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CONSTRUCTION OF DIFFERENT PECULIAR STRUCTURES OF 4-REGULAR PLANAR GRAPH WITH ODD REGIONS AND EVEN REGIONS AND ITS APPLICATION

ATOWAR UL ISLAM[1,*], SANKAR HALOI[2]

[1]Dept of Computer Science and IT, Cotton University, Guwahati, Assam, Pin -781001, India

[2]Dept of Mathematics, Cotton University, Guwahati, Assam, Pin-781001, India

Abstract: In this article we have proposed the construction of a structure of the 4-regular planar graphs for G(2m+2,4m+4) where m≥2. Based on the proposed structure we have specified two theorems on odd regions and total regions of 4-regular planar graphs. The experimental results and proof of the specified theorems have also been provided. Maximum region covered by odd region is also discussed using the structures of the 4-regular graph. Finally an application is given in region base map coloring and GSM network coloring.

Keywords: 4-regular planar graphs; odd regions, even regions; total regions, map color; GSM color.

2010 AMS Subject Classification: 05C10.

1. INTRODUCTION

Various authors have been working on Regular planar graphs from different perception. Jin and Wei [1] have suggested that a graph G is k- choosable if it has an L-coloring whenever L is a list assignment such that $|L(v)| \geq k$ for all $v \in V(G)$. For 3-choosability of plane graphs they have provided two sufficient conditions Therese [2] have proposed that all edge lengths are integers,

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*Corresponding author
E-mail address: atowar91626@gmail.com
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drawings of planar graphs. Dutta Dutta, Kalita and Baruah [3] have discussed different application of regular planar sub graphs of complete graphs. Islam, Dutta, Choudhury and Kalita [4] have discussed the construction of the structures with odd region and even region of three regular planner graphs from the graph G (2m+2, 3m+3) for $m \geq 2$. They have also developed an algorithm and provided an application based of its region. The minimum vertex cover of a class of regular planar sub-graphs H(2m+2,3m+3), K(2m+2,4m+4) for $m \geq 2$ and J(2m+2,5m+5) for $m \geq 5$ obtained from the complete graph K2m+2 had already been discussed by Islam, Kalita and dutta [5]. They have developed an algorithm to find the minimum vertex cover of these types of regular planar sub-graphs. They have also provided the application of the same on minimum vertex cover to reduce the power consumption of sensor network. Zhang, Liu and Li [6] have proved that planar graph G with maximum degree $\Delta \geq 12$ that the (2,1)-total labeling number $\lambda 2(G)$ is at most $\Delta + 2$. Ackerman , Keszegh and Vizer [7] have proved that if an n-vertex graph G can be drawn in the plane such that each pair of crossing edges is independent and there is a crossing-free edge that connects their endpoints, then G has O(n) edges. Graphs that admit such drawings are related to quasi-planar graphs and to maximal 1-planar and fan-planar graphs. Couch, Daniel, Guidry and Wright [8] have discussed the construction of a homing tour is known to be NP-complete. They have focused on split Euler tours (SETs) in 3-connected, 4-regular, planar graphs (tfps). The various results rely heavily on the structure of such graphs as determined by the Euler formula and on the construction of tfps from the octahedron. They have also construct a 2-connected 4-regular planar graph that does not have a SET. Lu and Wang [9] have obtained a sharp result that for any even $n \geq 34$, every { Dn , Dn +1} regular graph of order n contains (n/4) disjoint perfect matchings, where Dn = 2[n/4]-1. As a consequence, for any integer $D \geq Dn$, every {D, D +1} regular graph of order n contains (D-[n/4]+1) disjoint perfect matching's. In most of the works, it is seen that almost all the works have been done on regular planar graphs and planar graphs. On 3-regular planar graph and 4-regular planar graphs, very few works are seen to be done. But in the research paper hardly any work is seen on even region and odd region of 4-regular planar graph. But the regions are used in different Application in coloring and biological diversity. Therefore in the present work we propose two new theorems on the odd region of 4 – regular planar graphs. Section 1 includes the introduction which contains the works of other researcher. Section 2 includes the definition. Section 3 contains two theorems which are stated and proved. Section 4 includes discussion of odd region which covered by only three edges and Section 5 includes the Application and Section 6 includes the conclusion.

2. DEFINITION

Region: A region of a Graph (V, E) is define an area of a plane covered by or bounded by number of edges and number vertices. A region is two types –inner region and outer region which is also called infinite region. The inner region are two types – odd region and even region. In this article we discuss only inner regions.

Odd Region: If a region of a Graph (V, E) is covered by odd number of vertices and odd number of edges than it is called odd region. Fig-1 is an example of odd region graph.

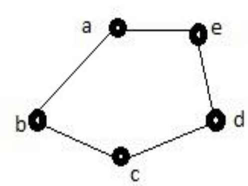

Figure 1: (bounded by odd number of edges)

Even Region: If a region of a Graph (V, E) is covered by even number vertices and even number edges, called even region. Fig-2 is an example of even region graph.

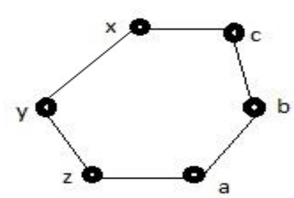

Figure 2: (bounded by Even number of edges)

There are numerous structures of 4 regular planar graphs. Here we focus only one type of peculiar structure for discussing the odd region and even region of the graph.

3. OUR WORK

We construct a structure for the graph G(2m+2,4m+4) where m≥2, the 4-regular planar graphs in the following ways.

A peculiar structure of 4 Regular planar graph:

Let G be a graph having 2m +2 vertices and 4m+4 edges for m≥2. For m=2, G contains six vertices, the vertex set is {v_1, v_2, v_3, v_4, v_5, v_6} and 12 edges. Let us join the six vertices by twelve edges using the following formulae.

$$\beta(v_w) = \begin{cases} v_{w+1} & for\ 1 \le w \le 5 \\ v_1 & for\ w = 6 \end{cases}$$

$$\beta(v_x) = \begin{cases} v_1 & for\ x = 3, 5 \\ v_2 & for\ x = 4 \end{cases}$$

$$\beta(v_{y+1}) = v_{7-y} \text{ for } 1 \le y \le 2$$

$$\beta(v_{z+2}) = v_{7-z}\ for\ 1 \le z \le 1$$

Using above formulae we construct the graph as shown in Figure-3, which is planar and regular graph of degree four and the edge set is {$v_1 v_2$, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1, v_1v_5, v_2v_4, v_4v_1, v_2v_6, v_3v_5, v_3v_6}.

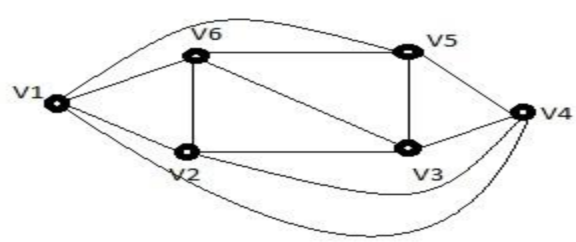

Figure-3 Four regular planar graph for m=2

For m=3, the Graph (G) contains eight vertices and sixteen edges and the vertex set is {v_1,v_2,v_3,v_3,v_4,v_6,v_7,v_8}. Let us join eight vertices with sixteen edges using the following formulae.

$$\beta(v_w) = \begin{cases} v_{w+1} & for\ 1 \le w \le 7 \\ v_1 & for\ w = 8 \end{cases}$$

$$\beta(v_x) = \begin{cases} v_1 & for\ x = 5,6 \\ v_2 & for\ x = 5 \end{cases}$$

$$\beta(v_{y+1}) = v_{9-y} \quad for\ 1 \le y \le 3$$

$$\beta(v_{z+2}) = v_{9-z} \quad for\ 1 \le z \le 2$$

Using above formulae we construct the graph as shown in Figure-4, which is planar and regular graph of degree four and edge set is - $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_1v_8, v_2v_8, v_3v_7, v_4v_6, v_1v_6, v_1v_5, v_2v_5, v_3v_8, v_4v_7\}$.

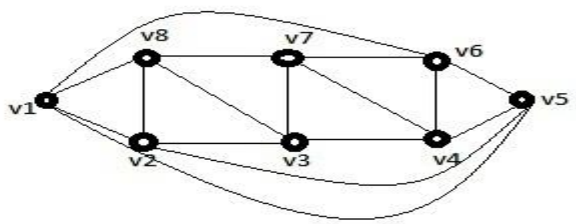

Figure-4: Four regular planar graph for m=3

For m=4, the Graph (G) contains ten vertices and twenty edges and the vertex set is $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$. Let us join eight vertices with twelve edges using the following formulae.

$$\beta(v_w) = \begin{cases} v_{w+1} & for \ \ 1 \le w \le 9 \\ v_1 & for \ w = 10 \end{cases}$$

$$\beta(v_x) = \begin{cases} v_1 & for \ x = 6,7 \\ v_2 & for \ x = 6 \end{cases}$$

$$\beta(v_{y+1}) = v_{11-y} \quad for \ 1 \le y \le 4$$

$$\beta(v_{z+2}) = v_{11-z} \quad for \ 1 \le z \le 3$$

Using above formulae we construct the graph as shown in Figure-5, which is planar and regular graph of degree four and the edge set is $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_8v_9, v_9v_{10}$ $v_1v_{10}, v_2v_{10}, v_3v_9, v_4v_8, v_5v_7, v_1v_7, v_1v_6, v_2v_6, v_3v_{10}, v_4v_9, v_5v_8\}$

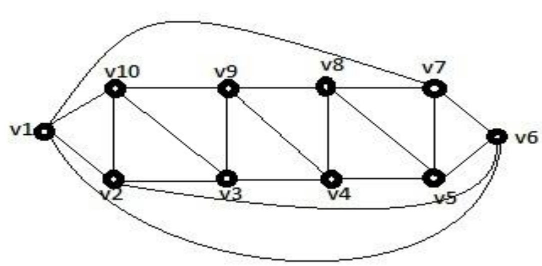

Figure-5 Four regular planar graph for m=4

In the same way, for m=6,7,8,9,10 -------- we can construct regular planar graph of degree four with odd and even regions and hence for constructing the different peculiar structure of the graph G, we can generalize the above cases by the following formulae. For the graph G having 2m+2 number of vertices and 4(m+1) edges for m≥2, we define

$\beta: V_G \rightarrow V_G$ such that

$$\beta(v_w) = \begin{cases} v_{w+1} & for\ 1 \le w \le 2m+1 \\ v_1 & for\ w = 2m+2 \end{cases}$$

$$\beta(v_x) = \begin{cases} v_1 & for\ x = m+3 \\ v_1 & for\ x = m+3-1 \\ v_2 & for\ x = m+2 \end{cases}$$

$$\beta(v_{y+2}) = v_{2m+3-y} \quad for\ 1 \le y \le (m-1)$$

$$\beta(v_{z+1}) = v_{2m+3-z}\ for\ 1 \le z \le m$$

The experimental results o of the above structure of four regular planar graph G(2m+2,4m+4) for different values of m≥2 are shown in Table-1.

A	B	C	D	E	F	G
2	6,12	7	NIL	7	3	NIL
3	8,16	7	2	9	3	4
4	10,20	11	NIL	11	3,5	NIL
5	12,24	11	2	13	3	6
6	14,28	15	NIL	15	3,7	NIL
7	16,32	15	2	17	3	8
8	18,36	19	NIL	19	3,9	NIL
9	20,40	19	2	21	3	10
10	22,44	23	NIL	23	3,11	NIL
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.

Table1- (Odd and Even region of four regular planar graph)

In the Table-1 A=Value of m B=Graph(V,E) C=No of Odd region D= No of Even region E=Total Region(Excluding outer region) F=No of edges odd region covered G= No of edges even region covered

We find a theorem from the above experimental results of Table-1.

Theorem1: The odd region of 4-Regular planar graph G(2m+2,4m+4) is 2m+3 when m=2n for n≥1 and 2m+1 when m=2n+1 for n≥1.

Using mathematical induction we have proof the Theorem-1

Proof: It has been proved [3] that the sub-graph H(2m+2,4m+4) for of the complete graph K(2m+2) is planar and regular of the degree 4. Here we proceed to prove that the regular planar graph H(2m+2,4m+4) has odd region 2m+3 for m =2n and 2m+1 for m=2n+1 for n ≥1. It is obtain that the experimental result is true when m=2 and m=3.That is, when m=2 for n=1, the graph H(6,12) has odd region 7 and the graph H(8,16) has also odd region 7 when m=3 for n=1 which is shown from figure-6 and figure-7.

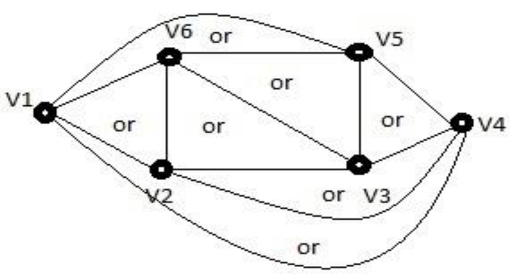

Fig-6: For m=2 (odd region is 7)

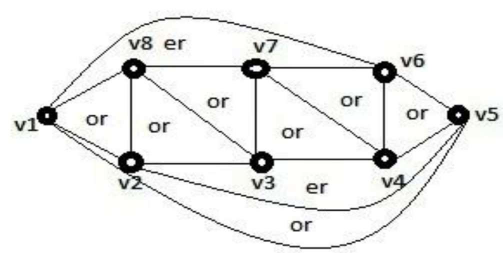

Fig -7: For m=3 (odd region is 7)

(For both Fig 6 and 7 or means odd region and ev means –even region)

The 1st part of the theorem claims for the even value of m is that if m=2n and n>=1 then Odd Region OR(m) = 2m+3 and for two consecutive values of n the difference between the odd regions is always 4.

Now, OR(m) =2m + 3

Therefore, when n =1,2, we have m=2n ,So

$OR_{n=1}$ (2×1=2) = $OR_{n=1}$ (2) = 2×2 + 3 = 7

$OR_{n=2}$ (2×2=4) = $OR_{n=2}$ (4) =2×4 + 3 = 11

$OR_{n=3}$ (2×3=6) = $OR_{n=3}$ (6)= 2×6 + 3 = 15

Proceeding in the same manner, we get,

OR_{n-1} (2(n-1)) = 2(2n-2)+3=4n-1

OR_{n} (2n) = 2×2n+3=4n+3

From above results, the differences of the successive odd-regions is found as,

$OR_{n=2} – OR_{n=1}$ = 11 – 7 = 4

$OR_{n=3} – OR_{n=2}$ = 15 – 11= 4

Proceeding in the same manner, we get,

$OR_{n=n} – OR_{n=n-1}$ = (4n+3) - (4n-1)

= 4n+3 – 4n+1

= 4

Hence the theorem is true for all n≥1 but had to prove for n+1 getting the generalization,

Now

$OR_{n=n+1}$(2×(n+1)= 2(2n+2)+3

= 4n+4+3

= 4n+7

Now

$OR_{n=n+1} – OR_{n=n}$ = (4n+7) – (4n+3)

= 4n+7-4n-3

= 4

Since the theorem in true for n and for n+1 and it evidently proved true for n=1, 2, 3. Hence the 1st part of theorem is true for any value of $n \geq 1$.

The 2nd part of the theorem claims for the even value of m is that if m=2n+1 and n>=1 then Odd Region OR(m) = 2m+1 and for two consecutive values of n the difference between the odd regions is always 4

Now,

$OR_{(m)} = 2m + 1$ Therefore, when k =1,2, we have m=2n +1,

So, $OR_{n=1}(2\times1+1=3) = OR_{n=1}(3) = 2\times3 + 1 = 7$

$$OR_{n=2}(2\times2+1=5) = OR_{n=2}(5) = 2\times5 + 1 = 11$$

$$OR_{n=3}(2\times3+1=7) = OR_{n=3}(7) = 2\times7 + 1 = 15$$

Proceeding in the same manner, we get,

$$OR_{n=n-1}(2(n-1)+1) = OR_{n=n-1}(2n-1)$$

$$=2(2n-1)+3 = 4n-2+3 = 4n+1$$

$$OR_n(2n+1) = OR_{n=n}(2n+1) = 2(2n+1)+3 = 4n+2+3 = 4n+5$$

From above results, the differences of the successive odd-regions is found as,

$$OR_{n=2} - OR_{n=1} = 11 - 7 = 4$$

$$OR_{n=3} - OR_{n=2} = 15 - 11 = 4$$

Proceeding in the same manner, we get,

$$OR_{n=n} - OR_{n=n-1} = (4n+5) - (4n+1) = 4n+5 - 4n-1 = 4$$

Hence the theorem is true for all $n \geq 1$but had to prove for n+1 getting the generalization, Now,

$$OR_{n=n+1}(2\times(n+1)+1) = OR_{n=n+1}(2n+3) = 2(2n+3)+3 = 4n+6+3 = 4n+9$$

Now again

$$OR_{k=k+1} - OR_{k=k} = (4n+9) - (4n+5) = 4n+9 - 4n-5 = 4$$

Since the theorem in true for n and for n+1 and it evidently proved true for n=1, 2, 3. Hence the 2nd part of theorem is true for any value of $n \geq 1$.

Theorem 2: The Total region of 4-Regular planar graph G(2m+2,4m+4) is 2m+3 when m=p+1 for $p \geq 1$.

Proof of the theorem by using mathematical induction:

Proof: It has been proved [3] that the sub-graph H(2m+2,4m+4) for of the complete graph K_{2m+2} is planar and regular of the degree 4.We now proceed to prove that the regular planar graph H(2m+2,4m+4) has Total region 2m+3 for m =p+1 for $p \geq 1$. It is found that the result is true when m=2 and 3.That is, when m=2 for p=1, the graph H(6,12) has Total region 7 and the graph H(8,16) has a Total region 9 when m=3 for p=1. The theorem claims for the value of m is that if m=p+1 and p>=1 then total region TR(m) = 2m+3 and for two consecutive values of p the difference between the regions is always 2.

Now, TR(m) =2m + 3

So, when p =1,2, we have m=p+1 and then

$TR_{p=1} (1+1=2) = TR_{p=1} (2) = 2\times2 + 3 = 7$

$TR_{p=2} (2+1=3) = TR_{p=2} (3) = 2\times3 + 3 = 9$

$TR_{p=3} (3+1=4) = TR_{p=3} (4) = 2\times4 + 3 = 11$

Continuing in the similar method, we get,

$TR_{p-1} ((p-1)+1) = 2p+3$

$TR_{p} (p+1) = 2(p+1)+3=2p+5$

From above equations, the differences of the successive Total regions is found as,

$TR_{p=2} - TR_{p=1} = 9 - 7 = 2$

$TR_{p=3} - TR_{p=2} = 11 - 9 = 2$

Continuing in the similar method, we get,

$TR_{p=p} - TR_{p=p-1} = (2p+5) - (2p+3)$

$= 2p+5 - 2p-3 = 2$

Hence the theorem is true for all p≥1but had to prove for p+1 getting the generalization, Now $TR_{p=p+1}((p+1)+1)= 2((p+1)+1)+3 = 2p+4+3$

$= 2p+7$

Now

$TR_{p=p+1} - TR_{p=p} = (2p+7) - (2p+5)$

$= 2p+7-2p-5 = 2$

Since the theorem is true for p and for p+1 and it evidently proved and true for p=1,2,3. Hence the theorem is true for any value of p≥1.

4. DISCUSSION

Maximum Regions covered by three edges:

The experimental results of the above peculiar structures of four regular planar graph G(2m+2,4m+4) for different values of m≥2 are shown in Table-2.

P	Q	R	S	T	U	V
2	6,12	7	0	7	7	0
3	8,16	7	2	9	7	2
4	10,20	11	0	11	10	1
5	12,24	11	2	13	11	2
6	14,28	15	0	15	14	1
7	16,32	15	2	17	15	2
8	18,36	19	0	19	18	1
9	20,40	19	2	21	19	2
10	22,48	23	0	23	22	1
11	24,48	23	2	25	23	2
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.

Table -2: Maximum edges covered by three edges.

In the Table-1 P=Value of m Q=Graph(V,E) R=No of Odd region S= No of Even region T=Total Region(Excluding outer region) U= No of Regions covered by three edges V= No of Regions(odd or even) covered by other than 3 edges

From the above Table-2 it is seen that the maximum regions are covered by three edges in the peculiar structures of the graph. For m equal to two, in the sub-graph, the regions more than three edges covered by zero (i.e. no regions are covered by more than three edges) and the numbers of regions covered by exactly three edges equal to seven . Again for m equal to three, in the sub-graph, numbers of regions more than three edges covered by two and the numbers of regions covered by exactly three edges equal to seven. Again for m equal to four, in the sub-graph numbers of regions are covered by more than three edges equal to one and numbers of regions covered by exactly three edges equal to ten. From the above results, it is clear that the number of regions covered by the three edges change with the change of the values of 'm', let it be f(p), and it is seen that number of other regions covered by other edges remain oscillated between one and two irrespective the values of m. Hence the total regions obtained by the edges will be f(p) + n, where n is either one or two. But if the graph is very large and many edges are involved at that situation, the regions covered by the three edges will be distinctly high but n remains at one or two, which can be neglected as it becomes so small as compared to the large value of m and f(p). So if we ignored the region which are covered by more than three edges then we can state that the structure of the graph G(2m+2,4m+4) region are covered by three edges.

5. APPLICATION

Graph region used in Biology:

Graph theory is used in conservation efforts and biology where a cycle represents regions where certain species exist and the edges represent movement between the regions or migration path. This information is important when looking at tracking the spread of disease, parasites and to study the impact of migration that affects other species or breeding patterns. Graph theoretical ideas are highly applied by computer science applications.

Graph Region based color also used in Map and GSM Network:

The famous four colors theorem asserts that it is always possible to properly color the regions of the map such that no two adjacent regions are assigned the same color, using at most four distinct colors[10][11][12][113]. But the map structure in Fig-8 and the dual graph of map structure in Fig-9 are contains odd regions except one even region. The odd regions of the duel graph in Fig -9 is covered by three edges and also planar. The structure of our graphs are planar and also contain odd region (maximum graph region are odd region except one or two) which is state in the discussion part that the regions are covered by three edges. So that our graph regions structure is similar with the duel graph map regions structure and maximum regions are covered by three edges. So odd regions and even regions can be used for map regions coloring.

Fig-8-The Map of India

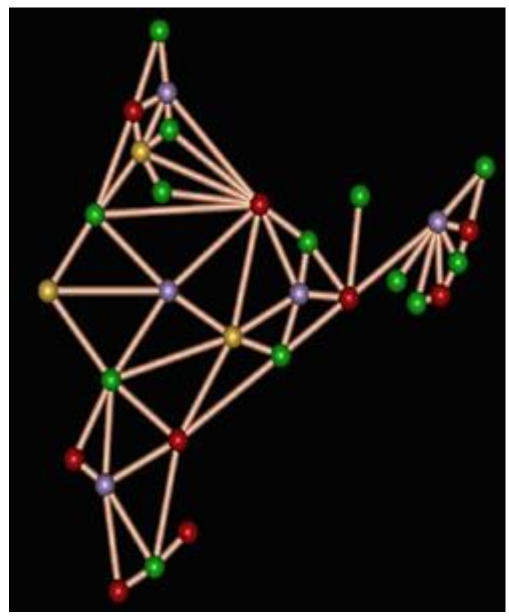

Fig-9-The Duel graph of the Map of India

Today, in the world GSM is the most popular standard for mobile phones. GSM is a cellular network with its entire geographical range divided into hexagonal cells. Within the cell each cell has a communication tower which connects with mobile phones. All mobile phones connect to the GSM network by searching for cells in the immediate vicinity. GSM networks operate in only four different frequency ranges the map of the cellular regions can be properly colored by using only four different colors. The duel graph of GSM network also resembles with the structure of our graphs regions which is covered by three edges. For example Fig-10 shows the regions structure which are odd regions and covered by three edges. So that odd regions can be used in GSM Network and also can be used in GSM cellular coloring.

Fig 10-The cells and dual graph of a GSM mobile phone network

6. Conclusion

The above two theorems discussed here justified our claims that the odd region of 4-Regular planar graph G (2m+2,4m+4) is 2m+3 when m=2n for n≥1 and 2m+1 when m=2n+1 for n≥1 and also the total region of 4-Regular planar graph G(2m+2,4m+4) is 2m+3 when m=n+1 for n≥1. The above discussion is useful for the researchers of different fields like: region-based map coloring, region-based segmentation, mathematical science and biological diversity etc.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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