Available online at http://scik.org

J. Math. Comput. Sci. 11 (2021), No. 2, 2287-2300

https://doi.org/10.28919/jmcs/5491

ISSN: 1927-5307

$\mathfrak{N}w$–FILTERS OF LATTICE WAJSBERG ALGEBRAS

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Abstract: In this paper, we define the $\mathfrak{N}w$ – filters of Lattice wajsberg algebras. We prove that every truth – favorite set of $\mathfrak{N}w$ – filter is also $\mathfrak{N}w$ – filter and falsity - favorite set need not to be a $\mathfrak{N}w$ – filter. After that we prove that some properties of $\mathfrak{N}w$ – filters. Further we show the cut set of Truth-value based neutrosophic set are also $\mathfrak{N}w$ – filters. Finally we define the $\mathfrak{N}w$ – lattice filters of Lattice wajsberg algebra and example is given. We prove that every $\mathfrak{N}w$ – filter is a $\mathfrak{N}w$ – lattice filter of Lattice wajsberg algebra and converse need not be true.

Keywords: lattice wajsberg algebras; truth-value based neutrosophic sets; $\mathfrak{N}w$ – filters; $\mathfrak{N}w$ – lattice filter; implicative filter.

2010 AMS Subject Classification: 16P70, 16D25.

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Received January 27, 2021

1. INTRODUCTION

In 1935, Wajsberg [17] introduced the concept of Wajsberg algebra. In 1984, Front, Antonio and Torrens [6] led the lattice wajsberg algebra and define filters and obtain some properties of filters. A. Ibrahim and Saravan [1] introduced the strong implicative filters of lattice Wajsberg algebras and derived some properties. After that B. Ahamed introduced [2] the concept fuzzy implicative filter and obtained some properties. Ideals and filters play vital role in algebras. Several researchers [8, 10, 13] worked on ideals and filters on different algebras and derived the properties.

At first L.A. Zadeh introduced the Fuzzy sets to handle the real life problems with uncertainty. After that several researchers [3,4,11,12,14,16] applied the fuzzy theory to different algebras, differential equations and derived some results. Later Gaw derived the vague set as a generalization of fuzzy set. Vague theory applied to several streams by researchers [15]. After that Smarandache [7] introduced the concept of neutrosophic sets. After that Monoranjan and Madhumangal [9] recall some definitions and introduced the truth value based neutrosophic sets and neutrosophic sets and define new operations with examples.

In this paper we consider the truth value based neutrosophic sets($\mathcal{N}_T^s$) defined by Monoranjan and Madhumangal and introduce the $\mathfrak{N}w$ - filter ($\mathfrak{N}w\mathfrak{F}$) of Lattice wajsberg algebra and obtain some results of $\mathfrak{N}w$ - filters.

For further information of lattice wajsberg algebra refer the Wajsberg algebra [6,17] by Front, Antonio and Torrens and for the truth value based neutrosophic sets Intuitionstic Neutrosophic Set by Monoranjan and Madhumangal [9].

2. PRELIMINARIES

Definition 2.1 [17]: Let (w, $\curvearrowright$, ´, 1_w) be a wajsberg algebra if it satisfies the following axioms for all $\mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w \in w$

1. $1_w \curvearrowright \mathfrak{x}_w = \mathfrak{x}_w$
2. $(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w) \curvearrowright ((\mathfrak{y}_w \curvearrowright \mathfrak{z}_w) \curvearrowright (\mathfrak{x}_w \curvearrowright \mathfrak{z}_w)) = 1_w$
3. $(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w) \curvearrowright \mathfrak{y}_w = (\mathfrak{y}_w \curvearrowright \mathfrak{x}_w) \curvearrowright \mathfrak{x}_w$

4. $(\mathfrak{x}'_w \curvearrowright \mathfrak{y}'_w) \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{x}_w) = 1_w$

Definition 2.2 [17]: The wajsberg algebra w is called a lattice Wajsberg algebra with the bounds $0_w, 1_w$ if it satisfies the following axioms for all $\mathfrak{x}_w, \mathfrak{y}_w \in w$

A partial ordering' $\leq$ 'on w, such that

$\mathfrak{x}_w \leq \mathfrak{y}_w$ if and only if $\mathfrak{x}_w \curvearrowright \mathfrak{y}_w = 1_w$, $(\mathfrak{x}_w \vee \mathfrak{y}_w) = (\mathfrak{x}_w \curvearrowright \mathfrak{y}_w) \curvearrowright \mathfrak{y}_w$

and $(\mathfrak{x}_w \wedge \mathfrak{y}_w) = ((\mathfrak{x}'_w \curvearrowright \mathfrak{y}'_w) \curvearrowright \mathfrak{y}'_w)'$.

Definition 2.3 [9]: A neutrosophic set $A_w = \{< \mathfrak{y}_w, w_T^A(\mathfrak{y}_w), w_I^A(\mathfrak{y}_w), w_F^A(\mathfrak{y}_w)>, \mathfrak{y}_w \in \mathbb{X}\}$ where w_T^A is truth membership function, w_I^A is an indeterminate membership function and w_F^A is false membership function, on a nonempty set w is a Truth –valued neutrosophic set ($\mathcal{N}_T^S$) if $w_T^A(\mathfrak{y}_w), w_I^A(\mathfrak{y}_w), w_F^A(\mathfrak{y}_w) \rightarrow [0,1]$ and $0 \leq w_T^A(\mathfrak{y}_w) + w_I^A(\mathfrak{y}_w) + w_F^A(\mathfrak{y}_w) \leq 3$. The set $\mathcal{N}_T^S$ is simply denoted by $A_w = (w_T^A, w_I^A, w_F^A)$.

Throughout this paper' w' refers the lattice wajsberg algebra and '$\mathcal{N}_T^S$' refers the truth –valued neutrosophic set.

3. $\mathfrak{N}w$- FILTERS ($\mathfrak{N}w\mathfrak{F}$)

Definition3.1: A $\mathcal{N}_T^S$ set $A_w = (w_T^A, w_I^A, w_F^A)$ on w is called a $\mathfrak{N}w$ - filter ($\mathfrak{N}w\mathfrak{F}$) if it satisfies $\forall\, \mathfrak{x}_w, \mathfrak{y}_w \in w$

(2.1) $w_T^A(1_w) \geq w_T^A(\mathfrak{x}_w)$, $w_I^A(1_w) \geq w_I^A(\mathfrak{x}_w)\}$, $w_F^A(1_w) \leq w_F^A(\mathfrak{x}_w)\}$.

(2.2) $w_T^A(\mathfrak{y}_w) \geq \min\{w_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_T^A(\mathfrak{x}_w)\}$

$w_I^A(\mathfrak{y}_w) \geq \min\{w_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_I^A(\mathfrak{x}_w)\}$

$w_F^A(\mathfrak{y}_w) \leq \max\{w_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_F^A(\mathfrak{x}_w)\}$.

Example 3.2: Let $w = \{0_w, \mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w, \mathfrak{v}_w, 1_w\}$ with the binary operation $\curvearrowright$ as follows:

$\mathfrak{z}$	$w_T^A(\mathfrak{z})$	$w_I^A(\mathfrak{z})$	$w_F^A(\mathfrak{z})$
0_w	.451	.557	. 51
$\mathfrak{x}_w$	.671	.641	. 345
$\mathfrak{y}_w$	.451	.557	. 51
$\mathfrak{z}_w$	.451	.557	. 51
$\mathfrak{v}_w$	.451	.557	. 51
1_w	.671	.641	. 345

The $\mathcal{N}_T^S$ sets $A_w = (\ \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}},\ \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}},\ \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}})$ and $B_w = (\ \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{B}},\ \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{B}},\ \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{B}}\ )$ defined on $\mathcal{w}$ as follows are $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$.

$\curvearrowright$	0_W	$\mathfrak{x}_W$	$\mathfrak{y}_W$	$\mathfrak{z}_W$	$\mathfrak{v}_W$	1_W
0_W	1_W	1_W	1_W	1_W	1_W	1_W
$\mathfrak{x}_W$	$\mathfrak{z}_W$	1_W	$\mathfrak{y}_W$	$\mathfrak{z}_W$	$\mathfrak{y}_W$	1_W
$\mathfrak{y}_W$	$\mathfrak{v}_W$	$\mathfrak{x}_W$	1_W	$\mathfrak{y}_W$	$\mathfrak{x}_W$	1_W
$\mathfrak{z}_W$	$\mathfrak{x}_W$	$\mathfrak{x}_W$	1_W	1_W	$\mathfrak{x}_W$	1_W
$\mathfrak{v}_W$	$\mathfrak{y}_W$	1_W	1_W	$\mathfrak{y}_W$	1_W	1_W
1_W	0_W	$\mathfrak{x}_W$	$\mathfrak{y}_W$	$\mathfrak{z}_W$	$\mathfrak{v}_W$	1_W

$\mathfrak{z}$	$\mathcal{w}_T^B(\mathfrak{z})$	$\mathcal{w}_I^B(\mathfrak{z})$	$\mathcal{w}_F^B(\mathfrak{z})$
0_w	.751	. 145	.602
$\mathfrak{x}_w$	.751	. 145	.602
$\mathfrak{y}_w$	.851	. 23	.601
$\mathfrak{z}_w$	.851	. 23	.601
$\mathfrak{v}_w$	.751	. 145	.602
1_w	.851	. 23	.601

Lemma 3.3: If the set $\mathcal{N}_T^S$ is a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$ then the truth- favorite set of $\mathcal{N}_T^S$ is also $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$.

Proof: Let the $\mathcal{N}_T^S$ set $A_w = (\ \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}},\ \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}},\ \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}})$ is a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$.

The set $A_{*w} = (\ \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}*},\ \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}*},\ \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}*}) = (\ \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}*}, 0,\ \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}})$ is the truth- favorite set of A_w, where $\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}*}(\mathfrak{x}_w) = \min\{\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_w) + \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_w), 1\}\ \forall\ \mathfrak{x}_w \in \mathcal{w}$.

Case 1.1: If $\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_w) + \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_w) < 1$ then $\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}*}(\mathfrak{x}_w) = \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_w) + \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_w)$

$$\leq \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(1_w) + \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(1_w) \quad \text{(from 2.1)}$$

$$\leq \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}*}(1_w)$$

Case1.2: If $\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_w) + \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_w) \geq 1$ then $\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}*}(\mathfrak{x}_w) = 1$.

From equation (2.1), $w_{T}^{A}(1_w)+ w_{I}^{A}(1_w) \geq 1$, so clearly $w_{T}^{A*}(1_w) = 1 = w_{T}^{A*}(\mathfrak{x}_w)$

That is $w_{T}^{A*}(1_w) \geq w_{T}^{A*}(\mathfrak{x}_w)$ for all $\forall\, \mathfrak{x}_w \in w$.

So $A_{*w} = (w_{T}^{A*}, w_{I}^{A*}, w_{F}^{A*}) = (w_{T}^{A*}, 0, w_{F}^{A})$ satisfies the condition (2.1).

Since A_w is a $\mathfrak{N}w\mathfrak{F}$ of w, we have $w_{T}^{A}(\mathfrak{y}_w) \geq \min \{ w_{T}^{A}(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_{T}^{A}(\mathfrak{x}_w)\}$

and $w_{I}^{A}(\mathfrak{y}_w) \geq \min \{ w_{I}^{A}(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_{I}^{A}(\mathfrak{x}_w)\}$

suppose $\mathfrak{x}_w \curvearrowright \mathfrak{y}_w = \mathfrak{z}_w$ for some $\mathfrak{z}_w \in w$, then $w_{T}^{A}(\mathfrak{y}_w) \geq \min \{ w_{T}^{A}(\mathfrak{z}_w), w_{T}^{A}(\mathfrak{x}_w)\}$

and $w_{I}^{A}(\mathfrak{y}_w) \geq \min \{ w_{I}^{A}(\mathfrak{z}_w), w_{i}^{A}(\mathfrak{x}_w)\}$.

Case 2.1: $\min \{ w_{T}^{A}(\mathfrak{z}_w), w_{T}^{A}(\mathfrak{x}_w)\} = w_{T}^{A}(\mathfrak{z}_w)$ and $\min \{ w_{I}^{A}(\mathfrak{z}_w), w_{I}^{A}(\mathfrak{x}_w)\} = w_{I}^{A}(\mathfrak{z}_w)$

That is $w_{T}^{A}(\mathfrak{y}_w) \geq w_{T}^{A}(\mathfrak{z}_w)$ and $w_{I}^{A}(\mathfrak{y}_w) \geq w_{I}^{A}(\mathfrak{z}_w)$.

Then clearly $w_{T}^{A*}(\mathfrak{y}_w) = \min\{w_{T}^{A}(\mathfrak{y}_w) + w_{I}^{A}(\mathfrak{y}_w), 1\} \geq \min \{w_{T}^{A}(\mathfrak{z}_w) + w_{I}^{A}(\mathfrak{z}_w),1\}$

$= w_{T}^{A*}(\mathfrak{z}_w)$.

Case 2.2: $\min \{ w_{T}^{A}(\mathfrak{z}_w), w_{T}^{A}(\mathfrak{x}_w)\} = w_{T}^{A}(\mathfrak{x}_w)$ and $\min \{ w_{I}^{A}(\mathfrak{z}_w), w_{I}^{A}(\mathfrak{x}_w)\} = w_{I}^{A}(\mathfrak{x}_w)$

That is $w_{T}^{A}(\mathfrak{y}_w) \geq w_{T}^{A}(\mathfrak{x}_w)$ and $w_{I}^{A}(\mathfrak{y}_w) \geq w_{I}^{A}(\mathfrak{x}_w)$.

Then clearly $w_{T}^{A*}(\mathfrak{y}_w) = \min\{w_{T}^{A}(\mathfrak{y}_w) + w_{I}^{A}(\mathfrak{y}_w), 1\} \geq \min \{w_{T}^{A}(\mathfrak{x}_w) + w_{I}^{A}(\mathfrak{x}_w),1\}$

$= w_{T}^{A*}(\mathfrak{x}_w)$.

Clearly $A_{*w} = (w_{T}^{A*}, w_{I}^{A*}, w_{F}^{A*}) = (w_{T}^{A*}, 0, w_{F}^{A})$ satisfies the condition (2.2).

Hence A_{*w} is a $\mathfrak{N}w\mathfrak{F}$ of w.

Remark 3.4: The false- favorite set of a $\mathcal{N}_{T}^{S}$ set on w may or may not be a $\mathfrak{N}w\mathfrak{F}$ of w.

$A_{\wedge w} = (w_{T}^{A\wedge}, w_{I}^{A\wedge}, w_{F}^{A\wedge}) = (w_{T}^{A}, 0, w_{F}^{A\wedge})$ is the false - favorite set of A_w, where

$w_{F}^{A\wedge}(\mathfrak{x}_w) = \min \{w_{F}^{A}(\mathfrak{x}_w) + w_{I}^{A}(\mathfrak{x}_w), 1\}\ \forall\, \mathfrak{x}_w \in w$.

Clearly the false- favorite set $A_{\wedge w}$ of A_w in the example 3.2, is a $\mathfrak{N}w\mathfrak{F}$ of w.

But the false- favorite set of $\mathfrak{N}w\mathfrak{F}$ $B_w = (w_{T}^{B}, w_{I}^{B}, w_{F}^{B})$ in the example 3.2 is not a $\mathfrak{N}w\mathfrak{F}$ of w

Because $w_{F}^{B\wedge}(1_w) = \min \{w_{I}^{B}(1_w)+ w_{F}^{B}(1_w), 1\} = \min\{.831,1\} = .831$

$w_{F}^{B\wedge}(0_w) = \min \{w_{I}^{B}(0_w)+ w_{F}^{B}(0_w), 1\} = \min\{.747,1\} = .747$

So $w_{F}^{B\wedge}(1_w) > w_{F}^{B\wedge}(0_w)$, it does not satisfies the condition (2.1).$B_{\wedge w}$ is not a $\mathfrak{N}w\mathfrak{F}$ of w.

If the false- favorite set of $\mathcal{N}_T^S$ set on $\mathcal{w}$ satisfies the condition (2.1) then it is a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$.

Theorem 3.5: Let $\mathcal{N}_T^S$ set $A_w= (\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}})$ is $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$. If $\mathfrak{x}_w \leq \mathfrak{y}_w$ then

$$\{\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_w) \leq \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{y}_w),\ \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_w) \leq \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{y}_w) \text{ and } \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{x}_w) \geq \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{y}_w)\}\ \forall\, \mathfrak{x}_w, \mathfrak{y}_w \in \mathcal{w}.$$

Proof: Since $\mathfrak{x}_w \leq \mathfrak{y}_w$, then $\mathfrak{x}_w \curvearrowright \mathfrak{y}_w = 1$.

By A_w is $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$, we have

$\mathcal{w}_T^A(\mathfrak{y}w) \geq \min \{ \mathcal{w}_T^A(\mathfrak{x}w \curvearrowright \mathfrak{y}w), \mathcal{w}_T^A(\mathfrak{x}w)\} = \min \{ \mathcal{w}_T^A(1w), \mathcal{w}_T^A(\mathfrak{x}w)\} = \mathcal{w}_T^A(\mathfrak{x}w)$,

$\mathcal{w}_I^A(\mathfrak{y}w) \geq \min \{\mathcal{w}_I^A(\mathfrak{x}w \curvearrowright \mathfrak{y}w), \mathcal{w}_I^A(\mathfrak{x}w)\} = \min \{ \mathcal{w}_I^A(1w), \mathcal{w}_I^A(\mathfrak{x}w)\} = \mathcal{w}_I^A(\mathfrak{x}w)$

and $\mathcal{w}_F^A(\mathfrak{y}_w) \leq \max \{\mathcal{w}_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathcal{w}_F^A(\mathfrak{x}_w)\} = \max \{\mathcal{w}_F^A(1_w), \mathcal{w}_F^A(\mathfrak{x}_w)\} = \mathcal{w}_F^A(\mathfrak{x}_w)$.

Theorem 3.6: A $\mathcal{N}_T^S$ set $A_w= (\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}})$ is $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$ if and only if it holds (2.1) and

(2.3) $\mathcal{w}_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{z}_w) \geq \min \{\mathcal{w}_T^A (\mathfrak{y}_w \curvearrowright (\mathfrak{x}_w \curvearrowright \mathfrak{z}_w)), \mathcal{w}_T^A(\mathfrak{y}_w)\}$,

$\mathcal{w}_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{z}_w) \geq \min \{\mathcal{w}_I^A (\mathfrak{y}_w \curvearrowright (\mathfrak{x}_w \curvearrowright \mathfrak{z}_w)), \mathcal{w}_I^A(\mathfrak{y}_w)\}$

and $\mathcal{w}_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{z}_w) \leq \max \{\mathcal{w}_F^A (\mathfrak{y}_w \curvearrowright (\mathfrak{x}_w \curvearrowright \mathfrak{z}_w)), \mathcal{w}_F^A(\mathfrak{y}_w)\}\ \forall\, \mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w \in \mathcal{w}$.

Proof: Let A_w is a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$, obviously it hold (2.1) and (2.3).

Conversely suppose that A_w is a $\mathcal{N}_T^S$ set with (2.1) and (2.3).

Taking $\mathfrak{x}_w = 1_w$ in (2.4), we get

$\mathcal{w}_T^A(1_w \curvearrowright \mathfrak{z}_w) \geq \min \{\mathcal{w}_T^A (\mathfrak{y}_w \curvearrowright (1_w \curvearrowright \mathfrak{z}_w)), \mathcal{w}_T^A(\mathfrak{y}_w)\}$

$\mathcal{w}_T^A(\mathfrak{z}_w) \geq \min \{\mathcal{w}_T^A (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)), \mathcal{w}_T^A(\mathfrak{y}_w)\}$.

$\mathcal{w}_I^A(1_w \curvearrowright \mathfrak{z}_w) \geq \min \{\mathcal{w}_I^A (\mathfrak{y}_w \curvearrowright (1_w \curvearrowright \mathfrak{z}_w)), \mathcal{w}_I^A(\mathfrak{y}_w)\}$

$\mathcal{w}_I^A(\mathfrak{z}_w) \geq \min \{\mathcal{w}_I^A (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)), \mathcal{w}_I^A(\mathfrak{y}_w)\}$.

and $\mathcal{w}_F^A(1_w \curvearrowright \mathfrak{z}_w) \leq \max \{\mathcal{w}_F^A (\mathfrak{y}_w \curvearrowright (1_w \curvearrowright \mathfrak{z}_w))$

$\mathcal{w}_F^A(\mathfrak{z}_w) \leq \max \{\mathcal{w}_F^A (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w))$.

So A_w is a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$.

Theorem 3.7: A $\mathcal{N}_T^S$ set $A_w= (\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}})$ is $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$ if and only if it hold (2.1) and

(2.4) $\mathcal{w}_T^A((\mathfrak{x}_w \curvearrowright(\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright \mathfrak{z}_w) \geq \min \{\mathcal{w}_T^A (\mathfrak{x}_w), \mathcal{w}_T^A(\mathfrak{y}_w)\}$,

$$\mathscr{w}_I^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright \mathfrak{z}_w) \geq \min\{\mathscr{w}_I^A(\mathfrak{x}_w), \mathscr{w}_I^A(\mathfrak{y}_w)\}$$

and $\mathscr{w}_F^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright \mathfrak{z}_w) \leq \max\{\mathscr{w}_F^A(\mathfrak{x}_w), \mathscr{w}_F^A(\mathfrak{y}_w)\}, \forall\, \mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w \in \mathscr{w}$.

Proof: Suppose that A_w is a $\mathfrak{N}\mathscr{w}\mathfrak{F}$ of $\mathscr{w}$ and $\mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w \in \mathscr{w}$.

Clearly $\mathscr{w}_T^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright \mathfrak{z}_w) \geq \min\{\mathscr{w}_T^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)),$

$\mathscr{w}_T^A(\mathfrak{y}_w)\}$-------(i)

For any $\mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w \in \mathscr{w}$, we have $((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w) = \mathfrak{x}_w \vee (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)$

$\geq \mathfrak{x}_w.$

So $\mathscr{w}_T^A(((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \geq \mathscr{w}_T^A(\mathfrak{x}_w)$ ---------------- (ii)

From (i) and (ii), $\mathscr{w}_T^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright \mathfrak{z}_w) \geq \min\{\mathscr{w}_T^A(\mathfrak{x}_w), \mathscr{w}_T^A(\mathfrak{y}_w)\}$.

Clearly $\mathscr{w}_I^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright \mathfrak{z}_w) \geq \min\{\mathscr{w}_I^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)),$

$\mathscr{w}_I^A(\mathfrak{y}_w)\}$

and $\mathscr{w}_I^A(((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \geq \mathscr{w}_I^A(\mathfrak{x}_w)$

So $\mathscr{w}_I^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright \mathfrak{z}_w) \geq \min\{\mathscr{w}_I^A(\mathfrak{x}_w), \mathscr{w}_I^A(\mathfrak{y}_w)\}$.

Clearly $\mathscr{w}_F^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright \mathfrak{z}_w) \leq \max\{\mathscr{w}_F^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)),$

$\mathscr{w}_F^A(\mathfrak{y}_w)\}$

and $\mathscr{w}_F^A(((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \leq \mathscr{w}_F^A(\mathfrak{x}_w)$

From (i) and (iv), $\mathscr{w}_F^A((\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)) \curvearrowright \mathfrak{z}_w) \leq \max\{\mathscr{w}_F^A(\mathfrak{x}_w), \mathscr{w}_F^A(\mathfrak{y}_w)\}$.

Conversely suppose that A_w is a $\mathcal{N}_T^S$ with (2.1) and (2.4).

$$\begin{aligned}
\mathscr{w}_T^A(\mathfrak{y}_w) = \mathscr{w}_T^A(1_w \curvearrowright \mathfrak{y}_w) &= \mathscr{w}_T^A(((\mathfrak{x}_w \curvearrowright \mathfrak{y}_w) \curvearrowright (\mathfrak{x}_w \curvearrowright \mathfrak{y}_w)) \curvearrowright \mathfrak{y}_w) \\
&\geq \min\{\mathscr{w}_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathscr{w}_T^A(\mathfrak{x}_w)\}. \\
\mathscr{w}_I^A(\mathfrak{y}_w) = \mathscr{w}_I^A(1_w \curvearrowright \mathfrak{y}_w) &= \mathscr{w}_I^A(((\mathfrak{x}_w \curvearrowright \mathfrak{y}_w) \curvearrowright (\mathfrak{x}_w \curvearrowright \mathfrak{y}_w)) \curvearrowright \mathfrak{y}_w) \\
&\geq \min\{\mathscr{w}_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathscr{w}_I^A(\mathfrak{x}_w)\}. \\
\mathscr{w}_F^A(\mathfrak{y}_w) = \mathscr{w}_F^A(1_w \curvearrowright \mathfrak{y}_w) &= \mathscr{w}_F^A(((\mathfrak{x}_w \curvearrowright \mathfrak{y}_w) \curvearrowright (\mathfrak{x}_w \curvearrowright \mathfrak{y}_w)) \curvearrowright \mathfrak{y}_w) \\
&\leq \max\{\mathscr{w}_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathscr{w}_F^A(\mathfrak{x}_w)\}.
\end{aligned}$$

So A_w is a $\mathfrak{N}\mathscr{w}\mathfrak{F}$ of $\mathscr{w}$.

Theorem 3.8: Every $\mathfrak{N}\mathscr{w}\mathfrak{F}$ $A_w = (\mathscr{w}_T^A, \mathscr{w}_I^A, \mathscr{w}_F^A)$ fulfills the following result:

(2.5) If $\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w) = 1_w$ then

$w_T^A(\mathfrak{z}_w) \geq \min\{w_T^A(\mathfrak{x}_w), w_T^A(\mathfrak{y}_w)\}, w_I^A(\mathfrak{z}_w) \geq \min\{w_I^A(\mathfrak{x}_w), w_I^A(\mathfrak{y}_w)\}$

and $w_F^A(\mathfrak{z}_w) \leq \max\{w_F^A(\mathfrak{x}_w)), w_F^A(\mathfrak{y}_w)\}\ \forall\, \mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w \in w$.

Proof: Suppose A_w is a $\mathfrak{N}w\mathfrak{F}$ of w and $\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w) = 1_w$ for all $\mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w \in w$.

We get $w_T^A(\mathfrak{z}_w) \geq \min\{w_T^A(\mathfrak{y}_w \curvearrowright \mathfrak{z}_w), w_T^A(\mathfrak{y}_w)\}$

$\geq \min\{\min\{w_T^A(\mathfrak{x}_w), w_T^A(\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w))\}, w_T^A(\mathfrak{y}_w)\}$

$\geq \min\{\min\{w_T^A(\mathfrak{x}_w), w_T^A(1_w)\}, w_T^A(\mathfrak{y}_w)\}$

$\geq \min\{w_T^A(\mathfrak{x}_w), w_T^A(\mathfrak{y}_w)\}$,

$w_I^A(\mathfrak{z}_w) \geq \min\{w_I^A(\mathfrak{y}_w \curvearrowright \mathfrak{z}_w), w_I^A(\mathfrak{y}_w)\}$

$\geq \min\{\min\{w_I^A(\mathfrak{x}_w), w_I^A(\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w))\}, w_I^A(\mathfrak{y}_w)\}$

$\geq \min\{\min\{w_I^A(\mathfrak{x}_w), w_I^A(1_w)\}, w_I^A(\mathfrak{y}_w)\}$

$\geq \min\{w_I^A(\mathfrak{x}_w), w_I^A(\mathfrak{y}_w)\}$

and $w_F^A(\mathfrak{z}_w) \leq \max\{w_F^A(\mathfrak{y}_w \curvearrowright \mathfrak{z}_w), w_F^A(\mathfrak{y}_w)\}$

$\leq \max\{\max\{w_F^A(\mathfrak{x}_w), w_F^A(\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w))\}, w_F^A(\mathfrak{y}_w)\}$

$\leq \max\{\max\{w_F^A(\mathfrak{x}_w), w_F^A(1_w)\}, w_F^A(\mathfrak{y}_w)\}$

$\leq \max\{w_F^A(\mathfrak{x}_w), w_F^A(\mathfrak{y}_w)\}$.

Lemma 3.9: Every $\mathfrak{N}w\mathfrak{F}$ $A_w = (w_T^A, w_I^A, w_F^A)$ of w fulfills the following result:

If $\mathfrak{x}_{n_w} \curvearrowright (\mathfrak{x}_{(n-1)_w} \curvearrowright$ --------- $(\mathfrak{x}_{1_w} \curvearrowright \mathfrak{y}_w)) = 1_w$ then

$w_T^A(\mathfrak{y}_w) \geq \min\{w_T^A(\mathfrak{x}_{n_w}),\text{-----------}, w_T^A(\mathfrak{x}_{1_w})\}$,

$w_I^A(\mathfrak{y}_w) \geq \min\{w_I^A(\mathfrak{x}_{n_w}), \text{------------}, w_I^A(\mathfrak{x}_{1_w})\}$

and $w_F^A(\mathfrak{y}_w) \leq \max\{w_F^A(\mathfrak{x}_{n_w}),\text{----------}, w_F^A(\mathfrak{x}_{1_w})\}\ \forall\, \mathfrak{x}_{n_w},\text{----------}, \mathfrak{x}_{1_w}\, \mathfrak{y}_w \in w$.

Theorem 3.10: Let A_w and B_w are two $\mathfrak{N}w\mathfrak{F}$ of w, then $A_w \cap B_w$ is a $\mathfrak{N}w\mathfrak{F}$ of w.

Proof: Let $\mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w \in w$ such that $\mathfrak{x}_w \leq (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w)$, then $\mathfrak{x}_w \curvearrowright (\mathfrak{y}_w \curvearrowright \mathfrak{z}_w) = 1_w$.

Since A_w and B_w are two $\mathfrak{N}w\mathfrak{F}$ of w then by (2.5), we have

$w_T^A(\mathfrak{z}_w) \geq \min\{w_T^A(\mathfrak{x}_w), w_T^A(\mathfrak{y}_w)\}$,

$w_I^A(\mathfrak{z}_w) \geq \min\{w_I^A(\mathfrak{x}_w), w_I^A(\mathfrak{y}_w)\}$

and $w_F^A(\mathfrak{z}_w) \leq \max\{w_F^A(\mathfrak{x}_w)), w_F^A(\mathfrak{y}_w)\}$.

$\mathcal{w}_T^B(\mathfrak{z}_w) \geq \min\{\mathcal{w}_T^B(\mathfrak{x}_w), \mathcal{w}_T^B(\mathfrak{y}_w)\}$,

$\mathcal{w}_I^B(\mathfrak{z}_w) \geq \min\{\mathcal{w}_I^B(\mathfrak{x}_w), \mathcal{w}_I^B(\mathfrak{y}_w)\}$

and $\mathcal{w}_F^B(\mathfrak{z}_w) \leq \max\{\mathcal{w}_F^B(\mathfrak{x}_w)), \mathcal{w}_F^B(\mathfrak{y}_w)\}$.

$\mathcal{w}_T^{A\cap B}(\mathfrak{z}_w) = \min\{\mathcal{w}_T^A(\mathfrak{z}_w), \mathcal{w}_T^B(\mathfrak{z}_w)\}$

$= \min\{\min\{\mathcal{w}_T^A(\mathfrak{x}_w), \mathcal{w}_T^A(\mathfrak{y}_w)\}, \min\{\mathcal{w}_T^B(\mathfrak{x}_w), \mathcal{w}_T^B(\mathfrak{y}_w)\}$

$= \min\{\min\{\mathcal{w}_T^A(\mathfrak{x}_w), \mathcal{w}_T^B(\mathfrak{x}_w)\}, \min\{\mathcal{w}_T^A(\mathfrak{y}_w), \mathcal{w}_T^B(\mathfrak{y}_w)\}$

$= \min\{\mathcal{w}_T^{A\cap B}(\mathfrak{x}_w), \mathcal{w}_T^{A\cap B}(\mathfrak{y}_w)\}$.

$\mathcal{w}_I^{A\cap B}(\mathfrak{z}_w) = \min\{\mathcal{w}_I^A(\mathfrak{z}_w), \mathcal{w}_I^B(\mathfrak{z}_w)\}$

$= \min\{\min\{\mathcal{w}_I^A(\mathfrak{x}_w), \mathcal{w}_I^A(\mathfrak{y}_w)\}, \min\{\mathcal{w}_I^B(\mathfrak{x}_w), \mathcal{w}_I^B(\mathfrak{y}_w)\}$

$= \min\{\min\{\mathcal{w}_I^A(\mathfrak{x}_w), \mathcal{w}_I^B(\mathfrak{x}_w)\}, \min\{\mathcal{w}_I^A(\mathfrak{y}_w), \mathcal{w}_I^B(\mathfrak{y}_w)\}$

$= \min\{\mathcal{w}_I^{A\cap B}(\mathfrak{x}_w), \mathcal{w}_I^{A\cap B}(\mathfrak{y}_w)\}$.

$\mathcal{w}_F^{A\cap B}(\mathfrak{z}_w) = \max\{\mathcal{w}_F^A(\mathfrak{z}_w), \mathcal{w}_F^B(\mathfrak{z}_w)\}$

$= \max\{\max\{\mathcal{w}_F^A(\mathfrak{x}_w), \mathcal{w}_F^A(\mathfrak{y}_w)\}, \max\{\mathcal{w}_F^B(\mathfrak{x}_w), \mathcal{w}_F^B(\mathfrak{y}_w)\}$

$= \max\{\max\{\mathcal{w}_F^A(\mathfrak{x}_w), \mathcal{w}_F^B(\mathfrak{x}_w)\}, \max\{\mathcal{w}_F^A(\mathfrak{y}_w), \mathcal{w}_F^B(\mathfrak{y}_w)\}$

$= \max\{\mathcal{w}_F^{A\cap B}(\mathfrak{x}_w), \mathcal{w}_F^{A\cap B}(\mathfrak{y}_w)\}$.

So $A_w \cap B_w$ is a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$.

For the $\mathcal{N}_T^s$ set $A_w = (\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}})$ on $\mathcal{w}$, we define the $\mathcal{N}_T^s$cut sets as follows:

$\mathcal{w}_T^{A\alpha} = \{\mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_w) \geq \alpha\}$, $\mathcal{w}_I^{A\beta} = \{\mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_w) \geq \beta\}$

and $\mathcal{w}_F^{A\gamma} = \{\mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{x}_w) \leq \gamma\}$, $\alpha, \beta, \gamma \in [0,1]$.

Theorem 3.11: A $\mathcal{N}_T^s$ set $A_w = (\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}, \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}})$ is a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$ if and only if its nonempty $\mathcal{N}_T^s$cut sets $\mathcal{w}_T^{A\alpha}$, $\mathcal{w}_I^{A\beta}$ and $\mathcal{w}_F^{A\gamma}$ are implicative filters of $\mathcal{w}$ for all $\alpha, \beta, \gamma \in [0,1]$.

Proof: Suppose A_w is $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$ and $\alpha, \beta, \gamma \in [0,1]$.

Let $\mathcal{w}_T^{A\alpha}$, $\mathcal{w}_I^{A\beta}$ and $\mathcal{w}_F^{A\gamma}$ are nonempty.

Clearly $1_w \in \mathcal{w}_T^{A\alpha}$, $1_w \in \mathcal{w}_I^{A\beta}$ and $1_w \in \mathcal{w}_F^{A\gamma}$.

$\mathfrak{x}_{1_w}$, $\mathfrak{x}_{2_w}$, $\mathfrak{y}_{1_w}$, $\mathfrak{y}_{2_w}$, $\mathfrak{z}_{1_w}$ and $\mathfrak{z}_{2_w} \in \mathcal{w}$ such that $(\mathfrak{x}_{1_w} \curvearrowright \mathfrak{x}_{2_w}$, $\mathfrak{x}_{1_w} \in \mathcal{w}_T^{A\alpha})$, $(\mathfrak{y}_{1_w} \curvearrowright \mathfrak{y}_{2_w}$,

$\mathfrak{y}_{1_w} \in \mathcal{w}_I^{A\beta})$ and

$(\mathfrak{z} \curvearrowright \mathfrak{z}_{2_w}$, $\mathfrak{z}_{1_w} \in \mathcal{w}_F^{A\gamma})$.

Then $\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_{2_w}) \geq \min\{\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w} \curvearrowright \mathfrak{x}_{2_w}), \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w})\} \geq \alpha$ implies $\mathfrak{x}_{2_w} \in \mathcal{w}_T^{A\alpha}$.

$\mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{y}_{2_w}) \geq \min\{\mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{y}_{1_w} \curvearrowright \mathfrak{y}_{I_w}), \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{y}_{1_w})\} \geq \beta$ implies $\mathfrak{y}_{2_w} \in \mathcal{w}_I^{A\beta}$.

$\mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{z}_{2_w}) \leq \max\{\mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{z}_{1_w} \curvearrowright \mathfrak{z}_{I_w}), \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{z}_{1_w})\} \leq \gamma$ implies $\mathfrak{z}_{2_w} \in \mathcal{w}_F^{A\gamma}$.

$\mathcal{w}_T^{A\alpha}$, $\mathcal{w}_I^{A\beta}$ and $\mathcal{w}_F^{A\gamma}$ are implicative filters of $\mathcal{w}$.

Conversely suppose that A_w is $\mathcal{N}_T^S$ and $\mathcal{w}_T^{A\alpha}$, $\mathcal{w}_I^{A\beta}$ and $\mathcal{w}_F^{A\gamma}$ are implicative filters of ,

$\alpha, \beta, \gamma \in [0,1]$.

For any $\mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w \in \mathcal{w}$ such that $\mathcal{w}_T^A(\mathfrak{x}_w) = \alpha$, $\mathcal{w}_I^A(\mathfrak{y}_w) = \beta$ and $\mathcal{w}_F^A(\mathfrak{z}_w) = \gamma$.

Then $\mathfrak{x}_w \in \mathcal{w}_T^{A\alpha}$, $\mathfrak{y}_w \in \mathcal{w}_I^{A\beta}$ and $\mathfrak{z}_w \in \mathcal{w}_F^{A\gamma}$, so $\mathcal{w}_T^{A\alpha}$, $\mathcal{w}_I^{A\beta}$ and $\mathcal{w}_F^{A\gamma}$ are nonempty.

For any $\mathfrak{x}_{1_w}$, $\mathfrak{x}_{2_w} \in \mathcal{w}$, let

$\alpha = \min\{\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w} \curvearrowright \mathfrak{x}_{2_w}), \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w})\}$, $\beta = \min\{\mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w} \curvearrowright \mathfrak{x}_{2_w}), \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w})\}$ and

$\gamma = \{\mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w} \curvearrowright \mathfrak{x}_{2_w}), \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w}$

Then clearly $\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_{2_w}) \geq \alpha = \min\{\mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w} \curvearrowright \mathfrak{x}_{2_w}), \mathcal{w}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w})\}$,

$\mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{y}_{2_w}) \geq \beta = \min\{\mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w} \curvearrowright \mathfrak{x}_{2_w}), \mathcal{w}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w})\}$

and $\mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{z}_{2_w}) \leq \gamma = \max\{\mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w} \curvearrowright \mathfrak{x}_{2_w}), \mathcal{w}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{x}_{1_w})\}$.

So A_w is a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$.

Lemma 3.12: If A_w is a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$ then $\mathcal{w}_T^{A\alpha} \cap \mathcal{w}_I^{A\beta} \cap \mathcal{w}_F^{A\gamma}$ are implicative filters of $\mathcal{w}$.

Theorem 3.13: Any implicative filter A of $\mathcal{w}$ is a (α, α, α) cut- $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$.

Proof: Let A is implicative filter of $\mathcal{w}$ and $\alpha \in [0,1]$.

Consider a $\mathcal{N}_T^S$ set $A_w = (w_T^A(\mathfrak{y}_w), w_I^A(\mathfrak{y}_w), w_F^A(\mathfrak{y}_w)) = (\alpha, \alpha, \alpha)$ if $\mathfrak{y}_w \in A_w$

$= (0_w, 0_w, 0_w)$ if $\mathfrak{y}_w \notin A_w$

Clearly $1_w \in A_w$, $A_w = (w_T^A(1_w), w_I^A(1_w), w_F^A(1_w))$

$= (\alpha, \alpha, \alpha)$

$\geq (w_T^A(\mathfrak{y}_w), w_I^A(\mathfrak{y}_w), w_F^A(\mathfrak{y}_w))$.

Let $\mathfrak{x}_w, \mathfrak{y}_w \in w$. If $\mathfrak{y}_w \in A$ then $w_T^A(\mathfrak{y}_w) = \alpha \geq \min\{w_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_T^A(\mathfrak{x}_w)\}$,

$w_I^A(\mathfrak{y}_w) = \alpha \geq \min\{w_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_I^A(\mathfrak{x}_w)\}$

and $w_F^A(\mathfrak{y}_w) = \alpha \leq \max\{w_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_F^A(\mathfrak{x}_w)\}$.

Suppose $\mathfrak{y}_w \notin A$ then $\mathfrak{x} \notin A$ or $\mathfrak{x}_w \curvearrowright \mathfrak{y}_w \notin A$.

So $w_T^A(\mathfrak{y}_w) = 0_w = \min\{w_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_T^A(\mathfrak{x}_w)\}$,

$w_I^A(\mathfrak{y}_w) = 0_w = \min\{w_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_I^A(\mathfrak{x}_w)\}$

and $w_F^A(\mathfrak{y}_w) = 0_w = \max\{w_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_F^A(\mathfrak{x}_w)\}$.

So A_w is $\mathfrak{N}w\mathfrak{F}$ of w.

Theorem 3.14: If A_w is $\mathfrak{N}w\mathfrak{F}$ of w then the $A = \{\mathfrak{x}_w \in w / (w_T^A(\mathfrak{y}_w), w_I^A(\mathfrak{y}_w), w_F^A(\mathfrak{y}_w)) = (w_T^A(1_w), w_I^A(1_w), w_F^A(1_w)\}$ is a implicative filter of w.

Proof: Since $A = \{\mathfrak{x}_w \in w / (w_T^A(\mathfrak{y}_w), w_I^A(\mathfrak{y}_w), w_F^A(\mathfrak{y}_w)) = (w_T^A(1_w), w_I^A(1_w), w_F^A(1_w))\}$

Clearly $1_w \in A$. Let $\mathfrak{x}_w, \mathfrak{y}_w \in w$ such that $\mathfrak{x}_w$, $\mathfrak{x}_w \curvearrowright \mathfrak{y}_w \in A$.

Then $w_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_T^A(\mathfrak{x}_w) = w_T^A(1_w)$,

$w_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_I^A(\mathfrak{x}_w) = w_I^A(1_w)$

and $w_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_F^A(\mathfrak{x}_w) = w_F^A(1_w)$.

So, $w_T^A(\mathfrak{y}_w) \geq \min\{w_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_T^A(\mathfrak{x}_w)\} = w_T^A(1_w)$,

$w_I^A(\mathfrak{y}_w) \geq \min\{w_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_I^A(\mathfrak{x}_w)\} = w_I^A(1_w)$

and $w_F^A(\mathfrak{y}_w) \leq \max\{w_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_F^A(\mathfrak{x}_w)\} = w_F^A(1_w)$.

That is $\mathfrak{y}_w \in A$.

So A a implicative filter of w.

Definition 3.15: A $\mathcal{N}_T^S$ set $A_w = (w_T^A, w_I^A, w_F^A)$ is on w is called a $\mathfrak{N}w$ -lattice filter ($\mathfrak{N}w\mathfrak{L}\mathfrak{F}$) if it satisfies $\forall\, \mathfrak{x}_w, \mathfrak{y}_w \in w$, (2.5) $\mathcal{W}_T^A\ (\mathfrak{x}_w \ \wedge \mathfrak{y}_w) \ \geq \ \min\{ \ \mathcal{W}_T^A\ (\mathfrak{x}_w), \ \ \mathcal{W}_{\boldsymbol{T}}^{\boldsymbol{A}}(\mathfrak{y}_w) \}$

$\mathcal{W}_I^A\ (\mathfrak{x}_w \ \wedge \mathfrak{y}_w) \ \geq \ \min\{ \ \mathcal{W}_I^A\ (\mathfrak{x}_w), \ \ \mathcal{W}_{\boldsymbol{I}}^{\boldsymbol{A}}(\mathfrak{y}_w)\}$

and $\mathcal{W}_F^A\ (\mathfrak{x}_w \ \wedge \mathfrak{y}_w) \ \leq \ \max\{ \ \mathcal{W}_F^A\ (\mathfrak{x}_w), \ \ \mathcal{W}_{\boldsymbol{F}}^{\boldsymbol{A}}(\mathfrak{y}_w) \}$

Example 3.16: The $\mathcal{N}_T^S$ set $A_w = (\ \mathcal{W}_{\boldsymbol{T}}^{\boldsymbol{A}}, \ \mathcal{W}_{\boldsymbol{I}}^{\boldsymbol{A}}, \ \mathcal{W}_{\boldsymbol{F}}^{\boldsymbol{A}})$ defined on $\mathcal{W}$ in example 3.3 as follows is $\mathfrak{N}\mathcal{W}\mathfrak{L}\mathfrak{F}$ of $\mathcal{W}$.

$\mathfrak{z}$	$\mathcal{W}_T^A(\mathfrak{z})$	$\mathcal{W}_I^A\ (\mathfrak{z})$	$\mathcal{W}_F^A(\mathfrak{z})$
0_w	.547	.557	.451
$\mathfrak{x}_w$	.547	.557	.451
$\mathfrak{y}_w$	.721	.561	.331
$\mathfrak{z}_w$	.547	.557	.451
$\mathfrak{v}_w$	.547	.557	.451
1_w	.721	.561	.331

Theorem 3.17: Every $\mathfrak{N}\mathcal{W}\mathfrak{F}$ A_w of $\mathcal{W}$ is $\mathfrak{N}\mathcal{W}\mathfrak{L}\mathfrak{F}$ of $\mathcal{W}$.

Proof: Let A_w is a $\mathfrak{N}\mathcal{W}\mathfrak{F}$ of $\mathcal{W}$.

By(2.2) $\mathcal{W}_T^A\ (\mathfrak{x}_w \ \wedge \mathfrak{y}_w) \ \geq \ \min\{ \ \mathcal{W}_T^A\ (\ \mathfrak{x}_w \ \curvearrowright (\ \mathfrak{x}_w \ \wedge \mathfrak{y}_w)), \ \ \mathcal{W}_T^A\ (\mathfrak{x}_w)\}$

$= \min\{ \ \mathcal{W}_T^A\ (\ \mathfrak{x}_w \ \curvearrowright \ \mathfrak{y}_w), \ \ \mathcal{W}_T^A\ (\mathfrak{x}_w)\}$

$\geq \ \min\{\min\{ \ \mathcal{W}_T^A\ (\ \mathfrak{y}_w \ \curvearrowright (\ \mathfrak{x}_w \ \wedge \mathfrak{y}_w)), \ \ \mathcal{W}_T^A\ (\mathfrak{y}_w)\}, \ \mathcal{W}_T^A\ (\mathfrak{x}_w)\}$

$\geq \ \min\{\min\{ \ \mathcal{W}_T^A\ (1_w), \ \ \mathcal{W}_T^A\ (\mathfrak{y}_w)\}, \ \mathcal{W}_T^A\ (\mathfrak{x}_w)\}$

$= \ \ \min\{ \ \mathcal{W}_T^A\ (\mathfrak{y}_w), \ \mathcal{W}_T^A\ (\mathfrak{x}_w) \}$

$\mathcal{W}_I^A\ (\mathfrak{x}_w \ \wedge \mathfrak{y}_w) \ \geq \ \min\{ \ \mathcal{W}_I^A\ (\ \mathfrak{x}_w \ \curvearrowright (\ \mathfrak{x}_w \ \wedge \mathfrak{y}_w)), \ \ \mathcal{W}_I^A\ (\mathfrak{x}_w)\}$

$= \min\{ \ \mathcal{W}_I^A\ (\ \mathfrak{x}_w \ \curvearrowright \ \mathfrak{y}_w), \ \ \mathcal{W}_I^A\ (\mathfrak{x}_w)\}$

$\geq \ \min\{\min\{ \ \mathcal{W}_I^A\ (\ \mathfrak{y}_w \ \curvearrowright (\ \mathfrak{x}_w \ \wedge \mathfrak{y}_w)), \ \ \mathcal{W}_I^A\ (\mathfrak{y}_w)\}, \ \mathcal{W}_T^A\ (\mathfrak{x}_w)\}$

$\geq \ \min\{\min\{ \ \mathcal{W}_I^A\ (1_w), \ \ \mathcal{W}_I^A\ (\mathfrak{y}_w)\}, \ \mathcal{W}_I^A\ (\mathfrak{x}_w)\}$

$= \ \geq \ \min\{ \ \mathcal{W}_I^A\ (\mathfrak{y}_w), \ \mathcal{W}_I^A\ (\mathfrak{x}_w) \}$

$\mathcal{W}_F^A\ (\mathfrak{x}_w \ \wedge \mathfrak{y}_w) \ \leq \ \max\{ \ \mathcal{W}_F^A\ (\ \mathfrak{x}_w \ \curvearrowright (\ \mathfrak{x}_w \ \wedge \mathfrak{y}_w)), \ \ \mathcal{W}_F^A\ (\mathfrak{x}_w)\}$

$= \max\{ \ \mathcal{W}_F^A\ (\ \mathfrak{x}_w \ \curvearrowright \ \mathfrak{y}_w), \ \ \mathcal{W}_F^A\ (\mathfrak{x}_w)\}$

$\leq$ max {max{ $\mathcal{w}_F^A$ ($\mathfrak{y}_w \curvearrowright$ ($\mathfrak{x}_w \wedge \mathfrak{y}_w$)), $\mathcal{w}_F^A$ ($\mathfrak{y}_w$)}, $\mathcal{w}_T^A$ ($\mathfrak{x}_w$)}

$\leq$ max {max{ $\mathcal{w}_F^A$ (1_w), $\mathcal{w}_F^A$ ($\mathfrak{y}_w$)}, $\mathcal{w}_F^A$ ($\mathfrak{x}_w$)}

= max { $\mathcal{w}_F^A$ ($\mathfrak{y}_w$), $\mathcal{w}_F^A$ ($\mathfrak{x}_w$) }.

So A_w of $\mathcal{w}$ is $\mathfrak{N}\mathcal{w}\mathfrak{L}\mathfrak{F}$ of $\mathcal{w}$.

Remark 3.18: The $\mathfrak{N}\mathcal{w}\mathfrak{L}\mathfrak{F}$ of $\mathcal{w}$ is need not to be a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$. For example the $\mathfrak{N}\mathcal{w}\mathfrak{L}\mathfrak{F}$ A_w of $\mathcal{w}$ is not a $\mathfrak{N}\mathcal{w}\mathfrak{F}$ of $\mathcal{w}$ because $\mathcal{w}_T^A$ ($\mathfrak{z}_w$) $\ngeq$ { $\mathcal{w}_T^A$ ($\mathfrak{y}_w \curvearrowright \mathfrak{z}_w$), $\mathcal{w}_T^A$ ($\mathfrak{y}_w$)}.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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