ON DIFFERENCE OPEN MULTISETS IN M-TOPOLOGICAL SPACES

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Abstract. In this paper, we introduce and study some of the topological properties of oD-mset, $o - \Delta$ mset by using the concept of open msets. Also, we present new separation axioms by using the notions of open msets, oD-mset, $o - \Delta$ mset, pre-open mset and semi-open mset and study some of its properties.

Keywords: multiset; M-topological space; oD-mset; $o - \Delta$ mset; separation axioms.

1. INTRODUCTION

N. Levine [11] introduced the notation of semi open sets in a topological space. In 1997, Caldas [3] defined a semi-Difference (briefly sD) sets and semi-D_i spaces for $i = 0, 1, 2$. Ashish Kar and Bhattacharya [10], in 1990, also defined pre-T_i spaces for $i = 0, 1, 2$. Another set of separation axioms analogous to the semi separation axioms defined in [13]. A set is a collection of distinct objects "well define". If there is a repetition in the items, this set is called multiset (mset, for short), is obtained [2,15]. For the sake of convenience a mset is write as $\{n_1/y_1, n_2/y_2, ..., n_l/y_l\}$ the element y_i occurs n_i times. n_i is a positive integer. Multiset have

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many uses, including the expression of gene sequences for DNA, as well as the study of mutations [4,5], and in the near future it will be of great importance in a study in next generation of sequence.Defining new sets of multiset and defining the separation axioms on them will help in the future in studying the repair of mutations, and this is what we are working on studying in future work.

In this research, we introduce and study topological properties of oD-multiset, $o - \Delta$ multiset by using the concept of open msets. We also present and study new separation axioms by using the notions of pre-open multiset,semi-open multiset, oD-multiset, $o - \Delta$ multiset.

2. PRELIMINARIES

In this section, a brief survey of some basic concepts of multiset, separation axioms on multiset topology and separation axioms.

Definition 2.1 (9)**.** Let Y be a support set and $[Y]^n$ be the multiset space defined over Y. For any muliset $S \in [Y]^n$, the complement $S^c of S$ in $[Y]^n$ is an element of $[Y]^n$ s.t.$C_S^c(y) = n - C_S(y) \forall y \in Y$.

Definition 2.2 (8)**.** A submultiset X of a M-topological space $Yin[U]^w$ is a closed if the multiset $Y \ominus X$ is open.

2.1. Separation axioms. That the topological properties that represent separation and firing rules are useful in applications where the biological through. And properties that affect the category of chromosomes can be controlled to separate parts of it under certain conditions. This is useful in changing the nature or sequence of the chromosome to avoid the emergence of diseases and the occurrence Mutation.

Sobhy El-Sheikh and et. al [7] introduced a whole M-singleton and $M - T_i$ spaces for $i = 0,1,2,3,4,5$. They study the relation between $M - T_i$ spaces.

Theorem 2.3. *Every M-T_1-space is M-T_0-space.*

Theorem 2.4. *Every M-T_2-space is a M-T_1-space.*

Caldas [3] defined a semi-Difference sets (briefly sD) by semi-open sets [11], and introduced the semi-D_i spaces for $i = 0,1,2$.

If we add X to the class of all semi open difference sets have not achieved any kind of spaces and where the class of all open difference sets did not achieve the union and intersection. So was defined open-Difference(briefly oD) sets by open sets [9], and Introduced the open-D_i set and open-D_i spaces for $i = 0, 1, 2$, as in [1] and study the relation between them.

Theorem 2.5. *For a space (Y, τ) the following are true:*

(1) *Every $O - D_i$ space is $O - D_{i-1}$ space, i = 1; 2.*

(2) *Every T_i space is $O - D_i$ space, i = 0; 1;2.*

(3) *If (Y, τ) is oD_0 space iff (Y, τ) is T_0 space.*

Monsef[1] defined an open- symmetric difference $(o\Delta)$ sets[1] by using the open sets, and introduced the $o - \Delta_i$ spaces for i = 0, 1, 2, as in [1]and study the relation between them.

Proposition 2.6.   (1) *Every $O - \Delta_i$ space is $O - \Delta_{i-1}$ space, $i = 1, 2$;*

(2) *Obviously, Every T_i space is $O - \Delta_i$ space, $i = 0, 1, 2$;*

(3) *Obviously, Every $o - D_i$ space is $O - \Delta_i$ space, $i = 0, 1, 2$.*

The notions of interior and closure of an M-set in M-topology have been introduced and studied by Jacob et al. [8]. The other topological structures like exterior and boundary have remain untouched by Mahanta and Das [12] introduce the concepts of exterior and boundary in multiset topology. Consider an M-topological space $(Y; \tau)$ in $[M]^w$.

El-Sheikh et al.[6] defined submsets of M- topological spaces.

Definition 2.7. Let (Y, τ) be a multiset topological space . A function $\gamma : P^*(Y) \to P^*Y)$ is called an operation on OM(Y), if $S \subseteq \gamma(S) \forall S \in OM(Y)$. The set of all$\gamma$ -open msets is denoted by, $OM(\gamma) = \{S : S subseteq \gamma(S); S \in P^*(Y)\}$.

El-Sheikh et al.[6] defined POM(X), PCM(X),SOM(X), SCM(X), $\alpha OM(X)$, $\alpha CM(X)$, $\beta CM(X)$, BOM(X) and BCM(X), studied the relation between them.

Theorem 2.8. *Let $((Y, \tau)$ be a M-topological space and $\gamma : P^*(Y) \to P^*Y))$ be an operation on OM(Y). If $\gamma \in \{int(cl); int(cl(int)); cl(int); cl(int(cl)); cl(int) \cup int(cl)\}$. Then,*

(1) *Arbitrary union of γ-open multisets is γ-open multiset.*

(2) *Arbitrary intersection of γ-closed multisets is γ-closed multiset.*

3. SEPARATION AXIOMS ON M-TOPOLOGICAL SPACE

Definition 3.1. A M-topological space (M, τ) is called:

(1) M- Pre-T_0 iff for every two M-singletons $\{k_1/y_1\}$, $\{k_2/y_2\} \subseteq M$ such that $y_1 \neq y_2$, there is a pre-open mset containing one of the M-singletons but not the other.

(2) M-Pre-T_1 iff for every two M-singletons $\{k_1/y_1\}$, $\{k_2/y_2\} \subseteq M$ such that $y_1 \neq y_2$, there is a pair of pre-open msets one containing $\{k_1/y_1\}$ but not $\{k_2/y_2\}$ and other containing $\{k_2/y_2\}$ but not $\{k_1/y_1\}$.

(3) M-Pre-T_2 iff to each pair of M-singletons $\{k_1/y_1\}$, $\{k_2/y_2\} \subseteq M$ such that $y_1 \neq y_2$, there is a pair of disjoint pre-open msets one containing $\{k_1/y_1\}$ and the other containing $\{k_1/y_1\}$.

Definition 3.2. A M-topological space is :-

(1) M-Semi-T_0 iff for every two M-singletons $\{k_1/y_1\}$, $\{k_2/y_2\} \subseteq M$ such that $y_1 \neq y_2$, there is a semi-open mset containing one of the M-singletons but not the other.

(2) M-Semi-T_1 iff for every two M-singletons $\{k_1/y_1\}$, $\{k_2/y_2\} \subseteq M$ such that $y_1 \neq y_2$, there is a pair of semi-open msets one containing $\{k_1/y_1\}$ but not $\{k_2/y_2\}$ and other containing $\{k_2/y_2\}$ but not $\{k_1/y_1\}$.

(3) M-Semi-T_2 ff to each pair of M-singletons $\{k_1/y_1\}$, $\{k_2/y_2\} \subseteq M$ such that $y_1 \neq y_2$, there is a pair of disjoint semi-open msets one containing $\{k_1/y_1\}$ and the other containing $\{k_1/y_1\}$.

Definition 3.3. A space X is a M- T_0'-space, if for every two M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq X$ such that $x_1 \neq x_2$, there exist an open mset $V \subseteq X$, $\{k_1/x_1\} \subseteq V$ and $\{k_2/x_2\} \subseteq bd(V)(k_2/x_2 \in bd(k_2/x_2)$ is a boundary point).The topology of a M- T_0'-space is called $M - T_0'$-topology.

Definition 3.4. A space X is a T_1'-space, if for each two M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq X$ such that $x_1 \neq x_2$, there is an open mset $V \subseteq X$, $\{k_1/x_1\} \subseteq V$ and $\{k_2/x_2\} \subseteq bd(V)$ and an open mset open mset $U \subseteq X$, $\{k_2/x_2\} \subseteq U$ to which $\{k_1/x_1\}$ does not subset. The topology of a $M - T_1'$-space is called a $M - T_1'$-topology.

Definition 3.5. A space X is a T_2'-space, if for every two M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq X$ such that $x_1 \neq x_2$, there exists two disjoint open msets $U, V \subseteq X$ such that $\{k_1/x_1\} \subseteq U$ and

$\{k_2/x_2\} \subseteq V$ such that $bd(U) \cap bd(V) = \emptyset$. The topology of a $M - T_2'$-space is called a $M - T_2'$-topology.

Definition 3.6. A submset A of M is a semi-difference mset (sD-mset) if there are two semi-open msets O_1, O_2 in M such that $O1 \neq M$ and $A = O_1 \setminus O_2$, $C_A(x) = \max\{C_{O_1}(x) - C_{O_2}(x), 0\}$.

It is clear that when the disease appears in the chromosome and we wanted this part of the patient and eliminate the right for the part and it is using the group and this gives a higher resolution if the injury at one end of chromosome.

Definition 3.7. A M-topological space is called:-

(1) M-Semi-D_0 if for $\{k_1/y_1\}$, $\{k_2/y_2\} \subseteq M$ such that $y_1 \neq y_2$, there exists a sD-mset of Y containing one of $\{k_1/y_1\}$ and $\{k_2/y_2\}$ but not the other.

(2) M-Semi-D_1 if for $\{k_1/y_1\}$, $\{k_2/y_2\} \subseteq M$ such that $y_1 \neq y_2$, there exists a pair of sD-sets one containing $\{k_1/y_1\}$ but not $\{k_2/y_2\}$ and other containing $\{k_2/y_2\}$ but not $\{k_1/y_1\}$.

4. OPEN-DIFFERENCE MSET

Add M to the class of all semi open difference msets have not achieved any kind of spaces and where the class of all open difference msets did not achieve the union and intersection. So they will know We defined an open-difference mset (oD-mset) by using the open msets, and introduced the M-0D_i spaces for i = 0, 1, 2.

Definition 4.1. A submset A of a M-topological space X in $[U]^w$ is called a open-difference mset (in short oD-mset) if there are two open msets O_1, O_2 in X such that $O_1 \neq X$ and $A = O_1 \setminus O_2$, $C_A(x) = \max\{C_{O_1}(x) - C_{O_2}(x), 0\} \, for all x \in X$

Proposition 4.2. *The intersection of two open- difference msets is open- difference mset.*

Proof. Let A and B are open- difference msets such that $A = a_1 \setminus a_2$. In addition, $B = b_1 \setminus b_2$. where $a_1, a_2. b_1, b_2$ are open msets.
$A \cap B = (a_1 \setminus a_2) \cap (b_1 \setminus b_2) = (a_1 \cap \{a_2\}^c) \cap (b_1 \cap \{b_2\}^c) = (a_1 \cap b_1) \cap (\{a_2\}^c \cap \{b_2\}^c) = (a_1 \cap b_1) \cap (a_2 \cup b_2)^c = (a_1 \cap b_1) \setminus (a_2 \cup b_2)$ is open- difference mset.   □

Example 4.3. *Let* $X = \{2/x, 3/y, 1/z\}$ *be a mset and* $\tau = \{X, \emptyset, \{2/x\}, \{3/y\}, \{1/z\}$
, $\{2/x, 3/y\}, \{2/x, 1/z\}, \{3/y, 1/z\}\}$ *be a M-topological space on X. Then The classes of all*
open-difference msets are denoted by $OM_d = \{\emptyset, \{2/x\}, \{3/y\}, \{1/z\}, \{2/x, 3/y\}, \{2/x, 1/z\}, \{3/y, 1/z\}\}$,
since $\{2/x\} \cup \{3/y, 1/z\}\} = X \notin OM_d$. *The classes of all open-difference msets* Om_d *addition*
to X are denoted by $DM = \{3/y, 1/z\}\}$.

Remark 4.4.   The properties of open- difference mset are :-

   (1) Open- difference mset may be open mset or closed mset or not open mset and closed
       mset.

   (2) The intersection of two open- difference msets is open- difference mset.

   (3) It is true that every open mset $U \neq X$ is an oD-mset since $U = U \setminus \emptyset$.

   (4) The classes of all open-difference msets added for X construct infra M-topological
       space.

   (5) If $O_1 = X$, O_1, O_2 are two open msets and $A = O_1 \setminus O_2$. Then the classes of all open-
       difference msets are closed msets construct M-topological space.

   (6) The classes of all open-difference msets is a subbase for a M-topological space.

   (7) The union of two open- difference msets is not open- difference mset.

Definition 4.5. A M-topological space (M, τ) is called

   (1) $M - oD_0$ if for every two M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq M$ such that $x_1 \neq x_2$, there
       exists an $oD - mset$ of M containing one of the M-singletons and not the other.

   (2) $M - oD_1$ if for every two M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq M$ such that $x_1 \neq x_2$, then
       there exists a pair of $oD - msets$ one containing $\{k_1/x_1\}$ but not $\{k_2/x_2\}$ and other
       containing $\{k_2/x_2\}$ but not $\{k_1/x_1\}$.

   (3) $M - oD_2$ if for each pair of M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq M$ such that $x_1 \neq x_2$, there
       exists two disjoint open-difference msets $U, V \subseteq M$ s.t. $\{k_1/x_1\} \in U, \{k_2/x_2\} \in V$.

Theorem 4.6. *For a space* (M, τ) *the following are satisfied:*

   (1) Every $M - 0D_i$ *space is* $M - oD_{i-1}$ *space, i = 1;2.*

   (2) Every $M - T_i$,*space is* $M - oD_i$ *space, i = 0; 1; 2.*

   (3) If $((M, \tau)$ *is* $M - oD_0$ *space iff* (M, τ) *is* $M - T_0$ *space.*

Proof.   (1) and (2)Straightforward.

The necessity condition for(3) Let (M, τ) be $OM - D_0$ so that for any distinct pair of M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq M$ such that $x_1 \neq x_2$ at least one belongs to an oD mset A. Therefore, we choose $\{k_1/x_1\} \subseteq A$ and $\{k_2/x_2\} \nsubseteq A$. Suppose $A = O_1 \setminus O_2$ for which $O_1 \neq M$ and O_1 and O_2 are Open msets in M. This implies that $\{k_1/x_1\} \subseteq O_1$. For the case, that$\{k_2/x_2\} \nsubseteq A$ we have:

(1) $\{k_2/x_2\} \nsubseteq O_1$, (2) $\{k_2/x_2\} \subseteq O_1$ and $\{k_2/x_2\} \subseteq O_2$. For (1), the space X is $M - T_0$ since $\{k_1/x_1\} \subseteq O_1$ and $\{k_2/x_2\} \nsubseteq O_1$. For (2), the space X is also $M - T_0$ since $\{k_2/x_2\} \subseteq O_2$ but $\{k_1/x_1\} \nsubseteq O_2$.   □

Proposition 4.7.   *(1) If (M, τ) is $M - oD_1$ space implies $M - T_0$ space;*

(2) If (M, τ) is $M - oD_0$ space implies M- Semi-D_0 space.

Proof.   obvious.   □

5. OPEN-SYMMETRIC DIFFERENCE MSET

We defined an open- symmetric difference msets ($o\Delta$-mset) by using the open msets, and defined the the $M - o\Delta_i$ spaces for $i = 0, 1, 2$.

Definition 5.1. A submset A of a M-topological space X in $[U]^w$ is called a open-symmetric difference mset (in short $o\Delta$-mset) if there are two open msets O_1, O_2 in X such that $O_1 \neq X$ and $A = O_1 \Delta O_2$, $C_A(x) = |C_{O_1}(x) - C_{O_2}(x)| \forall x \in X$.

Example 5.2. *Let $X = \{2/x, 3/y, 1/z\}$ be a mset and $\tau = \{X, \emptyset, \{2/x\}, \{3/y\}, \{1/z\}$,$\{2/x, 3/y\}, \{2/x, 1/z\}, \{3/y, 1/z\}\}$ be a M-topological space on X. Then The classes of all open-difference msets are denoted by $OM_d = \{\emptyset, \{2/x\}, \{3/y\}, \{1/z\}, \{2/x, 3/y\}, \{2/x, 1/z\}, \{3/y, 1/z\}\}$. The classes of all open-symmetric difference msets are denoted by $om\Delta = \{\emptyset, \{2/x\}, \{3/y\}, \{1/z\}, \{2/x, 3/y\}, \{2/x, 1/z\}, \{3/y, 1/z\}, X\}$ is M-topological space on X.*

Proposition 5.3. *The intersection of two open- symmetric difference msets is open- symmetric difference mset.*

Proof. Let A and B are open- Let A and B are open- symmetric difference msets such$A = a_1 \Delta a_2$. In addition, $B = b_1 \Delta b_2$.where $a_1, a_2.b_1, b_2$ are open msets.

$A \cap B = (a_1 \Delta a_2) \cap (b_1 \Delta b_2) = \{(a_1 \cup a_2) - (a_1 \cap a_2)\} \cap \{(b_1 \cup b_2) - (b_1 \cap b_2)\} = \{(a_1 \cup a_2) \cap (a_1 \cap a_2)^c\} \cap \{(b_1 \cup b_2) \cap (b_1 \cap b_2)^c\} = \{(a_1 \cup a_2) \cap (b_1 \cup b_2)\} \cap (b_1 \cap a_2)^c\} \cap (b_1 \cap b_2)^c\} = \{(a_1 \cup a_2) \cap (b_1 \cup b_2)\} \cap ((a_1 \cap a_2) \cap (b_1 \cap b_2))^c = \{(a_1 \cup a_2) \cap (b_1 \cup b_2)\} - \{(a_1 \cap a_2) \cap (b_1 \cap b_2)\}$

is open- symmetric difference mset. $\square$

Remark 5.4. The properties of open- symmetric difference msets are :-

(1) Open- symmetric difference mset may be open mset or closed mset or not open mset and closed mset.

(2) The intersection of two open- symmetric difference msets is open- symmetric difference mset.

(3) It is true that every open mset $U \neq X$ is an $o\Delta$-mset since $U = U \Delta \emptyset$.

(4) The classes of all open-symmetric difference msets added for X (a space X is not connected)construct infra M-topological space.

(5) The classes of all open-symmetric difference msets is a subbase for a M-topological space.

Lemma 5.5. *Every oD-mset is oΔ-mset.*

Proof. Let $E \in OM_d$ implies there exist two open msets P_1, P_2 such that $P_1 \neq X$ since $A - B = A\Delta(A\Delta B)$. Hence, $E = P_1 - P_2 = P_1 \Delta (P_1 \Delta P_2)$, is open- symmetric difference mset. $\square$

Appeared many ways to separate the disease (part proper for the sick, injured or suspected of having), but was for this method many conditions which reduce the importance and when there is a way to show this chapter anywhere in the chromosome, this method is high-resolution.

Definition 5.6. A M-topological space (M, τ) is called:-

(1) $M - o\Delta_0$ if for every two M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq M$ such that $x_1 \neq x_2$, there exists an $o\Delta$-mset of M containing one of the M-singletons and not the other

(2) $M - o\Delta_1$ if for every two M-singletons $\{k_1/x_1\}$, $\{k_2/x_2\} \subseteq M$ such that $x_1 \neq x_2$, then there exists a pair of $o\Delta$-msets one containing $\{k_1/x_1\}$ but not $\{k_2/x_2\}$ and other containing $\{k_2/x_2\}$ but not $\{k_1/x_1\}$.

(3) $M - o\Delta_2$ if for each pair of M-singletons $\{k_1/x_1\}, \{k_2/x_2\} \subseteq M$ such that $x_1 \neq x_2$, there exists two disjoint open- symmetric difference msets $U, V \subseteq M$ such that $\{k_1/x_1\} \in U, \{k_2/x_2\} \in V$.

Theorem 5.7. *For a space (M, τ) the following statements are true:*

(1) Every $M - o\Delta_i$ space is $M - o\Delta_{i-1}$ space, i = 1;2.

(2) Every $M - oD_i$ space is $M - o\Delta_i$ pace, i = 1;2.

(3) Every $M - T_i$ space is $M - o\Delta_i$ space, i = 0; 1; 2.

(4) Every $M - o\Delta_0$ space is $M - semiD_0$ space.

Proof.   Straightforward.   □

We all know that mathematical applications and solving life problems are among the most important issues that all or most researchers are interested in. But this does not leave us far from the theory that we will need in the future as we are currently using the previous theories, for example the multiple group exists from 1986 [15] and it was used in many applications in different fields, but we in 2018[4,5] used it differently to express the gene and reveal Mutations. In this research, we worked on establishing new types of multiple groups and new separation axioms and theories on them.

6. CONCLUSIONS

In this paper we introduce the separation axioms on mset topological spaces and difference mset topological spaces based on the singleton mset $\{m/x\}$, oD-mset, $o - \Delta$ mset, pre-open mset and semi-open mset. In the future, we study another topological property such as connected, some types of submsets and mappings on these spaces, we will use a new separation axioms in repair mutations and treat diseases.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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