau_6-x}{\tau_6-\tau_5}\right), & \tau_5 \leq x \leq \tau_6 \\ 0, & x \geq \tau_6 \end{cases}$$

3. PROPOSED FUZZIFICATION APPROACHES

In a real life environment with uncertainty, the data is observed as the approximate numbers. Such approximate numbers are probably represented in the form of intervals. In an uncertain environment, the cost or time of the assignment problem can only be observed in the form of interval numbers. So to solve such interval numbers based assignment problem using fuzzy set theory, it has to be converted as a fuzzy number of any one of the standard shape. In this section, we proposed

Let us denote the interval data as (l,u). The penta, hexa and hepta section of this interval is taken respectively as $d_1 = \frac{(u-l)}{5}$, $d_2 = \frac{(u-l)}{6}$, $d_3 = \frac{(u-l)}{7}$.

Therefore the trapezoidal, pentagonal and hexagonal fuzzy numbers will be respectively as of the symmetric forms

$$(l, l+2d_1, l+3d_1, u) \tag{1}$$

$$(l, l+2d_2, l+3d_2, l+4d_2, u) \tag{2}$$

$$(l, l+2d_3, l+3d_3, l+4d_3, l+5d_3, u) \tag{3}$$

4. PROPOSED CENTROIDS OF FUZZY NUMBERS

4.1. Gravity Point of Pentagonal Fuzzy Number.

The gravity point is the balancing point of the fuzzy numbers for defining ranking function to its ordering. In this section, we propose a new gravity point of pentagonal fuzzy number using the concept of Euler line of a triangle. Now, we proceed for calculating the gravity point of generalized pentagonal fuzzy number $\tilde{A} = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5; w)$ in the following way.

The pentagon of the pentagonal fuzzy number $\tilde{A} = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5; w)$ is divided into two plane figures. The two plane figures are triangle (BCD) and trapezoid (ABDE). In order to find the balancing point for defuzzification of the pentagonal fuzzy numbers, average of the balancing points of these two plane figures is to be evaluated for introducing proposed ranking function. The balancing of the triangle is evaluated by intersecting the Euler line and the median of the triangle BCD. The Euler Point of the triangle (BCD) is $\left(\frac{\tau_2+\tau_3+\tau_4}{3}, \frac{2\omega}{3}\right)$. It is denoted by

G'.

$$\text{(i.e) } G' = \left(\frac{\tau_2 + \tau_3 + \tau_4}{3}, \frac{2\omega}{3} \right)$$

For finding the balancing point of the trapezoid ABDE, the trapezoid is divided into three plane figures, triangle (ABP), rectangle (BDQP) and again triangle (DEQ). Subsequently, the centroid of the three plane figures ABP, BDQP and DEQ of the trapezoid ABDE are evaluated as $G_1 = \left(\frac{\tau_1+2\tau_2}{3}, \frac{\omega}{6} \right)$, $G_2 = \left(\frac{\tau_2+\tau_4}{2}, \frac{\omega}{4} \right)$ and $G_3 = \left(\frac{2\tau_4+\tau_5}{3}, \frac{\omega}{6} \right)$ respectively. These centroid points are non collinear. So the points might form a triangle. It is denoted by $\Delta G_1 G_2 G_3$. Furthermore, we find the Euler line of the triangle with the vertices G_1, G_2 and G_3 of the trapezoidal ABDE as

$$\begin{aligned} y = {} & \frac{5\omega}{24} - \frac{(-3\tau_2 + \tau_4 + 2\tau_5)(-2\tau_1 - \tau_2 + 3\tau_4)}{6\omega} \\ & + \left(\frac{\omega^2 - 12(-3\tau_2 + \tau_4 + 2\tau_5)(-2\tau_1 - \tau_2 + 3\tau_4)}{4\omega(\tau_1 - \tau_2 - \tau_4 + \tau_5)} \right) \left(x - \frac{\tau_1 + 2\tau_2 + 2\tau_4 + \tau_5}{6} \right) \end{aligned} \tag{4}$$

and the equation of the median for the trapezoid ABDE is

$$y = \frac{\omega}{2} - \left(\frac{\omega}{\tau_1 - \tau_2 - \tau_4 + \tau_5} \right) \left(x - \frac{\tau_2 + \tau_4}{2} \right) \tag{5}$$

These two lines are balancing lines of the trapezoid ABDE. So the intersection of these two lines will be the balancing point of trapezoid. It is derived as

$$\begin{aligned} G'' = \Bigg(& \frac{\omega^2(8\tau_1 + 7\tau_2 + 7\tau_4 + 8\tau_5) + 4(-3\tau_2 + \tau_4 + 2\tau_5)(-2\tau_1 - \tau_2 + 3\tau_4)(-2\tau_1 - 7\tau_2 - 7\tau_4 - 2\tau_5)}{6[5\omega^2 - 12(-3\tau_2 + \tau_4 + 2\tau_5)(-2\tau_1 - \tau_2 + 3\tau_4)]}, \\ & \frac{\omega}{2} - \frac{4\omega}{3} \left[\frac{\omega^2 - (-3\tau_2 + \tau_4 + 2\tau_5)(-2\tau_1 - \tau_2 + 3\tau_4)}{5\omega^2 - 12(-3\tau_2 + \tau_4 + 2\tau_5)(-2\tau_1 - \tau_2 + 3\tau_4)} \right] \Bigg) \end{aligned}$$

The average of these two balancing points G' and G'' of the plane figures BCD and ABDE of the pentagonal fuzzy number $\tilde{A} = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5; w)$ will be the better balancing point for defuzzication. It is evaluated as follows:

(6)

$$G=(x_0,y_0)=\left(\frac{\begin{array}{l}\omega^2(8\tau_1+17\tau_2+10\tau_3+17\tau_4+8\tau_5)\\+4(-3\tau_2+\tau_4+2\tau_5)\\(-2\tau_1-\tau_2+3\tau_4)\\(-2\tau_1-13\tau_2-6\tau_3-13\tau_4-2\tau_5)\end{array}}{\begin{array}{l}12\Big[5\omega^2-12(-3\tau_2+\tau_4+2\tau_5)\\(-2\tau_1-\tau_2+3\tau_4)\Big]\end{array}},\frac{\begin{array}{l}\omega\Big[27\omega^2-76(-3\tau_2+\tau_4+2\tau_5)\\(-2\tau_1-\tau_2+3\tau_4)\Big]\end{array}}{\begin{array}{l}12\Big[5\omega^2-12(-3\tau_2+\tau_4+2\tau_5)\\(-2\tau_1-\tau_2+3\tau_4)\Big]\end{array}}\right)$$

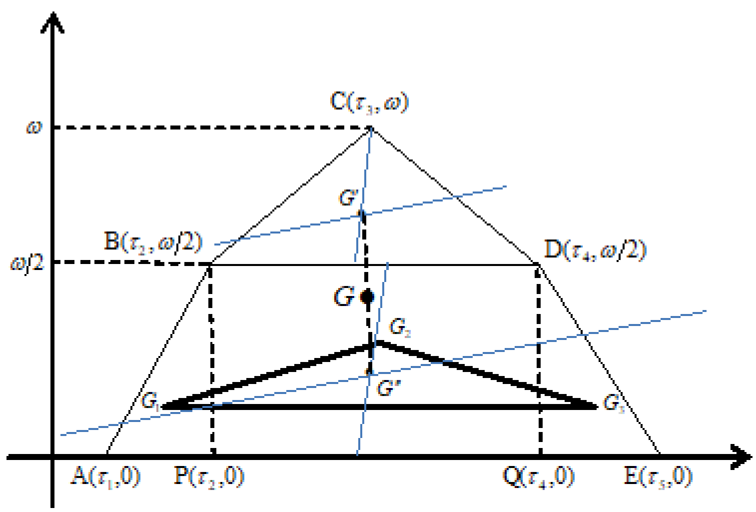

FIGURE 1. Centroid of Pentagonal Fuzzy Number $\tilde{A}=(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5;w)$.

5. Gravity Point of Hexagonal Fuzzy Number

Similarly, we proceed for calculating the gravity point of generalized hexagonal fuzzy number $\tilde{A}=(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5;w)$ in the following way. The hexagon of the hexagonal fuzzy number $\tilde{A}=(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5,\tau_6;\omega)$ is divided into two plane figures. The two plane figures are trapezoids ABEF & BCDE. In order to find the balancing point for defuzzification of the hexagonal fuzzy numbers, average of the balancing points of these two plane figures is to be evaluated for introducing proposed ranking function.

For finding the balancing point of the first trapezoid ABEF, the trapezoid is divided into three plane figures, triangle ABP, rectangle BESP and again triangle EFS. Subsequently, the centroid of the three plane figures ABP, BESP and EFS of the trapezoid ABEF are evaluated as $G_1' = \left(\frac{\tau_1+2\tau_2}{3}, \frac{\omega}{6}\right), G_2' = \left(\frac{\tau_2+\tau_5}{2}, \frac{\omega}{4}\right)$ and $G_3' = \left(\frac{2\tau_5+\tau_6}{3}, \frac{\omega}{6}\right)$ respectively. These centroid points are non collinear. So the points might form a triangle. It is denoted by $\Delta G_1' G_2' G_3'$. Furthermore, we find the Euler line of the triangle with the vertices G_1', G_2' and G_3' of the trapezoidal ABEF as

$$\begin{aligned} y = \frac{5\omega}{24} - \frac{(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)}{6\omega} \\ + \left(\frac{\omega^2 - 12(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)}{4\omega(\tau_1-\tau_2-\tau_5+\tau_6)}\right)\left(x - \frac{\tau_1+2\tau_2+2\tau_5+\tau_6}{6}\right) \end{aligned} \tag{7}$$

and the equation of the median for the trapezoid ABDE is

$$y = \frac{\omega}{2} - \left(\frac{\omega}{\tau_1-\tau_2-\tau_5+\tau_6}\right)\left(x - \frac{\tau_2+\tau_5}{2}\right) \tag{8}$$

These two lines are balancing lines of the trapezoid ABDE. So the intersection of these two lines will be the balancing point of trapezoid. It is derived as

$$G'' = \left(\frac{\omega^2(8\tau_1+7\tau_2+7\tau_5+8\tau_6) - 4(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)(2\tau_1+7\tau_2+7\tau_5+2\tau_6)}{6[5\omega^2 - 12(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)]}, \right.$$
$$\left. \frac{\omega}{2} - \frac{4\omega}{3}\left[\frac{\omega^2 - (-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)}{5\omega^2 - 12(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)}\right]\right)$$

For finding the balancing point of the second trapezoid BCDE, the trapezoid is divided into three plane figures, triangle (BCT), rectangle (CDUT) and again triangle (DEU). Subsequently, the centroid of the three plane figures BCT, CDUT and DEU of the trapezoid ABDE are evaluated as $G_1 = \left(\frac{\tau_1+2\tau_2}{3}, \frac{\omega}{6}\right), G_2 = \left(\frac{\tau_2+\tau_4}{2}, \frac{\omega}{4}\right)$ and $G_3 = \left(\frac{2\tau_4+\tau_5}{3}, \frac{\omega}{6}\right)$ respectively. These centroid points are non collinear. So the points might form a triangle. It is denoted by $\Delta G_1'' G_2'' G_3''$. Furthermore, we find the Euler line of the triangle with the vertices G_1'', G_2'' and G_3'' of the trapezoidal BCDE as

$$\begin{aligned} y = \frac{17\omega}{24} + \frac{(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)}{6\omega} \\ + \left(\frac{\omega^2 + 12(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)}{4\omega(\tau_2-\tau_3-\tau_4+\tau_5)}\right)\left(x - \frac{\tau_2+2\tau_3+2\tau_4+\tau_5}{6}\right) \end{aligned} \tag{9}$$

and the equation of the median for the trapezoid BCDE is

$$y=\frac{\omega}{2}-\left(\frac{\omega}{\tau_2-\tau_3-\tau_4+\tau_5}\right)\left(x-\frac{\tau_2+\tau_5}{2}\right) \tag{10}$$

These two lines are balancing lines of the trapezoid BCDE. So the intersection of these two lines will be the balancing point of trapezoid. It is derived as

$$G''=\left(\frac{\omega^2(8\tau_1+7\tau_3+7\tau_4+8\tau_5)+4(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)(2\tau_2+7\tau_3+7\tau_4+2\tau_5)}{6[5\omega^2+12(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)]},\right.$$
$$\left.\frac{\omega}{2}+\frac{7\omega}{6}\left[\frac{\omega^2+4(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)}{5\omega^2+12(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)}\right]\right)$$

The average of these two balancing points G' and G'' of the plane figures ABEF and BCDE of the hexagonal fuzzy number $\tilde{A}=(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5,\tau_6;\omega)$ will be the better balancing point for defuzzication. It is evaluated as follows:

(11)

$$G=(x_0,y_0)$$

$$=\left(\frac{\begin{bmatrix}5\omega^4(8\tau_1+15\tau_2+7\tau_3+7\tau_4+15\tau_5+8\tau_6)+4\omega^2\Big[(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)\\(24\tau_1+31\tau_2+35\tau_3+35\tau_4+31\tau_5+24\tau_6)-(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)\\(10\tau_1+59\tau_2+21\tau_3+21\tau_4+59\tau_5+10\tau_6)\Big]-48(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)\\(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)(2\tau_1+9\tau_2+7\tau_3+7\tau_4+9\tau_5+2\tau_6)\end{bmatrix}}{\begin{bmatrix}12\left[5\omega^2+12(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)\right]\\\left[5\omega^2-12(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)\right]\end{bmatrix}},\right.$$
$$\left.\frac{\begin{bmatrix}145\omega^5+404\omega^3\Big[(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)\\-(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)\Big]\\-1104\omega\Big[(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)\Big]\end{bmatrix}}{\begin{bmatrix}12\Big[5\omega^2+12(2\tau_2+\tau_3-3\tau_4)(-3\tau_3+\tau_4+2\tau_5)\Big]\\\Big[5\omega^2-12(-3\tau_2+\tau_5+2\tau_6)(-2\tau_1-\tau_2+3\tau_5)\Big]\end{bmatrix}}\right)$$

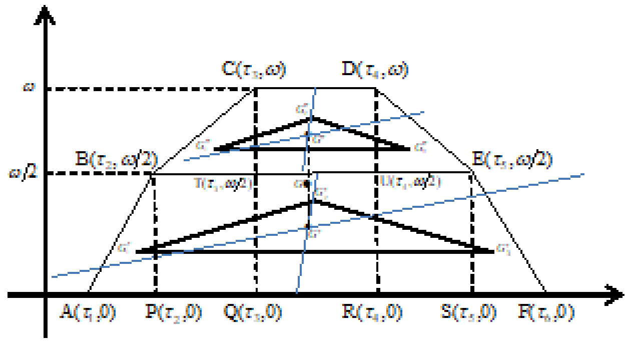

FIGURE 2. Centroid of Hexagonal Fuzzy Number $\tilde{A} = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6; \omega)$

Remark 1. The balancing point of trapezoidal fuzzy number $\tilde{A} = (\tau_1, \tau_2, \tau_3, \tau_4; w)$ based on Euler line of centroids introduced us [7] in 2017. It is given as follows:

$$G = (x_0, y_0) = \left(\frac{(2\tau_1+\tau_2-3\tau_3)(\tau_3-3\tau_2+2\tau_4)(2\tau_1+7\tau_2+7\tau_3+2\tau_4) + w^2(8\tau_1+7\tau_2+7\tau_3+8\tau_4)}{6[3(2\tau_1+\tau_2-3\tau_3)(\tau_3-3\tau_2+2\tau_4)+5w^2]}, \frac{7w}{3}\left(\frac{(2\tau_1+\tau_2-3\tau_3)(\tau_3-3\tau_2+2\tau_4)+w^2}{3(2\tau_1+\tau_2-3\tau_3)(\tau_3-3\tau_2+2\tau_4)+5w^2} \right) \right) \tag{12}$$

6. Ranking Function for Fuzzy Numbers

In the real world, most of the environments are fuzzy in nature. So the quantitative information cannot be observed in the form of crisp number. In such situations, the information is observed in the form of fuzzy number. But, the fuzzy numbers are of many shapes in fuzzy set theory. In optimization theory, many traditional crisp methods can be flexible for converting as fuzzy methods due to the difficulties in defining the unit values. So that the fuzzy problems are forced to convert as a crisp problems based on defuzzification techniques to find its optimum solutions using traditional crisp methods. For this purpose, the ranking functions have to be introduced for arriving a crisp numbers from fuzzy numbers. In this section, we introduce a ranking functions of various fuzzy quantities based on the distance between the respective proposed centroid points and the original point. It is defined and denoted as

$$R(\tilde{A}) = \sqrt{x_0^2 + y_0^2} \tag{13}$$

7. SINGLE AND MULTI OBJECTIVE INTERVAL DATA BASED ASSIGNMENT MODEL

In this section, we introduce the mathematical formulation of single and multi-objective Interval Data based Assignment Model.

In a general Assignment Problem, n jobs to be performed by n persons depending on their efficiency to do the job in one–one basis such that the assignment cost is minimum or maximum. If the objective of the assignment problems is to minimize only assignment cost or fuzzy time or fuzzy quantity or etc., then this type of fuzzy assignment problem is treated as single objective fuzzy assignment problem. Otherwise, if its objective is to minimize all the parameters such as cost, time, quantity and so on, then it will be treated as multi–objective interval based assignment problem.

Single and multi–objective interval data based assignment problems are represented explicitly as the following $n \times n$ fuzzy matrices respectively.

TABLE 1. Single Objective Interval Data based Assignment Model

	Jobs				
Workers	1	2	3	$\dots$	n
1	$[c^l_{11}, c^u_{11}]$	$[c^l_{12}, c^u_{12}]$	$[c^l_{13}, c^u_{13}]$	$\dots$	$[c^l_{1n}, c^u_{1n}]$
2	$[c^l_{21}, c^u_{21}]$	$[c^l_{22}, c^u_{22}]$	$[c^l_{23}, c^u_{23}]$	$\dots$	$[c^l_{2n}, c^u_{2n}]$
3	$[c^l_{31}, c^u_{31}]$	$[c^l_{32}, c^u_{32}]$	$[c^l_{33}, c^u_{33}]$	$\dots$	$[c^l_{3n}, c^u_{3n}]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$[c^l_{n1}, c^u_{n1}]$	$[c^l_{n2}, c^u_{n2}]$	$[c^l_{n3}, c^u_{n3}]$	$\dots$	$[c^l_{nn}, c^u_{nn}]$

Mathematically, the above single and multi-objective Assignment Problems can be stated respectively as

Single Objective Interval Data based Assignment Problem

$$\min z = \sum_{i=1}^{n} \sum_{j=1}^{n} [c^l_{ij}, c^u_{ij}] x_{ij}$$

TABLE 2. Multi Objective Fuzzy Assignment Model.

	Jobs			
Workers	1	2	$\dots$	n
1	$[c^l_{11}, c^u_{11}], [t^l_{11}, t^u_{11}]$, $[q^l_{11}, q^u_{11}], \dots$	$[c^l_{12}, c^u_{12}], [t^l_{12}, t^u_{12}]$, $[q^l_{12}, q^u_{12}], \dots$	$\dots$	$[c^l_{1n}, c^u_{1n}], [t^l_{1n}, t^u_{1n}]$, $[q^l_{1n}, q^u_{1n}], \dots$
2	$[c^l_{21}, c^u_{21}], [t^l_{21}, t^u_{21}]$, $[q^l_{21}, q^u_{21}], \dots$	$[c^l_{22}, c^u_{22}], [t^l_{22}, t^u_{22}]$, $[q^l_{22}, q^u_{22}], \dots$	$\dots$	$[c^l_{2n}, c^u_{2n}], [t^l_{2n}, t^u_{2n}]$, $[q^l_{2n}, q^u_{2n}], \dots$
3	$[c^l_{31}, c^u_{31}], [t^l_{31}, t^u_{31}]$, $[q^l_{31}, q^u_{31}], \dots$	$[c^l_{32}, c^u_{32}], [t^l_{32}, t^u_{32}]$, $[q^l_{32}, q^u_{32}], \dots$	$\dots$	$[c^l_{3n}, c^u_{3n}], [t^l_{3n}, t^u_{3n}]$, $[q^l_{3n}, q^u_{3n}], \dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$[c^l_{n1}, c^u_{n1}], [t^l_{n1}, t^u_{n1}]$, $[q^l_{n1}, q^u_{n1}], \dots$	$[c^l_{n2}, c^u_{n2}], [t^l_{n2}, t^u_{n2}]$, $[q^l_{n2}, q^u_{n2}], \dots$	$\dots$	$[c^l_{nn}, c^u_{nn}], [t^l_{nn}, t^u_{nn}]$, $[q^l_{nn}, q^u_{nn}], \dots$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = 1, j = 1, 2, \dots, n$$
$$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \dots, n$$

Multi Objective Interval Data based Assignment Problem

$$\min z = w_1 \sum_{i=1}^{n} \sum_{j=1}^{n} [c^l_{ij}, c^u_{ij}] x_{ij} + w_2 \sum_{i=1}^{n} \sum_{j=1}^{n} [t^l_{ij}, t^u_{ij}] x_{ij} + w_3 \sum_{i=1}^{n} \sum_{j=1}^{n} [q^l_{ij}, q^u_{ij}] x_{ij} + \cdots$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = 1, j = 1, 2, \dots, n$$
$$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \dots, n$$

and $w_1 + w_2 + w_3 + \dots = 1$, where $w_1, w_2, w_3, \dots$ are the weights of the objective which are given based on its priorities.

8. SINGLE AND MULTI OBJECTIVE FUZZY ASSIGNMENT MODEL

In this section, we introduce the mathematical formulation of single and multi-objective Fuzzy Assignment Model.

In a general Assignment Problem, n jobs to be performed by n persons depending on their efficiency to do the job in one–one basis such that the assignment cost is minimum or maximum. If the objective of the assignment problems is to minimize only fuzzy cost or fuzzy time or fuzzy quantity or etc., then this type of fuzzy assignment problem is treated as single objective fuzzy assignment problem. Otherwise, if its objective is to minimize all the fuzzy parameters such as fuzzy cost, fuzzy time, and fuzzy quantity and so on, then it will be treated as multi–objective fuzzy assignment problem.

Single and multi-objective fuzzy assignment problems are represented explicitly as the following $n \times n$ fuzzy matrices respectively.

TABLE 3. Single Objective Fuzzy Assignment Model

	Jobs				
Workers	1	2	3		n
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\tilde{c}_{13}$	$\dots$	$\tilde{c}_{1n}$
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\tilde{c}_{23}$	$\dots$	$\tilde{c}_{2n}$
3	$\tilde{c}_{31}$	$\tilde{c}_{32}$	$\tilde{c}_{33}$	$\dots$	$\tilde{c}_{3n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	$\tilde{c}_{n3}$	$\dots$	$\tilde{c}_{nn}$

TABLE 4. Multi Objective Fuzzy Assignment Model

	Jobs				
Workers	1	2	3		n
1	$\tilde{c}_{11},\tilde{t}_{11},\tilde{q}_{11},\dots$	$\tilde{c}_{12},\tilde{t}_{12},\tilde{q}_{12},\dots$	$\tilde{c}_{13},\tilde{t}_{13},\tilde{q}_{13},\dots$	$\dots$	$\tilde{c}_{1n},\tilde{t}_{1n},\tilde{q}_{1n},\dots$
2	$\tilde{c}_{21},\tilde{t}_{21},\tilde{q}_{21},\dots$	$\tilde{c}_{22},\tilde{t}_{22},\tilde{q}_{22},\dots$	$\tilde{c}_{23},\tilde{t}_{23},\tilde{q}_{23},\dots$	$\dots$	$\tilde{c}_{2n},\tilde{t}_{2n},\tilde{q}_{2n},\dots$
3	$\tilde{c}_{31},\tilde{t}_{31},\tilde{q}_{31},\dots$	$\tilde{c}_{32},\tilde{t}_{32},\tilde{q}_{32},\dots$	$\tilde{c}_{33},\tilde{t}_{33},\tilde{q}_{33},\dots$	$\dots$	$\tilde{c}_{3n},\tilde{t}_{3n},\tilde{q}_{3n},\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$\tilde{c}_{n1},\tilde{t}_{n1},\tilde{q}_{n1},\dots$	$\tilde{c}_{n2},\tilde{t}_{n2},\tilde{q}_{n2},\dots$	$\tilde{c}_{n3},\tilde{t}_{n3},\tilde{q}_{n3},\dots$	$\dots$	$\tilde{c}_{nn},\tilde{t}_{nn},\tilde{q}_{nn},\dots$

Mathematically, the above single and multi-objective Assignment Problems can be stated respectively as

Single Objective Fuzzy Assignment Problem

$$\min \tilde{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = 1, j = 1,2,\ldots,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2,\ldots,n$$

Multi Objective Fuzzy Assignment Problem

$$\min \tilde{z} = w_1 \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij} + w_2 \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{t}_{ij} x_{ij} + w_3 \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{q}_{ij} x_{ij} + \ldots\ldots$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = 1, j = 1,2,\ldots,n$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2,\ldots,n$$

and $w_1 + w_2 + w_3 + \ldots. = 1$, where $w_1, w_2, w_3, \ldots$ are the weights of the objective which are given based on its priorities.

9. Method for solving Interval Data based Assignment Problem

9.1 Algorithm

Step 1: Convert the given matrix of an interval data based assignment problem (IDAP) as a various matrices of fuzzy data-based assignment problems(FDAP_1, FDAP_2, …,FDAP_n) due to the different shapes of membership functions using fuzzification techniques.

Step 2: Convert the various fuzzy assignment problems (FDAP_1, FDAP_2, …,FDAP_n) obtained from step 1 into the crisp assignment problems (CAP_1, CAP_2, …, CAP_n) using method of defuzzification.

Step 3: If the crisp assignment problems are multi objective (i), then it should be converted as single objective crisp assignment problems. Otherwise, goto step 4.

Step 4: Solve the crisp assignment problems (CAP_1, CAP_2, …,CAP_n) using any one of the traditional methods like Hungarian Method, Simplex, etc. Optimum assignment values of the crisp assignment problems are denoted as $OV_1^i, OV_2^i, \ldots, OV_n^i$.

Step 5: Compare the optimum solutions obtained from various crisp assignment problems to find the minimum optimum assignment cost or time and its schedule for the given interval data based assignment problem

$$\text{i.e.,} \begin{pmatrix} \text{Optimum Assignment} \\ \text{Crisp Values for IBFAP} \end{pmatrix} = \min\left\{OV_1^i, \mathrm{OV}_2^i, \ldots, OV_n^i\right\} \tag{14}$$

Illustration 1

Suggest optimal assignment of the workers to jobs if the completion time (in hours) of different jobs by different workers is as given below:

	Workers			
Jobs	W	X	Y	Z
A	[4,12]	[0, 14]	[3,15]	[7,19]
B	[5,28]	[1,17]	[7,16]	[5,10]
C	[5,18]	[2,13]	[5,12]	[2,20]
D	[7,13]	[3,24]	[7,25]	[6,22]

Solution

Step 1: Given Interval based Assignment Problem is converted as following Fuzzy Assignment Problems using proposed fuzzification techniques (1), (2) and (3).

(4, 7.2, 8.8, 12)	(0, 5.6, 8.4, 14)	(3, 7.8, 10.2, 15)	(7, 11.8, 14.2, 19)
(5, 14.2, 18.8, 28)	(1, 7.4, 10.6, 17)	(7, 10.6, 12.4, 16)	(5, 7, 8, 10)
(5, 10.2, 12.8, 18)	(2, 6.4, 8.6, 13)	(5, 7.8, 9.2, 12)	(2, 9.2, 12.8, 20)
(7, 9.4, 10.6, 13)	(3, 11.4, 15.6, 24)	(7, 14.2, 17.8, 25)	(6, 12.4, 15.6, 22)

(4, 6.67, 8, 9.33, 12)	(0, 4.67, 7, 9.33, 14)	(3, 7, 9, 11, 15)	(7, 11, 13, 15, 19)
(5, 12.67, 16.5, 0.33, 28)	(1, 6.33, 9, 11.67, 17)	(7, 10, 11.5, 13, 16)	(5, 6.67, 7.5, 8.33, 10)
(5, 9.33, 11.5, 13.67, 18)	(2, 5.67, 7.5, 9.33, 13)	(5, 7.33, 8.5, 9.67, 12)	(2, 8, 11, 14, 20)
(7, 9, 10, 11, 13)	(3, 10, 13.5, 17, 24)	(7, 13, 16, 19, 25)	(6, 11.33, 14, 16.67, 22)

(4, 6.29, 7.43, 8.57, 9.71, 12)	(0, 4, 6, 8, 10, 14)	(3, 6.43, 8.14, 9.86, 11.57, 15)	(7, 10.43, 12.14, 13.86, 15.57, 19)
(5, 11.57, 14.86, 18.14, 21.43, 28)	(1, 5.57, 7.86, 10.14, 12.43, 17)	(7, 9.57, 10.86, 12.14, 13.43, 16)	(5, 6.43, 7.14, 7.86, 8.57, 10)
(5, 8.71, 10.57, 12.43, 14.29, 18)	(2, 5.14, 6.71, 8.29, 9.86, 13)	(5, 7, 8, 9, 10, 12)	(2, 7.14, 9.71, 12.29, 14.86, 20)
(7, 8.71, 9.57, 10.43, 11.29, 13)	(3, 9, 12, 15, 18, 24)	(7, 12.14, 14.71, 17.29, 19.86, 25)	(6, 10.57, 12.86, 15.14, 17.43, 22)

Step 2: The above Fuzzy Assignment Problems are converted respectively as following crisp assignment problems using Distance based Ranking Function (13) of Proposed Centroids (12), (6) and (11).

	Workers			
Jobs	W	X	Y	Z
A	8.0381	7.0432	9.0337	13.0234
B	16.5183	9.0336	11.5265	7.5414
C	11.5264	7.5404	8.5360	11.0275
D	10.0308	13.5224	16.0189	14.0216

	Workers			
Jobs	W	X	Y	Z
A	8.0174	7.0199	9.0155	13.0107
B	16.5084	9.0155	11.5121	7.5186
C	11.5121	7.5186	8.5164	11.0127
D	10.0139	13.5103	16.0087	14.0099

	Workers			
Jobs	W	X	Y	Z
A	8.0256	7.0291	9.0227	13.0157
B	16.5124	9.0227	11.5178	7.5274
C	11.5178	7.5272	8.5241	11.0186
D	10.0205	13.5151	16.0128	14.0146

Step 3: The above problems are single objective assignment problems. So the optimum solutions of the above problems are obtained using any one of the traditional method such as Hungarian Method as per step 4 as follows.

Solution of Crisp Assignment Problem 1:

	Workers			
Jobs	W	X	Y	Z
A	0.9949	[0]	0.9949	5.9802
B	8.9769	1.4922	2.9895	[0]
C	3.9860	0	[0]	3.4871
D	[0]	3.4916	4.9925	3.9908

The optimum assignment is

$$A \to X,\ B \to Z,\ C \to Y,\ D \to W$$

The minimum total time for the assignment schedule is

$= 7.0432 + 7.5414 + 8.5360 + 10.0308 = 33.1514.$

Solution of Crisp Assignment Problem 2:

	Workers			
Jobs	W	X	Y	Z
A	0.9975	[0]	0.9978	5.9908
B	8.9898	1.4969	2.9957	[0]
C	3.9935	0	[0]	3.4941
D	[0]	3.4964	4.9970	3.9960

The optimum assignment is

$$A \to X,\ B \to Z,\ C \to Y,\ D \to W$$

The minimum total time for the assignment schedule is

$= 7.0199 + 7.5186 + 8.5164 + 10.0139 = 33.0688.$

Solution of Crisp Assignment Problem 3:

	Workers			
Jobs	W	X	Y	Z
A	0.9965	[0]	0.9967	5.9866
B	8.9850	1.4953	2.9935	[0]
C	3.9906	0	[0]	3.4914
D	[0]	3.4946	4.9954	3.9941

The optimum assignment is

$$A \to X,\ B \to Z,\ C \to Y,\ D \to W$$

The minimum total time for the assignment schedule is

$= 7.0291 + 7.5274 + 8.5241 + 10.0205 = 33.1011.$

TABLE 5. Optimum Solutions of IDAP

	Optimum Solution of IDAP
Trapezoidal Fuzzy Number	33.1514
Pentagonal Fuzzy Number	33.0688
Hexagoal Fuzzy Number	33.1011

Step 4: After comparing the optimum solutions obtained from step 4, the better optimum solution of the given interval data based assignment problem is obtained using (14) as follows:

$$\begin{pmatrix} \text{Optimum Assignment} \\ \text{Cost or Time for IBFAP} \end{pmatrix} = \min\{33.1514, 33.0688, 33.1011\} = 33.0688.$$

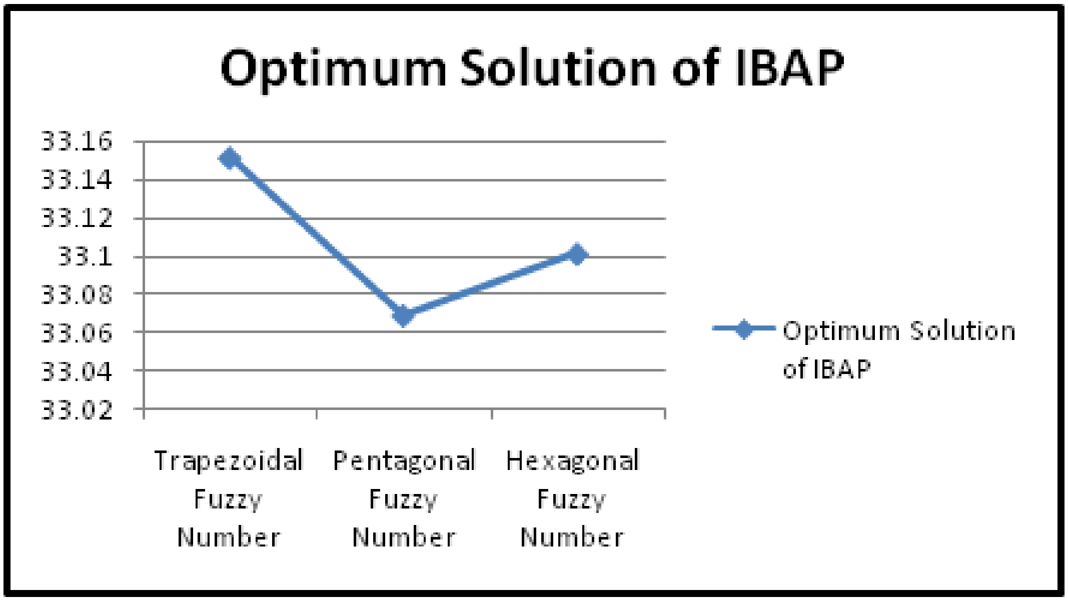

FIGURE 3. Optimum Solutions of Fuzzy Assignment Problems

Illustration 2

There are 4 jobs A, B, C and D and these are to be performed on four machine centres I, II, III and IV. One job is to be allocated to a machine centre, through each machine is capable of doing any job, different costs and time given by the matrix below:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{pmatrix} [13,17],[0,2] & [12,16],[3,5] & [10,14],[5,7] & [14,18],[2,4] \\ [21,25],[8,10] & [20,24],[5,9] & [23,27],[9,11] & [22,26],[8,10] \\ [29,33],[3,5] & [32,36],[4,6] & [30,34],[10,12] & [31,35],[6,8] \\ [19,23],[7,9] & [30,34],[6,8] & [42,46],[7,9] & [51,55],[4,6] \end{pmatrix} \end{array}$$

Find the optimum assignment that will result in minimum cost and time needed.

Solution Step 1: Given Interval data based Assignment Problem is converted as following

Fuzzy Assignment Problems using proposed fuzzification techniques (1), (2) and (3).

$$
\begin{array}{c}
\begin{array}{cccc} \quad\mathrm{I} & \quad\quad\mathrm{II} & \quad\quad\mathrm{III} & \quad\quad\mathrm{IV} \end{array} \\
\begin{array}{c} A \\ \\ B \\ \\ C \\ \\ D \\ \end{array}
\begin{pmatrix}
(13,14.6,15.4,17), & (12,13.6,14.4,16), & (10,11.6,12.4,14), & (14,15.6,16.4,18), \\
(0,0.8,1.2,2) & (3,3.8,4.2,5) & (5,5.8,6.2,7) & (2,2.8,3.2,4) \\
(21,22.6,23.4,25), & (20,21.6,22.4,24), & (23,24.6,25,4,27), & (22,23.6,24.4,26), \\
(8,8.8,9.2,10) & (5,5.6,7.4,9) & (9,9.8,10.2,11) & (8,8.8,9.2,10) \\
(29,30.6,31.4,33), & (32,33.6,34.4,36), & (30,31.6,32.4,34), & (31,32.6,33.4,35), \\
(3,3.8,4.2,5) & (4,4.8,5.2,6) & (10,10.8,11.2,12) & (6,6.8,7.2,8) \\
(19,20.6,21.4,23), & (30,31.6,32.4,34), & (42,43.6,44.4,46) & (51,52.6,52.4,55), \\
(7,7.8,8.2,9) & (6,6.8,7.2,8) & (7,7.8,8.2,9) & (4,4.8,5.2,6)
\end{pmatrix}
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cccc} \quad\mathrm{I} & \quad\quad\mathrm{II} & \quad\quad\mathrm{III} & \quad\quad\mathrm{IV} \end{array} \\
\begin{array}{c} A \\ \\ B \\ \\ C \\ \\ D \\ \end{array}
\begin{pmatrix}
(13,14.33,15,15.67,17), & (12,13.33,14,14.67,16), & (10,11.33,12,12.67,14) & (14,15.33,16,16.67,18), \\
(0,0.67,1,1.33,2) & (3,3.67,4,4.33,5) & (5,5.67,6,6.33,7) & (2,2.67,3,3.33,4) \\
(21,22.33,23,23.67,25), & (20,21.33,22,22.67,24), & (23,24.33,25,25.67,27), & (22,23.33,24,24.67,26), \\
(8,8.67,9,9.33,10) & (5,6.33,7,7.67,9) & (9,9.67,10,10.33,11) & (8,8.67,9,9.33,10) \\
(29,30.33,31,31.67,33), & (32,33.33,34,34.67,36), & (30,31.33,32,32.67,34), & (31,32.33,33,33.67,35), \\
(3,3.67,4,4.33,5) & (4,4.67,5,5.33,6) & (10,10.67,11,11.33,12) & (6,6.67,7,7.33,8) \\
(19,20.33,21,21.67,23), & (30,31.33,32,32.67,34), & (42,43.33,44,44.67,46), & (51,52.33,52,52.67,55), \\
(7,7.67,8,8.33,9) & (6,6.67,87,7.33,8) & (7,7.67,8,8.33,9) & (4,4.67,5,5.33,6)
\end{pmatrix}
\end{array}
$$

Step 2: The above Fuzzy Assignment Problems are converted respectively as following crisp assignment problems using distance based Ranking Function (13) of Proposed Centroids (12), (6) and (11).

$$
\begin{array}{c}
\begin{array}{cccc} \quad\mathrm{I} & \quad\quad\mathrm{II} & \quad\quad\mathrm{III} & \quad\quad\mathrm{IV} \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array}
\begin{pmatrix}
15.0211,1.3201 & 14.0226,4.0918 & 12.0263,6.0616 & 16.0198,3.1213 \\
23.0137,9.0412 & 22.0144,7.0450 & 25.0126,10.0371 & 24.0132,9.0412 \\
31.0102,4.0918 & 34.0093,5.0737 & 32.0099,11.0337 & 33.0096,7.0528 \\
21.0151,8.0463 & 32.0099,7.0528 & 44.0072,8.0463 & 53.0060,5.0737
\end{pmatrix}
\end{array}
$$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{pmatrix} 15.0093, 1.1321 & 14.0100, 4.0351 & 12.0116, 6.0234 & 16.0087, 3.0466 \\ 23.0061, 9.0156 & 22.0063, 7.0199 & 25.0056, 10.0141 & 24.0058, 9.0156 \\ 31.0045, 4.0351 & 34.0041, 5.0281 & 32.0044, 11.0128 & 33.0042, 7.0201 \\ 21.0066, 8.0176 & 32.0044, 7.0201 & 44.0032, 8.0176 & 53.0026, 5.0281 \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{pmatrix} 15.0138, 1.1989 & 14.0148, 4.0543 & 12.0173, 6.0363 & 16.0129, 3.0720 \\ 23.0090, 9.0243 & 22.0094, 7.0295 & 25.0083, 10.0218 & 24.0086, 9.0243 \\ 31.0067, 4.0543 & 34.0061, 5.0435 & 32.0065, 11.0199 & 33.0063, 7.0312 \\ 21.0099, 8.0273 & 32.0065, 7.0312 & 44.0047, 8.0273 & 53.0039, 5.0435 \end{pmatrix} \end{array}$$

Step 3: The above problems are multi objective assignment problems. So the problems have to be converted as single objective assignment problems as follows. Subsequently, optimum solutions are obtained using any one of the traditional method such as Hungarian Method as per step 4 as follows.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{pmatrix} 8.1706 & 9.0572 & 9.0440 & 9.5706 \\ 16.0275 & 14.5297 & 17.6918 & 16.5272 \\ 17.5510 & 19.5415 & 21.5218 & 20.0312 \\ 14.5307 & 19.5314 & 26.0268 & 29.0399 \end{pmatrix} \end{array}$$

Optimum assignments are

$$A \to III,\ B \to II,\ C \to IV,\ D \to I$$

The optimal value $= 9.0440 + 14.5297 + 20.0312 + 14.5307 = 58.1356$.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{pmatrix} 8.0707 & 9.0226 & 9.0175 & 9.5277 \\ 16.0109 & 14.5131 & 17.5099 & 16.5107 \\ 17.5198 & 19.5161 & 21.5086 & 20.0122 \\ 14.5121 & 19.5123 & 26.0104 & 29.0154 \end{pmatrix} \end{array}$$

Optimum assignments are

$$A \to III,\ B \to II,\ C \to IV,\ D \to I$$

The optimal value $= 9.0175 + 14.5131 + 20.0122 + 14.5121 = 58.0549$.

$$\begin{array}{c c}
 & \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 8.1064 & 9.0346 & 9.0268 & 9.5425 \\ 16.0167 & 14.5195 & 17.5151 & 16.5165 \\ 17.5305 & 19.5248 & 21.5132 & 20.0188 \\ 14.5186 & 19.5189 & 26.0160 & 29.0237 \end{pmatrix}
\end{array}$$

Optimum assignments are

$$A \to III,\ B \to II,\ C \to IV,\ D \to I$$

The optimal value $= 9.0268 + 14.5195 + 20.0188 + 14.5186 = 58.0837$.

TABLE 6. Optimum Solutions of IDMOAP

	Optimum Solution of IDMOAP
Trapezoidal Fuzzy Number	58.1356
Pentagonal Fuzzy Number	58.0549
Hexagoal Fuzzy Number	58.0837

Step 4: After comparing the optimum solutions obtained from step 4, the better optimum solution of the given interval data based assignment problem is obtained using (14) as follows:

$$\begin{pmatrix} \text{Optimum Assignment} \\ \text{Cost or Time for IVBMOFAP} \end{pmatrix} = \min\{58.1356, 58.0549, 58.0837\} = 58.0549.$$

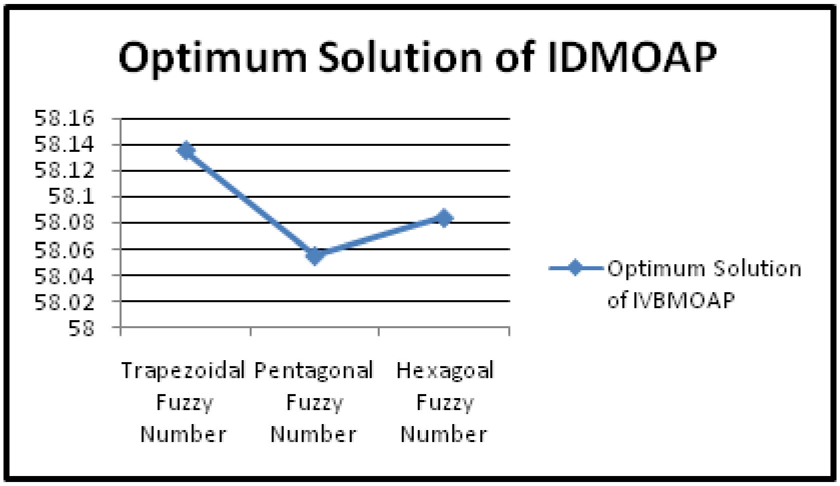

FIGURE 4. Optimum Solutions of Fuzzy Assignment Problems

10. CONCLUSION

This research work introduced an algorithm for solving IDAP using fuzzy set theory. The proposed algorithm has given the solution to the researchers for their confusion in selecting suitable shape of fuzzy quantity among its various shapes like trapezoidal, pentagonal and hexagonal etc., for obtaining the optimum solution of IDAP under fuzzy environment. The numerical illustrations have given the information that the pentagonal fuzzy quantity is more suitable for obtaining the better optimum solution of IDAP comparing with the other shapes of fuzzy quantities as per the proposed fuzzification and defuzzification techniques. Moreover, we came to know from this research that some other shape of fuzzy quantity also may give the better optimum solutions when we utilize some other fuzzification and defuzzification techniques.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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