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MAPPINGS AND PRODUCTS IN SOFT L-TOPOLOGICAL SPACES

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Abstract. Fuzzy set, soft set and their extensions have been successful in being a raapproachment between precise classical mathematics and imprecise real world. In particular, soft lattices as a generalization of soft set is a new mathematical approach to study uncertainity. Soft L-topological spaces are defined over a soft lattice L with a fixed set of parameter P and the continuity of mappings of soft L-topological spaces has also been studied. In this paper, we introduce the concept of soft L-continuous mapping between two soft L-topological spaces. Further some results based on soft L-homeomorphism are also obtained. Finally, the concept of cartesian product of soft L-sets are defined and explored some results relating to this.

Keywords: soft L-continuity; soft L-homeomorphism; soft L-cartesian product; soft L-product topology.
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1. INTRODUCTION

The concept of soft set theory begins with Molodtsov [1, 3] in the year 1999. It is completely new approach for modelling, vagueness and uncertainties. Few applications in many directions of soft set theory have been shown by Molodtsov in [1, 3]. Also Maji et.al [2, 3] studied soft sets introduced by Molodtsov [1, 3] and gave the definitions based on equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft

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set with examples and basic properties are also defined. The algebraic structure of set theory dealing with uncertainties has also been studied by some authors [4, 5, 6, 7, 8]. The concept of soft set has been extended to soft lattices and soft fuzzy sets by Li.F [9] in the year 2010. Soft lattices can also expressed in terms of algebraical and set theoretical manner. Cagman et al. [10] did a deep study on these two concepts and came to the conclusion that algebraical and set theoretical definitions are equivalent or same. Cagman et al. (2011)[10] presented the related properties of soft topology on a soft set. We follow the approach of M. Shabir and M. Naz [11] who introduced the concept of soft topological spaces in the year 2011 and studied some basic properties. In our work, we use the notion of soft set initiated by Molodtsov [1, **?**] and extend this idea to the field of soft lattices [9] and obtain the topological properties of soft lattices. In 2016, Cigdem Gunduz Aras, Ayse Sonmez and Huseyin Cakalli [12] introduced soft continuous mappings. Some of its properties are studied by many authors [16, 17, 18]. In 2012, H.Hazra, P.Majumdar and S.K.Samanta [13] gave the definition of continuity of soft mappings with their properties. In 2015, Yang et al. [14] first proposed the concept of soft continuous mapping between two soft topological spaces. In 2013, E. Peyghana, B. Samadia and A. Tayebib [15] discussed cartesian product of soft sets and soft product topology.

Soft Lattice topological spaces (Soft L-topological spaces or Soft L-space) [19] are introduced with a fixed set of parameters P over an initial universe X. We have defined some basic properties of soft L-topological spaces and also gave the definition of soft L-open and soft L-closed sets. The soft L-closure of a soft lattice is also defined which is a generalization of closure of a set. The concept of parameters plays a major role with the set of parameterized topologies on the initial universe. We define a topological space corresponding to each parameter, and it is more essential. We show that a soft topological space gives a parameterized family of topologies on the initial universe. Converse need not be true. It means if we are given some topologies for each parameter, it is not possible to construct a soft topological space.

We introduced soft L-continuous mappings [20] which are defined over an initial universe set with a fixed set of parameters. Further we discuss some algebraic properties of soft L-mappings such as injectivity, surjectivity, bijectivity and composition of soft L-mappings and study their continuity properties under soft L-topology. The continuity of mappings of soft

L-topological spaces has defined and its properties has investigated. Also, soft open and soft closed L-mappings, soft L-homeomorphism are defined and some interesting results are obtained.

In this paper, the concept of soft L-continuous mapping between two soft L-topological spaces is proposed and some results are proved. Also, we have proved some theorems based on soft L-homeomorphism. Finally, cartesian product for soft L-sets are defined and some results are discussed.

2. Preliminaries and Basic Definitions

Throughout this paper, we consider L as a complete lattice and we denote universal bounds as $\perp$ and $\top$. Our assumption is L is consistent i.e. floor is different from top. Therefore, $\perp \leq \alpha \leq \top$ for every $\alpha \in L$. Also $\vee\phi = \perp$ and $\wedge\phi = \top$. The two point lattice $\{\perp, \top\}$ is denoted by 2. A unary operation $\prime : L \longrightarrow L$ is quasi complementation. It is an involution (i.e., $\alpha'' = \alpha$ for all $\alpha \in L$) that inverts the ordering. (i.e., $\alpha \leq \beta \Longrightarrow \beta' \leq \alpha'$). De Morgan's laws also hold in $(L, \prime)$. (i.e., $(\vee A)' = \wedge\{\alpha' : \alpha \in A\}$ and $(\wedge A)' = \vee\{\alpha' : \alpha \in A\}$ for every $A \subset L$). In addition, $\perp' = \top$ and $\top' = \perp$. Based on these concepts, we use a completely distributive lattice $(L, \prime)$ as a complete lattice equipped with an order reserving involution in this paper.

Definition 2.1. [1] *Assume X as an initial universe set and P be a set of parameters. The power set of X is denoted as $\wp(X)$ and $B \subset P$. Then a pair (P, B) is said to be a soft set over X, where the mapping P is given by $P : B \to \wp(X)$.*
i.e., a soft set over X is regarded as a parametrized family of subsets of the universe X. For $b \in B$, the set of approximate elements of the soft set (P, B) denoted by $P(b)$.

Definition 2.2. [9] *Consider $M = (f, X, L)$, where L is a complete lattice, $f : X \longrightarrow \wp(L)$ is a mapping, X is a universe set, then M is called the soft lattice denoted by f_P^L.*
ie, for every $x \in X$, f_P^L is a soft lattice over L, if $f(x)$ is a sub lattice of L.

Definition 2.3. [19] *The relative complement of a soft lattice f_P^L is denoted by $(f_P^L)'$ and is defined as $(f_P^L)' = (f_P'^L)$ where $f' : P \longrightarrow \wp(L)$ is a mapping given by $f'(\alpha) = L - f(\alpha)$ for all $\alpha \in P$.*

Definition 2.4. [19] *Consider X as an initial universe set and P as the non-empty set of parameters.*
Let τ be the set of complete, uniquely complemented soft lattices over L, then τ is said to be a soft lattice topology on L if;
$(i)\phi, L$ belongs to τ.
(ii) The arbitrary union of soft lattices in τ belongs to τ.
(iii) The finite intersection of soft lattices in τ belongs to τ.
Then (L,τ,P) is called a soft lattice topological space (soft topological lattice space or soft L -space) over L.

Definition 2.5. [19] *Consider (L,τ,P) as a soft lattice topological space over L, then the members of τ are called as soft L-open sets in L.*

Definition 2.6. [19] *Let (L,τ,P) be a soft lattice topological space over L. A soft lattice f_P^L over L is said to be a soft L-closed set in L, if its relative complement $(f_P^L)'$ belongs to τ.*

Definition 2.7. [19] *We consider L as a lattice, P be the set of parameters and $\tau = \{\phi, L\}$. Then τ is called the soft indiscrete lattice topology on L and (L,τ,P) is said to be a soft indiscrete lattice topological space over L.*

Definition 2.8. [19] *Consider L be a lattice, P be the set of parameters and let τ be the collection of all soft lattices which can be defined over L. Then τ is called the soft discrete lattice topology on L and (L,τ,P) is said to be a soft discrete lattice topological space over L.*

Definition 2.9. [19] *We consider (L,τ,P) as a soft lattice topological space over L and f_P^L be a soft lattice over L. Then the soft lattice closure of f_P^L, denoted by $\overline{f}_P^L$, is the intersection of all soft L-closed super sets of f_P^L.*

Definition 2.10. [19] *Let (L,τ,P) be a soft lattice topological space over L and f_P^L be a soft lattice over L. Then we associate with f_P^L, a soft lattice L, denoted by $\overline{f}_P^L$ and defined as $\overline{f}(\alpha) = \overline{f(\alpha)}$, where $\overline{f(\alpha)}$ is the soft L-closure of $f(\alpha)$ in τ_α for each $\alpha \in P$.*

Definition 2.11. [19] *Consider (L,τ,P) as a soft lattice topological space over L, g_P^L be a soft lattice over L and $x \in L$. Then x is said to be a soft L-interior point of g_P^L if there exists a soft L-open set f_P^L such that $x \in f_P^L \subset g_P^L$.It is denoted by $(f_P^L)^o$.*

Definition 2.12. [19] *Let (L,τ,P) be a soft lattice topological space over L, g_P^L be a soft lattice over L and $x \in L$. Then g_P^L is said to be a soft lattice neighbourhood of x if there exists a soft L-open set f_P^L such that $x \in f_P^L \subset g_P^L$.*

Proposition 2.13. [19] *Let (L,τ,P) be a soft L- space over L . Then the set $\tau_a = \{f(a)|f_P^L \in \tau\}$ for all $a \in P$ gives a topology on L.*

Definition 2.14. [20] *Consider f_P^L as a soft lattice over L. The soft lattice f_P^L is called a soft L-point, denoted by (l_p,P), for the element $p \in P$, $f(p) = \{l\}$ and $f(p') = \phi$ for all $p' \in P - \{l\}$.*

Definition 2.15. [20] *Let (L_1,τ_1,P) and (L_2,τ_2,P) be two soft lattice topological spaces. The mapping f_g is called a soft L-mapping from L_1 to L_2 denoted by $f_g\colon (L_1,\tau_1,P) \longrightarrow (L_2,\tau_2,P)$, where $f\colon L_1 \longrightarrow L_2$ and $g\colon P \longrightarrow P$ are two mappings. For each soft L-neighbourhood g_P^L of $(f(l)_p,P)$, if there exist a soft L-neighbourhood f_P^L of (l_p,P) such that $f_g(f_P^L \subset g_P^L)$, then f_g is said to be soft L-continuous mapping at (l_p,P).*
If f_g is soft L-continuous mapping for all (l_p,P), then f_g is called soft L-continuous mapping.

Definition 2.16. [20] *Let (L_1,τ_1,P) and (L_2,τ_2,P) be two soft lattice topological spaces, $f_g\colon (L_1,\tau_1,P) \longrightarrow (L_2,\tau_2,P)$ be a mapping. Then*
(a) If the image $f_g(f_P^L)$ of each soft L-open set f_P^L over L_1 is a soft L-open set in L_2, then f_g is said to be a soft L-open mapping.
(b) If the image $f_g(h_P^L)$ of each soft L-closed set h_P^L over L_1 is a soft L-closed set in L_2, then f_g is said to be a soft L-closed mapping.

Theorem 2.17. [20] *We know that (L_1,τ_1,P) and (L_2,τ_2,P) are two soft lattice topological spaces, $f_g\colon (L_1,\tau_1,P) \longrightarrow (L_2,\tau_2,P)$ be a mapping. Then the following conditions are equivalent:*
(1) *$f_g\colon (L_1,\tau_1,P) \longrightarrow (L_2,\tau_2,P)$ is a soft L-continuous mapping.*
(2) *For each soft L-open set G_P^L over L_2, $f_g^{-1}(g_P^L)$ is a soft L-open set over L_1.*

(3) *For each soft L-closed set* H_P^L *over* L_2, $f_g^{-1}(h_P^L)$ *is a soft L-closed set over* L_1.

(4) *For each soft L-set* F_P^L *over* L_1, $f_g\overline{(f_P^L)} \subset \overline{f_g(f_P^L)}$.

(5) *For each soft L-set* G_P^L *over* L_2, $\overline{f_g^{-1}(g_P^L)} \subset f_g\overline{(g_P^L)}$.

(6) *For each soft L-set* g_P^L *over* L_2, $f_g^{-1}((g_P^L)^o) \subset (f_g^{-1}(g_P^L))^o$.

Theorem 2.18. [20] *If* $f_g\colon (L_1,\tau_1,P) \longrightarrow (L_2,\tau_2,P)$ *is a soft L-continuous mapping, then for each* $\alpha \in P$, $f_{g\alpha}\colon (L_1,\tau_{1\alpha}) \longrightarrow (L_2,\tau_{2\alpha})$ *is a soft continuous mapping.*

Proposition 2.19. [20] *If* $f_{g\alpha}\colon (L_1,\tau_{1\alpha}) \longrightarrow (L_2,\tau_{2\alpha})$ *is soft L-open(closed) mapping, then for each* $\alpha \in P$, $f_{g\alpha}\colon (L_1,\tau_{1\alpha}) \longrightarrow (L_2,\tau_{2\alpha})$ *is an soft open(closed) mapping.*

Theorem 2.20. [20] *Let* (L_1,τ_1,P) *and* (L_2,τ_2,P) *be two soft lattice topological spaces,*
$f_g\colon (L_1,\tau_1,P) \longrightarrow (L_2,\tau_2,P)$ *be a mapping. Then*

(a) f_g *is a soft L-open mapping if for each soft L-set* f_P^L *over* L_1,
$f_g((f_P^L)^o) \subset (f_g(f_P^L))^o$ *is satisfied.*

(b) f_g *is a soft L-closed mapping if for each soft L-set* f_P^L *over* L_1,
$\overline{f_g((f_P^L))} \subset f_g\overline{(f_P^L)}$ *is satisfied.*

Definition 2.21. [20] *Consider* (L_1,τ_1,P) *and* (L_2,τ_2,P) *as two soft lattice topological spaces,* $f_g\colon (L_1,\tau_1,P) \longrightarrow (L_2,\tau_2,P)$ *be a mapping. If* f_g *is a bijection, soft L-continuous and* f_g^{-1} *is a soft L-continuous mapping, then* f_g *is said to be soft L-homeomorphism from* L_1 *to* L_2.
When a soft homeomorphism f_g *exists between* L_1 *and* L_2*, we say that* L_1 *is soft L-homeomorphic to* L_2.

Theorem 2.22. [20] *Let* (L_1,τ_1,P) *and* (L_2,τ_2,P) *be two soft lattice topological spaces,*
$f_g\colon (L_1,\tau_1,P) \longrightarrow (L_2,\tau_2,P)$ *be a bijection mapping. Then the following conditions are equivalent:*

(1) f_g *is a homeomorphism on soft L-topological space,*

(2) f_g *is a continuous and closed mapping on soft L-topological space,*

(3) f_g *is a continuous and open mapping on soft L-topological space.*

3. Soft Lattice Continuous Mapping Between Soft L-Topological Spaces

In this subsection, we discuss the concept of Soft lattice continuous mapping between two soft L-topological spaces with their related properties.

Consider the two initial universe sets be X and Y and let P be a non-empty parameter. The set of all soft L-sets over X is denoted by $S_{L_1}(X)$. Similarly, the set of all soft L-sets over Y is denoted by $S_{L_2}(Y)$.

Definition 3.1. *Let f_g be a mapping from X to Y. Then*

(1) *The soft L-set mapping induced by f_g, denoted $f_g^{\longrightarrow}$ is a soft mapping from $S_{L_1}(X)$ to $S_{L_2}(Y)$ that maps f_P^L to $f_g^{\longrightarrow}(f_P^L) = (f_g^{\longrightarrow}(f^L), P)$, where $f_g^{\longrightarrow}(f_P^L)$ is defined by $f_g^{\longrightarrow}(f^L)(\alpha) = \{f_g(l) | l \in f^L)(\alpha)\} \forall \alpha \in P$.*

(2) *The inverse soft L-set mapping induced by f_g, denoted by the notation $f_g^{\longleftarrow}$ is a soft mapping from $S_{L_2}(Y)$ to $S_{L_1}(X)$ that maps g_P^L to $f_g^{\longleftarrow}(g_P^L) = (f_g^{\longleftarrow}(g^L), P)$, where $f_g^{\longleftarrow}(g_P^L)$ is defined by $f_g^{\longleftarrow}(g^L)(\alpha) = \{l | f_g(l) \in g^L)(\alpha)\} \forall \alpha \in P$.*

Example 3.2. *Suppose $L_1 = \{l_1, l_2, l_3\}, L_2 = \{h_1, h_2\}, P = \{p_1, p_2\}$. The mapping f_g is given by $f_g(l_1) = h_1, f_g(l_2) = h_1, f_g(l_3) = h_2$.*

(1) *If $f_P^L \in S_{L_1}(X)$ is defined by $\{f(p_1) = \{l_1, l_2\}, f(p_2) = \{l_2, l_3\}\}$, then $f_g^{\longrightarrow}(f_P^L) = (f_g^{\longrightarrow}(f^L), P) = \{f_g^{\longrightarrow} f(p_1) = \{h_1\}, f_g^{\longrightarrow} f(p_2) = L_2\} \in S_{L_2}(Y)$.*

(2) *If $g_P^L \in S_{L_2}(Y)$ is defined by $\{g(p_1) = \{h_2\}, g(p_2) = \{h_1\}\}$, then $f_g^{\longleftarrow}(g_P^L) = (f_g^{\longleftarrow}(g^L), P) = \{f_g^{\longleftarrow} g(p_1) = \{l_3\}, f_g^{\longleftarrow} g(p_2) = \{l_1, l_2\}\} \in S_{L_1}(X)$.*

Proposition 3.3. *Let us consider f_g to be a mapping from $X$to Y, $f_{1P}^L, f_{2P}^L \in S_{L_1}(X)$. Then*

(1) $f_g^{\longrightarrow}(\phi) = \phi$.

(2) $f_{1P}^L \subset f_{2P}^L \Rightarrow f_g^{\longrightarrow}(F_{1P}^L) \subset f_g^{\longrightarrow}(f_{2P}^L)$

(3) $f_g^{\longrightarrow}(f_{1P}^L \cup f_{2P}^L) = f_g^{\longrightarrow}(f_{1P}^L) \cup f_g^{\longrightarrow}(f_{2P}^L)$

(4) $f_g^{\longrightarrow}(f_{1P}^L \cap f_{2P}^L) \subset f_g^{\longrightarrow}(f_{1P}^L) \cap f_g^{\longrightarrow}(f_{2P}^L)$.

Proposition 3.4. *When f_g be a mapping from X to Y, $g_{1P}^L, g_{2P}^L \in S_{L_2}(Y)$. Then*

(1) $f_g^{\longleftarrow}(\phi) = \phi$, $f_g^{\longleftarrow}(Y) = X$

(2) $g_{1P}^L \subset g_{2P}^L \Rightarrow f_g^{\longleftarrow}(g_{1P}^L) \subset f_g^{\longleftarrow}(g_{2P}^L)$

(3) $f_g^{\longleftarrow}(g^L_{1P} \cup g^L_{2P}) = f_g^{\longleftarrow}(g^L_{1P}) \cup f_g^{\longleftarrow}(g^L_{2P})$

(4) $f_g^{\longleftarrow}(g^L_{1P} \cap g^L_{2P}) = f_g^{\longleftarrow}(g^L_{1P}) \cap f_g^{\longleftarrow}(g^L_{2P})$

(5) $f_g^{\longleftarrow}(g^L_{1P})' = (f_g^{\longleftarrow}(g^L_{1P}))'$.

Proposition 3.5. *Consider f_g be a mapping from Xto Y, $f^L_P \in S_{L_1}(X)$ and $g^L_P \in S_{L_2}(Y)$. Then*

(1) $f_g^{\longleftarrow}(f_g^{\longrightarrow}(f^L_P)) \supset f^L_P$. If f_g is one to one, then $f_g^{\longleftarrow}(f_g^{\longrightarrow}(f^L_P)) = f^L_P$.

(2) $f_g^{\longrightarrow}(f_g^{\longleftarrow}(g^L_P)) \subset g^L_P$. If f_g is surjective, then $f_g^{\longrightarrow}(f_g^{\longleftarrow}(g^L_P)) = g^L_P$.

Proof. (1) Let $f_g^{\longrightarrow}(g^L_P) = g^L_P$. Then $\forall \alpha \in P$,

$f_g^{\longleftarrow}(g^L)(\alpha) = \{l | f_g(l) \in g^L(\alpha)\} = \{l | f_g(l) \in \{f_g(t) | t \in f^L(\alpha)\} \supset f^L(\alpha)$,

which implies $f_g^{\longleftarrow}(f_g^{\longrightarrow}(f^L_P)) \supset f^L_P$.

If f_g is one to one, then $\{l | f_g(l) \in \{f_g(t) | t \in f^L(\alpha)\} = f^L(\alpha)$, thus $f_g^{\longleftarrow}(f_g^{\longrightarrow}(f^L_P)) = f^L_P$.

(2) Let $f_g^{\longrightarrow}(g^L_P) = f^L_P$. Then $\forall \alpha \in P$,

$f_g^{\longleftarrow}(f^L)(\alpha) = \{f_g(l) | l \in f^L)(\alpha)\} = \{f_g(l) | l \in \{f_g(t) | t \in g^L(\alpha)\} \supset g^L(\alpha)$,

which implies $f_g^{\longleftarrow}(f_g^{\longrightarrow}(g^L_P)) \subset g^L_P$.

If f_g is surjective, then $\{f_g(l) | l \in \{f_g(t) | t \in g^L(\alpha)\} = g^L(\alpha)$, thus $f_g^{\longrightarrow}(f_g^{\longleftarrow}(g^L_P)) = g^L_P$. □

Definition 3.6. *Let (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces over X and Y respectively and f_g be a mapping from X and Y. If $\forall g^L_P \in \tau_2, f_g^{\longleftarrow}(g^L_P) \in \tau_1$, then f_g is called soft L-continuous mapping from (L_1, τ_1, P) to (L_2, τ_2, P).*

Example 3.7. *Suppose $L_1 = \{l_1, l_2, l_3\}, L_2 = \{h_1, h_2, h_3\}, P = \{p_1, p_2\}$ and $\tau_1 = \{\phi, L_1, f^L_{1P}, f^L_{2P}\}$ is a soft L-topological space over X, where f^L_{1P}, f^L_{2P} are soft lattices over X defined by*

$f_1(p_1) = \{l_2\}, f_1(p_2) = \{l_1\}$,

$f_2(p_1) = \{l_2, l_3\}, f_2(p_2) = \{l_1, l_2\}$.

Then τ_1 is a soft L-topology on X and hence (L_1, τ_1, P) is a soft lattice topological spaces over X.

Also $\tau_2 = \{\phi, L_2, g^L_{1P}, g^L_{2P}\}$ is a soft L-topological space over Y, where g^L_{1P}, g^L_{2P} are soft lattices over Y, defined as

$g_1(p_1) = \{h_1\}, g_1(p_2) = \{h_2\}$,

$g_2(p_1) = \{h_1, h_3\}, g_2(p_2) = \{h_1, h_2\}$,

If $f_g : L_1 \longrightarrow L_2$ as $f_g(l_1) = h_2, f_g(l_2) = h_1, f_g(l_3) = h_3$.

Now $f_g^{\longleftarrow}(g_P^L) \in \tau_1$ *for all* $g_P^L \in \tau_2$.

Thus f_g *is a soft L-continuous mapping from* (L_1, τ_1, P) *to* (L_2, τ_2, P).

Proposition 3.8. *We have* (L_1, τ_1, P) *and* (L_2, τ_2, P) *as the two soft lattice topological spaces over X and Y respectively. If* f_g *is soft L-continuous mapping from* (L_1, τ_1, P) *to* (L_2, τ_2, P)*, then* f_g *is a soft continuous mapping from* f_g *to* $(L_1, \tau_{1\alpha})$ *to* $(L_2, \tau_{2\alpha})$ *for all* $\alpha \in P$.

Proof. Using proposition 2.17, $(L_1, \tau_{1\alpha})$ and $(L_2, \tau_{2\alpha})$ are two soft lattice topological spaces for all $\alpha \in P$. If $A \in \tau_{2\alpha}$, then there exists a soft L-set $g_P^L \in \tau_2$ such that $A = g(\alpha)$.

Since f_g is soft L-continuous mapping from (L_1, τ_1, P) to (L_2, τ_2, P), then $f_g^{\longleftarrow}(g_P^L) \in \tau_1$. Thus $f_g^{-1}(A) = f_g^{-1}(g(\alpha)) = \{l | f_g(l) \in g(\alpha)\} = f_g^{\longleftarrow}(g(\alpha)) \in \tau_{1\alpha}$.

By the definition of soft continuous, f_g is a soft continuous mapping from $(L_1, \tau_{1\alpha}) \longrightarrow (L_2, \tau_{2\alpha})$ for all $\alpha \in P$. This proposition tells that a soft L-continuous mapping gives a parameterized family of soft continuous mapping. □

Example 3.9. *Suppose* $L_1 = \{l_1, l_2, l_3\}, L_2 = \{h_1, h_2, h_3\}, P = \{p_1, p_2\}$ *and* $\tau_1 = \{\phi, L_1, f_{1P}^L, f_{2P}^L\}$ *is a soft L-topological space over X, where* f_{1P}^L, f_{2P}^L *are soft lattices over X defined by*

$f_1(p_1) = \{l_2\}, f_1(p_2) = \{l_1\}$,

$f_2(p_1) = \{l_2, l_3\}, f_2(p_2) = \{l_1, l_2\}$.

Then τ_1 *is a soft L-topology on X and hence* (L_1, τ_1, P) *is a soft lattice topological spaces over* X.

Also $\tau_2 = \{\phi, L_2, g_{1P}^L, g_{2P}^L\}$ *is a soft L-topological space over* Y*, where* g_{1P}^L, g_{2P}^L *are soft lattices over* Y*, defined as*

$g_1(p_1) = \{h_1\}, g_1(p_2) = \{h_2\}$,

$g_2(p_1) = \{h_1, h_3\}, g_2(p_2) = \{h_1, h_2\}$,

If $f_g : X \longrightarrow Y$ *as* $f_g(l_1) = h_2, f_g(l_2) = h_1, f_g(l_3) = h_3$.

Now $f_g^{\longleftarrow}(g_P^L) \in \tau_1$ *for all* $g_P^L \in \tau_2$.

Thus f_g *is a soft L-continuous mapping from* (L_1, τ_1, P) *to* (L_2, τ_2, P).

Here by proposition 2.17, $\tau_{1p_1} = \{\phi, L_1, \{l_2\}, \{l_2, l_3\}\}$ *and* $\tau_{1p_2} = \{\phi, L_1, \{l_1\}, \{l_1, l_2\}\}$ *are two topologies on* X.

$\tau_{2p_1} = \{\phi, L_2, \{h_1\}, \{h_1, h_3\}\}$ *and* $\tau_{2p_2} = \{\phi, L_2, \{h_1, h_2\}\}$ *are two topologies on* Y.

Hence f_g is a soft continuous mapping from (L_1, τ_{1p_1}) to (L_2, τ_{2p_1}) and also a soft continuous mapping from (L_1, τ_{1p_2}) to (L_2, τ_{2p_2}).

The following example shows that the inverse of proposition 3.8 does not hold in general.

Example 3.10. *Suppose $L_1 = \{l_1, l_2, l_3\}, L_2 = \{h_1, h_2, h_3\}, P = \{p_1, p_2\}$ and $\tau_1 = \{\phi, L_1, f_{1P}^L, f_{2P}^L\}$ is a soft L-topological space over X, where f_{1P}^L, f_{2P}^L are soft lattices over X defined by*

$f_1(p_1) = \{l_2\}, f_1(p_2) = \{l_1\}$,

$f_2(p_1) = \{l_2, l_3\}, f_2(p_2) = \{l_1, l_2\}$.

Then τ_1 is a soft L-topology on X and hence (L, τ_1, P) is a soft lattice topological spaces over X.

Let $Y = \{h_1, h_2, h_3\}, \tau_2 = \{\phi, L_2, g_{3P}^L\}$, where the soft L-set g_{3P}^L over Y defined by $g_3(p_1) = \{h_1\}, g_3(p_2) = \{h_1, h_2\}$.

If f_g is a mapping from (L_1, τ_1, P) to (L_2, τ_2, P).

Here by proposition 2.17, $\tau_{1p_1} = \{\phi, L_1, \{l_2\}, \{l_2, l_3\}\}$ and $\tau_{1p_2} = \{\phi, L_1, \{l_1\}, \{l_1, l_2\}\}$ are two topologies on X.

Also $\tau_{2p_1} = \{\phi, L_2, \{h_1\}, \{h_1, h_3\}\}$ and $\tau_{2p_2} = \{\phi, L_2, \{h_1, h_2\}\}$ are two topologies on Y.

Hence f_g is a continuous mapping from (L_1, τ_{1p_1}) to (L_2, τ_{2p_1}) and also from (L_1, τ_{1p_2}) to (L_2, τ_{2p_2}).

However, $f_g^{\longleftarrow}(g_{3P}^L) = \{f_g^{\longleftarrow}(g_3(p_1) = \{l_2\}, f_g^{\longleftarrow}(g_3(p_2) = \{l_1, l_2\}\} \notin \tau_1$

This implies f_g is not a soft L-continuous mapping from (L_1, τ_1, P) to (L_2, τ_2, P). The following proposition gives some equivalence characterizations of soft L-continuous mapping.

Proposition 3.11. *We take (L_1, τ_1, P) and (L_2, τ_2, P) as two soft lattice topological spaces over X and Y respectively and $f_g : X \longrightarrow Y$. The following conditions are equivalent:*

(1) *f_g is a soft L-topological mapping from (L_1, τ_1, P) to (L_2, τ_2, P).*

(2) *For each soft L-closed set g_P^L in Y, $f_g^{\longleftarrow}(g_P^L)$ is a soft L-closed set in X.*

(3) *For each soft L-set f_P^L in X, $f_g^{\longrightarrow}\overline{(f_P^L)} \subset \overline{f_g^{\longrightarrow}(f_P^L)}$.*

(4) *For each soft L-set g_P^L in Y, $f_g^{\longleftarrow}\overline{(g_P^L)} \supset \overline{f_g^{\longleftarrow}(g_P^L)}$.*

Proof. $\underline{(1) \Rightarrow (2)}$: Let g_P^L be a soft L-closed set in Y. Then $(g_P^L)'$ is a soft L-closed set in Y. By (1) and Proposition 3.4, $f_g^{\longleftarrow}((g_P^L)') = (f_g^{\longleftarrow}(g_P^L))'$ is a soft L-closed set in X.

Hence $f_g^{\longleftarrow}(g_P^L)$ is a soft L-closed set in X.

(2) $\Rightarrow$ (3): Let f_P^L be a soft L-set in X.

By theorem 2.16, $f_g^{\longrightarrow}(f_P^L) \subset \overline{f_g^{\longrightarrow}(f_P^L)}$.

Then by Proposition 3.4 and Proposition 3.5, $f_P^L \subset f_g^{\longleftarrow}(f_g^{\longrightarrow}(f_P^L)) \subset \overline{f_g^{\longleftarrow}(f_g^{\longrightarrow}(f_P^L))}$.

Since $\overline{f_g^{\longrightarrow}(f_P^L)}$ is a soft L-closed set in Y, then by (2), $\overline{f_g^{\longleftarrow}(f_g^{\longrightarrow}(f_P^L))}$ is a soft L-closed set in X.

Thus $\overline{(f_P^L)} \subset f_g^{\longleftarrow}(\overline{f_g^{\longrightarrow}(f_P^L)})$.

Also by Proposition 3.3 and Proposition 3.5,

$f_g^{\longrightarrow}\overline{(f_P^L)}) \subset f_g^{\longrightarrow}(f_g^{\longleftarrow}(\overline{f_g^{\longrightarrow}(f_P^L)})) \subset \overline{f_g^{\longrightarrow}(f_P^L)}$.

So $f_g^{\longrightarrow}\overline{(f_P^L)}) \subset \overline{f_g^{\longrightarrow}(f_P^L)}$.

(3) $\Rightarrow$ (4): Let g_P^L be a soft L-set in Y.

By (3), Proposition 3.5 and theorem 2.16,

$f_g^{\longrightarrow}\overline{(f_g^{\longleftarrow}(g_P^L))} \subset \overline{f_g^{\longrightarrow}(f_g^{\longleftarrow}(g_P^L))} \subset \overline{(g_P^L)}$.

Then by Proposition 3.4 and Proposition 3.5,

$f_g^{\longleftarrow}\overline{(g_P^L)} \supset f_g^{\longleftarrow}(f_g^{\longrightarrow}\overline{(f_g^{\longleftarrow}(g_P^L))}) \supset \overline{f_g^{\longleftarrow}(g_P^L)}$.

So $f_g^{\longleftarrow}\overline{(g_P^L)} \supset \overline{f_g^{\longleftarrow}(g_P^L)}$.

(4) $\Rightarrow$ (1): Let g_P^L be a soft L-closed set in Y. Then $(g_P^L)'$ is a soft L-closed set in Y.

By (4) and theorem 2.16, $\overline{f_g^{\longleftarrow}((g_P^L)')} \subset f_g^{\longleftarrow}((g_P^L)')$.

Obviously, $\overline{f_g^{\longleftarrow}((g_P^L)')} \supset f_g^{\longleftarrow}(g_P^L)')$.

Thus $\overline{f_g^{\longleftarrow}((g_P^L)')} = f_g^{\longleftarrow}((g_P^L)') = (f_g^{\longleftarrow}(g_P^L))'$ [By 2] which soft L-closed set in X.

Therefore $f_g^{\longleftarrow}(g_P^L)$ is a soft L-closed set in X.

Hence f_g is a soft L-topological mapping from (L_1, τ_1, P) to (L_2, τ_2, P). □

4. Soft L-Continuous Mapping - Soft L-Homeomorphism

Theorem 4.1. *We consider (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces, $f_g\colon (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a mapping. Then f_g is soft L-continuous if and only if $f_g\overline{(f_P^L)} \subset \overline{f_g((f_P^L))}$.*

Proof. Consider f_g be soft L-continuous. Since $\overline{f_g((f_P^L))}$ is a soft L-closed set in L_2, $f_g^{-1}\overline{f_g((f_P^L))}$ is soft L-closed set in L_1 containing f_P^L.

Also, $\overline{(f_P^L)}$ is the smallest soft L-closed set in L_1 containing f_P^L.

Therefore, $\overline{(f_P^L)} \subset f_g^{-1}\overline{f_g((f_P^L))}$.

Hence $f_g\overline{(f_P^L)} \subset \overline{f_g((f_P^L))}$.

Conversely, let $f_g\overline{(f_P^L)} \subset \overline{f_g((f_P^L))}$.

Let f_P^L be a soft L-closed set in L_2. Then

$f_g\overline{f_g^{-1}(f_P^L)} \subset \overline{f_g f_g^{-1}(f_P^L)} \subset \overline{(f_P^L)} = f_P^L$.

Hence $\overline{f_g^{-1}(f_P^L)} \subset f_g^{-1}(f_P^L)$.

Therefore $f_g^{-1}(f_P^L)$ is soft L-closed set.

Thus f_g be soft L-continuous. □

Theorem 4.2. *A bijection soft L-continuous mapping f_g is a soft L-homeomorphism if and only if $\overline{f_g(f_P^L(\alpha))} = f_g\overline{(f_P^L)}(\alpha) \forall \alpha \in P$.*

Proof. Let f_g be soft L-homeomorphism.

Then by theorem 4.1, f_g is soft L-continuous and soft L-closed mapping.

By theorem 2.19, if f_g is soft L-closed mapping if for each soft L-set f_P^L over L and for every $\alpha \in P$, $\overline{f_g(f_P^L(\alpha))} \subset f_g\overline{(f_P^L)}(\alpha)$ is satisfied.

Now we need to show that $f_g\overline{(f_P^L)}(\alpha) \subset \overline{f_g(f_P^L(\alpha))}$.

Since $\overline{(f_P^L)}$ is a soft L-closed set in L_1 and f_g is soft L-closed mapping, $f_g\overline{(f_P^L)}(\alpha)$ is a soft L-closed set in L_2 which is containing $f_g(f_P^L)$.

Since $\overline{f_g(f_P^L)}$ is the smallest soft L-closed set containing $f_g(f_P^L)$, we have $f_g(f_P^L) = \overline{f_g(f_P^L)}$.

Conversely, if f_g is bijective and the condition holds.

i.e., $\overline{f_g(f_P^L(\alpha))} = f_g\overline{(f_P^L)}(\alpha) \forall \alpha \in P$.

Then by theorem 2.21, f_g is soft L-continuous.

Let f_P^L be a soft L-closed set in L_1. Then $\overline{(f_P^L)} = f_P^L$.

Therefore, $f_g\overline{(f_P^L)} = f_g(f_P^L)$.

Then by the given condition, $f_g(f_P^L) = \overline{f_g(f_P^L)}$.

Hence f_P^L be a soft L-closed set in L_2. □

Theorem 4.3. *Let us consider* (L_1,τ_1,P) *and* (L_2,τ_2,P) *to be two soft lattice topological spaces. Then* f_g *is a soft L-homeomorphism if and only if* $f_g\colon (L_1,\tau_1,P)\longrightarrow (L_2,\tau_2,P)$ *is a soft homeomorphism.*

Proof. The proof follows from theorem 2.21. □

5. Cartesian Product of Soft L-Sets and Soft L-Product Topology

Definition 5.1. *Let* $SS(L)_P$ *be the collection of all soft L-sets with a set of parameter* P *over* L *and* A *and* B *are subsets of* P.
The cartesian product of soft L-sets $f^L_P\in SS(L_1)_A$ *and* $g^L_P\in SS(L_2)_B$ *is a soft L-set* $(f^L_P\times g^L_P, A\times B)$ *in* $SS(L_1\times L_2)_{A\times B}$, *where*
$f^L_P\times g^L_P\colon A\times B\longrightarrow P(L_1)\times P(L_2)$ *is a mapping given by*
$(f^L_P\times g^L_P)(a,b)=f^L_P(a)\times g^L_P(b)$ *for each* $(a,b)\in A\times B$.

Definition 5.2. *Let* $f^L_{P_1}$, $f^L_{P_2}$ *be soft L-sets in* $SS(L)_{P_1}$ *and* $SS(L)_{P_2}$ *respectively, where* P_1 *and* P_2 *are two different parameters. Then the cartesian product of* $f^L_{P_1}$ *and* $f^L_{P_2}$ *denoted by* $f^L_{P_1}\times f^L_{P_2}$ *in* $SS(L)_{P_1\times P_2}$ *is defined as* $(f^L_{P_1}\times f^L_{P_2})(p_1,p_2)=f^L_{P_1}(p_1)\times f^L_{P_2}(p_2)$.

Definition 5.3. *Let* (L_1,τ_1,P_1) *and* (L_2,τ_2,P_2) *be two soft lattice topological spaces. The soft lattice topological space* $(L_1\times L_2,\tau,P_1\times P_2)$, *where* τ *is the collection of all soft lattice unions of elements of* $\{f^L_{P_1}\times g^L_{P_2}\colon f^L_{P_1}\in\tau_1, g^L_{P_2}\in\tau_2\}$ *is called soft L-product topological space over* $L_1\times L_2$.
Symbolically, we write $\tau=\tau_1\times\tau_2$.

Proposition 5.4. *Let* $f^L_{1_{P_1}}$, $g^L_{1_{P_1}}\in SS(L)_{P_1}$ *and* $f^L_{2_{P_2}}$, $g^L_{2_{P_2}}\in SS(L)_{P_2}$. *Then*
$(i)\phi^L_{P_1}\times f^L_{2_{P_2}}=f^L_{1_{P_1}}\times\phi^L_{P_2}=\phi^L_{P_1\times P_2}$
$(ii)(f^L_{1_{P_1}}\times f^L_{2_{P_2}})\cap(g^L_{1_{P_1}}\times g^L_{2_{P_2}})=(f^L_{1_{P_1}}\cap g^L_{1_{P_1}})\times(f^L_{2_{P_2}}\cap g^L_{2_{P_2}})$

Proof. (i) Let $\phi^L_1=\phi^L_{1_{P_1}},\phi^L_2=\phi^L_{2_{P_2}}$ and $f^L_1=f^L_{1_{P_1}},f^L_2=f^L_{2_{P_2}}$. Then we have
$(f^L_1\times\phi^L_2)(p_1,p_2)=f^L_1(p_1)\times\phi^L_2(p_2)$
$=f^L_1(p_1)\times\phi^L$
$=\phi^L$

$= \phi^L \times f_2^L(p_2)$

$= \phi^L(p_1) \times f_2^L(p_2)$

$= (\phi_1^L \times f_2^L)(p_1, p_2)$

This implies (i).

(ii) Let $(f_1^L \times f_2^L, P_1 \times P_2) \cap (g_1^L \times g_2^L, P_1 \times P_2) = (h^L, P_1 \times P_2)$, $(f_{1_{P_1}}^L \cap g_{1_{P_1}}^L) = i_{P_1}^L$ and $(f_{2_{P_2}}^L \cap g_{2_{P_2}}^L) = j_{P_2}^L$. Then

$h^L(p_1, p_2) = (f_1^L \times f_2^L)(p_1, p_2) \cap (g_1^L \times g_2^L)(p_1, p_2)$

$= (f_1^L(p_1) \times f_2^L(p_2)) \cap (g_1^L(p_1) \times g_2^L(p_2))$

$= (f_1^L(p_1) \cap f_2^L(p_2)) \times (g_1^L(p_1) \cap g_2^L(p_2))$

$= i^L(p_1) \times j^L(p_2)$

$= (i^L \times j^L)(p_1, p_2)$

Hence $(h^L, P_1 \times P_2) \times j_{P_2}^L$. □

Proposition 5.5. *Let (L_1, τ_1, P_1) and (L_2, τ_2, P_2) be two soft lattice topological spaces. Let $B = \{f_{P_1}^L \times g_{P_2}^L | f_{P_1}^L \in \tau_1, g_{P_2}^L \in \tau_2\}$ and τ be the collection of all arbitrary union of elements of B. Then τ is a soft L- topology over $L_1 \times L_2$.*

Proof. We have

$\phi_1^L = \phi_{1_{P_1}}^L \in \tau_1, \phi_2^L = \phi_{2_{P_2}}^L \in \tau_2$.

Then by proposition 5.12;

$\phi_{1_{P_1}}^L \times \phi_{2_{P_2}}^L = \phi_{P_1 \times P_2}^L$.

Moreover $L_1 = L_{1_{P_1}} \in \tau_1$ and $L_2 = L_{2_{P_2}} \in \tau_2$.

Then $L_1 \times L_2 = (L_{1_{P_1}} \times L_{2_{P_2}}, P_1 \times P_2)$

such that the following holds:

$(L_{1_{P_1}} \times L_{2_{P_2}})(p_1, p_2) = L_{1_{P_1}}(p_1) \times L_{2_{P_2}}(p_2)$

$= L_{1_{P_1}} \times L_{2_{P_2}}$, for each $(p_1, p_2) \in P_1 \times P_2$.

Therefore $L_1 \times L_2 \in \tau$.

Let $f_{P_1 \times P_2}^L, g_{P_1 \times P_2}^L \in \tau$. Then $\exists$ the elements $f_{\alpha_{P_1}}^L \times g_{\beta_{P_2}}^L, f_{\beta_{P_1}}^L \times g_{\alpha_{P_2}}^L, \alpha \in i^L, \beta \in j^L$ of B such that

$f_{P_1 \times P_2}^L = \cup_{\alpha \in i^L} (f_\alpha^L \times g_\alpha^L, P_1 \times P_2)$,

$g_{P_1 \times P_2}^L = \cup_{\beta \in j^L} (f_\beta^L \times g_\beta^L, P_1 \times P_2)$,

Let $h_{P_1 \times P_2}^L = f_{P_1 \times P_2}^L \cap g_{P_1 \times P_2}^L$. Then we have

$h^L(p_1,p_2) = f^L(p_1,p_2) \cap g^L(p_1,p_2)$

$= [\cup_{\alpha\in i^L}(f^L_\alpha(p_1)\times g^L_\alpha(p_2)] \cap [\cup_{\beta\in j^L}(f^L_\beta(p_1)\times g^L_\beta(p_2)]$

$= \cup_{\beta\in j^L}\cup_{\alpha\in i^L}[[f^L_\alpha(p_1)\times g^L_\alpha(p_2)]\cap[f^L_\beta(p_1)\times g^L_\beta(p_2)]]$

$= \cup_{\beta\in j^L}\cup_{\alpha\in i^L}[[f^L_\alpha(p_1)\cap g^L_\alpha(p_2)]\times[f^L_\beta(p_1)\cap g^L_\beta(p_2)]]$

$= \cup_{\alpha\in i^L}\cup_{\beta\in j^L}[(f^L_\alpha\cap f^L_\beta)(p_1)\times(g^L_\alpha\cap g^L_\beta)(p_2)]$

$= \cup_{\alpha\in i^L}\cup_{\beta\in j^L}(f^L_\alpha\cap f^L_\beta\times g^L_\alpha\cap g^L_\beta)(p_1,p_2)$

Hence $h^L_{P_1\times P_2} = \cup_{\alpha\in i^L,\beta\in j^L}(f^L_\alpha\cap f^L_\beta\times g^L_\alpha\cap g^L_\beta)_{P_1\times P_2}$

$h^L_{P_1\times P_2} = \cup_{\alpha\in i^L,\beta\in j^L}(f^L_\alpha\cap f^L_\beta)_{P_1}\times(g^L_\alpha\cap g^L_\beta)_{P_2}$

$\Longrightarrow h^L_{P_1\times P_2}\in\tau.$

Thus an arbitrary union of elements of τ is an elements in τ. □

Proposition 5.6. *Let Let $f^L_{P_1}$ and $g^L_{P_2}$ be soft lattices in $SS(L_1)_{P_1}$ and $SS(L_2)_{P_2}$ respectively. Then $(f^L_{P_1}\times g^L_{P_2})' = (f^{L'}_{P_1}\times L_2)\cup(L_1\times g^{L'}_{P_2})$.*

Proof. Let $[(f^L\times g^L)_{P_1\times P_2}]' = (f^L\times g^L)'_{P_1\times P_2}$. Then

$(f^L\times g^L)'(p_1,p_2) = (L_1\times L_2) - [(f^L\times g^L)(p_1,p_2)]$

$= (L_1\times L_2) - [(f^L(p_1)\times g^L(p_2)]$

$= [(L_1 - f^L(p_1)\times L_2)]\cup[L_1\times(L_2 - g^L(p_2))]$

Also $(f^{L'}_{P_1}\times L_2)\cup(L_1\times g^{L'}_{P_2}) = (f^{L'}\times L_2)_{P_1\times P_2}\cup(L_1\times g^{L'})_{P_1\times P_2}$

Let us take soft lattice as $h^L_{P_1\times P_2}$. Then

$h^L(p_1,p_2) = (f^{L'}\times L_2)(p_1,p_2)\cup(L_1\times g^{L'})(p_1,p_2)$

$= [f^{L'}(p_1)\times L_2]\cup[L_1\times g^{L'}(p_2)]$

$= [(L_1 - f^L(p_1)\times L_2))]\cup[L_1\times(L_2 - g^L(p_2))].$ □

Corollary 5.7. *Let $f^L_{P_1}$ and $g^L_{P_2}$ be soft L-closed set in soft lattice topological spaces (L_1,τ_1,P_1) and (L_2,τ_2,P_2) respectively. Then $f^L_{P_1}\times g^L_{P_2}$ is soft L-closed set in soft L-product space $(L_1\times L_2,\tau,P_1\times P_2)$.*

Proof. It is obvious that $f^{L'}_{P_1}$ and L_1 are soft L-open sets in (L_1,τ_1,P_1) and $g^{L'}_{P_2}$ and L_2 are soft L-open sets in (L_2,τ_2,P_2).

Now by Proposition 5.6; $(f^L_{P_1}\times g^L_{P_2})'$ is soft L-open in $(L_1\times L_2,\tau,P_1\times P_2)$.

Hence $f^L_{P_1}\times g^L_{P_2}$ is soft L-closed set in soft L-product space $(L_1\times L_2,\tau,P_1\times P_2)$. □

6. Conclusion

Topological structures on soft sets are more generalized methods and they can be useful for measuring the similarities and dissimilarities between the objects in a universe which are soft sets.The concept of soft L-topological spaces are defined over a soft lattice with a fixed set of parameter. Also soft L-continuous mappings are defined over an initial universe set with a fixed set of parameters. This paper deals with the mappings and cartesian products in soft L-topological spaces. The concept of soft L-continuous mapping between two soft L-topological spaces is first proposed. Moreover, some results based on soft L-homeomorphism are also proved. The concept of cartesian product of soft L-sets are defined and some interesting results are obtained in the last section.

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Conflict of Interests

The author(s) declare that there is no conflict of interests.

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