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DAILY STOCK PRICES PREDICTION USING VARIANCE GAMMA MODEL

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Abstract: The Black-Scholes-Merton model is often used for modeling company assets, which requires normally distributed data. The company's asset price fluctuates greatly, causing the form of data distribution to allow for heavy tails, asymmetry and excess kurtosis. Therefore we need a model that can capture this phenomenon. The model is the Variance Gamma model, abbreviated as VG. The assumption of normality in the Black-Sholes-Merton theory is unable to capture the heavy tail and the asymmetry that exists in the log returns asset. This form of density usually that is too high compared to normal density is known as excess kurtosis. The additional parameters in the VG process can overcome heavy tail, asymmetry and excess kurtosis. These parameters can control the kurtosis, asymmetry, and adjust the slope of the log asset density. The VG process can be obtained from two approaches. First, the VG process as a change in Brownian motion to Gamma time, abbreviated as VG1. Second, the VG process is obtained by the difference of the two Gamma processes, abbreviated as VG2. In this study we applied the VG model to predict the daily stock price of PT Bank Negara Indonesia (Persero) Tbk. The results obtained show that the VG model provides accurate daily stock price prediction results.

Keywords: excess kurtosis; heavy tail; variance gamma; stock price.

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1. INTRODUCTION

A stock can be defined as a sign of a person or party (business entity) in a company or capital participation in a company or limited liability company. By including this capital, the party has a claim on company income, a claim on company asset, claims on company asset, and the right to attend the General Meeting of Shareholders (GMS) [1].

Basically, there are two advantages that investor get by buying or owning stock, namely dividend and capital gain. Dividend is a stock of the profit given by the company and comes from the profit generated by the company. Capital Gain is the difference between the purchase price and the selling price. Apart from profit, stock has risks in the form of capital loss. Capital loss is a condition in which investor sells stock at a lower price than the purchase price.

One of the ways that can be done to find out how much profit is obtained from stock trading activity is by looking at the value of stock returns. According to [2] returns is the rate of returns on the result earned as a result of investing. Based on [3] by looking at the returns value, the investor can find out the change in the price of a stock, how much profit or loss will be received, so that it can be used as a guide to decide whether to invest in the stock or not. The stock price often have unpredictable changes, so it can increase or decrease at any time. The changes in stock price that can increase and decrease at any time cause uncertainty of the returns value to be received, so that investor can not has certainty whether to gain or lose. Regarding to the uncertainty of stock price changes, a mathematical model is needed to predict future stock prices based on existing stock price data. According to [4], one of the mathematical models that can be used to model and predict stock prices with normal distribution stock returns conditions is the Geometric Brownian Motion (GBM) model.

The GBM model assumes that the returns from asset is normally distributed. The research using data from the price of traded asset in Indonesia showed the existence of excess kurtosis and tail in the returns distribution so that the performance of the Geometric Brownian Motion model was not good enough to describe the dynamics of asset price. The existence of excess kurtosis and tail caused the distribution of the data to be abnormal. One of models which is suitable for data that has the advantage of kurtosis and tail is the Variance Gamma model. A three-parameter stochastic process, called the Variance Gamma process, which generalized Brownian motion was developed as a model for asset price dynamic. This process is obtained by moving Brown's motion with drift at random times given by the Gamma process. [5] have proposed a Variance Gamma (VG) approach which has the advantage of having a parameter added to the distribution of log returns to control volatility and kurtosis in the distribution of log returns. it has also been done by [6]. Then, this VG model was generalized by [7] by developing into the VG process three parameters, namely the addition of a parameter that controls skewness. There are papers supporting the use of process variance-gamma in finance such as the paper by [8], [9], where the variance-gamma distribution is confirmed to be an excellent model for dealing with financial data. There are several procedures that can be used for computation in the Variance-Gamma model. See,[10],[11]. The closed form is presented in [7] and later developed by[12],[13].

2. PRELIMINARIES

2.1. Variance Gamma Density Function

According to [7] density function from $z = \ln\left(\frac{S(t)}{S(0)}\right)$ following the process of Variance Gamma (VG) is:

$$h(z) = \frac{2\exp\left(\frac{\theta x}{\sigma^2}\right)}{v^{\frac{t}{v}}\sqrt{2\pi}\,\sigma\Gamma\left(\frac{t}{v}\right)}\left(\frac{x^2}{\frac{2\sigma^2}{v}+\theta^2}\right)^{\frac{t}{2v}-\frac{1}{4}} . K_{\frac{t}{v}-\frac{1}{2}}\left(\frac{1}{\sigma^2}\sqrt{x^2\left(\frac{2\sigma^2}{v}+\theta^2\right)}\right) \tag{1}$$

Where K is a modification of the second kind of Bessel function,

$x = z - mt - \frac{t}{v}\ln\left(1 - \theta v - \frac{\sigma^2 v}{2}\right)$

Where:

- $S(t)$: value asset at the time t,
- $S(0)$: value asset at the time $t = t_o$
- m : mean from log returns asset
- θ : parameter that controls skewness
- v : parameter that controls kurtosis

σ : volatility

The density function of equation (1) was obtained from the integral with the lower limit equal to zero and the upper limit equal to infinity from the product of the conditional z normal distribution g or $h(z|g)$, with the marginal distribution g or written

$$h(z) = \int_0^\infty \frac{\exp\left(-\frac{1}{2\sigma^2 g}\left(z-mt-\frac{t}{v}\ln\left(1-\theta v-\frac{\sigma^2 v}{2}\right)-\theta g\right)^2\right)}{\sigma\sqrt{2\pi g}} \cdot \frac{g^{\frac{t}{v}-1}\exp\left(-\frac{g}{v}\right)}{v^{t/v}\Gamma\left(\frac{t}{v}\right)} \tag{2}$$

z is normally distributed with the mean = $mt + \frac{t}{v}\ln\left(1 - \theta v - \frac{\sigma^2 v}{2}\right)$ and variance = $\sigma^2 g$.

Using the integral proposed by [14] section 3.471.9 then equation (2) produces equation (1).

2.2. VG Distribution Fit Test

The distribution fit test uses the *Chi-square* test by making k sub-data intervals. The *Chi-square* statistical value is determined as follows:

$$\chi^2 = \sum_{i=1}^{k} \frac{(o_i - Np_i)^2}{Np_i}$$

Where o_i is the observed value in the i-th sub interval, N is the sample mesure and $p_i$is the probability obtained randomly in the i-th sub interval. The value of the Chi-square statistical value is compared with the value of $\chi^2_{\alpha-1-m}$, where α is the level of significance and m is the number of parameters in the VG model [15].

2.3. Estimated Parameter

One method of estimating the Variance Gamma parameter is the moment method. This method is easy to do and has a closed form. According to [7], $X(t)$ at time interval t is a random variable VG with normal distribution with mean θg and variant $\sigma\sqrt{g}$ written as follows:

$$X_{VG1}(t) = \theta g + \sigma\sqrt{g}z \tag{3}$$

Where z is a random variable with a standard Normal distribution that is independent with a random variable g with a Gamma distribution with mean t and variance νt.

The initial step taken to estimate the VG parameter is to determine the first four moments (m) of $X(t)$ as follows:

1. $m_1 = E\big(X(t)\big)$

$$= E(\theta g + \sigma\sqrt{g}z\ )$$

$$= \theta t$$

2. $m_2 = E[\left(X(t) - E\left(X(t)\right)\right)^2]$

By assuming $x = X(t) - E\left(X(t)\right)$ that is $x = (g-t)\theta + \sigma\sqrt{g}z$, so that $x^2 = \left[(g-t)\theta + \sigma\sqrt{g}z\right]^2$. Then elaborate in the form $\left[(g-t)\theta + \sigma\sqrt{g}z\right]^2$ then calculate the expected value the result obtained

$$m_2 = E(x^2)$$

$$= (\theta^2 v + \sigma^2)t$$

3. As in step (2) where $x^3 = x^2 . x$ so that

$x^3 = \left[(g-t)\theta + \sigma\sqrt{g}z\right]^2 . [(g-t)\theta + \sigma\sqrt{g}z]$, by elaborating this form and calculating the expectation obtained

$$m_3 = E(x^3)$$

$$= (2\theta^3 v^2 + 3\sigma^2\theta v)t$$

4. The same steps are carried out, that is $x^4 = x^2 . x^2$, so that

$x^4 = \left[(g-t)\theta + \sigma\sqrt{g}z\right]^2 . \left[(g-t)\theta + \sigma\sqrt{g}z\right]^2$ by elaborating this form and calculating the expectation obtained

$$m_4 = E(x^4)$$

$$= (3\acute{o}^4 v + 12\sigma^2\theta^2 v^2 + 6\theta^4 v^3)t + (3\sigma^4 + 6\sigma^2\theta^2 v + 3\theta^2 v^2)t^2$$

According to [16], value $\theta^2 \approx \theta^3 \approx \theta^4 \approx 0$, so that the Variance Gamma parameters are estimated as follows::

1. Estimated Parameter σ :

$$Var(X) = E(x^2)$$

$$= \theta^2 v + \sigma^2$$

$$= (0)v + \sigma^2$$

$$= \sigma^2$$

So that $\hat{\sigma} = \sqrt{Var(X)}$

2. Estimated Parameter θ :

Skewness $(X) = \frac{m_3}{\sigma^3}$

$$= \frac{E(x^3)}{\sigma^3}$$

$$= \frac{2\theta^3 v^2 + 3\acute{o}^2 \theta v}{(\theta^2 v + \sigma^2)^{3/2}}$$

$$= \frac{2(0)v^2 + 3\sigma^2 \theta v}{((0)v + \sigma^2)^{3/2}}$$

$$= \frac{3\theta v}{\sigma}$$

So that

$$\hat{\theta} = \frac{\sigma\, Skewness(X)}{3v}$$

3. Estimated Parameter v :

Kurtosis $(X) = \frac{m_4}{\sigma^4}$

With the same step the result obtained as follows

Kurtosis(X) $= 3(v + 1)$

So that

$$\hat{v} = \frac{Kurtosis\ (X)}{3} - 1$$

2.4. Variance Gamma Stock Price Model

The stock pricing model that follows VG is

$$S(t) = S(0).exp[(\mu + \omega)t + X_{VG}(t)] \tag{4}$$

where:

$$\omega = \frac{1}{v}\ln\left(1 - \theta v - \frac{1}{2}\sigma^2 v\right)$$

μ: the average log returns asset

A Variance Gamma process $X_{VG}(t)$ can be determined in two ways:

a. According to the equation (3)

$$X_{VG1}(t) = \theta g + \sigma\sqrt{g}z$$

that is process VG obtained form the normal standard process of a Gamma process

b. The difference between two independent Gamma processes, namely $g_r(t)$ and $g_s(t)$ is,

$$X_{VG2}(t) = g_r(t) - g_s(t) \tag{5}$$

$g_r(t)$ and $g_s(t)$ are two independent Gamma processes with mean μ_r and μ_s and each variant v_r and v_s with

$$\mu_r = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{v}} + \frac{\theta}{2},$$

$$\mu_s = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{v}} - \frac{\theta}{2},$$

$$v_r = \mu_r^2 v,$$

$$v_s = \mu_s^2 v.$$

2.5. Monte Carlo Simulation for Variance Gamma Process

Monte Carlo simulation is a powerful engineering instrument for solving various problems in the field of probability and statistics. However, Monte Carlo simulation does not give exact result, because in essence Monte Carlo simulation is a method of numerical approximation. Like most numerical methods, Monte Carlo simulation requires an iterative process. Briefly, the VG process simulation procedure is as follows [11],

a. Simulation $X_{VG1}(t)$ as Brownian motion changes to Gamma time

Input: parameter VG that is : θ, σ, v; time changing $\Delta t_1, \ldots, \Delta \mathrm{t_n}$ where $\sum_{i=1}^{N} \Delta t_i = T$.

Inisialization: $X_{VG1}(t_0) = 0$

Loop: i = 1 to **N**:

1. Generate $\Delta g_i \sim \Gamma\left(\frac{\Delta t_i}{v}, v\right), Z_i \sim N(0,1)$ which are mutually independent

2. Return $X_{VG1}(t_i) = X_{VG1}(t_{i-1}) + \theta \Delta g_i + \sigma\sqrt{\Delta g_i}\, z_i$.

b. Simulation $X_{VG2}(t)$ difference of the two Gamma processes

Input: parameter VG that is: θ, σ, v; time changing $\Delta t_1, \ldots, \Delta \mathrm{t_n}$ where $\sum_{i=1}^{N} \Delta t_i = T$.

Inisialization: $X_{VG2}(t_0) = 0$

Loop: i = 1 to N:

1. Generate $g_{ri} \sim \Gamma\left(\frac{\Delta t_i}{v}, v\mu_r\right),\ g_i \sim \Gamma\left(\frac{\Delta t_i}{v}, v\mu_s\right)$ which are mutually independent
2. Return $X_{VG2}(t_i) = X_{VG2}(t_{i-1}) + g_{ri} - g_{si}$

2.6. Materials and Methods

According to [7] the density function VG is the density function for random variable log returns in asset prices. So the first step is to calculate the log returns for stocks using the equation $r_t = \ln \frac{S_t}{S_{t-1}}, \quad t = 1, \dots, n$. The next stages are as follows,

a. Data exploration. This stage was done by making a histogram to determine the type of the distribution.
b. Testing the normality distribution, kurtosis testing and VG distribution testing
c. Estimating the VG parameter
d. Doing simulation to get the VG process
e. VG modeling
f. Making prediction using data out sample
g. Calculating MAPE

In this research, the above steps are applied to daily stock empirical data from PT Bank Negara Indonesia (Persero) Tbk (stock code: BBNI.JK). This study used secondary data on the daily closing price of BNI Bank of 200 observations. The data used was the data for the period December 13^{th}, 2019 to October 9^{th}, 2020 which was divided into in-sample and out-sample data. Data sharing was done with a percentage of 80% data in sample and 20% data out sample. The data source was taken from [17]. The data were processed using r4.0.2 software. The package used in the software is "VarianceGamma".

3. MAIN RESULTS

The data used in this study were secondary data on the daily closing price of BNI Bank for the period December 13^{th}, 2019 to October 09^{th}, 2020. The first step was to explore descriptive data and statistics. Exploration of data on the daily stock log returns of in-sample data is done by making a histogram.

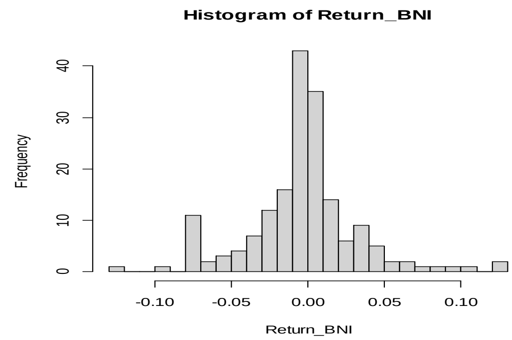

Figure 1. BNI Bank Daily Stock Log Returns Histogram

In Figure 1. presents a daily stock log returns histogram of BNI Bank which resembles a symmetrical shape (skewness = 0) and leptokurtic (kurtosis> 3). This result indicates that the data distribution is not normal. To ensure that the data distribution is not normal, hypothesis testing is carried out as follows:

H_0 : The log returns of BNI Bank Daily stock prices are normally distributed

H_1 : The log returns of BNI Bank Daily stock prices are not normally distributed

The test was carried out using ks.test with $\alpha = 5\%$, the value result obtained D = 0.12712 and p-value = 0.006147. Based on this, the null hypothesis decision is rejected, that is, the distribution is not normal.

To find out how the form and data spread, it can be seen from the value of skewness and kurtosis. The descriptive statistics as follows,

Table 1. Statistics descriptive log returns of BNI Bank Daily stock prices.

Minimum	-0.124642
Maximum	0.127927
Median	0.000000
Mean	-0.002523
Skewness	0.272279
Kurtosis	5.217211

Based on Table 1, Information obtained on the value of skewness = 0.272279 which indicates right-tailed distribution shape.

The shape of the distribution height can be seen from the kurtosis value. Kurtosis value = 5.217211 indicates the form of the leptokurtic distribution. The hypothesis test is as follow:

H_0 : kurtosis = 3

H_1 : kurtosis > 3

The test was carried out using the Anscombe-Glynn test [18] using α = 5%, $Z\alpha$ value obtained= 3.6210, p-value = 0.0001467. Based on this, the null hypothesis is rejected, namely the kurtosis value is more than 3.

Based on the data exploration result, descriptive statistics and hypothesis testing, it can be concluded that the data was not normally distributed with excess kurtosis (excess kurtosis), which is a leptokurtic form. So that modeling is carried out for data that is not normally distributed, namely the Gamma Variance model. Before estimating the parameters, the first step was exploration the data by plotting the Variance-Gamma probability.

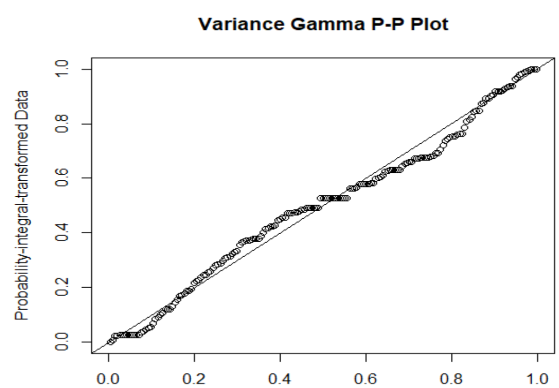

Figure 2 . The Probability Plot of Variance Gamma Distribution

In Figure 2 is a plot of the probability of the Variance Gamma distribution that shows a pattern forming a straight line, so that there is a match between the data and the distribution used, namely the Variance Gamma distribution.

The fit test of the VG distribution model as follows:

H_0: The log returns of BNI Bank daily stock price are following VG distribution

H_1: The log returns of BNI Bank daily stock price are not following VG distribution

The test was carried out using the Chi-square test with $\alpha = 5$ %, the statistical value obtained was $\chi^2 = 3.82$ which was smaller than $\chi^2_{0.05;3} = 7.81$. Based on these result, gave a decision that the null hypothesis is accepted, which is the distribution of the log returns of BNI Bank daily stock price with VG distribution.

The variance Gamma model parameter estimation was obtained by using the moment method approach. The results are as follows:

Table 2. The estimation result of the Gamma Variance model parameter

Stock Code	$\hat{\sigma}$	$\hat{v}$	$\hat{\theta}$
BBNI.JK	0.0324712	0.6188400	-0.0025302

Stock price modeling followed equation (4) through two approaches to the Variance Gamma process. The first approach used equation (3), namely the Variance Gamma process obtained from the Gamma process and the standard Normal process. Parameter shape = 0.00641 and scale = 0.61844.

The second approach used equation (5), namely the Gamma Variance process was obtained from the difference between the two Gamma processes. The shape and scale parameters for this process are:

Table 3. Parameter shape and scale of the first Variance Gamma process

Gamma Process	Parameter	
$g_r(t)$	$shape_1 = 0.00126$	$scale_1 = 22.14127$
$g_s(t)$	$shape_2 = 0.00150$	$scale_2 = 20.30327$

After the Variance Gamma process was obtained, the next step was modeling the stock price. This model was used to predict stock prices for the period of September 14^{th}, 2020 to October 09^{th}, 2020. Besides using the Variance Gamma model, predictions were made using other models, namely the Geometric Brownian Motion (GMB) model and the GBM model with jump diffusion as a comparison. Each of them carried out 10 prediction simulations for each model.

The results of the prediction simulation plots for each model are as follows:

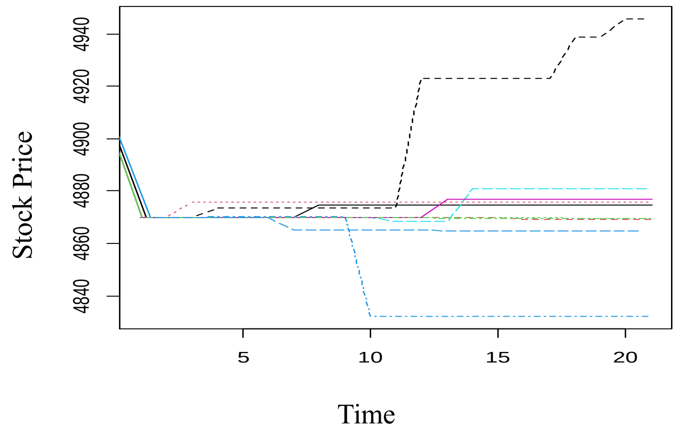

Figure 3. The Prediction Simulation Plot of VG1 Model

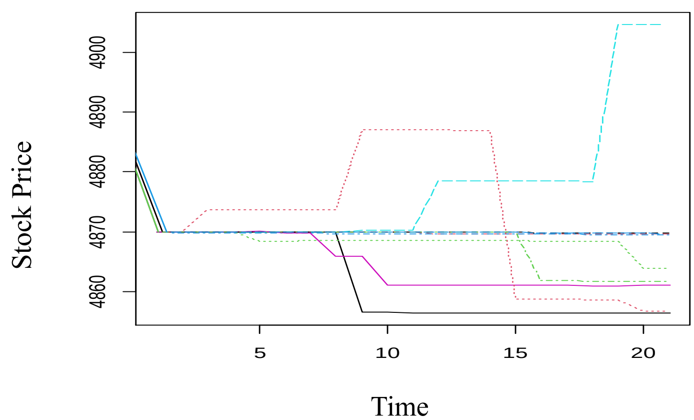

Figure 4. The Prediction Simulation Plot of VG2 Model

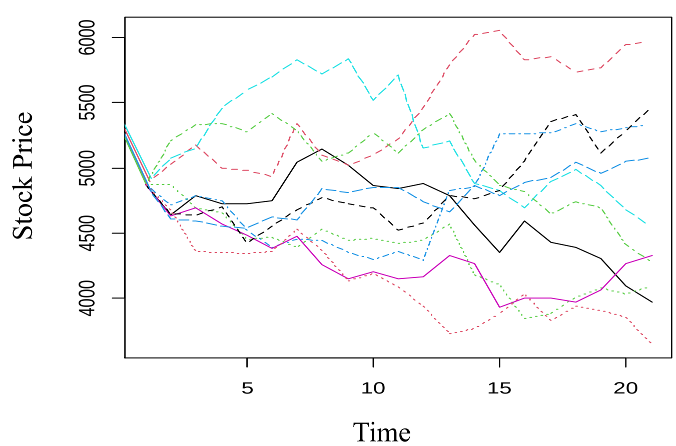

Figure 5. The Prediction Simulation Plot of GBM Model

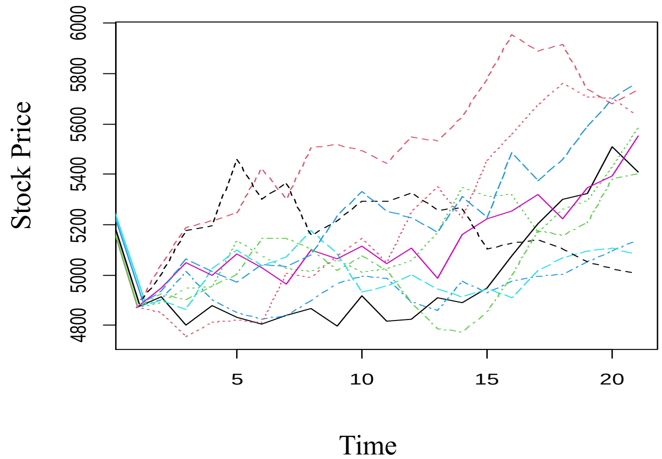

Figure 6. The Prediction Simulation Plot of GBM Jump Diffusion Model

Seen in the prediction simulation plot on the four figures, the interval between the predicted results of the GBM model is wider than the VG model. The daily stock prices prediction simulation plot in Figure 3 and Figure 4 (VG model) shows the prediction results are around the price of 4800 to 4900. While in Figure 5 and Figure 6 (GBM model), the prediction results are around the price of 4000 to 5900. This indicates that the stock price prediction using the VG model is more homogeneous than the GBM model.

To see the homogeneity of the forecasting results for each model, the variance value or coefficient of variance can be used. The following is the variance and variance coefficient of the prediction results for the four models

Table 4. Variance and Variance Coefficient Value

Model	Maximum Prediction	Minimum Prediction	Variance	Variance Coefficient
VG1	4902	4837	14.76	0.30%
VG2	4904	4815	12.73	0.26%
GBM	5917	3447	517.34	10.97%
GBM Jump Diffusion	5955	4753	257.93	5.01%

Based on Table 4. The variance value and variance coefficient value of the VG model are smaller than the GBM model. This informs that the VG model forecasting results are more homogeneous than the GBM model.

The accuracy of the prediction results using the MAPE value, the result is as follows:

Table 5. MAPE Model VG and GBM Value

MAPE			
VG1	VG2	GBM	GBM Jump Diffusion
5.817883%	5.753231%	9.708325%	11.771320 %

Based on the results calculation of the MAPE value, it can be concluded that the VG model has better prediction accuracy than the GBM model. This is indicated by the smaller MAPE value of the VG model than the GBM model. The small MAPE value indicates that the predicted value is close to the actual value. The MAPE value is under 10% indicating that the prediction result including criteria is very accurate.

4. CONCLUSION

This paper uses the VG process to model daily stock prices for data that are not normally distributed. The Gamma Variance Process is obtained by evaluating Brownian motion with drift at random times given by the Gamma process. Additional parameters in the VG model to control skewness and excess kurtosis. The VG model is applied to the daily stock data of PT Bank BNI Tbk. The VG model gives better results than the GBM and GBM jump diffusion models because it has a smaller MAPE value. In terms of accuracy criteria, the VG1, VG2 and GBM models have MAPE values under 10%. This criterion informs that the three models have very accurate prediction result.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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