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RELIABILITY ANALYSIS OF A COMPLEX REPAIRABLE SYSTEM IN SERIES CONFIGURATION WITH SWITCH AND CATASTROPHIC FAILURE USING COPULA REPAIR

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Abstract: This paper describes an analytical framework for in-depth investigation of a complex system consisting of two subsystems (namely L and M) in series configuration. Subsystem-L is composed of three identical units in parallel configuration that are working under 1-out-of-3: G policy, while subsystem-M has two non-identical units that are working under 1-out-of-2: G: policy. In subsystem-M, priority in operation is given to M1 unit whereas M2 unit put into cold standby mode if not in use. Moreover, both the subsystems are connected with controllers that may be perfect or imperfect at the time of need. We have considered a catastrophic failure due to frequent change in environmental conditions or man-made disruption. Failure rates of units in both the subsystems are constant and assumed to follow exponential distribution, but their repair supports two types of distributions namely general distribution and Gumbel-Hougaard family copula distribution. The system is studied by using the supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula to derive differential equations and obtain important reliability

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indexes such as availability of the system, reliability of the system and profit analysis. The results have shown by tables and graphs. Conclusive part have been discussed in the last section of this study.

Keywords: k-out-of-n: G system; availability; catastrophic failure; cost analysis; Gumbel-Hougaard family copula distribution.

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1. INTRODUCTION

The goal of reliability and availability is to estimate errors in measurement and to suggest ways of improving so that errors are minimized. They play substantial roles in product quality and services. While reliability is defined as the probability of a system or a component to perform its intended functions for a specified period of time, availability is usually concerned with repairable systems and is defined as the probability of the system to work at a specific time, apart from the crashes and repairs it had before. Redundant systems, which have been widely used in practice, such as space shuttles, communication satellites, a dishwasher, a hybrid car, a cargo ship, or a fighter plane are frequently discussed in research literature. Redundancy is a technique commonly used to improve system reliability and availability. It is used in the form of identical components connected in such a way that when one component fails, the others will keep the system functioning. In general, there are three types of standbys: (i) Cold standby in which the standby unit is only called upon when the primary or operating unit fails. In this inactive components have a zero failure rate and cannot fail while in standby state; (ii) Hot standby in which the standby unit has the same failure rate as when it is run with the operating unit; (iii) Warm standby in which the standby unit runs in the background of operating unit. It can fail in this state, but its failure rate is less than that of operating unit. Moreover, redundancy is highly cost effective in achieving a certain reliability level of the system. Therefore, in order to enhance reliability k-out-of-n system structure in which at least k components out of n must be functioning for the system to be operational play a vital role. In this, a series system can be regarded as an n-out-of-n: G system, while a parallel system as a 1-out-of-n: G system. k-out-of-n warm standby systems have found application in various fields including medical diagnosis, redundant-system testing, network design, power generation and transmission system and so on. Availability of the system, reliability of the system, mean time to failure and cost analysis studied by several authors for k-out-of-n: G redundant system under various conditions such as repairable system by Kullstam [9], redundancy optimization under common cause failure by Bai, Yun and Chung [3], consecutive k-out-of-n using r repairman by Wu and Guan [25], two stage weighted with components in common by Chen and Yang [24], warm standby system with two category of units by Zhang, Xie and Horigome [15], generalized multi-state system by Zuo and Tian [6], single unit M|G|1 system model with helping unit by Kumar and Gupta [8], exact reliability formula for consecutive system by Liang, Xiong and Li [23], non-identical components considering shut-off rules using quasi-birth-death process by Moghaddass, Zuo and Wang [12], with and without repair with three failure modes by Kumar and Sirohi [7], real example of sliding window system by Levitin and Dai [4], generalized block replacement policy with respect to a threshold number of failed components and risk costs by Park and Pham [5], full system equipped with a single warm standby component by Erylmaz [14] and standby with multiple working vacations by Sharma and Kumar [13].

Initially reliability models were based on the assumption that only one k-out-of-n system, in which all units are arranged in parallel, is possible for any complex engineering system. However, there are many situations when two or more k-out-of-n type system in series configuration is possible, and, when this happen, the results are awesome in case of reliability. In such cases, we can divide the complete system into two subsystems in series configuration. Many researchers have extensively studied the use of series systems. Bao and Cui [21] studied a series Markov repairable system and presented availability indices as measure of reliability using time interval omission problem concept. Singh, Ram and Rawal [18] presented a novel method for cost analysis of an engineering system, which consists of two subsystems, viz. subsystem-1 and subsystem-2 with controllers in series. Subsystem-1 works under the k-out-of-n: good policy and subsystem-2 consists of three identical units in parallel configuration that connected in series. The controllers control both the subsystems and the operator may fail the system deliberately. The system is studied by supplementary variable technique and Laplace transforms. Jia, Shen and Xing [22] studied two-unit series multistate Markov repairable systems with repair time omission is developed based on a model for single-unit multistate Markov repairable systems. It is worth noticing that we may employ general repair if the system is in operation and running under minor or major partial failure mode, but whenever the system is in complete failure mode, the system is to be repaired using copula distribution. Singh and Rawal [16] evaluated reliability characteristics of two subsystems in a series configuration under different failure and repair discipline with controllers using Gumbel-Hougaard family copula distribution. Lado and Singh [1] proposed a series system with two subsystems operated by human operator. In this, each subsystem has two identical units in parallel. The paper has studied via two types of repair viz. copula repair and general repair and concluded that copula repair is more reliable compare to general repair. Singh and Poonia [17] studied two-non identical unit system by regenerative point technique using correlation concept. Singh, Poonia and Abdullahi [19] studied a complex engineering system in series using copula repair and catastrophic failure. Singh, Poonia and Rawal [20] analyzed a computer network using copula repair under 2-out-of-3: G policy and evaluated various reliability physiognomies. Some recently published articles under copula methodology and catastrophic failure may be seen in Dhruv et al. [2], Sirohi and Poonia [10], and Poonia, Sirohi and Kumar [11].

However, situations in the real world are increasingly complicated, which cannot be covered by simple engineering models. Therefore, in this paper, a complex system consisting of two subsystems (namely L and M) in series configuration is studied. Subsystem-L is composed of three identical units in parallel configuration that are working under 1-out-of-3: G policy, while subsystem-M has two non-identical units that are working under 1-out-of-2: G: policy. In subsystem-M, priority in operation is given to M1 unit whereas M2 unit put into cold standby mode if not in use. Then, based on the behavior of the whole system, all the system states can also be classified into three subsets as follows.

Classification I: The system operates perfectly; in this situation, all the components in both the subsystems are in the perfect functioning state.

Classification II: The system is partially working; in this situation, at least one component in one or both the subsystems is in failure state, and the remainder is perfect functioning.

Classification III: The system is completely failed; in this situation, either subsystem L or M is in the complete failure state. Further, system may be completely failed due to controller (s) or catastrophic failure.

Therefore, the system remains working until one of the subsystems is completely failed. Based on all the assumptions above, the system could be modeled by a continuous-time stochastic process. Some reliability indexes, for example reliability of the system, availability of the system, and cost analysis are obtained using supplementary variable technique, Laplace transforms and copula repair. The remainder of this paper is organized as follows. In Section 2, notations, assumptions for the model, system configuration, transition diagram and state description of the system is given. In section 3, we have developed mathematical modelling using differential equation. In section 4, some reliability indexes, such as the availability, reliability, and cost analysis, are obtained for systems for various values of failure and repair rates. Finally, the discussions and conclusions are given in Section-5. Explicit expressions for reliability, availability, and cost analysis functions are obtained with help of Maple-17. Tables and graphs present a comparative analysis of results. The system configuration and transition state diagram of the designed model are shown in fig 1(a) and 1(b) respectively.

2. NOTATION, ASSUMPTION AND DESCRIPTION OF STATES

2.1 NOTATIONS

s, t	Laplace transform / Time scale variable
$\lambda_1 / \mu_1(x)$	Failure rate / Repair rate of each unit in subsystem-1.
λ_2, λ_3	Failure rate of first / second unit in subsystem-2.
$\mu_2(x) / \mu_3(x)$	Repair rate of first / second unit in subsystem-2.

λ_E Deliberate failure rate when two units in subsystem-1 and first unit in subsystem-2 failed.

$\lambda_{S1} / \lambda_{S2}$ Failure rate of control device between units for subsystem-1/subsystem-2.

λ_C Failure rate related to catastrophic failure mode.

$P_0(t)$ The state transition probability that the system is in S_i state at an instant for $i = 0$.

$\bar{P}(s)$ Laplace transformation of the state transition probability $P(t)$.

$P_i(x,t)$ The Probability that the system is in state S_i for $i = 1$ to 12 and the system is under repair with elapsed repair time is x, t. x is repaired variable and t is time variable.

$E_p(t)$ Expected profit in the interval $[0,t)$.

K_1, K_2 Revenue generated and service cost per unit time respectively.

$\mu_0(x)$ An expression of the joint probability from failed state S_i to good state S_0 according to Gumbel-Hougaard family copula is given as $\mu_0(x) = C_\theta\{u_1(x), u_2(x)\} = \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}$ where $u_1(x) = \phi(x)$ and $u_2(x) = e^x$. Here θ is the parameter $1 < \theta < \infty$.

2.2 ASSUMPTION

The following assumptions have been made throughout the study of the model:

1. Initially the system is in state S_0, and all the units of subsystem-L and M are in good working conditions.
2. The subsystem-L works successfully if minimum one unit is in good working condition i.e. 1-out-of-3: G policy.
3. The subsystem-M having two unit's namely main unit and cold standby in parallel configuration. It works successfully if at least one unit is operating. Main unit is more efficient

so preference in operation will be given to it as compared to cold standby unit. The activation time for cold standby unit is negligible.

4. As soon as repair of a unit in subsystem-L completed, it again becomes operational (as good as new), while in subsystem-M main unit replaces standby unit. The replaced unit is in cold standby again if the system can function normally. No damage reported due to repair of the system.
5. Whenever there is a failure in two units of subsystem-L and main unit in subsystem-M, the system goes to perilous state where system has to stop functioning deliberately to avoid further failures with emergency failure rate λ_E .
6. One repairperson is available full time with the system and may be called as soon as the system reaches to partially or completely failed state.
7. All failure rates are constant and follows the exponential distribution.
8. The failure rate and repair rate of each unit in subsystem-L is same, while in subsystem-M, it is different for both the units.
9. Both the subsystems are connected via controllers, which in the system is unreliable at the time of need, and the function of the switch is: "as long as the switch fails, the whole system fails immediately".
10. The complete failed system needs repair immediately. For this Gumbel-Hougaard, family of copula can be employed to restore the system.

2.3 SYSTEM CONFIGURATION AND STATE TRANSITION DIAGRAM

System configuration shown in Fig 1 (a) while transition diagram in Fig 1 (b). In transition diagram, S_0 is perfect state, S_1, S_2, S_3, S_4 and S_5 partial failed/degraded and S_6, S_7, S_8, S_9, S_{10}, S_{11} and S_{12} are complete failed states. Due to failure in any unit in the subsystem 1 and in subsystem 2, the transitions approaches to partially failed states S_1, S_2, S_3 S_4 and S_5 respectively. The state S_6, S_7 and S_8 are complete failed states due to failure of units in both the subsystems, while S_9 is completely failed state due to deliberate failure. The states S_{10} and S_{11} are complete failed states due to controller and S_{12} due to catastrophic failure.

Figure 1 (a) System configuration

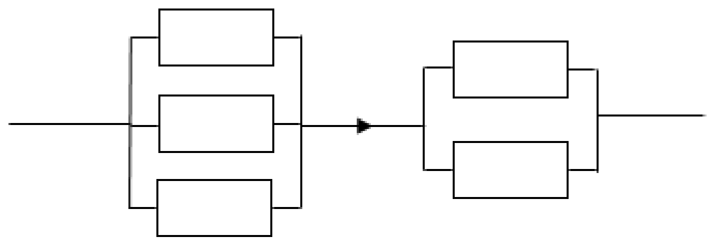
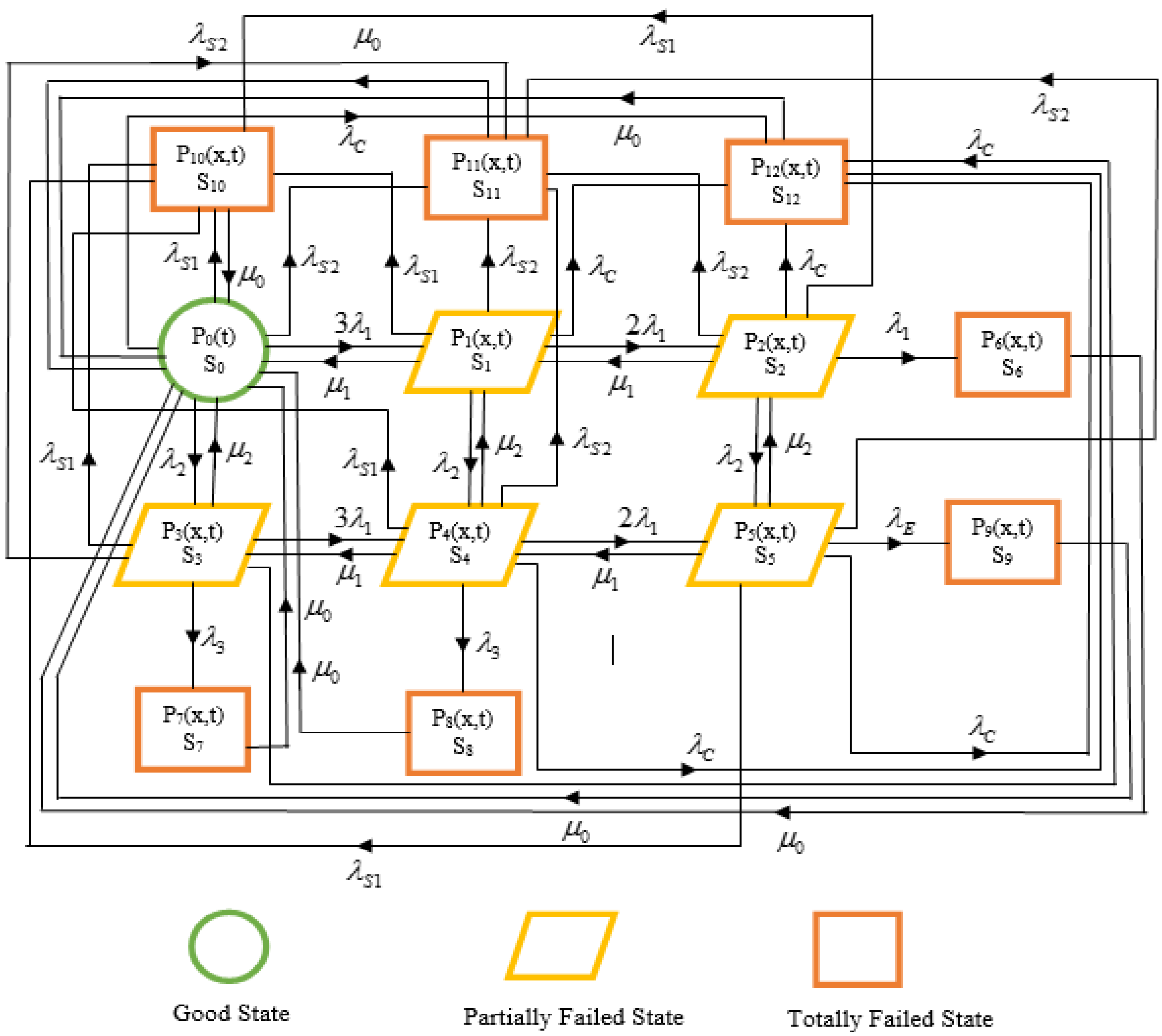


Figure 1 (b) State transition diagram of the model



2.4 STATE DESCRIPTION OF THE SYSTEM

The state description of the model highlights that S_0 is a state where both the subsystems are in good working condition. S_1, S_2, S_3, S_4 and S_5 are the states where the system is in partially failure mode and the repair is employed. States S_6, S_7, S_8, S_9, S_{10}, S_{11} and S_{12} are the states where the system is in the totally failure mode. Repair is being applied using Gumbel-Hougaard family copula distribution.

Table 1 State Description

State	Description
S_0	This is a perfect state and all units of subsystem-1 and subsystem-2 are in good working condition.
S_1	The indicated state is degraded but is in operational mode after the failure of the any one unit in subsystem-1 but both units of subsystem-2 are in the good operational state. The system is under repair.
S_2	The indicated state is degraded but is in operational mode after the failure of the any two units in subsystem-1 but both units of subsystem-2 are in the good operational state. The system is under repair.
S_3	The indicated state is degraded but is in operational mode after the failure of the first unit in subsystem-2 and all units of subsystem-1 are in the good operational state. The system is under repair.
S_4	The indicated state is degraded but is in operational mode after the failure of any one unit in subsystem-1 and first unit of subsystem-2. The system is under repair.
S_5	The indicated state is degraded but is in operational mode after the failure of any two units in subsystem-1 and first unit of subsystem-2. The system is under repair.
S_9	The states represent that the system is in complete failure mode due to deliberate failure and the system is under repair using Gumbel-Hougaard family copula distribution.
S_6, S_7 S_8, S_{10} S_{11}, S_{12}	The states represent that the system is in completely failure mode and the system is under repair using Gumbel-Hougaard family copula distribution.

3. FORMULATION OF MATHEMATICAL MODEL

By probability of considerations and continuity arguments, we can obtain the set of difference-differential equations associated with the present mathematical model. The state transition probability of the system are calculated under the presumption that the system is in state S_0, will remain in the state S_0 during the time $[t, t+\Delta t]$ and it will not move to any other state and if it in failed state then after repair it will approach to state S_0. If the failure rate to move the state S_1, S_3, S_{10}, S_{11} and S_{12} during the time $[t, t+\Delta t]$ is $3\lambda_1\Delta t,\ \lambda_2\Delta t,\ \lambda_{S_1}\Delta t,\ \lambda_{S_2}\Delta t$ and $\lambda_C\Delta t$, then the rate that it will not move to the states will be $(1-3\lambda_1\Delta t),\ (1\text{-}\lambda_2\Delta t),\ (1\text{-}\lambda_{S_1}\Delta t),\ (1\text{-}\lambda_{S_2}\Delta t)$ and $(1\text{-}\lambda_C\Delta t)$. The state transition probability that the system is in state S_0 during t and $[t+\Delta t]$ is

$$P_0(t+\Delta t)=(1-3\lambda_1\Delta t)(1\text{-}\lambda_2\Delta t)\left(1\text{-}\lambda_{S_1}\Delta t\right)\left(1\text{-}\lambda_{S_2}\Delta t\right)(1\text{-}\lambda_C\Delta t)P_0(t)+\left[\int_0^\infty \mu_1(x)P_1(x,t)\,dx\Delta t\right.$$

$$+\int_0^\infty \mu_2(x)P_3(x,t)\,dx\Delta t+\int_0^\infty \mu_0(x)P_6(x,t)\,dx\Delta t+\int_0^\infty \mu_0(x)P_7(x,t)\,dx\Delta t+\int_0^\infty \mu_0(x)P_8(x,t)\,dx\Delta t$$

$$\left.+\int_0^\infty \mu_0(x)P_9(x,t)\,dx\Delta t+\int_0^\infty \mu_0(x)P_{10}(x,t)\,dx\Delta t+\int_0^\infty \mu_0(x)P_{11}(x,t)\,dx\Delta t+\int_0^\infty \mu_0(x)P_{12}(x,t)\,dx\Delta t\right]$$

$$P_0(t+\Delta t)=\left\{\left(1-3\lambda_1-\lambda_2-\lambda_{s_1}-\lambda_{s_2}-\lambda_C\right)\Delta t+(\text{Product of two terms})(\Delta t)^2+\ldots\right\}P_0(t)+$$

$$\left[\int_0^\infty \mu_1(x)P_1(x,t)\,dx\Delta t+\int_0^\infty \mu_2(x)P_3(x,t)\,dx\Delta t+\sum_k\int_0^\infty \exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}P_k(x,t)\,dx\{k=6\text{ to }12\}\right.$$

$$\lim_{\Delta t\to 0}\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t}+\left(3\lambda_1+\lambda_2+\lambda_{s_1}+\lambda_{s_2}+\lambda_C\right)P_0(t)=\left[\int_0^\infty \mu_1(x)P_1(x,t)\,dx+\int_0^\infty \mu_2(x)P_3(x,t)\,dx\right.$$

$$+\sum_k\int_0^\infty \exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}P_k(x,t)\,dx\{k=6,7,8,9,10,11,12\}$$

$$\left[\frac{\partial}{\partial t}+3\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right]P_0(t)=\int_0^\infty \mu_1(x)P_1(x,t)\,dx+\int_0^\infty \mu_2(x)P_3(x,t)\,dx$$

$$+\sum_k\int_0^\infty \exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}P_k(x,t)\,dx\{k=6,7,8,9,10,11,12\} \tag{1}$$

Similarly,

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)\right]P_1(x,t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)\right]P_2(x,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_2(x)\right]P_3(x,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)+\mu_2(x)\right]P_4(x,t)=0 \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_E+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)+\mu_2(x)\right]P_5(x,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp\left[x^\theta+\left\{\log\phi(x)\right\}^\theta\right]^{1/\theta}\right]P_k(x,t)=0\ \ \{k=6,7,8,9,10,11,12\} \tag{7}$$

Boundary conditions

$$P_1(0,t)=3\lambda_1P_0(t) \tag{8}$$

$$P_2(0,t)=2\lambda_1P_1(0,t)=6\lambda_1^2P_0(t) \tag{9}$$

$$P_3(0,t)=\lambda_2P_0(t) \tag{10}$$

$$P_4(0,t)=3\lambda_1P_3(0,t)+\lambda_2P_1(0,t)=6\lambda_1\lambda_2P_0(t) \tag{11}$$

$$P_5(0,t)=2\lambda_1P_4(0,t)+\lambda_2P_2(0,t)=18\lambda_1^2\lambda_2P_0(t) \tag{12}$$

$$P_6(0,t)=\lambda_1P_2(0,t)=6\lambda_1^3P_0(t) \tag{13}$$

$$P_7(0,t)=\lambda_3P_3(0,t)=\lambda_2\lambda_3P_0(t) \tag{14}$$

$$P_8(0,t)=\lambda_3P_4(0,t)=6\lambda_1\lambda_2\lambda_3P_0(t) \tag{15}$$

$$P_9(0,t)=\lambda_EP_5(0,t)=18\lambda_1^2\lambda_2\lambda_EP_0(t) \tag{16}$$

$$P_{10}(0,t)=\lambda_{S_1}\left[P_0(t)+P_1(0,t)+P_2(0,t)+P_3(0,t)+P_4(0,t)+P_5(0,t)\right] \tag{17}$$

$$P_{11}(0,t)=\lambda_{S_2}\left[P_0(t)+P_1(0,t)+P_2(0,t)+P_3(0,t)+P_4(0,t)+P_5(0,t)\right] \tag{18}$$

$$P_{12}(0,t)=\lambda_C\left[P_0(t)+P_1(0,t)+P_2(0,t)+P_3(0,t)+P_4(0,t)+P_5(0,t)\right] \tag{19}$$

Initial conditions

$P_0(0)=1$, and other state probabilities are zero at $t=0$ (20)

Solution of the model

Taking Laplace transformation of equations (1) to (19) and using equation (20), we obtain

$$\left[s+3\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right]\bar{P}_0(s)=1+\int_0^\infty \mu_1(x)\bar{P}_1(x,s)\,dx+\int_0^\infty \mu_2(x)\bar{P}_3(x,s)\,dx$$

$$+\sum_k \int_0^\infty \exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}\bar{P}_k(x,s)\,dx\{k=6,7,8,9,10,11,12\} \quad (21)$$

$$\left[s+\frac{\partial}{\partial x}+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)\right]\bar{P}_1(x,s)=0 \quad (22)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)\right]\bar{P}_2(x,s)=0 \quad (23)$$

$$\left[s+\frac{\partial}{\partial x}+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_2(x)\right]\bar{P}_3(x,s)=0 \quad (24)$$

$$\left[s+\frac{\partial}{\partial x}+2\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)+\mu_2(x)\right]\bar{P}_4(x,s)=0 \quad (25)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_E+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)+\mu_2(x)\right]\bar{P}_5(x,s)=0 \quad (26)$$

$$\left[s+\frac{\partial}{\partial x}+\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}\right]\bar{P}_k(x,s)=0 \ \{k=6,7,8,9,10,11,12\} \quad (27)$$

Boundary conditions

$$\bar{P}_1(0,s)=3\lambda_1\bar{P}_0(s) \quad (28)$$

$$\bar{P}_2(0,s)=2\lambda_1\bar{P}_1(0,s)=6\lambda_1^2\bar{P}_0(s) \quad (29)$$

$$\bar{P}_3(0,s)=\lambda_2\bar{P}_0(s) \quad (30)$$

$$\bar{P}_4(0,s)=3\lambda_1\bar{P}_3(0,s)+\lambda_2\bar{P}_1(0,s)=6\lambda_1\lambda_2\bar{P}_0(s) \quad (31)$$

$$\bar{P}_5(0,s)=2\lambda_1\bar{P}_4(0,s)+\lambda_2\bar{P}_2(0,s)=18\lambda_1^2\lambda_2\bar{P}_0(s) \quad (32)$$

$$\bar{P}_6(0,s)=\lambda_1\bar{P}_2(0,s)=6\lambda_1^3\bar{P}_0(s) \tag{33}$$

$$\bar{P}_7(0,s)=\lambda_3\bar{P}_3(0,s)=\lambda_2\lambda_3\bar{P}_0(s) \tag{34}$$

$$\bar{P}_8(0,s)=\lambda_3\bar{P}_4(0,s)=6\lambda_1\lambda_2\lambda_3\bar{P}_0(s) \tag{35}$$

$$\bar{P}_9(0,s)=\lambda_E\bar{P}_5(0,s)=18\lambda_1^2\lambda_2\lambda_E\bar{P}_0(s) \tag{36}$$

$$\bar{P}_{10}(0,s)=\lambda_{S_1}\left[1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right]\bar{P}_0(s) \tag{37}$$

$$\bar{P}_{11}(0,s)=\lambda_{S_2}\left[1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right]\bar{P}_0(s) \tag{38}$$

$$\bar{P}_{12}(0,s)=\lambda_C\left[1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right]\bar{P}_0(s) \tag{39}$$

Laplace transformation of boundary conditions after repair.

$$\bar{P}_1(0,s)=3\lambda_1\bar{P}_0(s)+\int_0^\infty\mu_1(x)\bar{P}_2(x,s)dx+\int_0^\infty\mu_2(x)\bar{P}_4(x,s)dx \tag{40}$$

$$\bar{P}_2(0,s)=2\lambda_1\bar{P}_1(x,s)+\int_0^\infty\mu_2(x)\bar{P}_2(x,s)dx \tag{41}$$

$$\bar{P}_3(0,s)=\lambda_2\bar{P}_0(s)+\int_0^\infty\mu_1(x)\bar{P}_4(x,s)dx \tag{42}$$

$$\bar{P}_4(0,s)=3\lambda_1\bar{P}_3(x,s)+\lambda_2\bar{P}_1(x,s)+\int_0^\infty\mu_1(x)\bar{P}_5(x,s)dx \tag{43}$$

No change noticed for the rest conditions.

Now solving all the equations with the boundary conditions, one may get

$$\bar{P}_0(s)=\frac{1}{D(s)} \tag{44}$$

$$\bar{P}_1(s)=\frac{3\lambda_1}{D(s)}\frac{1-\bar{S}_{\mu_1}\left(s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{\left(s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}=\frac{3\lambda_1}{D(s)}\frac{1-P}{\left(s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)} \tag{45}$$

$$\bar{P}_2(s)=\frac{6\lambda_1^2}{D(s)}\frac{1-\bar{S}_{\mu_1}\left(s+\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{\left(s+\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)} \tag{46}$$

$$\bar{P}_3(s)=\frac{\lambda_2}{D(s)}\frac{1-\bar{S}_{\mu_2}\left(s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C}=\frac{\lambda_2}{D(s)}\frac{1-Q}{s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C} \tag{47}$$

$$\bar{P}_4(s)=\frac{6\lambda_1\lambda_2}{D(s)}\frac{1-\bar{S}_{\mu_3}\left(s+2\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{s+2\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C} \tag{48}$$

$$\bar{P}_5(s)=\frac{18\lambda_1^2\lambda_2}{D(s)}\frac{1-\bar{S}_{\mu_3}\left(s+\lambda_E+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{s+\lambda_E+\lambda_{S_1}+\lambda_{S_2}+\lambda_C} \tag{49}$$

$$\bar{P}_6(s)=\frac{6\lambda_1^3}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s} \tag{50}$$

$$\bar{P}_7(s)=\frac{\lambda_2\lambda_3}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s} \tag{51}$$

$$\bar{P}_8(s)=\frac{6\lambda_1\lambda_2\lambda_3}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s} \tag{52}$$

$$\bar{P}_9(s)=\frac{18\lambda_1^2\lambda_2\lambda_E}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s} \tag{53}$$

$$\bar{P}_{10}(s)=\frac{\lambda_{S_1}}{D(s)}\left[1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right]\frac{1-R}{s}=\frac{\lambda_{S_1}}{D(s)}\frac{U(1-R)}{s} \tag{54}$$

$$\bar{P}_{11}(s)=\frac{\lambda_{S_2}}{D(s)}\left[1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right]\frac{1-R}{s}=\frac{\lambda_{S_2}}{D(s)}\frac{U(1-R)}{s} \tag{55}$$

$$\bar{P}_{12}(s)=\frac{\lambda_{S_2}}{D(s)}\left[1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right]\frac{1-R}{s}=\frac{\lambda_{S_2}}{D(s)}\frac{U(1-R)}{s} \tag{56}$$

where $D(s)=s+3\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C-3\lambda_1P-\lambda_2Q-RU$

$$P=\bar{S}_{\mu_1}\left(s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)=\frac{\mu_1}{s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1}$$

$$Q=\bar{S}_{\mu_2}\left(s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)=\frac{\mu_2}{s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_2}$$

$$R=\bar{S}_{\mu_0}(s)=\frac{\mu_0}{s+\mu_0}$$

and $U = 6\lambda_1^3 + \lambda_2\lambda_3 + 6\lambda_1\lambda_2\lambda_3 + 18\lambda_1^2\lambda_2\lambda_E + \left(1 + 3\lambda_1 + \lambda_2 + 6\lambda_1\lambda_2 + 6\lambda_1^2 + 18\lambda_1^2\lambda_2\right)\left(\lambda_{S_1} + \lambda_{S_2} + \lambda_C\right)$

Sum of Laplace transformations of the state transitions, where the system is in operational mode and failed state at any time, is as follows

$$\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) + \bar{P}_3(s) + \bar{P}_4(s) + \bar{P}_5(s) \quad (57)$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s) \quad (58)$$

4. Analytical Study

4.1 AVAILABILITY ANALYSIS

When repair follows general and Gumbel-Hougaard family copula distribution, we have

$$\bar{S}_{\mu_0}(s) = \bar{S}_{\exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}}(s) = \frac{\exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}}{s + \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}} = \frac{\mu_0(x)}{s + \mu_0(x)}$$

setting $\bar{S}_{\alpha_i}(s) = \frac{\alpha_i}{s + \alpha_i}, i = 1, 2, 3$ and $\bar{S}_\phi(s) = \frac{\phi}{s + \phi}$. Taking the values of different parameters as

$\lambda_1 = 0.030, \lambda_2 = 0.035, \lambda_3 = 0.040, \lambda_E = 0.50, \lambda_{S_1} = 0.021, \lambda_{S_2} = 0.022, \lambda_C = 0.035, \theta = 1, x = 1, \mu_i = 1$

$(i = 1, 2, 3)$ in (57), then taking inverse Laplace transform, we obtain the availability of the system.

Here we have considered following particular cases:

(a) Availability of the system when failure rates follow exponential distribution and repair follows two types of distribution general distribution and Gumbel–Hougaard family copula distribution.

$$P_{up}(t) = 0.000401e^{-1.1780t} + 0.033716e^{-2.8150t} - 0.025111e^{-1.2853t} - 0.001537e^{-1.1956t}$$
$$+0.993857e^{-0.0060t} - 0.001165e^{-1.1430t} - 0.000161e^{-1.1280t} \quad (59)$$

(b) Availability of the system when failure rates for both the units in subsystem 2 are same and follow exponential distribution.

$$P_{up}(t) = -0.000781e^{-1.1780t} - 0.002097e^{-1.1730t} + 0.035638e^{-2.8219t} - 0.035185e^{-1.3309t}$$
$$-0.000028e^{-1.2051t} + 1.002818e^{-0.0089t} - 0.000364e^{-1.1280t} \quad (60)$$

(c) Availability of the system when all types of failure rates are same and follow exponential distribution.

$$P_{up}(t) = 0.034954e^{-2.8195t} - 0.032330e^{-1.3161t} + 1.000332e^{-0.0082t} - 0.002654e^{-1.1680t} - 0.000303e^{-1.128t} \quad (61)$$

(d) Repair follow two types of distribution but no controller in the subsystem-2.

$$P_{up}(t) = 0.024749e^{-2.7883t} - 0.023318e^{-1.2669t} - 0.001408e^{-1.1737t} + 1.000917e^{-0.0069t} + 0.000400e^{-1.1560t} - 0.000136e^{-1.1060t} - 0.001176e^{-1.1210t} \quad (62)$$

(e) Repair follow two types of distribution but no controller in the both the subsystems.

$$P_{up}(t) = 0.000399e^{-1.1350t} - 0.000165e^{-0.0850t} - 0.001187e^{-1.1000t} + 0.015948e^{-2.7628t} - 0.021333e^{-1.2495t} + 0.001289e^{-1.1528t} - 1.007629e^{-0.0078t} \quad (63)$$

(f) Repair follow two types of distribution, but catastrophic failure is ignored.

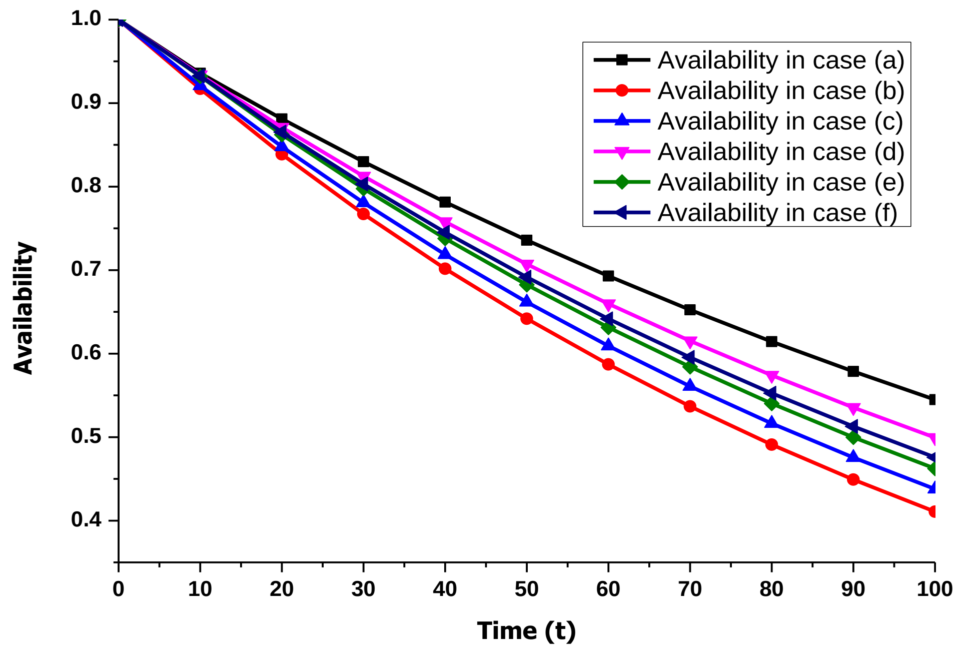
$$P_{up}(t) = -0.000164e^{-1.0930t} - 0.001183e^{-1.1080t} + 0.000399e^{-1.1430t} + 0.019329e^{-2.7725t} - 0.022123e^{-1.2561t} - 0.001334e^{-1.1608t} + 1.005076e^{-0.0075t} \quad (64)$$

For different values of time variable $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, one may get different values of $P_{up}(t)$ with the help of (59-64) as shown in table-2 and the corresponding figure-2.

Table 2 Variation of availability with respect to time in various cases

Time (t)	a	b	c	d	e	f
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0.9358	0.9172	0.9209	0.9337	0.9321	0.9326
20	0.8812	0.8388	0.8479	0.8710	0.8622	0.8654
30	0.8298	0.7672	0.7807	0.8125	0.7976	0.8031
40	0.7814	0.7017	0.7187	0.7579	0.7378	0.7452
50	0.7358	0.6418	0.6617	0.7071	0.6825	0.6915
60	0.6929	0.5870	0.6092	0.6596	0.6314	0.6417
70	0.6525	0.5369	0.5609	0.6153	0.5841	0.5955
80	0.6144	0.4910	0.5164	0.5740	0.5403	0.5526
90	0.5786	0.4491	0.4755	0.5354	0.4998	0.5128
100	0.5448	0.4108	0.4378	0.4995	0.4623	0.4758

Figure 2 Availability as a function of time



4.2 RELIABILITY ANALYSIS

Taking all repair rates equal to zero and obtain inverse Laplace transform in (57), we get an expression for the reliability of the system after taking the failure rates as $\lambda_1 = 0.030, \lambda_2 = 0.035, \lambda_3 = 0.040, \lambda_E = 0.50, \lambda_{S_1} = 0.021, \lambda_{S_2} = 0.022, \lambda_C = 0.035$. Here we have considered only two cases as rest cases are giving almost same output:

(a) Reliability of the system when failure rates follow exponential distribution:

$$P_{up}(t) = 0.018381e^{-2.17692t} - 0.519446e^{-0.1260t} - 1.885541e^{-0.1100t} + 3.689836e^{-0.0788t} - 0.108105e^{-0.1060t} + 0.014969e^{-0.0670t} - 0.210095e^{-0.0900t} \quad (65)$$

(b) Reliability of the system when all types of failure rates are same and follow exponential distribution:

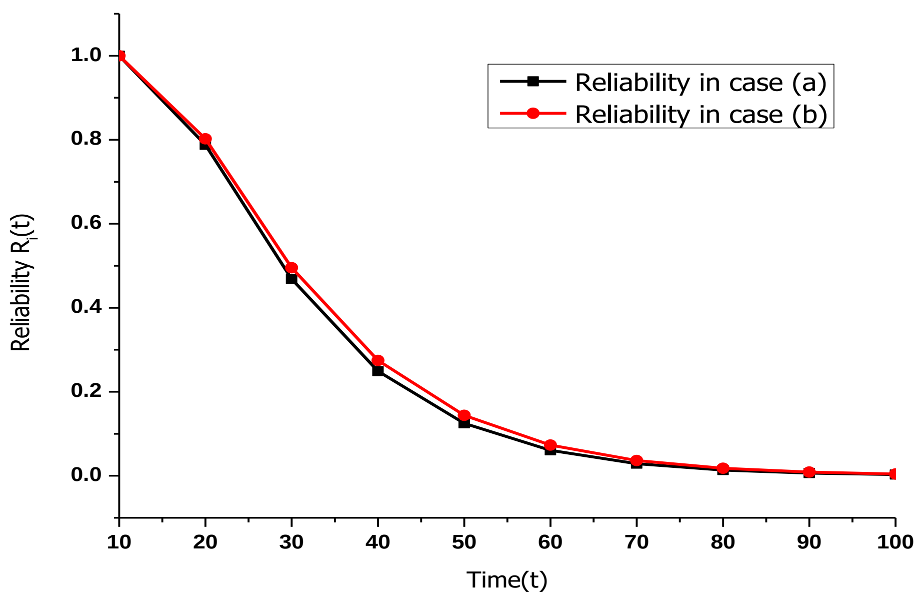
$$P_{up}(t) = 0.015210e^{-0.0650t} + 0.018218e^{-2.7687t} - 0.386880e^{-0.1250t} - 1.992909e^{-0.1050t} + 3.566179e^{-0.0742t} - 0.219818e^{-0.0850t} \quad (66)$$

For different values of time variable $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, one may get different values of reliability $R(t)$ with the help of (65-66) as shown in table-3 and the corresponding figure-3.

Table 3 Computed values of reliability corresponding to the different cases

Time (t)	a	b
0	1.0000	1.0000
10	0.7878	0.8023
20	0.4686	0.4952
30	0.2490	0.2744
40	0.1250	0.1439
50	0.0607	0.0731
60	0.0289	0.0364
70	0.0135	0.0179
80	0.0063	0.0087
90	0.0029	0.0042
100	0.0013	0.0020

Figure 3 Reliability as a function of time



4.3 COST ANALYSIS

Let the service facility be always available, then expected profit during the interval $\left[0,t\right)$ is

$$E_p\left(t\right)=K_1\int_0^t P_{up}\left(t\right)dt-K_2t \qquad (67)$$

For same set of parameters defined in (57), one can obtain (68).

Therefore

$$E_p(t) = -165.359942e^{-0.0060t} + 0.001285e^{-1.1956t} - 0.011977e^{-2.8150t} + 0.019536e^{-1.2853t}$$

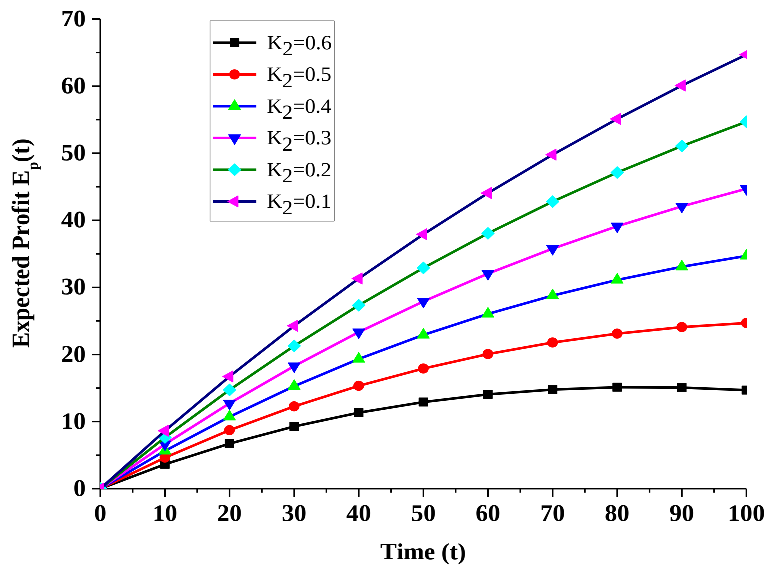
$$+0.001019e^{-1.1430t} - 0.000340e^{-1.1780t} + 0.000142e^{-1.1280t} + 165.350275 - K_2 t \qquad (68)$$

Setting $K_1 = 1$ and $K_2 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$ respectively and varying $t = 0, 10, 20, 30, 40,$ $50, 60, 70, 80, 90$ and 100 units of time, the results for expected profit can be obtain as per table-4 and figure-4.

Table 4 Profit computation for different vales of time

Time (t)	K_2					
	0.6	0.5	0.4	0.3	0.2	0.1
10	3.6361	4.6361	5.6361	6.6361	7.6361	8.6361
20	6.7192	8.7192	10.7192	12.7192	14.7192	16.7192
30	9.2725	12.2725	15.2725	18.2725	21.2725	24.2725
40	11.3269	15.3269	19.3269	23.3269	27.3269	31.3269
50	12.9114	17.9114	22.9114	27.9114	32.9114	37.9114
60	14.0535	20.0535	26.0535	32.0535	38.0535	44.0535
70	14.7790	21.7790	28.7790	35.7790	42.7790	49.7790
80	15.1122	23.1122	31.1122	39.1122	47.1122	55.1122
90	15.0760	24.0760	33.0760	42.0760	51.0760	60.0760
100	14.6919	24.6919	34.6919	44.6919	54.6919	64.6919

Figure 4 Expected profit as a function of time



5. CONCLUSION

This paper studies the reliability characteristics of a complex repairable standby system consisting of two subsystems in series configuration with controllers under catastrophic failure. First Subsystem-L is composed of three identical units in parallel configuration that are working under 1-out-of-3: G policy, while second subsystem-M has two non-identical units that are working under 1-out-of-2: G: policy. In subsystem-M, priority in operation is given to M1 unit whereas M2 unit put into cold standby mode if not in use. Explicit expressions have been derived using supplementary variable technique. Warm/cold-standby redundancy has been used as an effective technique for improving reliability of system design. The following conclusions may be drawn based on the study conducted in this paper:

1. Table-2 and Figure-2 gives the analysis of availability of the system in six different possibilities when failure rates are fixed at different values with respect to time. One can clearly observe that availability of the system decreases as the value of time t increases.
2. Table-3 and figure-3 gives information for reliability of the system at different values of time. As the reliability in all six cases (discussed for availability) is almost same, so we have considered only two cases. The graph showing a steep fall in reliability from top to lowermost in a very short period in both the cases based on failure rate of units.
3. From table-2 and 3, one can observe that corresponding values of availability are greater than the values of reliability, which highlights the requirement of systematic repair for any complex systems for healthier performance. Additionally, availability is more in case (a) as compared to other cases, while reliability is better in case (b).
4. An acute examination from table 4 and figure 4 reveals that expected profit increases as service cost K_2 decreases, while the revenue cost per unit time is fixed at K_1=1. The calculated expected profit is maximum for K_2= 0.1 and minimum for K_2=0.6. We observe that as service cost decreases, profit increase with variation of time. In general, for low service cost, the expected profit is high in comparison to high service cost.

Model developed in this paper found to be highly advantageous in proper maintenance analysis, decision, and evaluation of performances. Another possible future work is to evaluate maximum reliability and availability of the investigated system.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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