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GENERALIZATION OF ANTI P-FUZZY GROUP WITH ANTI P-FUZZY ALGEBRA FROM THE ALGEBRA A

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Abstract: The Algebras are generalized with Anti Partially ordered fuzzy groups. Our main objective of this work is to define Anti partially ordered algebra and its anti groups on the algebra A. Some Properties of Anti partially ordered algebra are explored. We also discuss about the condition for which Anti partially ordered algebra to be Anti Partially ordered fuzzy groups and similarly condition for Anti Partially ordered fuzzy groups to be Anti partially ordered algebra.

Keywords: anti P-fuzzy algebra (APFA); anti P-fuzzy group (APFG) on the algebra A; P-fuzzy set; P-fuzzy algebra; P-fuzzy subgroup.

AMS Classification Code: 03E72, 28E10, 08A72

1. INTRODUCTION

In 1965, Zadeh mathematically initiated the concept of fuzzy set [4], it opened a new path of thinking to many mathematicians, engineers, physicists, chemists and many others due to its diverse applications in various fields. The Fuzzy Algebraic structures play a important role on Fuzzy Mathematics with wide applications. Rosenfeld [7] in 1971 introduced the concept of fuzzy subgroups, which was the first fuzzification of any algebraic structures. Biswas [6] in 1994

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defined interval μ -valued fuzzy subgroups of the same nature of Rosenfeld's fuzzy subgroups and discussed some important results. In 1998, Lazlo filep[5] introduced the concept of P – Fuzzy Algebra. Biswas introduced the concept of Anti fuzzy set which is an extension of fuzzy set i.e., the complement of fuzzy set. K. H. Kim and Y. H. Yon [3] defined Anti fuzzy ideals and Anti fuzzy R – subgroups [2] using near rings. D. Y. Li, C. Y. Zhang and S. Q. Ma [1] generalized the Intuitionistic Anti-fuzzy Subgroup in Group G. Later Fuzzy set was developed into some special types of AFS which are widely used in artificial intelligence, computer science, medical science, control engineering, decision theory, expert systems, operations research, pattern recognition, robotics and other fields.

2. PRELIMINARIES

Definition 2.1. A fuzzy subset μ of the set X is a function $\mu: X \rightarrow [0,1]$, where X be a non-empty set.

Definition 2.2. Let A be a nonempty set and $P = (P, *, 1, \leq)$ where $*$ is minimum operation a $(2, 0)$ type ordered algebra, satisfies the property of monoid, poset and isotone.

Definition 2.3. A mapping $\mu: A \rightarrow P$ is a P-fuzzy subset of $A(P^A)$ where (P, \leq) is a partially ordered set and X is a nonvoid set.

Definition 2.4. A P – fuzzy set $\mu \in P^A$ along with n -ary and nullary operation is called a P – fuzzy algebra or fuzzy subalgebra on the algebra A .

(Note: If A is a group then nullary operation is consequence of n – ary operation)

Definition 2.5. Let G be a group. A fuzzy subset μ of a group G is called a fuzzy subgroup of the group G if

- $\mu(xy) \geq \min(\mu(x), \mu(y))$ for every $x, y \in G$ and
- $\mu(x^{-1}) = \mu(x)$ for every $x \in G$.

Definition 2.6. Let G be a group. A fuzzy subset μ of a group G is called a Anti fuzzy subgroup of the group G if

- $\mu(xy) \leq \max(\mu(x), \mu(y))$ for every $x, y \in G$ and
- $\mu(x^{-1}) = \mu(x)$ for every $x \in G$.

3. ANTI P-FUZZY ALGEBRA

Definition 3.1. A P – fuzzy set $\wp \in P^A$ is called a Anti P – fuzzy algebra or Anti fuzzy subalgebra on the algebra A, if

- For any n – ary($n \geq 1$) operation $f \in F$

$$\wp (f(x_1, \dots, x_n)) \leq \wp(x_1) * \dots * \wp(x_n) \text{ for all } x_1, \dots, x_n \in A$$
- For any constant (nullary operation) C

$$\wp (c) \leq \wp(x) \text{ for all } x \in A.$$

(Note: If A is a group then nullary operation is consequence of n – ary operation)

Example: 3.1 Let $A = \{1, 0, -1\}$, $f \in \{+, \cdot\}$ and

$$\wp(x) = \begin{cases} 0.3 & \text{when } x = -1, \\ 0.6 & \text{when } x = 0 \\ 1 & \text{when } x = 1 \end{cases}$$

Consider $x = -1$ and $x = 1$ under.

$$\begin{aligned} \wp (\cdot (-1, 1)) &\leq \max \{ \wp (-1), \wp (1) \} \\ \wp (-1) &\leq \max \{ 0.3, 1 \} \\ 0.3 &\leq 1 \end{aligned}$$

Consider $x = -1$ and $x = 1$ under +

$$\begin{aligned} \wp (+(-1, 1)) &\leq \max \{ \wp (-1), \wp (1) \} \\ \wp (0) &\leq \max \{ 0.3, 1 \} \\ 0.6 &= 1 \end{aligned}$$

Definition 3.2. Let G be a group from the algebra A. A P - fuzzy subset $\wp \in P^A$ of a group G is called a Anti P - fuzzy subgroup of the group G from the algebra A if

- $\wp (xy) \leq \max (\wp (x), \wp (y))$ for every $x, y \in G$ and
- $\wp (x^{-1}) = \wp (x)$ for every $x \in G$.

Example: 3.2

Let $G = \{i, -i, 1, -1\}$ be a group under multiplication

$$\wp(x) = \begin{cases} 0.5 & \text{when } x = -1, -i \\ 0.8 & \text{when } x = 0 \\ 1 & \text{when } x = 1, i \end{cases}$$

Consider $x = -1$ and $x = 1$ under multiplication

$$\wp(-1 \times 1) \leq \max \{ \wp(-1), \wp(1) \}$$

$$\wp(-1) \leq \max \{ 0.5, 1 \}$$

$$0.5 \leq 1$$

$$\wp(-1 \times 0) \leq \max \{ \wp(-1), \wp(0) \}$$

$$\wp(0) \leq \max \{ 0.5, 0.8 \}$$

$$0.8 = 0.8$$

Theorem 3.1: Let $\wp \in P^A$ and if there exist APFG for APFA then both APFG and APFA have unique identity element under same operation.

Proof: Consider $\wp(e)$ and $\wp(e')$ are the identities of APFG and APFA under operation multiplication.

Let $x \in A, \wp(x) \in [0, 1]$

$$\wp(xe) = \wp(x) = \wp(xe')$$

$$\Rightarrow \wp(x) \wp(e) = \wp(x) \wp(e')$$

$$\Rightarrow \wp(e) = \wp(e')$$

Theorem 3.2: Let $\wp \in P^A$ and if there exist APFG for APFA then for each $a \in A$ in APFG is same as the inverse of a from APFA under same operation.

Proof: Consider a' and a'' are the inverses of ' a ' from APFG and APFA under operation multiplication.

Let $a \in A, \wp(a) \in [0, 1]$, Also $a', a'' \in A$ and $\wp(a'), \wp(a'') \in [0, 1]$

By above theorem, both APFG and APFA have unique identity element under same operation.

$$\wp(a'a) = \wp(e) = \wp(a''a)$$

$$\Rightarrow \wp(a') \wp(a) = \wp(a'') \wp(a)$$

$$\Rightarrow \wp(a') = \wp(a'')$$

Theorem 3.3: Let $\wp \in P^A$ from APFA is a Anti P – Fuzzy group of the group G on the algebra A iff $\wp(x_1 y_2^{-1}) \leq \max \{ \wp_1(x), \wp_2(y) \}$

Proof:

Real part: Consider $\mu(x) \in P^A$ from Anti P - Fuzzy Algebra then

➤ For any n – ary ($n \geq 1$) operation $f \in F$

$$\wp(f(x_1, \dots, x_n)) \leq \wp(x_1) * \dots * \wp(x_n) \text{ for all } x_1, \dots, x_n \in A$$

to prove that

$$\wp(x_1 x_2^{-1}) \leq \max \{ \wp(x_1), \wp(x_2) \}$$

Let $x_1, x_2 \in A$ where $P \in [0, 1]$. Consider $x_1 = x$ and $x_2 = y^{-1}$

$$\Rightarrow \wp(f(x_1 x_2)) = \wp(f(x y^{-1}))$$

$$\leq \wp(x) * \wp(y^{-1})$$

$$\wp(x) * \wp(y^{-1}) = \wp(x * y^{-1})$$

$$\wp(x * y^{-1}) \leq \max \{ \wp(x), \wp(y) \}$$

$$\wp(x y^{-1}) \leq \max \{ \wp(x), \wp(y) \}$$

$\Rightarrow \mu \in P^A$ from Anti P - Fuzzy Algebra is Anti P - Fuzzy Group.

Converse part:

If $\wp(x y^{-1}) \leq \max \{ \wp(x), \wp(y) \}$ for every $x, y \in A$ where $P \in [0, 1]$ then APFG is a APFA. Consider $x_1 = x$ and $x_2 = y^{-1}$

$$\wp(x y^{-1}) \leq \max \{ \wp(x), \wp(y^{-1}) \}$$

$$\leq \max \{ \wp(x), \wp(y) \}$$

Since $\wp(x y^{-1}) = \wp(x * y^{-1})$.

$$\wp(x * y^{-1}) = \wp(x) * \wp(y^{-1})$$

$$= \wp(x) * \wp(y)$$

$$\geq \wp(f(x y)) \text{ (n-ary operation)}$$

This implies $\wp(f(x y)) = \wp(x * y^{-1}) \leq \max \{ \wp(x), \wp(y) \}$

$$\leq \wp(x) * \wp(y)$$

$\Rightarrow \wp(f(x y)) \leq \wp(x) * \wp(y)$ for all $x, y \in A$.

Theorem 3.4: let $\wp \in P^A$ from APFG of a group G from the algebra A and $x \in G$ then $\wp(xy) = \wp(y)$ for every $y \in G$ iff $\wp(x) = \wp(e)$.

Proof:

Real Part: let $\wp \in P^A$ from APFG of a group G from the algebra A and $\wp(xy) = \wp(y)$, $x, y \in G$

$$\wp(x) \wp(y) = \wp(y)$$

$$\wp(x) \wp(y) = \wp(y) \cdot \wp(e) \text{ where } \mu(e) \text{ is an identity element of APFG.}$$

$$\wp(x) = \wp(e).$$

Converse part:

Given that $\wp(x) = \wp(e)$.

$\wp(x) \wp(y) = \wp(e) \wp(y)$. (since $\wp(e)$ is an identity element)

$\wp(xy) = \wp(e) \wp(y)$.

$\wp(xy) = \wp(y)$.

Theorem 3.5: Consider $\wp_1 \in H$ and $\wp_2 \in K$ are APFG then $H \cap K$ is also a APFG.

Proof: Since $\wp(e) \in H \cap K$

$\Rightarrow H \cap K$ is non-empty.

Consider $\wp_1(x), \wp_2(x) \in H \cap K$

$\Rightarrow \wp_1(x), \wp_2(x) \in H$ and $\wp_1(x), \wp_2(x) \in K$

Since H and K are APFG, $\wp_1(x) \wp_2^{-1}(x) \in H$ and $\wp_1(x) \wp_2^{-1}(x) \in K$

$\Rightarrow \wp_1(x) \wp_2^{-1}(x) \in H \cap K$

$\Rightarrow H \cap K$ is a Anti P - Fuzzy group.

Theorem 3.6: Consider $\wp_1 \in H$ and $\wp_2 \in K$ are APFG then $H \cup K$ is also a APFG iff one is contained in the other.

Proof:

Real part: Let H and K be two APFG such that $H \subseteq K$.

\Rightarrow Hence either $H \subseteq K$ or $K \subseteq H$.

$\Rightarrow H \cup K = K$ or $H \cup K = H$.

\Rightarrow Hence $H \cup K$ is a APFG.

Converse part: Suppose $H \cup K$ is a APFG. To prove that $H \subseteq K$ or $K \subseteq H$.

Suppose that H is not contained in K and K is not contained in H . Then there exists element \wp_1, \wp_2 such that

$\wp_1(x) \in H$ and $\wp_1(x) \notin K$ ---(1)

$\wp_2(x) \in K$ and $\wp_2(x) \notin H$ ---(2)

Clearly $\wp_1(x), \wp_2(x) \in H \cup K$

Since $H \cup K$ is a APFG on the algebra A . $\wp_1(x) \wp_2(x) \in H$ or $\wp_1(x) \wp_2(x) \in K$

Case (i) Let $\wp_1(x) \wp_2(x) \in H$. Since $\wp_1(x) \in H$, $\wp_1^{-1}(x) \in H$.

Hence $\wp_1^{-1}(x) (\wp_1(x) \wp_2(x)) = \wp_2(x) \in H$ which contradicts (2).

Case (ii) Let $\wp_1(x) \wp_2(x) \in K$. Since $\wp_2(x) \in K$, $\wp_2^{-1}(x) \in K$.

Hence $\wp_2^{-1}(x) (\wp_2(x) \wp_1(x)) = \wp_1(x) \in K$ which contradicts (1).

Our assumption is wrong H is not contained in K and K is not contained in H .

$\Rightarrow H \subseteq K$ or $K \subseteq H$.

Theorem 3.7: Let $\wp_1 \in A$ and $\wp_2 \in B$ be two APFSG of a group G . Then AB is an APFSG of G for some A is a subset or not a subset of B .

Proof: Let A and B be two APFSG of a group G .

$$G = \{1, -1, i, -i\}$$

$$\wp_1(x) = \begin{cases} 0.7 & \text{for } x = -1 \\ 1 & \text{for } x = 1 \end{cases} \quad \text{and}$$

$$\wp_2(x) = \begin{cases} 0.5 & \text{for } x = -i \\ 0.6 & \text{for } x = i \\ 0.7 & \text{for } x = -1 \\ 1 & \text{for } x = 1 \end{cases}$$

$$\wp_1(x)\wp_2(x) = \begin{cases} 0.35 & \text{for } x = i \\ 0.42 & \text{for } x = -i \\ 0.5 & \text{for } x = 1 \\ 0.7 & \text{for } x = -1 \text{ and} \\ 0.5 & \text{for } x = -i \\ 0.6 & \text{for } x = i \\ 0.7 & \text{for } x = -1 \\ 1 & \text{for } x = 1 \end{cases}$$

$$= \begin{cases} \max(0.35, 0.6) & \text{for } x = i \\ \max(0.42, 0.5) & \text{for } x = -i \\ \max(0.5, 1) & \text{for } x = 1 \\ \max(0.7, 0.7) & \text{for } x = -1 \end{cases}$$

$$= \begin{cases} 0.6 & \text{for } x = i \\ 0.5 & \text{for } x = -i \\ 1 & \text{for } x = 1 \\ 0.7 & \text{for } x = -1 \end{cases}$$

Since A is a subset of AB and A is an APFSG.

So AB is an APFSG.

Assume in contrary A is not a subset of B .

Consider $G = S_3 = \{e, p_1, p_2, p_3, p_4, p_5\}$. Now the subsets

$A = \{e, p_1, p_2\}$ and $B = \{e, p_3\}$ are APFSG of a group G .

$AB = \{ee, ep_3, p_1e, p_1p_3, p_2e, p_2p_3\}$

$AB = \{e, p_3, p_1, p_4, p_2, p_5\} = G$

Therefore AB is an APFSG of group G .

Theorem 3.8: Let $\wp_1 \in A$ and $\wp_2 \in B$ be two APFSG of a group G . Then AB is an APFSG of G iff $AB = BA$.

Proof: Let $\wp_1 \in A$ and $\wp_2 \in B$ be two APFSG of a group G , then AB is an APFSG of G . To show that $AB = BA$

Consider an element $\wp(x) \in AB$. Since AB is an APFSG of G , therefore there exists inverse for $\wp(x)$, $\wp(x^{-1}) \in AB$.

Let $\wp(x^{-1}) = \wp(a) \wp(b) = \wp(ab)$ where $a \in A$ and $b \in B$.

Therefore $\wp(x) = \wp(ab)^{-1} = \wp(b^{-1}) \wp(a^{-1})$ Since A and B are APFSG of group G , $\wp(a^{-1}) \in A$ and $\wp(b^{-1}) \in B$.

Therefore $\wp(x) \in BA$

Hence $AB \subseteq BA$ ---(1)

Now, let $\wp(x) \in BA$, then $\wp(x) = ba$ where $b \in B$ and $a \in A$.

Therefore $x^{-1} = \wp(ba)^{-1} = \wp(a^{-1}) \wp(b^{-1}) \in AB$

Since AB is a APFSG of group G and $\wp(x^{-1}) \in AB$, so $\wp(x) \in AB$.

$BA \subseteq AB$ --- (2)

From equation (1) and (2)

$AB = BA$

Conversely,

Let $AB = BA$

To prove that AB is an APFSG of group G .

Clearly $\wp(e) \in AB$ and also AB is a non empty.

Let $\wp(x), \wp(y) \in AB$, then $\wp(x) = \wp(a_1) \wp(b_1)$ and $\wp(y) = \wp(a_2) \wp(b_2)$ where $\wp(a_1), \wp(a_2) \in A$ and $\wp(b_1), \wp(b_2) \in B$.

Therefore $\wp(xy^{-1}) = \wp(a_1b_1)\wp(a_2b_2)^{-1} = \wp(a_1)\wp(b_1)\wp(b_2^{-1})\wp(a_2^{-1})$

Now $\wp(b_2^{-1})\wp(a_2^{-1}) \in BA$

also $AB = BA$

So $\wp(b_2^{-1})\wp(a_2^{-1}) \in AB$

Therefore $\wp(b_2^{-1})\wp(a_2^{-1}) = \wp(a_3)\wp(b_3)$ where $\wp(a_3) \in A$ and $\wp(b_3) \in B$

So $\wp(xy^{-1}) = \wp(a_1)\wp(b_1)\wp(a_3)\wp(b_3)$.

Now $\wp(b_1)\wp(a_3) \in BA$

Since $BA = AB$, $\wp(b_1)\wp(a_3) \in AB$

So $\wp(b_1)\wp(a_3) = \wp(a_4)\wp(b_4)$ where $\wp(a_4) \in A$ and $\wp(b_4) \in B$.

$\wp(xy^{-1}) = \wp(a_1a_4)\wp(b_4b_3) \in A$

Therefore AB is an APFSG of group G .

Theorem 3.9: Let $\wp_1 \in A$ and $\wp_2 \in B$ be two APFSG of a group G , then AB is an APFSG of G .

Proof: Let $\wp(x) \in AB$, then $\wp(x) = \wp(ab)$ where $\wp(a) \in A$ and $\wp(b) \in B$.

Since G is abelian $\wp(ab) = \wp(ba)$

Therefore $\wp(x) \in BA$

Hence $AB \subseteq BA$.

Similarly Let $\wp(x) \in BA$, then $\wp(x) = \wp(ba)$ where $\wp(a) \in A$ and $\wp(b) \in B$.

Since G is abelian $\wp(ba) = \wp(ab)$

Therefore $\wp(x) \in AB$

Hence $BA \subseteq AB$.

Therefore $AB = BA$

Hence AB is a subgroup of G .

Theorem 3.10: Lagrange's Theorem:

Let μ be a APFA of order n and H be any APFG of G then the order of H divides order of G on the Algebra A .

Proof: Let \wp be a Anti P – Fuzzy Algebra of Algebra A with $\wp(e)$ as its identity element.

$\Rightarrow H = \{x \in G/\wp(x) = \wp(e)\}$ is a anti P - fuzzy group of group G from the algebra A , for P – level subset of the group G where $t = \wp(e)$.

By Lagrange's theorem $O(H)|O(G)$

\Rightarrow The order of H divides order of G.

\Rightarrow the order of APFG divides order of APFA on the Algebra A.

Theorem 3.11 Every P - level subsets of an APFG of a group G from APFA must be a fuzzy subgroup from APFA.

Proof: Let \wp be two APFSG of a group G from the APFA on the algebra A.

$G = \{1, -1, i, -i\}$

$$\wp(x) = \begin{cases} 0.4 & \text{for } x = -i \\ 0.6 & \text{for } x = i \\ 0.8 & \text{for } x = -1 \\ 1 & \text{for } x = 1 \end{cases}$$

(\wp, \cdot) is an APFSG of the group G under multiplication. Let P_t be the P – Level subset for $t = 0.6$, $P_t = \{-1, 1\}$. Since the sets are arranged in partially ordered, therefore every P - level subsets of an APFG of a group G from APFA must be a fuzzy subgroup.

Theorem 3.12: Every lower level subsets of an APFSG of the group G need not in general be a sub-group of G.

Proof: From the above example

Let \wp_t be the P – Level subset for $t = 0.4$, $\wp_t = \{i, -1, 1\}$.

It is clear that the above lower level set is not a Group.

CONCLUSION

The research work on P - anti Fuzzy subgroup is extended to Partially ordered fuzzy Right Algebra on the Algebra A using P – Fuzzy set. In future work it can be extended to anti P - Fuzzy bigroups, anti P - Fuzzy rings etc.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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