Available online at http://scik.org

J. Math. Comput. Sci. 11 (2021), No. 2, 2301-2340

https://doi.org/10.28919/jmcs/5511

ISSN: 1927-5307

MULTI-STAGE, MULTI-PRODUCT AND MULTI-PERIOD MILK SUPPLY CHAIN NETWORK DESIGN: A CASE OF COASTAL REGION OF ODISHA

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Abstract: Due to the complexity of supply chain networks in today's business environment, it is important to consider several strategic planning decisions along with the classical location-allocation decisions to achieve an integrated system. In this paper, a milk supply chain network design (SCND) problem of coastal region Odisha is considered. The proposed model is focussed on multi-stage, multi-product and multi-period supply chain network design of milk of a dairy industry. The network design includes the strategic decision-making approach of influence area, order quantity, replenishment cycle time, preservation factor and number of proposed distribution centre of milk products. This supply chain network design generic model of the milk products is developed as a mixed integer non-linear programming (MINLP) model. It is solved by using piecewise nonlinear optimization approach for location-allocation decisions of the clusters. The sensitivity analysis is also conducted to check the importance and influence of the model parameters on total supply chain network profit per unit time of the multi-milk product. Through a real-world dairy industrial case of multi products, this proposed SCND generic model is demonstrated and then customized model is developed for the case analysis. It is observed that the total supply chain network profit per unit time for the proposed model is more than that of existing model. Hence the strategic conditions like the service region must be greater than the influence area and imposing preservation effort for the multi-milk product can be a mechanism for revenue growth of the supply chain network design.

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Received February 5, 2021

Keywords: milk supply chain network design (SCND); multi-stage; multi-product; multi-period; coastal region of Odisha; location-allocation of DCs; mixed integer non-linear programming (MINLP); piecewise non-linear optimization.

2010 AMS Subject Classification: 90B10, 90C11, 90C30, 90B80, 90B30.

1. INTRODUCTION

SCND is one of the most vital planning problems in supply chain management (SCM). Currently, design decisions should be practicable enough to function under the complex, dynamic, uncertain and robust business environments for many decades. Due to the complexity of supply chain networks in today's business environment, it is significant to consider several planning decisions along with the classical location-allocation decisions such as number of proposed facility locations, influence area and facility location allocation to achieve an integrated system. In strategic level, several vital supply chain decisions to be made. This study is based on real-life case study of a SCND of multi-milk product (Premium milk, Toned milk, Gold milk) of leading milk producing organization in Odisha. This product is highly perishable in nature; SCND implicates decision-making at a strategic level. Network design is the basis for the efficient operation of supply chain and consequently one of the most important problems a supply chain manager is to solve. This study conducts a real-life case-based modeling to address the gap in the area of supply chain network design. We investigate the milk supply chain network design under preservation technology and propose a generic mathematical model for milk supply chain network design encompassing economic objective. A customized mathematical model is also developed for a leading milk producing organization in Odisha. Both of the models are formulated and solved by using piecewise nonlinear optimization. The remainder of the paper is organized as follows: in Section 2, introduces the related literature on facility location problem, preservation technology, piecewise mixed integer NLP modeling. Section 3 presents generic mathematical model followed by assumptions, notations and objectives of the SCND problem of multi-milk products. In Section 4, optimization of the proposed SCND model is tested through some mathematical properties and algorithms are developed for the solution of the proposed model. Section 5 applies the proposed model and algorithms to a case of multi-milk product in coastal region of Odisha and the results and analysis are discussed through comparative and sensitivity analysis. Section 6 provides the conclusions and possible future directions of research.

2. LITERATURE REVIEW

Deteriorating inventory had been studied in the past decades Dave and Patel [46], Kang and Kim [43], Wee [15], Lodree and Uzochukwu [21], Bhunia et al. [4], Chang et al. [44] and focused on: constant or variable deterioration rate; quantity discount; and supply chain co-ordination. However investing on preservation technology (PT) for reducing deterioration rate has received little attention in the past years. The consideration of PT can reduce the deterioration rate significantly. Moreover, sales, inventory and order quantities are very sensitive to the rate of deterioration, especially for fast deteriorating products. Yang and Wee [36], Johnny et al. [6] specified that the higher rate of deterioration would result in higher total annual relevant cost and a lower demand rate.

The mentioned studies on perishable items have considered deterioration an exogenous variable, which cannot be controlled by the company. In practice, however, specialized equipment or processes, such as freezer equipment and cold storage techniques, can be used to control deterioration, many enterprises have examined deterioration causes and have developed preservation technologies to control them and increase profit Dye and Hsieh [7]. Hsu et al. [35] determined replenishment and preservation technology investment policies under a constant deterioration rate and partial backlogging. Dye and Hsieh [7, 8] have considered the effect of preservation technology investment costs on preservation equipment for reducing deterioration rate under two-level forward financing. In the current study, both preservation technology cost and the deterioration rate function of preservation effort are considered.

Ouyang et al. [25,26] found that if the retailer can reduce effectively the deteriorating rate of item by improving the storage facility, the total annual relevant inventory cost will be reduced. Wang et al. [24] focused on deciding on resources portfolio and allocating resources to various orders in each production period. Many enterprises invest on equipments to reduce the deterioration rate and extending the product expiration date. For example, refrigeration equipments are used to reduce the deterioration rate of fruits, flowers and sea foods in the supermarket. Murr and Morris [9] showed that a lower temperature will increase the storage life and decrease decay. Moreover, during the vacuum technology are introduced to reduce the deterioration rate of medicine and food stuff. Zauberman et al. [14] developed a method for colour retention of Litchi fruits with SO_2 fumigation. The trade-off between the increased cost of investment and the increased profit due to decreased deterioration rate is the focus of this study.

In a multi-item distribution channel, considerable savings can be realized during the replenishment by coordinating the ordering of several different items. Multi-echelon multi-item replenishment strategies are already widely applied in the real world, for example, the supplying of parts for computers and for automotive assembly Hahm and Yano [16, 17] or refrigerated goods to supermarkets Hammer [29]. In these industries, a supplier normally produces different products for a single customer and ships to the customer simultaneously in a single truck. In the grocery supply industry or a fast moving consumer goods industry different types of refrigerated goods (General Mills yogurt, Derived Milk products etc.) can be shipped in the same truck to the same supermarket or retail store Hammer [29], Goyal [42], Kao [10], Graves [41], Ben-Khedher and Yano [34], Van Eijs [11], Rempala [39], Chen and Chen [18,19], Nilsson et al. [3], Tsao and Sheen [50], pattnaik and gahan[30,31],Axsater.et.al[40] and others have developed models and algorithms for solving multi-item replenishment problems for different constraints. Because multi-echelon coordination is frequently applied in current business practice, it is an essential component in supply chain model. Hence the multi-product multi-echelon supply chain is the focus of the present study.

Lee et al. [12] discussed a heuristic approach for solving serially distributed storage depots under general-integer policy. Paksoy et al. [44] focused on revised multi-choice goal programming for multi-period, multi-stage inventory controlled supply chain model with popup stores in Guerrilla marketing. Taxakis et al. [23] designed a model and a production distribution and inventory planning model in multi-product supply chain networks. Ackerman et al. [28] studied a green supply chain network design framework for the processed food industry with the application to the orange juice agro food cluster and Bourlakis.et.al [27] develops food supply chain. Govindan et al. [22] discussed a comprehensive review and future research directions of supply chain network design under uncertainty. Varesi et al. [32] developed sustainability of supply network design by a case of wine industry in Australia where Pham et al. [45] developed an optimal supply chain network design with process network and BOM under uncertainties for a case study in toothbrush industry. After reviewing the literature of related past works, the key points are summarized in Table 1.

Table 1 illustrates the contribution of relevant researchers related to the present study. The present study focused on developing both generic and customized supply chain network design models for integrated multi-product and multi-period supply chain. Using piecewise non-linear optimization approach all the decision variables are obtained for the generic model. This study gives the solution of the location-allocation problem for both theory and practice. The Some relevant properties are derived from the optimization approach. In this study some strategic conditions like the service region of multi-milk product must be greater than that of the influence area and preservation effort for the multi-product are imposed and compared with other existing models. Through sensitivity analysis the impact of model parameter on all the decision variables and the total supply chain network profit per unit time is studied.

Table 1: Comparisons with other Recent Researches

Research Parameters Authors	**Demand**	**Model**	**Echelon**	**Benefit**	**Preservation Effort**	**Replenishment**	**Product**	**Single/Multi Product**
Dasci et al. [1]	Deterministic	NLP	Four	Production-distribution system design	No	Infinite	Not specified	Single
Abad & Jaggi [38]	Price Dependent Exponential Demand	NLP	Two	No	No	Finite	Not specified	Single
Chen & Chen [18,19]	Constant	NLP	Two	Centralization	No	Finite	Not specified	Multi
Chen & Chen [20]	Price Dependent	NLP	Multi	Demand and Revenue Increments	No	Finite	Not specified	Multi
Tsao & Sheen [49]	Time and Price Dependent	NLP	One	No	Yes	Finite	Not specified	Single
Hsu et al. [37]	Constant	NLP	Single	Capital constraint	Yes	Finite	Not specified	Single
Tsao & Lu [51]	Poisson	Continuous approximation in NLP	Multi	Quantity discounts and distance discounts, SCND	No	Finite	Not specified	Single
Tsao & Teng [52]	Deterministic	Piecewise NLP	Multi	Supply Chain Network Design(SCND)	Yes	Finite	Fresh food	Multi
Alejandro [13]	Deterministic	IPP	Multi	No	No	Finite	Bakery	Multi
Varsei et al. [32]	Uncertain	Multi-objective	Four	Sustainable supply chain network design	Yes	Infinite	Wine	Single
Tsao [53]	Deterministic	Piecewise NLP	Multi	Supply Chain Network Design(SCND)	Yes	Finite	Agricultural product	Single
Macro et al. [2]	Deterministic	Multi-objective	Two	Green supply chain network design	Yes	Finite	Food (Orange juice)	Single
Present Study (2021)	Deterministic (Empirical)	Multi-stage Mixed integer NLP (Piecewise)	Multi (Three Tier)	Integrated Multi-product Multi-period Supply Chain Network Design(SCND)	Yes	Finite (Joint)	Milk [Premium milk, Toned milk, Gold milk]	Multi

3. MATHEMATICAL MODEL

3.1 ASSUMPTIONS AND NOTATIONS

This study discusses an integrated multi-echelon multi-milk product supply chain network design (SCND) with member producers selling highly perishable milk products to village diary as Cooperative Society, then it comes to the Chilling Center as District Milk Union then it delivered immediately to the Distribution Center (DC) for processing and marketing. Milk products are transported from DCs to retailers; therefore, the DCs are at the third level and helping to consolidate shipments arriving from district milk unions and awaiting delivery to retailers with preservation technology. Retailers from downstream meet demands from end-customers. The generic mathematical model of milk supply chain network of multi-product is developed on the basis of the following assumptions:

1. Service regions have somewhat irregular shapes compared with the circles, hexagons, and squares addressed in economics literature. This irregular service area has been demonstrated to have little effect on the optimal solution (Dasci and Verter, [1]).
2. Each DC is located at the center of the service area. (Dasci and Verter, [1]) and (Tsao, [53]) and (Geofrion et al., [5]).
3. Supply chain network profit is:

$$\pi_k(A_i, T_j, \tau_j) = \begin{cases} \pi_I(A_i, T_j, \tau_j), when\ C_i = A_i\ , \forall i = 1,2 \ldots N \\ \pi_{II}(A_i, T_j, \tau_j), when\ C_i \geq A_i, \forall j = 1,2..L \end{cases}$$

4. Number of existing DC in each cluster i is 1 i.e. no. of DC = no. of cluster = 1.
5. Optimum number of proposed DCs in each cluster i is $m_i = \frac{ith\ service\ area}{ith\ influence\ area} = \frac{C_i}{A_i}$ (integer). (Dasci and Verter, [1])
6. Each retailer is assigned to a particular DC and is served only by that DC. When the locations of retailers are within the influence area of a DC, those retailers within the influence area are assigned by the DC. (Tsao, [53])
7. When the milk federation enhances preservation technology, the quantity of deteriorated products decreases. In this study, as the milk product is taken into consideration, so the

deterioration rate is assumed to decrease exponentially as the preservation effort increases; i.e. $\theta_j = \beta e^{-\beta\tau_j}$, where, τ_j is preservation effort factor, β is a positive constant. (Tsao, [53])

8. When the milk federation increases preservation efforts, the cost of preservation efforts is also increased. It means the marginal increase of preservation effort also increases the preservation effort cost. The preservation effort cost is not only convex but also increases with preservation effort, $K(\tau_j) = a_1 + b\tau_j{}^2$, where $a_1 and\ b$ are positive constants (Tsao, [53]) .

Table 2 presents the notations used in the generic and customized mathematical models:

Table 2: Summary of Notations

N	:	The number of clusters considered
L	:	The number of products considered
i	:	The index of DCs, $i = 1,2, \dots \dots . N$
j	:	The index of products, $j = 1,2, \dots \dots . L$
k	:	The index of the cases, $k \in [I, II]$, case-I; $C_i = A_i$ {existing DC} and $case - II$; $C_i \geq A_i$ {proposed DC}
m_i	:	Number of proposed DCs, $m_i = \frac{C_i}{A_i}(decision\ variable)$; integer
A_i	:	Influence area for each DC in cluster $i\ and A_i \geq 0$ *$(decision\ variable)$* for the cases $k \in [I, II]$. It is a geographical area wherein a DC is directly capable of influencing or distributing product to its retailers by its own transport vehicle under its control. (Sq. Kilometre); A_i^* is the optimal value of A_i
T_j	:	Joint replenishment cycle time of product j *$(decision\ variable)$*, $\forall\ T_j \geq 0$ (days); T_j^* is the optimal value of T_j
τ_j	:	Preservation effort *$(decision\ variable)$* of product j, $\forall\ \tau_j \geq 0$. (Rs.); τ_j^* is the optimal value of τ_j
Q_{ij}	:	Ordering quantity of product j for each DC in cluster i for the cases $k \in [I, II]$ during each replenishment cycle *$(decision\ variable)$* (Litre)
$I_{ij}(t)$	:	Inventory level of product j for each DC in cluster i with respect to time t
θ_j	:	Deteriorating rate of the product j, $\theta_j = \beta e^{-\beta\tau_j}$, β is positive constant (Litre)
F	:	Facility cost of operating each DC in cluster i (Rs.)
μ	:	Planning horizon, $\sum_{j=1}^{L} T_j \leq \mu$ (days)

δ_i	:	Store density in cluster i
λ_{ij}	:	Daily demand rate (deterministic) for a retail store of product j in cluster i ('000 Litre)
C_f	:	Fixed cost per inbound shipment of product j (Rs.)
C_{vj}	:	Variable cost per item of product j for each inbound shipment (Rs.)
C_r	:	Outbound transportation cost per unit distance per item of product j (Rs.)
f_r	:	Constant that depends on the distance metric and shape of the DC service region
$K(\tau_j)$	:	Preservation effort cost per unit of product j, $K(\tau_j) = a_1 + b{\tau_j}^2$, where $a_1 and\, b$ are positive constants and τ_j is a preservation effort factor for DC in cluster i, (Rs.) (Tsao, 2016)
R	:	Ordering cost per order for DC in cluster i (Rs.)
h_j	:	Inventory holding cost per unit per unit of time of product j for DC in cluster i (Rs.)
C_i	:	Service area in cluster i (Sq. Kilometre)
p_j	:	Unit selling price of product j (Rs.)
c_j	:	Unit purchasing cost of product j (Rs.)
π_k	:	Total supply chain network profit per unit time for the case $k, k \in [I, II]$ (Rs.); π_k^* is the optimal value of π_k

3.2 OBJECTIVES OF THE MATHEMATICAL MODEL

The main objective of this study is to develop a mathematical model for milk products to obtain the optimal influence area for each cluster, joint replenishment cycle time of product j, preservation effort factor of product j, number of proposed DCs (integer) in each cluster, order quantity of product j for each DC and the total supply chain network profit per unit time. The objectives are outlined below to conduct the study in the context of a supply chain network design model for case-I and II for an in-depth mathematical analysis point of view:

1. to determine the optimal influence area; A_i for each distribution centers (DCs) of cluster $i, i = 1,2, \ldots..N$.
2. to determine the optimal joint replenishment cycle time of product j; $T_j, \sum_{j=1}^{L} T_j \leq \mu$.
3. to determine the optimal investment factor in preservation technology of product j; τ_j.
4. to determine the optimal number of proposed DCs (integer) in cluster i; m_i for case-II model.
5. to determine the optimal order quantity Q_{ij} of product j for each DC of cluster $i, i = 1,2, \ldots.N\ and\ j = 1,2..L$.

6. to determine the optimal total supply chain network profit per unit time $\pi_k, k \in [I, II]$.
7. to investigate the sensitivity of the optimal total supply chain network profit per unit time and optimum decision variables due to the change in model parameters.

3.3 GENERIC MATHEMATICAL MODEL

In this section, the generic mathematical model of a milk product is formulated for supply chain network design under preservation technology (Tsao, [53]).

$$\frac{dI_{ij}(t_j)}{dt_j} = -\theta_j I_{ij}(t_j) - \mu\lambda_{ij}\delta_i C_i, 0 \le t_j \le T_j \text{ , } i = 1,2..N \text{ and } j = 1,2,..L \quad (1)$$

$$\frac{dI_{ij}(t_j)}{dt_j} + \theta_j I_{ij}(t_j) = -\mu\lambda_{ij}\delta_i C_i$$

The integrating factor $(IF) = e^{\int \theta_j dt_j} = e^{t_j\theta_j}$ and multiplying this integrating factor with the equation (1) the equation is:

$$\frac{dI_{ij}(t_j)e^{t_j\theta_j}}{dt_j} = -\mu\lambda_{ij}\delta_i C_i e^{t_j\theta_j} \Rightarrow \int dI_{ij}(t_j)e^{t_j\theta_j} = -\int \mu\lambda_{ij}\delta_i C_i e^{t_j\theta_j} dt_j \Rightarrow I_{ij}(t_j)e^{t_j\theta_j} = \frac{-\mu\lambda_{ij}\delta_i C_i e^{t_j\theta_j}}{\theta_j} + c_1$$

$$\Rightarrow I_{ij}(t_j) = \frac{-\mu\lambda_{ij}\delta_i C_i}{\theta_j} + \frac{c_1}{e^{t_j\theta_j}}, \textit{where } c_1 \textit{ is an arbitrary constant} \quad (2)$$

Given that, $I_{ij}(T_j) = 0$ it means for $t_j = T_j, I_{ij}(t_j) = 0$.

So,$\Rightarrow 0 = \frac{-\mu\lambda_{ij}\delta_i C_i}{\theta_j} + \frac{c_1}{e^{T_j\theta_j}} \Rightarrow c_1 = \frac{\mu\lambda_{ij}\delta_i C_i e^{T_j\theta_j}}{\theta_j}$, putting the value of c_1 in the equation (2).

$$I_{ij}(t_j) = \frac{-\mu\lambda_{ij}\delta_i C_i}{\theta_j} + \frac{\mu\lambda_{ij}\delta_i C_i e^{T_j\theta_j}}{\theta_j e^{t_j\theta_j}} \Rightarrow I_{ij}(t_j) = \frac{\mu\lambda_{ij}\delta_i C_i}{\theta_j}\left[e^{\theta_j(T_j - t_j)} - 1\right], for\ 0 \le t_j \le T_j \quad (3)$$

$I_i(0)$ *is the initial inventory level of item from DC i* or the inventory level at the beginning of the replenishment cycle and it is also equivalent to the order quantity of the j milk product.

$$Q_{ij} = I_{ij}(0) = \frac{\mu\lambda_{ij}\delta_i C_i}{\theta_j}\left[e^{\theta_j T_j} - 1\right] = \frac{\mu\lambda_{ij}\delta_i C_i}{\beta e^{-\beta\tau_j}}\left[e^{\beta e^{-\beta\tau_j} T_j} - 1\right] \quad (4)$$

Derivation of Components of Total Supply Chain Network Profit per unit time:

Total sales revenue: $SR = \sum_{i=1}^{N}\sum_{j=1}^{L} \frac{p_j\mu\lambda_{ij}\delta_i C_i}{T_j}$

Total purchasing cost:$PC = \sum_{i=1}^{N}\sum_{j=1}^{L} c_j \frac{Q_{ij}}{T_j} = \sum_{i=1}^{N}\sum_{j=1}^{L} \frac{c_j \mu\lambda_{ij}\delta_i C_i [e^{\beta e^{-\beta\tau_j} T_j - 1}]}{\beta e^{-\beta\tau_j} T_j}$

Total inventory holding cost of all items is:

$HC = \sum_{i=1}^{N}\sum_{j=1}^{L} \frac{h_j}{T_j} \times \int_0^{T_j} I_i(t_j) dt_j = \sum_{i=1}^{N}\sum_{j=1}^{L} \frac{h_j}{T_j} \times \int_0^{T_j} \frac{\mu\lambda_{ij}\delta_i C_i}{\theta_j} \left[e^{\theta_j (T_j - t_j)} - 1\right] dt_j =$

$\sum_{i=1}^{N}\sum_{j=1}^{L} \frac{h_j \mu\lambda_{ij}\delta_i C_i}{\beta^2 e^{-2\beta\tau_j} T_j} \left[e^{\beta e^{-\beta\tau_j} T_j} - \beta e^{-\beta\tau_j} T_j - 1\right]$

Total facility cost: $FC = \sum_{i=1}^{N} F \frac{C_i}{A_i}$

Total inbound transportation cost: $ITC = \sum_{i=1}^{N}\sum_{j=1}^{L} \frac{[C_f + [C_{vj}\mu\lambda_{ij}\delta_i C_i \left[e^{\beta e^{-\beta\tau_j} T_j - 1}\right]]/\beta e^{-\beta\tau_j}]}{T_j}$

Total outbound transportation cost: $OTC = \sum_{i=1}^{N}\sum_{j=1}^{L} C_r f_r \sqrt{A_i} \mu\lambda_{ij}\delta_i C_i$

Total cost of preservation effort: $PEC = \sum_{i=1}^{N}\sum_{j=1}^{L} (a_1 + b\tau_j^2) \mu\lambda_{ij}\delta_i C_i$

Total ordering cost: $OC = \sum_{i=1}^{N}\sum_{j=1}^{L} \frac{R}{T_j} \frac{C_i}{A_i}$

The total supply chain network profit per unit time for the case $k, k \in [I, II]$; π_k is given by:

$\pi_k(A_1, A_2, A_{3,\dots\dots,} A_N, T_j, \tau_j) = \pi_k = \text{Total Sales Revenue} - \text{Total Purchasing Cost} -$
$\text{Total Inventory Holding Cost} - \text{Total Facility Cost} - \text{Total Inbound Transportation Cost} -$
$\text{Total Outbound Transportation Cost} - \text{Total Cost of Preservation Effort} -$
$\text{Total Ordering Cost} \Rightarrow \pi_k = SR - PC - HC - FC - ITC - OTC - PEC - OC$

$$\pi_k(A_1, A_2, A_{3,\dots\dots,} A_N, T_j, \tau_j) = \sum_{i=1}^{N}\sum_{j=1}^{L} \left[\frac{p_j \mu\lambda_{ij}\delta_i C_i}{T_j} - \frac{c_j \mu\lambda_{ij}\delta_i C_i [e^{\beta e^{-\beta\tau_j} T_j - 1}]}{\beta e^{-\beta\tau_j} T_j} - \frac{h_j \mu\lambda_{ij}\delta_i C_i}{\beta e^{-2\beta\tau_j} T_j} \left[e^{\beta e^{-\beta\tau_j} T_j} - \beta e^{-\beta\tau_j} T_j - 1\right] - F\frac{C_i}{A_i} - \frac{[C_f + [C_{vj}\mu\lambda_{ij}\delta_i C_i \left[e^{\beta e^{-\beta\tau_j} T_j - 1}\right]]/\beta e^{-\beta\tau_j}]}{T_j} - C_r f_r \sqrt{A_i} \mu\lambda_{ij}\delta_i C_i - (a_1 + b\tau_j^2) \mu\lambda_{ij}\delta_i C_i - \frac{R}{T_j}\frac{C_i}{A_i} \right] \quad (5)$$

4. Optimization

The problem is to determine the optimal influence area A_i^* in each DC of cluster i, the optimal preservation effort $\tau_j{}^*$, the optimal joint replenishment cycle time $T_j{}^*$,and to maximize the total annual network profit $\pi_k(A_1, A_2, A_{3,.....,}A_N, T_j, \tau_j), k \in [I, II]$which is a two-branch function with $N+2$ decision variables (Tsao, 2016). By using a truncated Taylor series expansion $e^{\theta_j T_j} = 1 + \theta_j T_j + \frac{1}{2!}(\theta_j T_j)^2$, the equation (5) can be written as:

$$\pi_k(A_1, A_2, A_{3,.....,}A_N, T_j, \tau_j) = \sum_{i=1}^{N}\sum_{j=1}^{L}\left[\frac{p_j\mu\lambda_{ij}\delta_i C_i}{T_j} - \frac{c_j\mu\lambda_{ij}\delta_i C_i[1-\beta\tau_j+\frac{\beta T_j}{2}]}{(1-\beta\tau_j)} - \frac{h_j\mu\lambda_{ij}\delta_i C_i}{2} - F\frac{C_i}{A_i} - \frac{[C_f+[C_{vj}\mu\lambda_{ij}\delta_i C_i[\beta T_j-\beta^2 T_j\tau_j]]/\beta(1-\beta\tau_j)]}{T_j} - C_r f_r\sqrt{A_i}\mu\lambda_{ij}\delta_i C_i - (a_1 + b\tau_j^{\,2})\mu\lambda_{ij}\delta_i C_i - \frac{R}{T_j}\frac{C_i}{A_i}\right] \quad (6)$$

The first order derivative of $\pi_k(A_1, A_2, A_{3,.....,}A_N, T_j, \tau_j)$ with respect to $A_i$is:

$$\frac{\partial\pi_k(A_1,A_2,A_{3,.....,}A_N,T_j,\tau_j)}{\partial A_i} = \sum_{j=1}^{L}\left(\frac{FC_i}{A_i^2}\right) + \left(\frac{RC_i}{T_j A_i^2}\right) - \left(\frac{C_r f_r\mu\lambda_{ij}\delta_i C_i}{2A_i^{1/2}}\right), i = 1,2,\ldots..N, j = 1,2,..L$$

Second-order derivative of $\pi_j(A_1, A_2, A_{3,.....,}A_N, T_j, \tau_j)$ with respect to A_i is:

$$\frac{\partial^2\pi_k(A_1,A_2,A_{3,.....,}A_N,T_j,\tau_j)}{\partial A_i^2} = -\sum_{j=1}^{L}\left(\frac{2FC_i}{A_i^3}\right) - \left(\frac{2RC_i}{T_j A_i^3}\right) + \left(\frac{C_r f_r\mu\lambda_{ij}\delta_i C_i}{4A_i^{3/2}}\right), i = 1,2,\ldots..N.$$

From $\frac{\partial\pi_k(A_1,A_2,A_{3,.....,}A_N,T_j,\tau_j)}{\partial A_i} = 0$, it can get the threshold of $F = \sum_{j=1}^{L}\left\{\frac{C_r f_r\mu\lambda_{ij}\delta_i A_i^{3/2}}{2} - \frac{R}{T_j}\right\}$.

This means $\frac{\partial^2\pi_k(A_1,A_2,A_{3,.....,}A_N,T_j,\tau_j)}{\partial A_i^2} < 0$ when, $F > \sum_{j=1}^{L}\left\{\frac{C_r f_r\mu\lambda_{ij}\delta_i A_i^{3/2}}{2} - \frac{R}{T_j}\right\}$.

Since, the facility opening cost F is very large, $\frac{\partial^2\pi_k(A_1,A_2,A_{3,.....,}A_{N,}\, T_j,\tau_j)}{\partial A_i^2} < 0$ is satisfied in general case and $\frac{\partial^2\pi_k(A_1,A_2,A_{3,.....,}A_N,T_j,\tau_j)}{\partial A_i\tau_j} = 0$ & $\frac{\partial^2\pi_k(A_1,A_2,A_{3,.....,}A_N,T_j,\tau_j)}{\partial A_i A_p}$=0. A_i is independent to A_p, where $i = 1,2\ldots..N; p = 1,2\ldots N\ and\ i \neq p$. The Hessian matrix is:

$$H_{N+1} = \begin{bmatrix} \frac{\partial^2\pi_k}{\partial A_1^2} & 0 & . & . & 0 & 0 \\ 0 & \frac{\partial^2\pi_k}{\partial A_2^2} & . & . & 0 & 0 \\ . & . & . & . & . & . \\ . & . & . & . & 0 & . \\ 0 & 0 & . & 0 & \frac{\partial^2\pi_k}{\partial A_N^2} & 0 \\ 0 & 0 & . & . & 0 & \frac{\partial^2\pi_k}{\partial\tau^2} \end{bmatrix} \quad (7)$$

Since $\frac{\partial^2 \pi_k}{\partial A_i^2} < 0$ and $\frac{\partial^2 \pi_k}{\partial \tau^2} < 0$ for $i = 1,2 \dots N$, it is known that $(-1)^l . |D_l| > 0$ is satisfied for all $l, l = 1,2 \dots . N+1$. From the maximum theorem, $\pi_k\left(A_1, A_2, A_{3,\dots\dots,} A_N, \tau_j \middle| T_j\right)$ is a concave function of $(A_1, A_2, \dots \dots, A_N)$. This means that the optimal $A_i\left(T_j\right), and\ \tau_i\left(T_j\right), i = 1,2, \dots . N$, can be obtained by solving $\frac{\partial \pi_k\left(A_1, A_2, A_{3,\dots\dots,} A_{N,} \tau_j \middle| T_j\right)}{\partial A_i} = 0$ and $\frac{\partial \pi_k\left(A_1, A_2, A_{3,\dots\dots,} A_N, \tau_j \middle| T_j\right)}{\partial \tau_j} = 0$ simultaneously leads to:

$$A_i^*\left(T_j\right) = \sum_{j=1}^{L} \left[\frac{2(F+RT_j)}{C_r f_r \mu \lambda_{ij} \delta_i T_j}\right]^{2/3}, i = 1,2 \dots N. \quad (8)$$

$$\tau_j^*\left(T_j\right) = \sum_{j=1}^{L} \frac{\beta^2 T_j (C_j + C_{vj})}{4b},\ i = 1,2 \dots N. \quad (9)$$

4.1 MATHEMATICAL PROPERTIES

Equations (8) and (9) lead to Properties 1 and 2:

Property 1: (a) The influence area A_i for each DC of cluster i increases as the planning horizon μ or the outbound transportation cost per unit distance per item C_r or constant f_r decreases.

(b) The influence area A_i for each DC of cluster i increases as the facility cost F or ordering cost per order R increases.

Property 2: (a) The preservation effort for product j τ_j increases as the deterioration parameter β increases.

(b) The preservation effort for product j τ_j increases as the preservation effort parameter b decreases.

Substituting $A_i^*\left(T_j\right)$ and $\tau_j^*\left(T_j\right), i = 1,2 \dots N$ into the corresponding $\pi_k\left(A_1, A_2, A_{3,\dots\dots,} A_N, T_j, \tau_j\right)$ reduces the model to a single variable function $\pi_k\left(T_j\right)$. Based on the Algorithm and Lingo 13.0 software package, the optimal values for $A_i^*, T_j^*\ and\ \tau_j^*$ of the given model are obtained.

4.2 ALGORITHM

In Table 3 algorithm of the original SCND problem which determines the optimal values of the decision variables with optimal supply chain network profit per unit time for the two cases; I and II respectively.

Table 3: Algorithm for the Original Problem

Case-I $\{C_i = A_i\}$	
Step 1.	Find an initial solution $T_{j,i=1}$. by inspection by solving Equation (6)
Step 2.	Calculate $\pi_I'(T_j)$ and $\pi_I''(T_j)$.
Step 3.	Let $T_{j,i+1} = T_{j,i} - \frac{\pi_I'(T_j)}{\pi_I''(T_j)}$.
Step 4.	If $\lvert T_{j,i+1} - T_{j,i} \rvert \leq \varepsilon$, where ε is the tolerance error, then go to Step 5; otherwise, let $i = i + 1$ and go to Step 3.
Step 5.	Let $T_j^* = T_{j,i+1}$and $\pi_I^*(T_j^*) = \pi_I(T_{j,i+1})$
Step 6	Determine $A_i^*(T_j^*)$and $\tau_j^*(T_j^*)$ by Equations (8) and (9) respectively.
Case-II$\{C_i \geq A_i\}$	
Step 1.	Find an initial solution $T_{j,i=1}$. by inspection by solving Equation (6)
Step 2.	Calculate $\pi_{II}'(T_j)$ and $\pi_{II}''(T_j)$.
Step 3.	Let $T_{j,i+1} = T_{j,i} - \frac{\pi_{II}'(T_j)}{\pi_{II}''(T_j)}$.
Step 4.	If $\lvert T_{j,i+1} - T_{j,i} \rvert \leq \varepsilon$, where ε is the tolerance error, then go to Step 5; otherwise, let $i = i + 1$ and go to Step 3.
Step 5.	Let $T_j^* = T_{j,i+1}$and $\pi_{II}^*(T_j^*) = \pi_{II}(T_{j,i+1})$
	Determine $A_i^*(T_j^*)$and $\tau_j^*(T_j^*)$ by Equations (8) and (9) respectively.

5. CASE STUDY

The leading milk producing organization of Odisha is taken as the real-life case study in this study. The main activity of this organization is procurement, processing and marketing of milk and milk products for the development of rural communities economically. The Milk Union of Odisha was established in the year 1978 and was registered in 1980 under National Dairy Development Board and became fully functional in 1981. Out of 4.6 Crore of population majority of the population in Odisha is dependent on agriculture and animal husbandry for its survival, but being one of the economically backward states its industrial development is quite slow, and to add to the worries poor irrigation facilities and occurrence of natural calamities have proved to be even more detrimental for the state and its population. This is where this federation came into picture and provided livelihood to the rural population and safe and processed milk to the urban population.

It came into existence after conceiving the novel idea to start a collective work with farmers/milk producers through milk generation and marketing. It aims to be a leading milk producing organization at international level of efficiency with wide and satisfied customer base, maximizing wealth of stakeholders and continuing to the state economy. The major objective of this federation is the advancement of dairying, encouraging and educating people, through mutual participation. It also aims to increase productivity and per capita consumption, promotion of clean milk production and distribution with enhanced technology, customer satisfaction with reliable, uninterrupted service and quality products, encourage a performance oriented culture encouraging innovation, promote work climate encouraging employees to participate and contribute for organizational growth, continuous up-gradation of skills and competence of employees and their career advancement and enrich quality of life of people and preserve ecological balance.

The Supply Chain of Leading Milk Producing Organization of Odisha

The backbone of the milk producing organization of Odisha is its supply chain, which is required for effective procurement of raw materials (raw milk) to delivering them to the end customers. It follows three tier systems for procuring the raw milk. There are three stages of the supply chain such as the village cooperative society, the district milk union and the milk federation.

The village cooperative society is the first tier of entire supply chain system and is a voluntary association of milk producers who are willing to sell milk on a collective basis. The members select a Committee and a Chairman too to better manage and handle things. The Committee then selects some more people to act as milk tester, secretary. Milk producers bring milk to the society every morning and evening. Sample of milk from each of producers is tested by Lactometer for quality. The society sells milk to the nearest District Milk Union (DMU).

The Milk Union then carries the collects the milk to their processing center, where the milk is chilled to 5°C, whose chairman is the District Collector. The other functions of the DMU are providing technical inputs to the milk union like Artificial insemination (AI), Embryo Transfer Technology (ETT) and Feed and Fodder Programs (FFP).

The chilled milk from the DMU is then sent for processing and marketing the milk and milk related products to the milk federation or distribution center (DC). An important feature of this milk supply chain is after marketing the products monetary realization is achieved is routed back through the supply chain network design to the producers once in 10 days. The milk and milk products are being marketed by authorized retailers of the leading milk producing organization throughout the state Odisha. Urban consumer is the last point of milk flow supply chain of the organization. This milk is supplied in standardized poly packs under preservation technology to authorized retail centers once every morning in small hired organization trucks. These trucks pick up milk from the nearest federation dairy and have to cover authorized retail outlets in particular service area of each cluster assigned to them. Table 4 presents the distribution center and marketing center of milk product of this leading milk federation in Coastal Region of Odisha. Figure 1 shows the flowchart of the milk supply chain network design of the milk industry. Figure 2 shows the view of Odisha in India; Western area and Coastal area in Odisha; Distribution centers (DCs) of milk product in Odisha; Distribution centers of milk product in Coastal area.

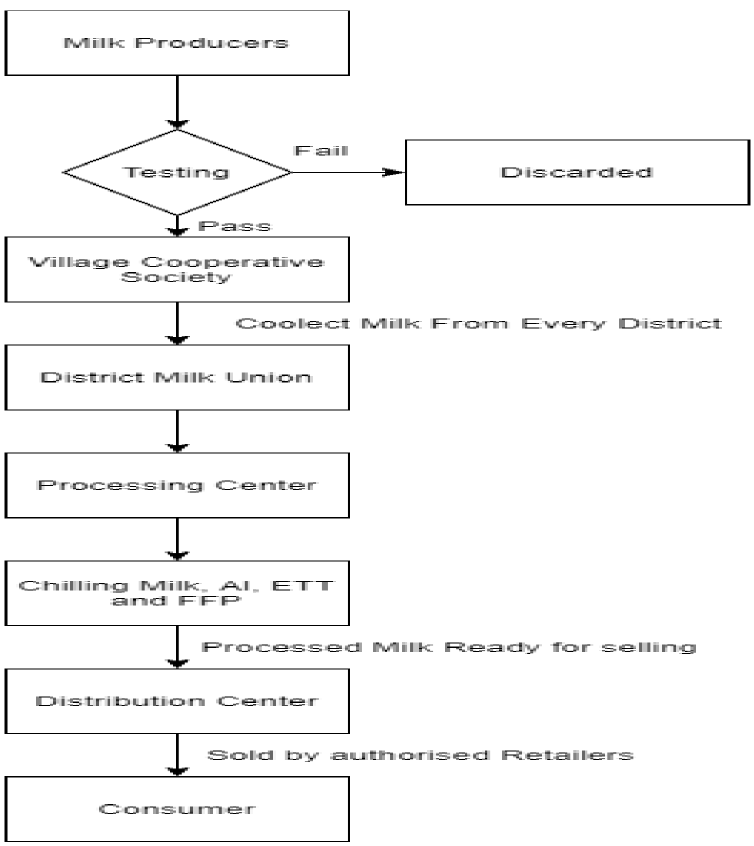


Figure 1: Flowchart of milk supply chain network design

Table 4: The Distribution Center and Marketing Center of Leading Milk Producing Organization in Odisha (Coastal Odisha)

Cluster (i)	Distribution Center	m_i (Existing no. of DCs)	Marketing Center
1	Balasore	1	Balasore, Remuna & Bhadrak
2	Tirtol	1	Tirtol, Jagatsinghpur & Kendrapada
3	Berhampur	1	Ganjam, Berhampur, Malkanagiri & Gajapati
4	Khurda	1	Khurda, Puri, Cuttack, Bhubaneswar & Jatni

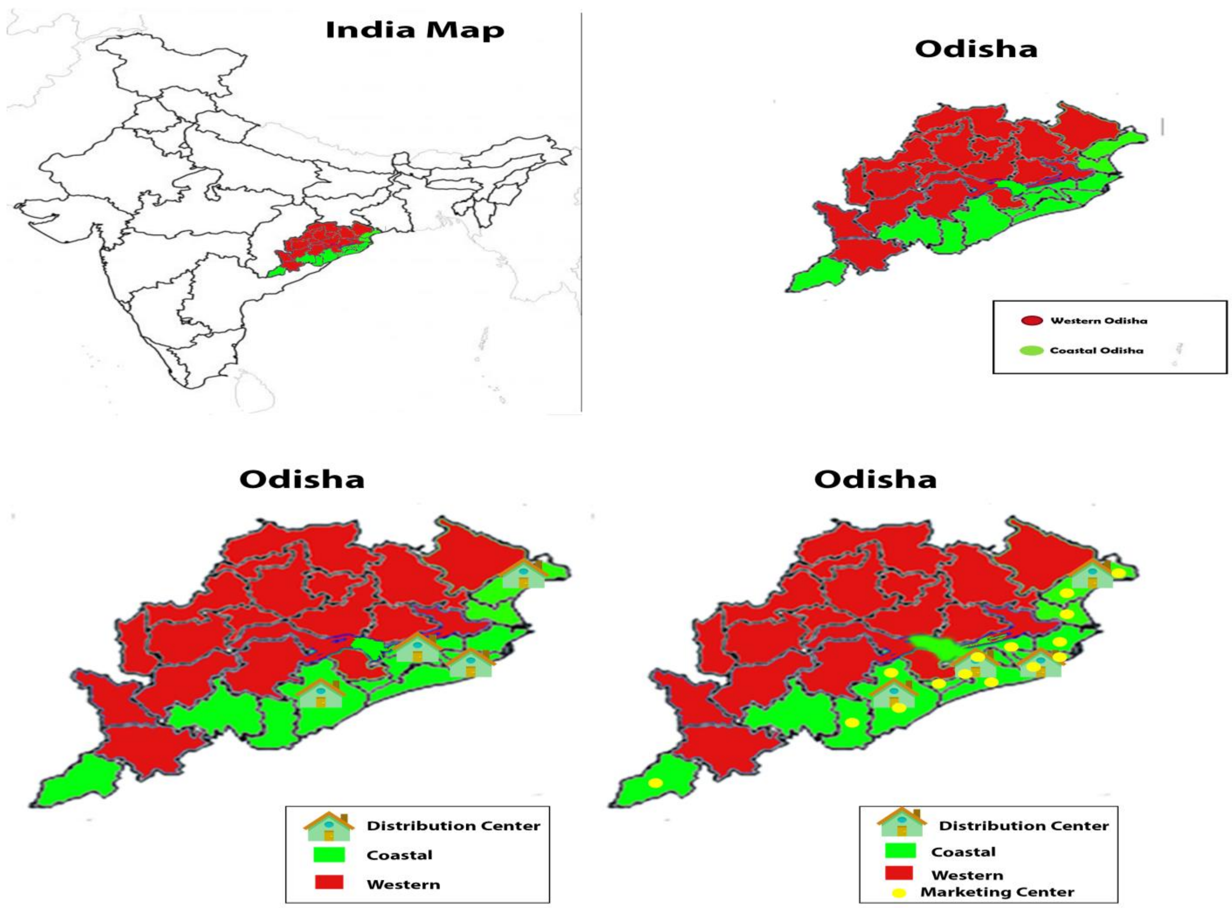


Figure 2: Odisha in India; Western Odisha and Coastal Odisha; Distribution Centers of Milk Product in Coastal Odisha; Networking of Distribution Centers and Marketing Centers of Milk Product in Coastal Area

5.1 CASE STUDY SETTINGS

The state Odisha has an area of 155,707 km^2, which is 4.87% of total area of India, and a coastline of 450 km. Odisha is divided into two geographical regions i.e. Western region and Coastal region. The Western region of Odisha has an area of 130569.21 km^2 and Coastal region of Odisha has an area of 25137.79 km^2. Leading Milk Federation in Odisha divides Western region with 9 clusters and Coastal region with 4 clusters. In 4 clusters like Balasore, Tirtol, Berhampur and Khurda, there is only one distribution centre (DC) for each cluster for distributing the milk products to the retailers. According to the demand rate of the three milk products [premium milk, Toned milk, Gold milk], it is sought to obtain the optimal influence area; A_i , optimal joint replenishment cycle time for product j; T_j , optimal investment in preservation technology for j milk product; τ_j, optimal number of proposed DCs (integer) in cluster i; m_i, optimal order quantity for product j; Q_{ij} for each DC of cluster i and optimal total supply chain network profit of milk products π_k for $i = 1, \ldots, 4$ and $j = 1,2,3$ and $k = I$ and II. Figure 3 depicts the schematic diagram of the operation of the supply chain of the milk product.

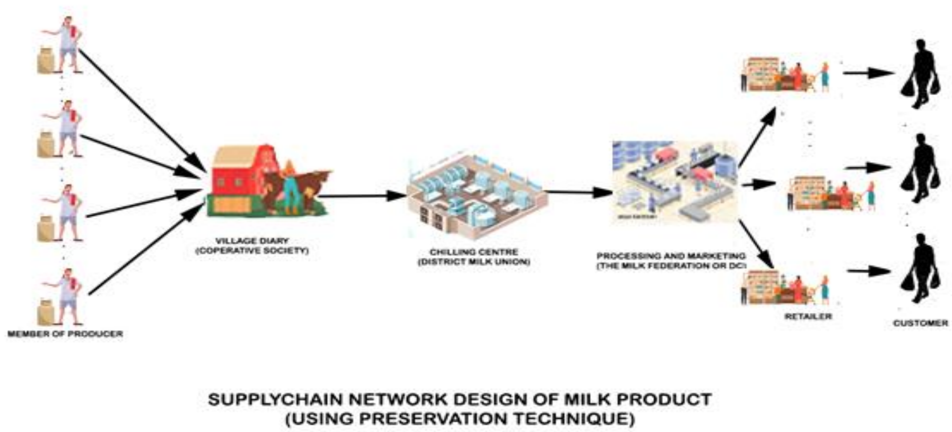


Figure 3: Schematic diagram of supply chain network design of milk product

5.2 OBJECTIVES OF THE CASE STUDY

The objectives are outlined in the previous section to conduct the case study in the context of a customized multi-milk product supply chain network design model in Coastal region of Odisha where, $\{cluster, i = 1,2,3,4 \ and \ product, j = 1,2,3\}$ for an in-depth mathematical analysis point of view.

5.3 CUSTOMIZED MATHEMATICAL MODEL FOR COASTAL REGION OF ODISHA

The total supply chain network profit per unit time for the two cases $k, k \in [I, II]$; π_k under preservation technology is given by:

$$\pi_k(A_1, A_2, A_{3,\ldots\ldots,} A_N, T_j, \tau_j) = Total\ Sales\ Revenue - Purchasing\ Cost - Inventory\ Holding\ Cost - Total\ Facility\ Cost - Total\ Inbound\ Transportation\ Cost - Total\ Outbound\ Transportation\ Cost - Total\ Cost\ of\ Preservation\ Effort - Total\ Ordering\ Cost \Rightarrow \pi_k = SR - PC - HC - FC - ITC - OTC - PEC - OC$$

$$\pi_k = \sum_{i=1}^{4}\sum_{j=1}^{3}\Bigg[\frac{p_j\mu\lambda_{ij}\delta_i C_i}{T_j} - \frac{c_j\mu\lambda_{ij}\delta_i C_i[e^{\beta e^{-\beta\tau_j}T_j} - 1]}{\beta e^{-\beta\tau_j}T_j} - \frac{h_j\mu\lambda_{ij}\delta_i C_i}{\beta e^{-2\beta\tau_j}T_j}\left[e^{\beta e^{-\beta\tau_j}T_j} - \beta e^{-\beta\tau_j}T_j - 1\right] - F\frac{C_i}{A_i} - \frac{[C_f + [C_{vj}\mu\lambda_{ij}\delta_i C_i\left[e^{\beta e^{-\beta\tau_j}T_j} - 1\right]]/\beta e^{-\beta\tau_j}]}{T_j} - C_r f_r\sqrt{A_i}\mu\lambda_{ij}\delta_i C_i - (a_1 + b\tau_j^{\,2})\mu\lambda_{ij}\delta_i C_i - \frac{R}{T_j}\frac{C_i}{A_i}\Bigg]$$

5.4 RESULTS AND ANALYSIS

The multi-milk product supply chain network profit model can be applied in solving the SCND of three milk products [Premium milk, Toned milk, Gold milk] of a milk federation of Coastal region of Odisha. Different cases like with preservation effort effects and without preservation effort effects are incorporated with the proposed model which is solved on a personal computer using Lingo 13.0 software package.

In this section, some managerial insights are identified by considering the following interesting issues:

- The annual supply chain network profit (attained from the case-I and -II) are compared and the appropriate value of influence area A_i for each DC in cluster i of Coastal region of Odisha, joint replenishment cycle time and the preservation effort T_j and τ_j of j milk product respectively and the number of proposed DCs (integer) in each cluster i for (case-II) are obtained.

- The impact of preservation effort on the total supply chain network profit per unit time is also studied.

Using the empirical data set (confidentiality) of Tables 5 and 6, the model is illustrated. Table 7 and Table 8 show the optimal values of the given models of case-I and -II respectively.

Table 5: Daily Demand Rate of Milk Products in of Coastal Odisha Distribution Centers

Cluster (i)	Distribution Center	C_i (Km^2)	Marketing Center	Daily Capacity ('000 litre)	λ_{ij}(Daily Demand Rate;'000 litre)		
					Premium Milk	Toned Milk	Gold Milk
1	Balasore	16557	Balasore Remuna Bhadrak	50	32.4	12.15	4.045
2	Tirtol	4403	Tirtol Jagatsinghpur Kendrapada	5	3.33	1.67	0
3	Berhampur	17712	Ganjam Berhampur Malkanagiri Gajapati	30	20.75	7.78	2.596
4	Bhubaneswar	6837	Khurda Puri Cuttack Bhubaneswar Jatni	200	116.18	43.57	14.526

Note: the data used here have been scaled to protect confidentiality

Table 6: Data set of the Milk Products in Coastal Odisha Distribution Centers

Parameters	Balasore [Premium milk, Toned milk, Gold milk]	Tirtol [Premium milk, Toned milk, Gold milk]	Berhampur [Premium milk, Toned milk, Gold milk]	Khurda [Premium milk, Toned milk, Gold milk]
C_i(Km^2)	16557	4403	17712	6837
λ_{ij} ('000 Litre)	[32.40, 12.15, 4.045]	[3.33, 1.67, 0.0]	[20.75, 7.78, 2.596]	[116.18, 43.57, 14.526]
μ (Days)	5	5	5	5
δ_i	0.02397	0.04747	0.02049	0.1916
F (Rs.)	6,000	6,000	6,000	6,000
p_j (Rs.)	[38, 36, 40]	[38, 36, 40]	[38, 36, 40]	[38, 36, 40]
c_j (Rs.)	[35.55, 26, 36.55]	[35.55, 26, 36.55]	[35.55, 26, 36.55]	[35.55, 26, 36.55]
h_j (Rs.)	[0.47, 0.47, 0.48]	[0.47,0.47,0.48]	[0.47,0.47,0.48]	[0.47,0.47,0.48]
R (Rs.)	30	30	30	30
C_r(Rs.)	0.75	0.75	0.75	0.75
f_r	0.01	0.01	0.01	0.01
C_f(Rs.)	17531	17531	17531	17531
C_{vj}(Rs.)	[0.83, 0.83, 0.83]	[0.83, 0.83, 0.83]	[0.83, 0.83, 0.83]	[0.83, 0.83, 0.83]
β	0.005	0.005	0.005	0.005
a_1	2	2	2	2
b	0.1	0.1	0.1	0.1

Note: the data used here have been scaled to protect confidentiality

Table 7: Numerical Results of the Milk Products [Premium milk, Toned milk, Gold milk] in Coastal Odisha Distribution Centers (Case-I)

Cluster (i)	Distribution Center	Iteration	C_i(Km^2)	A_i(Km^2)	m_i	Q_{ij} (Litre)	θ_j (Litre)	T_j(Days)	τ_j(Rs.)	PEC (Rs.)	π_I (Rs.)
1	Balasore	257 (0.38 sec)	16557	16557	1	[137363.4,66365.73,1130.386]	[0.0049 ,0.0049, 0.005]	[2.13, 2.73, 0.14]	[1.04, 1.24, 0.00]	2742460	1734815
2	Tirtol		4403	4403	1	[7435.126,4803.992,0.00]					
3	Berhampur		17712	17712	1	[80445.89,38860.38,663.3957]					
4	Khurda		6837	6837	1	[1625806,785536.8,13398.79]					

Table 8: Numerical Results of the Milk Products [Premium milk, Toned milk, Gold milk] in Coastal Odisha Distribution Centers (Case-II)

Cluster (i)	Distribution Center	Iteration	C_i(Km^2)	A_i(Km^2)	m_i	Q_{ij} (Litre)	θ_j (Litre)	T_j(Days)	τ_j(Rs.)	PEC (Rs.)	π_{II} (Rs.)
1	Balasore	1211 (1.31sec)	16557	4139.25	4	[137170.7,66436.21,1131.042]	[0.0049, 0.0049, 0.005]	[2.12, 2.74, 0.14]	[1.03, 1.24, 0.00]	2742250	2211758
2	Tirtol		4403	4403	1	[7424.697,4809.094,0.00]					
3	Berhampur		17712	5904	3	[80333.04,38901.65,663.781]					
4	Khurda		6837	455.8	15	[1623525,786371.1,13406.57]					

From Table 7 it is observed that, the optimum solutions of influence area of all the four clusters are equal to that of the service area. The optimum order quantity of the three products is maximum for the cluster Khurda [1625806, 785536.8, 13398.79] ltr. and minimum for the cluster Tirtol [7435.126, 4803.992, 0.00] ltr.. The optimum joint replenishment cycle time and preservation technology effort for the three products are [2.12, 2.74, 0.14] days and Rs. [1.03, 1.24, 0.00] respectively. The optimum total supply chain network profit per unit time of three milk products is Rs. 1734815 which is obtained after 257 iterations with total elapsed time 0.38 sec.

Similarly, from Table 8 it is observed that, the optimum solution of influence area of Balasore, Berhampur and Khurda are less than that of the service area whereas the influence area of Tirtol cluster is same with that of the service area. So, the optimum number of DCs in Balasore cluster is 4, in Berhampur cluster is 3, in Khurda cluster is 15 respectively and there is only one DC for Tirtol cluster of Coastal region of Odisha. There is only 4 no. of existing DCs are operating for distribution of milk and milk products in coastal region of Odisha but after getting the optimal solution, 19 no. of proposed DCs should be set up in 4 clusters, now it becomes 4+19=23 because of the demand rate of three milk products [Premium milk, Toned milk, Gold milk] in the particular cluster. The optimum order quantity of these three products is maximum for the cluster Khurda [1623525, 786371.1, 13406.57] ltr. and minimum for the cluster Tirtol [7424.694, 4809.094, 0.00] ltr. The optimum joint replenishment cycle time and preservation technology effort are [2.12, 2.74, 0.14] days and Rs. [1.03, 1.24, 0.00] respectively. The optimum total supply chain network profit per unit time of these three milk products is Rs. 2211758 which is obtained after 1211 iterations and with the elapsed time 1.31 sec. Hence, it is observed that total supply chain network profit per unit time of proposed model is Rs. 476943 more than that of existing model.

5.5 COMPARATIVE ANALYSIS

Table 9 presents the comparative analysis of the four different models of case-I and -II with respect to with and without implications of preservation technology for three milk products respectively.

Table 9: Comparative Analysis of Case-I and –II

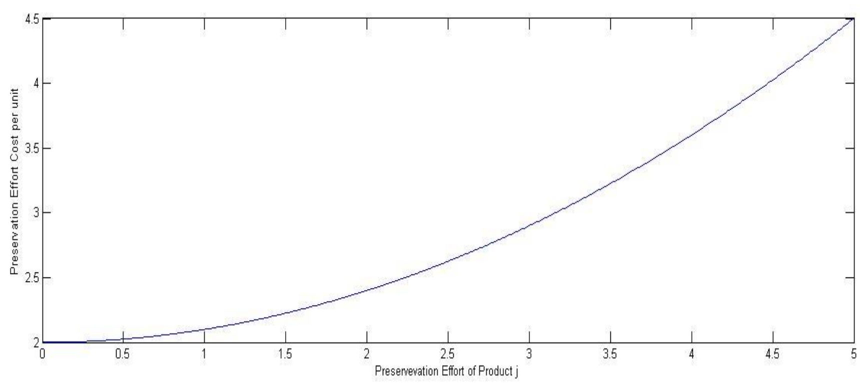
Cluster(i)	Distribution Center	Iteration	C_i(Km^2)	A_i(Km^2)	m_i	Q_{ij} (Litre)	θ_j (Litre)	T_j(Days)	τ_j(Rs.)	PEC (Rs.)	π_I (Rs.)
CASE-I ($C_i = A_i$)											
1	Balasore	257	16557	16557	1	[137363.4,66365.73,1130.386]	0.0049,	2.13,	1.04,	2742460	1734815
2	Tirtol	(0.38 sec)	4403	4403	1	[7435.126,4803.992,0.00]	0.0049,	2.73,	1.24,		
3	Berhampur		17712	17712	1	[80445.89,38860.38,663.3957]	0.005	0.14	0.00		
4	Khurda		6837	6837	1	[1625806,785536.8,13398.79]					
Cluster(i)	Distribution Center	Iteration	C_i(Km^2)	A_i(Km^2)	m_i	Q_{ij} (Litre)	θ_j (Litre)	T_j(Days)	τ_j(Rs.)	PEC (Rs.)	π_I (Rs.)
1	Balasore	173	16557	16557	1	[120009.4,72641.76,1217.204]	0.005,	1.86,	0.00,	2599234	1601005
2	Tirtol	(0.27 sec)	4403	4403	1	[6495.799,5258.293,0.00]	0.005,	2.99,	0.00,		
3	Berhampur		17712	17712	1	70282.65,42535.31,714.35]	0.005	0.15	0.00		
4	Khurda		6837	6837	1	1420407,859823.0,14427.87]					
CASE-II ($C_i \geq A_i$)											
Cluster(i)	Distribution Center	Iteration	C_i(Km^2)	A_i(Km^2)	m_i	Q_{ij} (Litre)	θ_j (Litre)	T_j(Days)	τ_j(Rs.)	PEC (Rs.)	π_{II} (Rs.)
1	Balasore	1211	16557	4139.25	4	[137170.7,66436.21,1131.042]	0.0049,	2.12,	1.03,1.24,	2742250	2211758
2	Tirtol	(1.31 sec)	4403	4403	1	[7424.697,4809.094,0.00]	0.0049,	2.74,	0.00		
3	Berhampur		17712	5904	3	[80333.04,38901.65,663.781]	0.005	0.14			
4	Khurda		6837	455.8	15	[1623525,786371.1,13406.57]					
Cluster(i)	Distribution Center	Iteration	C_i(Km^2)	A_i(Km^2)	m_i	Q_{ij} (Litre)	θ_j (Litre)	T_j(Days)	τ_j(Rs.)	PEC (Rs.)	π_{II} (Rs.)
1	Balasore	899	16557	4139	4	[119726.8,72743.49,1218.788]	0.005,	1.85,	0.00,	2599234	2078217
2	Tirtol	(0.80 sec)	4403	4403	1	[6480.51,5265.66,0.00]	0.005,	2.99,	0.00,		
3	Berhampur		17712	5904	3	[70117.19,42594.87,715.28]	0.005	0.15	0.00		
4	Khurda		6837	455	15	[1417063,861027.1,14446.65]					

Under case-I, there are two different models such as with and without preservation effort respectively. It is studied that, the total supply chain network profit per unit time with preservation effort is maximum of Rs. 1734815 where, it is minimum of Rs. 1601005 for the model of without preservation effort. It may be concluded that the preservation effort effects for selling the milk products enables to increase the total supply chain network profit. It results the preservation effort can be a mechanism for revenue expansion of the supply chain network design of three milk products.

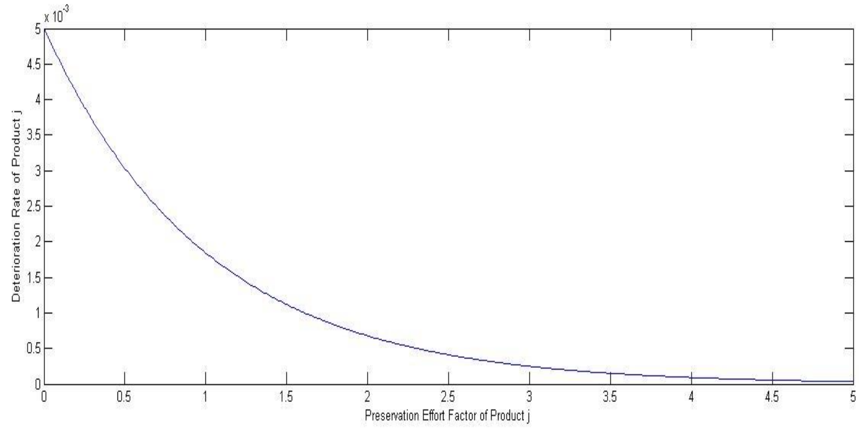
Under case-II, there are two different models such as with and without preservation effort respectively. It is studied that, the total supply chain network profit per unit time with preservation effort is maximum of Rs. 2211758 where, it is minimum of Rs. 2078217 for the model of without preservation effort. It may be concluded that the preservation effort effect for selling milk product enables to increase the total supply chain network profit per unit time under the condition of $C_i \geq A_i$. It is the best model because the number of DCs are also increased than one with the practical condition that the influence area must be less than or equal to the service area. It is also observed that, the optimum solution of influence area of Balasore, Berhampur and Khurda are less than that of the service area whereas the influence area of Tirtol cluster is same with that of the service area. So, the optimum number of DCs in Balasore cluster is 4, in Berhampur cluster is 3, in Khurda cluster is 15 respectively and there is only one DC for Tirtol cluster of Coastal region of Odisha. There is only 4 no. of existing DCs are operating for distribution of milk and milk products in coastal region of Odisha but after getting the optimal solution, 19 no. of proposed DCs should be set up in 4 clusters, now it becomes 4+19=23 because of the demand rate of three milk products [Premium milk, Toned milk, Gold milk] in the particular cluster.

For the model of case-II with preservation technology, the optimum number of distribution centre is 4, 1, 3 and 15 for the four clusters such as Balasore, Tirtol, Berhampur and Khurda respectively. In this proposed model the optimum order quantity of three milk products is maximum for the cluster Khurda and minimum for the cluster Tirtol. The optimum influence area of the four clusters [Balasore, Tirtol, Berhampur, Khurda] are [4139.25, 4403, 5904, 455.8] km^2 respectively. The optimum joint replenishment cycle time and preservation technology effort are [2.12, 2.74, 0.14] days and Rs. [1.03, 1.24, 0.00] respectively. The optimum total supply chain network profit per unit time of the three milk products is Rs. 2211758 which is obtained after 1211 iterations. It results the preservation effort can be a mechanism for revenue generation of the supply chain network design of three milk products in four clusters of coastal region of Odisha.

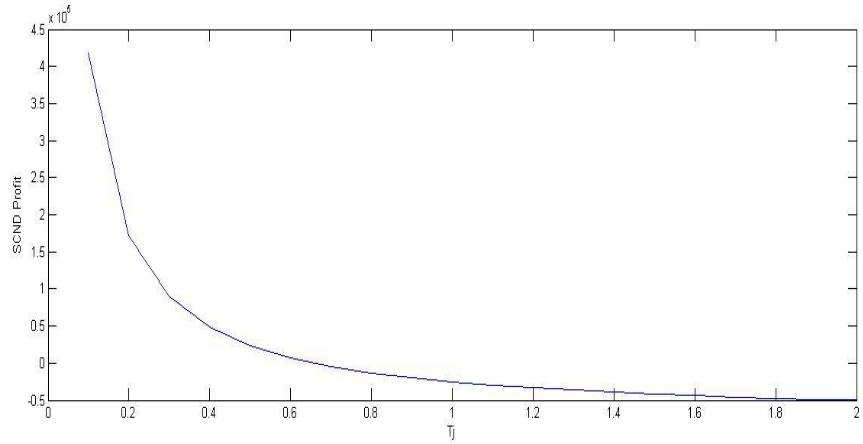
Figure 4 (i) shows the two dimensional plotting of preservation effort factor; τ_j and preservation effort cost per unit of product j. It indicates that the preservation effort cost is directly related to preservation effort factor. Figure 4 (ii) shows the two dimensional plotting of preservation effort factor; τ_j and deterioration rate of product j which shows deterioration rate is inversely related to preservation effort factor. Figure 4 (iii) shows the two dimensional plotting of optimal joint replenishment cycle time of product j; $T_j^*$and optimal total supply chain network profit per unit time shows it is inversely related to T_j^*. Figure 5 shows the plotting of existing distribution centre and marketing centres with respective influence area of 4 clusters of coastal region like Balasore, Tirtol, Berhampur and Khurda. Figure 6 depicts plotting of influence area of proposed (decision variable) distribution centres (23) like $\{C_1$: Balasore, Bhadrak, Remuna, Oupada$\}$, $\{C_2$: Tirtol$\}$, $\{C_3$: Berhampur, Gajapati, Malkanagiri$\}$, $\{C_4$: Khurda, Bhubaneswar, Cuttack, Puri, Bidanashi, Choudwar, Badamba, Bomikhal, Athagarh, Jatni, Konark, Pipli, Satasankha, Nimapara and Balugaon$\}$ and retailers of Coastal region of Odisha.



(i)



(ii)



(iii)

Figure 4: Two Dimensional Plotting of: (i) Preservation Effort Cost of Product j with Preservation Effort Factor of Product j, (ii) Deterioration Rate of Product j with Preservation Effort Factor of Product j and (iii) Total Supply Chain Network Profit with Joint Replenishment Cycle Time of Product j respectively

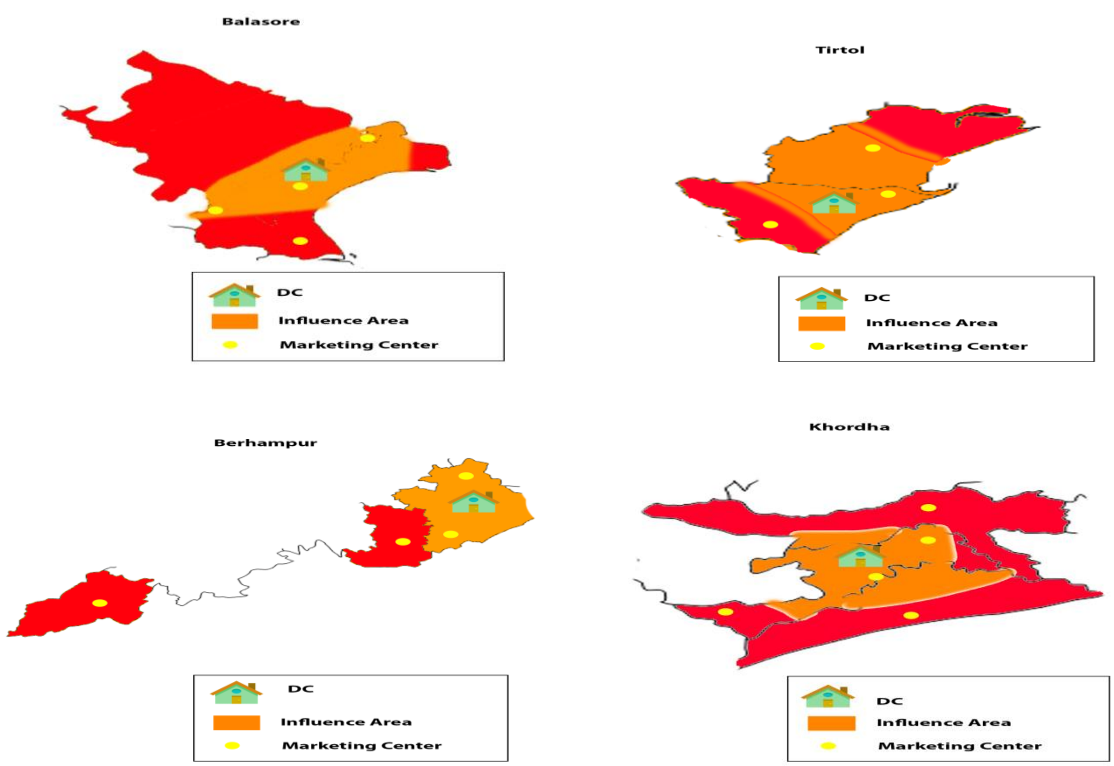


Figure 5: Plotting of Existing Distribution Centre and Marketing Centres with respective Influence Area of 4 Clusters of Coastal Region i.e. Balasore; Tirtol; Berhampur and Khurda

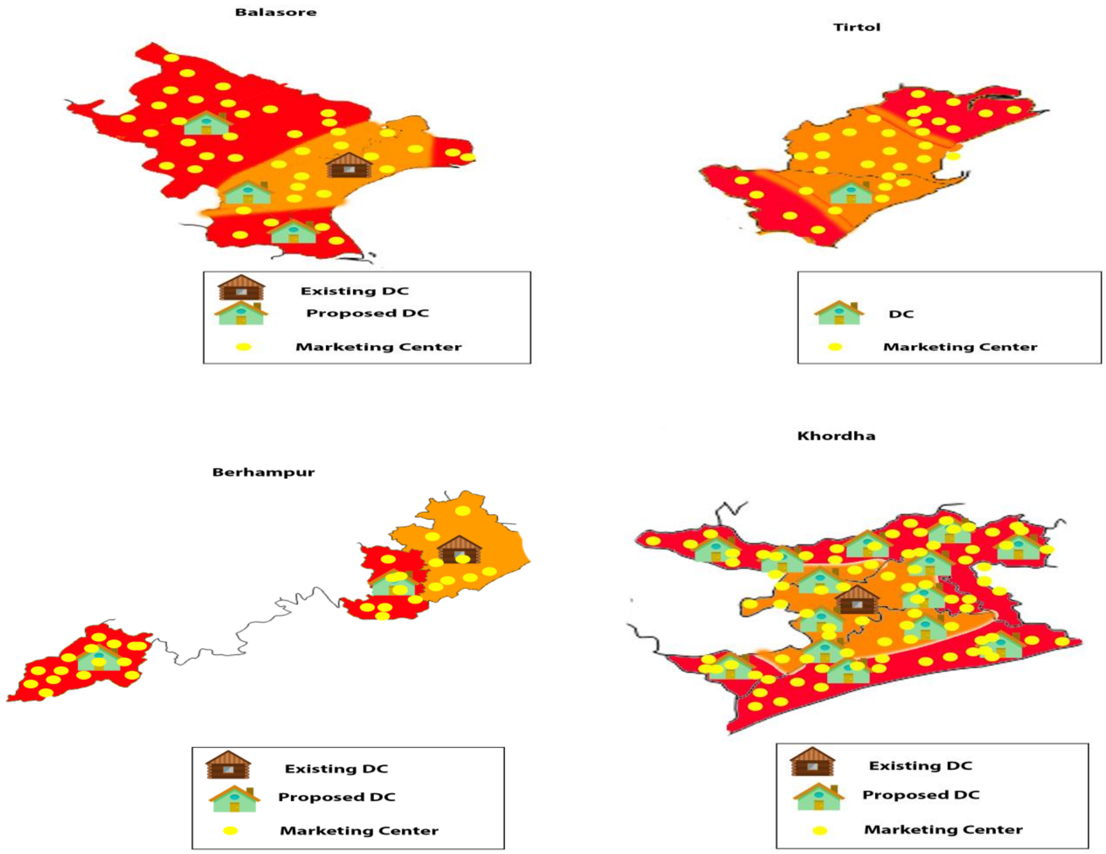


Figure 6: Plotting of Influence Area of Proposed (Decision variable) Distribution Centres (23) and Retailers of Coastal Region of Odisha: { C_1: Balasore, Bhadrak, Remuna, Oupada}, {C_2: Tirtol}, {C_3: Berhampur, Gajapati, Malkanagiri}, {C_4: Khurda, Bhubaneswar, Cuttack, Puri, Bidanashi, Choudwar, Badamba, Bomikhal, Athagarh, Jatni, Konark, Pipli, Satasankha, Nimapara and Balugaon}

5.6 SENSITIVITY ANALYSIS

It is interesting to investigate the influence of the model parameters, $\mu, F, h_j, R, C_r, f_r, C_f, C_{vj}, \beta, a_1 and\ b$ on the decision variables of SCND model. The computational results of Table A-1 and A-2 under case-I and -II models are summarized in Table 10 and 11 respectively. Critical sensitivity analysis of preservation effort parameters on the decision variables of SCND model under case-II are shown in Table A-3.

Table 10: Sensitivity Analysis Report of Table A-1 ($Case - I; C_i = A_i$)

Parameter	A_1^*	A_2^*	A_3^*	A_4^*	T_j^*	τ_j^*	PEC^*	m_i^*	π_1^*
μ	IS	IS	IS	IS	HS	HS	HS	IS	IS
F	IS	IS	IS	IS	IS	IS	IS	IS	MS
h_j	IS	IS	IS	IS	IS	IS	IS	IS	IS
R	IS	IS	IS	IS	IS	IS	MS	IS	MS
C_r	IS	IS	IS	IS	IS	IS	IS	IS	HS
f_r	IS	IS	IS	IS	IS	IS	IS	IS	HS
C_f	IS	IS	IS	IS	HS	HS	MS	IS	HS
C_{vj}	IS	IS	IS	IS	IS	IS	MS	IS	HS
β	IS	IS	IS	IS	HS	HS	MS	IS	MS
a_1	IS	IS	IS	IS	IS	IS	HS	IS	HS
b	IS	IS	IS	IS	HS	HS	MS	IS	S

Note: S-Sensitive, MS-Moderately Sensitive, HS-Highly Sensitive, IS-Insensitive

Table 11: Sensitivity Analysis Report of Table A-2 ($Case - II, C_i \geq A_i$)

Parameter	A_1^*	A_2^*	A_3^*	A_4^*	T_j^*	τ_j^*	PEC^*	m_i^*	π_1^*
μ	HS	HS	HS	HS	HS	HS	HS	HS	HS
F	HS	HS	HS	HS	IS	IS	MS	HS	MS
h_j	IS	IS	IS	IS	IS	IS	IS	IS	IS
R	IS	IS	IS	IS	IS	IS	MS	IS	MS
C_r	HS	HS	HS	HS	IS	IS	MS	HS	MS
f_r	HS	HS	HS	HS	IS	IS	MS	HS	MS
C_f	IS	IS	IS	IS	HS	HS	MS	IS	MS
C_{vj}	IS	IS	IS	IS	IS	IS	MS	IS	HS
β	IS	IS	IS	IS	HS	HS	HS	IS	MS
a_1	IS	IS	IS	IS	IS	IS	HS	IS	HS
b	IS	IS	IS	IS	HS	HS	JS	IS	MS

Note: S-Sensitive, MS-Moderately Sensitive, HS-Highly Sensitive, IS-Insensitive

It is investigated from Tables A-1 and 10 that the optimal influence area A_i^* and the optimal number of distribution centres in cluster i m_i^* are insensitive but the optimal influence area A_i^*, the optimal joint replenishment cycle time of product j $T_j^*$and the optimal preservation effort of product j τ_j^* are highly sensitive to the parameters $F, h_j, R, C_r, f_r C_{vj}$ and a_1 and are insensitive for the rest model parameters respectively. The optimal total supply chain network profit π_{II}^* is highly sensitive to the parameters $C_r, f_r C_f, C_{vj} and$ $a_1$but is moderately sensitive and insensitive to the rest of the model parameters.

The results of Table A-2 and 11 are as follows:

When the planning horizon μ increases, the optimal influence area A_i^*, the optimal joint replenishment cycle time of product j T_j^*, the optimal preservation effort of product j τ_j^*, the optimal number of distribution centres in cluster i m_i^* and the optimal total supply chain network profit π_{II}^* increase. This verifies Property 1 (a). When the planning horizon increases, it is reasonable to reduce the influence area for each cluster i.

When the facility cost F increases, the optimal influence area A_i^* increases, the optimal joint replenishment cycle time of product T_j^*, the optimal preservation effort of product j τ_j^*, are insensitive but the optimal number of distribution centres in cluster i m_i^* and the optimal total supply chain network profit π_{II}^* decrease with slow pace. This verifies Property 1 (b). When the facility cost increases, it is practical to expand the influence area for each cluster i.

When the inventory holding cost per unit of product j h_j increases, the optimal influence area A_i^*, the optimal joint replenishment cycle time of product j T_j^*, the optimal preservation effort of product j τ_j^*, the optimal number of distribution centres in cluster i m_i^* and the optimal total supply chain network profit π_{II}^* are all insensitive.

When the ordering cost per order R increases, the optimal influence area A_i^*, the optimal influence area A_i^*, the optimal joint replenishment cycle time of product j T_j^*, the optimal preservation effort of product j τ_j^*, the optimal number of distribution centres in cluster i m_i^* are all insensitive but the optimal total supply chain network profit π_{II}^* decrease with low pace. When the ordering cost per order R increases, it is practical that the optimal total supply chain network profit π_{II}^* will be reduced.

When the outbound transportation cost per unit C_r increases, the optimal influence area A_i^* decrease, the optimal joint replenishment cycle time of product j T_j^*, the optimal preservation effort of product j τ_j^* are insensitive, the optimal number of distribution centres in cluster i m_i^* increase but the optimal total supply chain network profit π_{II}^* decrease with low pace. This verifies Property 1 (a). When the outbound transportation cost per unit C_r increases, it is practical that the optimal influence area A_i^* and the total supply chain network profit π_{II}^* will be reduced.

When the constant of service region f_r increases, the optimal influence area A_i^* decrease, the optimal joint replenishment cycle time of product j T_j^*, the optimal preservation effort of product j $\tau_j^*$are insensitive, the optimal number of distribution centres in cluster i m_i^* increase but the optimal total supply chain network profit π_{II}^* decrease with low pace. This verifies Property 1 (a). When the constant of service region f_r increases, it is practical that the optimal influence area A_i^* and the total supply chain network profit π_{II}^* will be reduced.

When the fixed cost for inbound shipment C_f increases, the optimal influence area A_i^* and the optimal number of distribution centres in cluster i m_i^* are insensitive but the optimal joint replenishment cycle time of product j T_j^*, the optimal preservation effort of product j τ_j^*, and the optimal total supply chain network profit π_{II}^* decrease. When the fixed cost for inbound shipment C_f increases, it is logical that the total supply chain network profit π_{II}^* will be reduced.

When the variable cost of product j for inbound shipment C_{vj} increases, the optimal influence area A_i^*, the optimal joint replenishment cycle time of product j T_j^*, the optimal preservation effort of product j τ_j^*, and the optimal number of distribution centres in cluster i m_i^* are all insensitive but the optimal total supply chain network profit π_{II}^* decrease. When the variable cost of product j C_{vj} increases, it is practical that the total supply chain network profit π_{II}^* will be reduced.

When the deterioration input parameter β increases, the optimal influence area A_i^* and the optimal number of distribution centres in cluster i m_i^* are insensitive, and the optimal joint replenishment cycle time of product j $T_j^*$are all insensitive but the optimal preservation effort of product j τ_j^*, and the optimal total supply chain network profit π_{II}^* increase slowly. This verifies Property 2 (a). When the deterioration input parameter β increases, it is reasonable to increase the preservation effort for product j.

From Table A-3 it shows that when the fixed preservation technology effort a_1 increases, the optimal influence area $\forall i = 1,2,3,4$ A_i^* for each cluster, the optimal joint replenishment cycle time of product $j, \forall j = 1,2,3$ T_j^*, the optimal preservation effort of product j τ_j^* and the optimal number of distribution centres in cluster i, m_i^* are insensitive are all insensitive but the optimal total supply chain network profit π_{II}^* decreases. When the increment of preservation effort factor b increases the optimal influence area $\forall i = 1,2,3,4$ A_i^* for each cluster and the optimal number of distribution centres in cluster i, m_i^* are insensitive but the optimal joint replenishment cycle time of product, $\forall j = 1,2,3$ T_j^*, the optimal preservation effort of product j τ_j^* and the optimal total supply chain network profit π_{II}^* decrease.

Figures 7 and 8 show the impact of planning horizon; μ, outbound transportation cost; C_r, facility cost; F, and ordering cost; R on optimal influence area A_i^* of cluster i and the impact of preservation model parameter; b, deterioration model parameter; β on optimal preservation effort τ_j^* of product j respectively. It is observed that when the planning horizon μ and outbound transportation cost; C_r increase the optimal influence area A_i^* of cluster i decreases. It validates the Property 1 (a) whereas, when the facility cost; F and ordering cost; R increase the optimal influence area A_i^* of cluster i also increases. It validates the Property 1 (b). It is observed from that when the, deterioration model parameter; β increases the optimal preservation effort τ_j^* of product j increases but when the preservation input parameter b increases the optimal preservation effort τ_j^* of product j decreases. It validates the Property 2 (a) and (b) respectively.

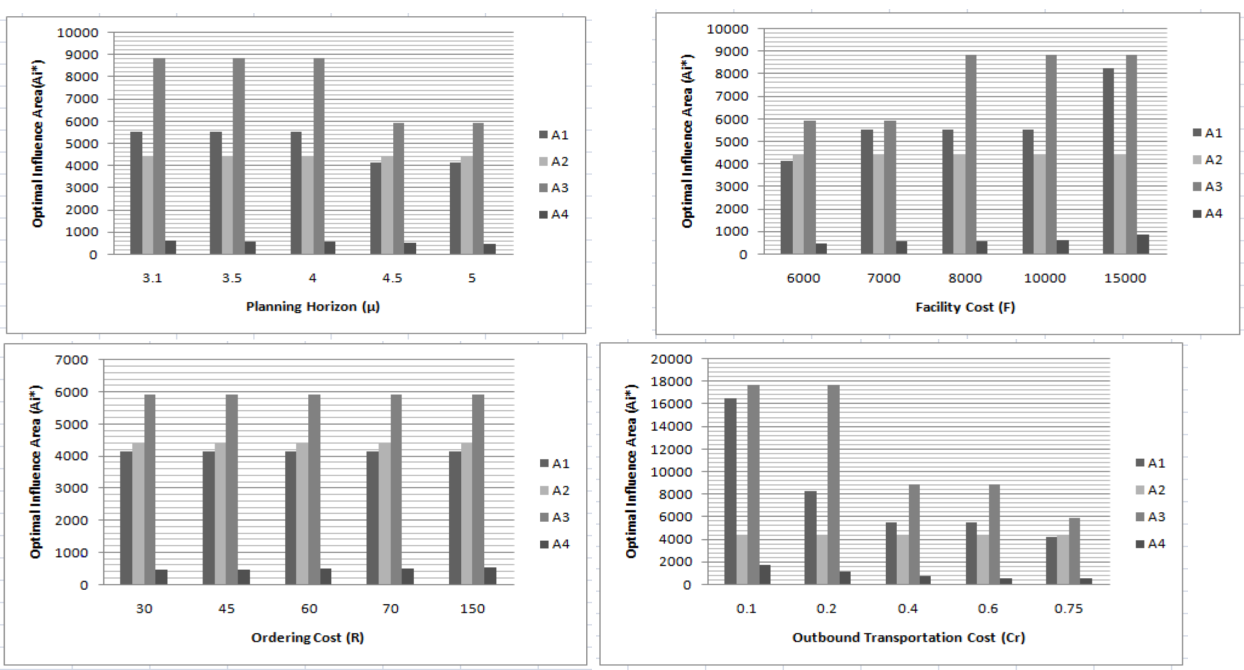


Figure 7: Impact of Planning Horizon;μ, Facility Cost;F, and Ordering Cost;R and Outbound Transportation Cost;C_r on Optimal Influence Area A_i^* of cluster i {Property 1: (a) & (b)}

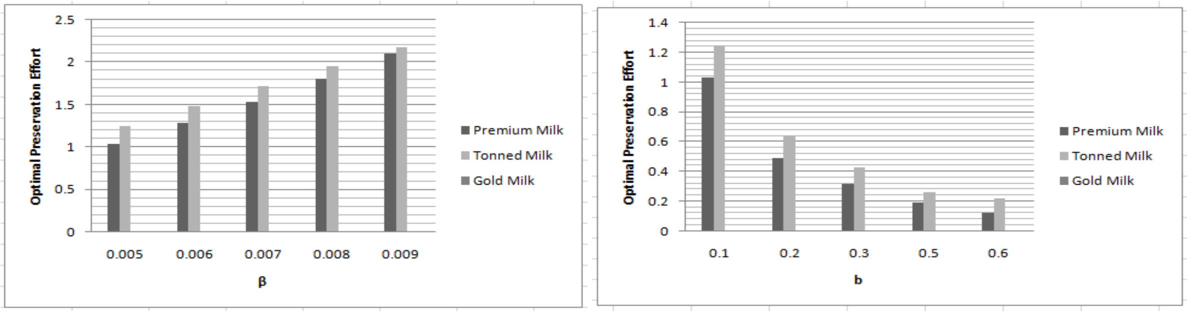


Figure 8: Impact of Deterioration Input Parameter;β and Preservation Input Parameter; b on Optimal Preservation Effort τ_j^* of Product j {Property 2: (a) & (b)}

5. CONCLUSIONS

This paper investigated supply chain network design problems especially location allocation of multi-milk product (Premium milk, Toned milk, Gold milk) in Coastal region of Odisha of leading milk producing organization of Odisha considering the inventory deterioration and preservation effort. Preservation technology cost is a function of preservation effort. Deterioration rate is the function of preservation effort factor as when preservation effort increases, the deterioration rate of the milk product decreases exponentially. The key outcomes enforce in determining the optimal influence area of DC; joint replenishment cycle time for three milk products, preservation effort and optimal proposed number of distribution centres to maximize the total SCND profit per unit of time. In addition to, order quantity of each milk product for each DC is also evaluated for Coastal Odisha. An algorithm based on piecewise nonlinear optimization is provided to solve supply chain network design problems efficiently.

In the proposed SCND model in Coastal Odisha there are 4 no. of existing DCs are in 4 clusters such as Balasore, Tirtol, Berhampur and Khurda but, it is not sufficient according to the demand rate and respective clusters. From the results it is found that, the total supply chain network profit per unit time with preservation effort is of Rs. 2211758 after 1211 iterations. It may be concluded that the preservation effort effect for selling milk product enables to increase the total supply chain network profit under the condition of $C_i \geq A_i$ as the total supply chain network profit per unit time under the condition $C_i = A_i$ is of Rs. 1734815. So, case-II is the best model because the number of DCs are also increased than one with the practical condition that the influence area must be less than or equal to the service area. It is also observed that, the optimum solution of influence area of Balasore, Berhampur and Khurda are less than that of the service area whereas the influence area of Tirtol cluster is same with that of the service area. So, the optimum number of DCs in Balasore cluster is 4, in Berhampur cluster is 3, in Khurda cluster is 15 respectively and there is only one DC for Tirtol cluster of coastal region of Odisha. There is only 4 no. of existing DCs but after getting the optimal solution, 19 no. of proposed DCs are added with 4 no. of existing DCs, now it becomes 19+4=23 because of the demand rate of the three milk products in the particular cluster. The optimum order quantity of premium milk is maximum for the cluster Khurda and minimum for the cluster Tirtol. The optimum replenishment cycle time and preservation technology investment are {2.12, 2.74, 0.14} days and Rs. {1.03, 1.24, 0.00} respectively. It results the preservation effort can be a catalyst for revenue growth of the supply chain network design of the multi-milk product [Premium milk, Toned milk, Gold milk] in Coastal region of Odisha.

Through sensitivity analysis, it is studied that maximum decision variables are insensitive only for the deterioration model parameters are responsible for sensitivity of the decision variables and total SCN profit per unit time. Hence, the proposed model provides an influential tool for analyzing potential changes in model parameters for strategic decision-making purpose in a supply chain system. Theoretically, a generic model is developed and for justifying purpose a customized model is framed for a milk industry.

Further research on this study may consider when the deterioration rate of the product is depending upon only time and also upon both preservation effort and time under stochastic demand of perishable item. It may extended in future by adding other products with more issues like minimizing freight cost, discount offering, shortages, unit lost sales, stochastic deterioration, partial backlogging etc.

APPENDIX

A-1: Sensitivity Analysis (Case-I)

Parameter	A_1^*	A_2^*	A_3^*	A_4^*	T_j^*	τ_j^*	PEC^*	π_1^*
$\mu = 2.5$	-	-	-	-	-	-	-	-
$\mu = 3.5$	16557	4403	17712	6837	1.35,1.93,0.22	.50,.93,0	1854699	881479.6
$\mu = 6.5$	16557	4403	17712	6837	2.75,3.63,.12	1.25,1.43,0	3642056	2611257
$\mu = 7.5$	16557	4403	17712	6837	3.10,4.28,.11	1.34,1.51,0	4243523	3190294
$F = 3000$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1746815
$F = 4200$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1742015
$F = 7800$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1727615
$F = 9000$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1722815
$h_j = .235, .235, .24$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1734815
$h_j = .329, .329, .336$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1734815
$h_j = .611, .611, .624$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1734815
$h_j = .705, .705, .72$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1734815
$R = 15$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742482	1735291
$R = 21$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742474	1735101
$R = 39$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742447	1734529
$R = 45$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742438	1734339
$C_r = .375$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	2164776
$C_r = .525$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1992792
$C_r = .975$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1476838
$C_r = 1.125$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1304854
$f_r = .005$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	2164776
$f_r = .007$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1992792
$f_r = .013$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1476838
$f_r = .015$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742460	1304854
$C_f = 8765.5$	16557	4403	17712	6837	2.28,2.59,.14	1.11,1.21,0	2753181	2016613
$C_f = 12271.7$	16557	4403	17712	6837	2.22,2.64,.14	1.08,1.22,0	2749337	1903163
$C_f = 22790.3$	16557	4403	17712	6837	2.00,2.86,.14	0.97,1.27,0	2733476	1569499
$C_f = 26296.5$	16557	4403	17712	6837	1.89,2.96,.15	0.91,1.29,0	2725460	1461765
$C_{vj} = .415, .415.415$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742576	2277001
$C_{vj} = .581, .581, .581$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742530	2060126
$C_{vj} = 1.079{,}1.079{,}1.079$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742391	1409504
$C_{vj} = 1.245{,}1.245{,}1.245$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	2742345	1192631
$\beta = 0.0025$	16557	4403	17712	6837	2.06,2.80,.14	0.50,0.62,0	2633341	1743051
$\beta = 0.0035$	16557	4403	17712	6837	2.07,2.78,.14	0.70,0.87,0	2666922	1731007
$\beta = 0.0065$	16557	4403	17712	6837	2.21,2.65,.14	1.40,1.60,0	2853683	1767276
$\beta = 0.0075$	16557	4403	17712	6837	2.27,2.59,.14	1.67,1.84,0	2950905	1806563
$a_1 = 1$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	1442843	3034432
$a_1 = 1.4$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	1962690	2514585
$a_1 = 2.6$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	3522231	955044.7
$a_1 = 3$	16557	4403	17712	6837	2.13,2.73,.14	1.04,1.24,0	4042077	435197.9
$b = 0.05$	16557	4403	17712	6837	2.32,2.54..13	2.26,2.40,0	2913957	1885485
$b = 0.07$	16557	4403	17712	6837	2.22,2.64,.14	1.54,1.75,0	2813299	1797651
$b = 0.13$	16557	4403	17712	6837	2.07,2.79,.14	0.78,0.96,0	2706327	1702218
$b = 0.15$	16557	4403	17712	6837	2.04,2.81,.14	0.66,0.84,0	2690787	1688036

A-2: Sensitivity Analysis (Case-II)

Parameter	A_1^*	A_2^*	A_3^*	A_4^*	T_j^*	τ_j^*	PEC^*	m_i^*	π_1^*
$\mu = 2.5$	-	-	-	-	-	-	-	-	-
$\mu = 3.5$	5519	4403	8856	569	1.35,1.93,.22	0.49,0.94,0	1854512	[3,1,2,12]	1186637
$\mu = 6.5$	3311.4	4403	5904	379.83	2.75,3.63,.12	1.25,1.43,0	3641817	[5,1,3,18]	3268328
$\mu = 7.5$	3311.4	4403	4428	359.84	3.10,4.28,.11	1.34,1.51,0	4243259	[5,1,4,19]	3971026
$F = 3000$	2759.5	4403	4428	297.26	2.12,2.74,.14	1.03,1.24,0	2742127	[6,1,4,23]	2293210
$F = 4200$	3311.4	4403	5904	359.84	2.12,2.74,.14	1.03,1.24,0	2742194	[5,1,3,19]	2256255
$F = 7800$	5519	4403	8856	569	2.12,2.74,.14	1.03,1.24,0	2742305	[3,1,2,12]	2174461
$F = 9000$	5519	4403	8856	621	2.12,2.74,.14	1.03,1.24,0	2742317	[3,1,2,11]	2153013
$h_j = .235, .235, .24$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742250	[4,1,3,15]	2211758
$h_j = .329, .329, .336$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742250	[4,1,3,15]	2211758
$h_j = .611, .611, .624$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742250	[4,1,3,15]	2211758
$h_j = .705, .705, .72$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742250	[4,1,3,15]	2211758
$R = 15$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742378	[4,1,3,15]	2214496
$R = 21$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742327	[4,1,3,15]	2213401
$R = 39$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742173	[4,1,3,15]	2210115
$R = 45$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742122	[4,1,3,15]	2209020
$C_r = .375$	8278.5	4403	8856	759.67	2.12,2.74,.14	1.03,1.24,0	2742350	[2,1,2,9]	2360241
$C_r = .525$	5519	4403	8856	569.75	2.12,2.74,.14	1.03,1.24,0	2742305	[3,1,2,12]	2297024
$C_r = .975$	3311.4	4403	5904	379.83	2.12,2.74,.14	1.03,1.24,0	2742205	[5,1,3,18]	2134558
$C_r = 1.125$	3311	4403	4428	359.84	2.12,2.74,.14	1.03,1.24,0	2742183	[5,1,4,19]	2086575
$f_r = .005$	8278.5	4403	8856	759.67	2.12,2.74,.14	1.03,1.24,0	2742350	[2,1,2,9]	2360241
$f_r = .007$	5519.3	4403	8856	569.75	2.12,2.74,.14	1.03,1.24,0	2742305	[3,1,2,12]	2297024
$f_r = .013$	3311.4	4403	5904	379.83	2.12,2.74,.14	1.03,1.24,0	2742205	[5,1,3,18]	2134558
$f_r = .015$	3311.4	4403	4428	359.84	2.12,2.74,.14	1.03,1.24,0	2742183	[5,1,4,19]	2086575
$C_f = 8765.5$	4139.25	4403	5904	455.8	2.27,2.59,.14	1.11,1.21,0	2753033	[4,1,3,15]	2493454
$C_f = 12271.7$	4139.25	4403	5904	455.8	2.22,2.64,.14	1.08,1.22,0	2749170	[4,1,3,15]	2380038
$C_f = 22790.3$	4139.25	4403	5904	455.8	1.10,2.86,.14	0.97,1.27,0	2733190	[4,1,3,15]	2046543
$C_f = 26296.5$	-	-	-	-	-	-	-	-	-
$C_{vj} = .415, .415.415$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742366	[4,1,3,15]	2753942
$C_{vj} = .581, .581, .581$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742320	[4,1,3,15]	2537068
$C_{vj} = 1.079, 1.079, 1.079$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742180	[4,1,3,15]	1886448
$C_{vj} = 1.245, 1.245, 1.245$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	2742134	[4,1,3,15]	1669575
$\beta = 0.0025$	-	-	-	-	-	-	-	-	-
$\beta = 0.0035$	4139.25	4403	5904	455.8	2.07,2.79,.14	0.70,0.87,0	2666803	[4,1,3,15]	2208010
$\beta = 0.0065$	4139.25	4403	5904	455.8	2.20,2.66,.14	1.40,1.60,0	2853389	[4,1,3,15]	2244133
$\beta = 0.0075$	4139.25	4403	5904	455.8	2.27,2.60,.14	1.67,1.84,0	2950564	[4,1,3,15]	2283354
$a_1 = 1$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	1442633	[4,1,3,15]	3511375
$a_1 = 1.4$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	1962480	[4,1,3,15]	2991528
$a_1 = 2.6$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	3522020	[4,1,3,15]	1431988
$a_1 = 3$	4139.25	4403	5904	455.8	2.12,2.74,.14	1.03,1.24,0	4041867	[4,1,3,15]	912140.8
$b = 0.05$	4139.25	4403	5904	455.8	2.32,2.54,.13	2.26,2.40,0	2913673	[4,1,3,15]	2362256
$b = 0.07$	4139.25	4403	5904	455.8	2.22,2.65,.14	1.54,1.75,0	2813049	[4,1,3,15]	2274512
$b = 0.13$	4139.25	4403	5904	455.8	2.07,2.79,.14	0.77,0.96,0	2706147	[4,1,3,15]	2179213
$b = 0.15$	-	-	-	-	-	-	-	-	-

A-3: Sensitivity Analysis: A Critical Study

Parameter	Optimal Solution	$b = 0.05$	$b = 0.1$	$b = 0.2$
$a_1 = 1$	A_1^*	4139.25	4139.25	-
	A_1^*	4403	4403	-
	A_1^*	5904	5904	-
	A_1^*	455.8	455.8	-
	m_i^*	[4,1,3,15]	[4,1,3,15]	-
	T_j^*	2.32,2.54,0.13	2.12,2.74,0.14	-
	τ_j^*	2.56,2.41,0.00	1.03,1.24,0.00	-
	PEC^*	1614056	1442633	-
	π_{II}^*	3661873	3511375	-
$a_1 = 2$	A_1^*	4139.25	4139.25	4139.25
	A_1^*	4403	4403	4403
	A_1^*	5904	5904	5904
	A_1^*	455.8	455.8	455.8
	m_i^*	[4,1,3,15]	[4,1,3,15]	[4,1,3,15]
	T_j^*	2.32,2.54,0.13	2.12,2.74,0.14	1.10,2.86,0.15
	τ_j^*	2.26,2.40,0.00	1.03,1.24,0.00	0.49,0.63,0.00
	PEC^*	2913673	2742250	2666151
	π_{II}^*	2362256	2211758	2142476
$a_1 = 3$	A_1^*	4139.25	4139.25	4139.25
	A_1^*	4403	4403	4403
	A_1^*	5904	5904	5904
	A_1^*	455.8	455.8	455.8
	m_i^*	[4,1,3,15]	[4,1,3,15]	[4,1,3,15]
	T_j^*	2.32,2.54,0.13	2.12,2.740.14	1.10,2.86,0.15
	τ_j^*	2.26,2.40,0.00	1.03,1.24,0.00	0.49,0.63,0.00
	PEC^*	4213290	4041867	3965768
	π_{II}^*	1062639	912140.8	842858.5

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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