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SIGNATURE OF INTERMITTENCY DURING EMISSION OF TARGET ASSOCIATED PARTICLES IN HEAVY ION COLLISIONS AT SPS ENERGIES

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Abstract: A study of intermittent type of fluctuations of target fragments produced in the interactions of ^{32}S-AgBr at 200 AGeV using the method of scaled factorial moments, F_q has been performed. An intermittent behaviour is observed for fast and slow target fragments for the experimental data in terms of new scaled variable $X(\text{Cos}\theta)$ suggested by Bialas and Gazdzicki. The variations of the anomalous fractal dimensions, d_q, and the generalized dimensions, D_q, with the order of the moments, q, are investigated with the help of F_q, moments. The anomalous dimension, d_q increases linearly with the order of moments, q, suggesting the multifractality with the production mechanism of target associated fragments.

Keywords: nucleus-nucleus collisions; intermittency; scaled factorial moments; anomalous dimensions; multifractals.

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1. INTRODUCTION

A general characteristic of spatial distribution of secondary charged particles produced in relativistic nucleus-nucleus collisions is that it exhibits fluctuations and has recently attracted a great deal of attention due to the possibility of extracting important information about the mechanism of multiparticle production in such collisions. Fluctuations in the multi-particle final state of heavy ion collisions may broadly be divided into two categories: statistical and non-statistical fluctuations or dynamical fluctuations. The statistical fluctuation arises due to finite particle multiplicity in the final state of nuclear collisions and dominates over the non-statistical fluctuation where the event multiplicity is low. The dynamical fluctuations on the other hand arises due to the underlying physical processes taking place in high energy nucleus-nucleus collisions and contributes significantly where the event multiplicity is relatively large.

Bialas and Peschanski [1, 2] are the pioneers who introduced the scaled factorial moments (SFMs) as a diagnostic tool for extracting dynamical fluctuations in particle distribution data of different types in high energy interactions. A power law growth of SFMs with decreasing phase space interval size indicates an intermittent type of fluctuation over a range of resolution scales. This intermittent type of non-statistical fluctuation was initially thought to be the result of the transition from the quark–gluon plasma to normal hadronic matter, and interest was centred on self-similarity studies. Various experimental data sets [3–8 and reference therein] are available in support of this intermittent behaviour. One of the possible characteristics of the scaled factorial moment analysis is that it can detect and characterize the dynamical fluctuation and it is also capable of filtering out the statistical noise. In this method, the scaled factorial moments, F_q, are computed as a function of decreasing phase space size. The values of F_q, for purely statistical fluctuation saturate with decreasing phase space size, whereas in dynamical fluctuation, F_q, moments are supposed to increase with decreasing phase-space size and exhibit power law behaviour of normalized factorial moments, F_q. However, ordinary multiplicity moments ($\langle n^q\rangle/\langle n\rangle^q$) method is used to demonstrate different features of multiplicity distributions and is unable to revel the existence of dynamical fluctuation due to significant contribution of the purely statistical fluctuations.

It is interesting to note that most of the investigations on dynamical fluctuations in high energy nuclear collisions have been performed on the studies of pions and little attention has been paid to the analysis of the target fragments. The first step in this direction was taken by Ploszajczak and Tucholski [9]. They investigated intermittent behaviour in nuclear fragmentation at intermediate energies by studying the bin size dependence of normalized factorial moments of the target fragments. It would be interesting to explore if the target evaporated particles also follow a multifractal structure. This will not only provide a unified description of the whole production process but also provide an additional parameter to understand the dynamics of particle production process. Also, the target fragmentation process may also carry important information of the forces and dynamics during and after nuclear impact. In our earlier papers [10, 11] we performed one dimensional intermittency of pions in ^{32}S-Em interactions at 200AGeV in pseudo rapidity (η) phase-space. In this paper, we report an investigation on the nature of dynamical fluctuations carried out in the target fragmentation of ^{32}S-AgBr interactions at 200 AGeV in the frame work of spatial phase space, (Cosθ).

2. EXPERIMENTAL DETAILS

In this experiment two stacks of Ilford G5 nuclear emulsion plates exposed horizontally to a ^{32}S-beam at 200 AGeV from Supper Proton Synchrotron, SPS at CERN have been utilized for data collection. The scanning of the plates is performed with the help of Leica DM2500M microscope with a 10X objective and 10X ocular lens provided with semi-automatic scanning stages. The method of line scanning was used to collect the inelastic ^{32}S-Em interactions. The interactions collected from line scanning were scrutinized under an optical microscope (Semi-Automatic Computerized, Leica DM6000M) with a total magnification of 10×100 using 10X eyepiece and 100X oil immersion objective. The measuring system associated with it has 1μm resolution along X and Y axes and 0.5 μm resolution along the Z-axis.

The tracks associated with the interactions are classified in accordance with their ionization, range and velocity [12, 13]. The tracks having specific ionization $g^*(= g/g_o) < 1.4$ and relative velocity $\beta > 0.7$ are taken as shower tracks, where g_o is the Fowler and Perkins parameter for plateau ionization of relativistic particles. The number of such tracks in an event is represented by 'N_S'. Shower tracks producing particles are mostly pions, with small admixture of charged K-mesons and fast protons. The secondary tracks having specific ionization in the interval $1.4 < g^* \leq 10$ are known as grey tracks. The numbers of such tracks in a star are designated by 'N_g'. This corresponds to protons with velocity in the interval $0.3 \leq \beta \leq 0.7$ and range ≥ 3.0 mm in emulsion. Grey tracks are associated with the recoiling protons and have energy range up to 400 MeV. Black tracks are mainly the fragments emitted from excited target. The secondary tracks having specific ionization $g^* > 10$ are classified as black tracks, which is represent by 'N_b'. This corresponds to protons of relative velocity $\beta < 0.3$ having a range in emulsion $R < 3.0$ mm. The particles producing black tracks are mainly the fragments emitted from the excited target. This ionization corresponds to protons with energy range < 30 MeV. Besides these tracks, there are a few projectile fragments as well. In high-energy nuclear collisions, the particles in the projectile beam that collide with the target nucleus also undergo fragmentation. These particles have constant ionization, long range and small emission angle. They generally lie within 3° with respect to the main beam direction. These projectile fragments must be identified with great care. The black and grey tracks taken together are said to be heavily ionizing particles or slow particles. Thus these tracks correspond to $g^* \geq 1.4$ or $\beta \leq 0.7$. Their number in a star, $N_h = (N_b + N_g)$ is a characteristics of the target.

There is a limitation with nuclear emulsion that the exact identification of target is not possible since the medium of the emulsion is heterogeneous and composed of H, C, N, O, Ag and Br nuclei. The events produced due to the collisions with different targets in nuclear emulsion are usually classified into three main categories on the basis of the multiplicity of heavily ionizing tracks in it [14, 15]. The events with $N_h \geq 1$ are classified as collisions with hydrogen (H, $A_T = 1$), $2 \leq N_h \leq 7$ are classified as collision with group of light nuclei (CNO, $<A_T> = 14$) and $N_h \geq 8$ are classified as collision with group of heavy nuclei (AgBr, $<A_T> = 94$) respectively. So, in order to ensure that the target in the emulsion is AgBr, we have considered the events with $N_h \geq 8$ only. According to this procedure 300 ^{32}S-AgBr events are selected in this investigation and the average multiplicities of black and grey particles are 3.14 ± 0.11 and 6.67 ± 0.14 respectively. Further details of the experimental data may be found in our earlier publications [16-19 and reference therein].

3. MATHEMATICAL FORMALISM

Particle Density Distribution

In order to investigate the contribution of dynamical fluctuations in the emission spectra of target associated particles (black and grey), Bialas and Grazdzicki [20] proposed that the single particle rapidity spectrum of Cosθ - space has been converted to a spectrum of new scaled variableX(Cosθ), where X(Cosθ) is defined as:

$$X(\cos\theta) = \frac{\int_{\cos\theta_{min}}^{\cos\theta} \rho \cos\theta . d\cos\theta}{\int_{\cos\theta_{min}}^{\cos\theta_{max}} \rho \cos\theta . d\cos\theta} \qquad (1)$$

Where $\text{Cos}\theta_{min}$ and $\text{Cos}\theta_{max}$, respectively equals to -1 and +1. The new variable X(Cosθ) corresponds to a single particle density distribution is uniformly distributed from 0 to +1 and the shape of the single particle density distribution spectrum in X(Cosθ) space is flat in nature. The single particle density distribution spectrum in X(Cosθ) space is shown in Fig.1, the best fitted lines for the data points are found to be flat in nature, with slope m = 0 and correlation coefficient R = 1.

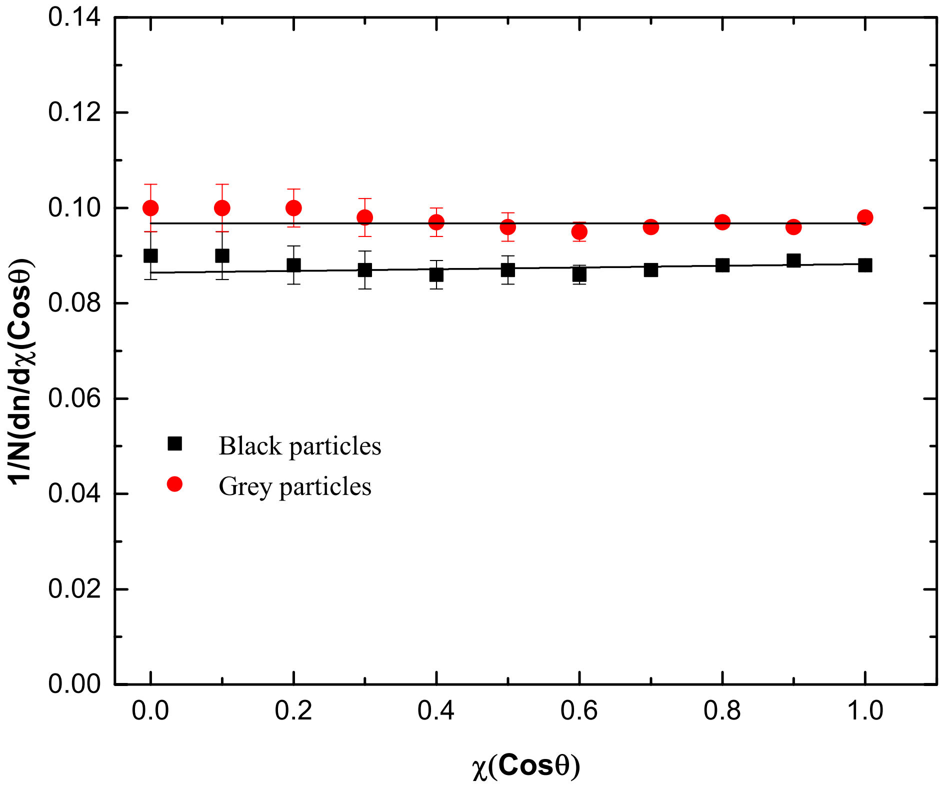

Fig. 1. Single-particle density distributions of the scaled variable X(Cosθ) for the ^{32}S projectile at 200 A GeV.

For the study of the non-statistical fluctuations in $Cos\theta$-distributions, of fast and slow target associated particles in a given $Cos\theta$ interval of total length $\Delta\text{Cos}\theta = \text{Cos}\theta_{\text{max}} - \text{Cos}\theta_{\text{min}}$, is divided into M bins of equal width, $\delta\text{Cos}\theta = \Delta\text{Cos}\theta/M$. Depending on the type of averaging, two types of moments are defined as:

(i) Horizontal Scaled Factorial Moments and

(ii) Vertical Scaled Factorial Moments.

The horizontal and vertical scaled factorial moments are identical if the single-particle angular distribution is flat. However, if the distribution is not flat, one should either consider vertically averaged moments or apply a correction factor [21, 22] to the horizontal moments.

Horizontal Scaled Factorial Moments

Horizontal scaled factorial moments (SFMs) of different orders q are defined as [1, 2, 23]:

$$F_q^H = M^{q-1} \sum_{m=1}^{M} \frac{n_m\,(n_m-1)\ldots\ldots\ldots\ldots(n_m-q+1)}{N(N-1)\ldots\ldots\ldots(N-q+1)} \tag{2}$$

where, n_m is the number of grey or black tracks in bin m (m = 1, 2, 3….) and N represents the total multiplicity of grey or black particles in a particular event in the Cosθ intervals ~ΔCosθ.

For an ensemble of events of varying multiplicity the above relation reduces to [24 - 26].

$$F_q^H = M^{q-1} \sum_{m=1}^{M} \frac{n_m\,(n_m-1)\ldots\ldots\ldots\ldots(n_m-q+1)}{<N>^q} \tag{3}$$

where, <N> represents the mean multiplicity of the grey or black tracks in the angular intervals ΔCosθ = Mδ Cosθ.

On averaging over the number of events in data sample, one can get:

$$< F_q^H >= \frac{M^{q-1}}{N_{ev}} \sum_{m=1}^{M} \frac{n_m\,(n_m-1)\ldots\ldots\ldots\ldots(n_m-q+1)}{<N>^q} \tag{4}$$

It has been shown [1] that for a smooth angular distribution of grey and black particles not exhibiting any fluctuations other than the statistical ones, $<F_q>$ is essentially independent of the angular bin width δCosθ in the limit $\delta\text{Cos}\theta \rightarrow 0$. However, if the fluctuations are dynamical in nature, then in the limit of small bin-size, the scaled factorial moments would obey the following power law:

$$< \mathrm{F}_q^H > = (\mathrm{M})^{\alpha_q} = \left[\frac{\Delta\text{Cos}\theta}{\delta\text{Cos}\theta}\right]^{\alpha_q}, \quad (\text{for } \delta\text{Cos}\theta \rightarrow 0) \tag{5}$$

The power-law dependence of the scaled factorial moments on the number of bins represented by the above relation is known as intermittency. Observation of such a power law may indicate a cascade mechanism of multiparticle production.

The power law predicts a characteristic linear rise of $\ln < \mathrm{F}_q >$ as a function of lnM, which is represented by the following relation:

$$\ln < F_q^H > = \alpha_q \ln M + C \tag{6}$$

where, α_q, which measures the strength of intermittency is called the intermittency exponent and C is a constant. The intermittency exponent, α_q, is obtained by performing best fits according to Eqn. (6).

$$\alpha_q = \frac{\Delta ln<F_q^H>}{\Delta lnM} \tag{7}$$

The intermittency exponent, α_q, increases with increasing order of q of the moments however, for a random uncorrelated particle production, $<F_q>$ should be constant for all values of q showing the absence of non-statistical fluctuations. The power law behaviour of the scaled factorial moments is predicted to be due to the fractal nature of the multi-particle spectra. Lipa and Buschbeck [27] have correlated the scaling behaviour of the factorial moments to the physics of fractal and multifractal objects through the relation:

$$d_q = \frac{\alpha_q}{q-1} \tag{8}$$

where d_q is called the anomalous dimension. It is used for the description of the fractal objects. Thus, using the above relation, the anomalous dimension d_q, can be calculated directly from the intermittency index α_q. The order independence of d_q indicates monofractal behaviour of multiparticle spectra, whereas an increase of d_q with q indicates the presence of multifractality in the emission spectrum.

The power law behaviour of $<F_q>$ on M reveals self-similarity and in general it indicates the existence of fractal properties which are called multifractals. According to the fractal theory, self-similar systems are characterized by infinite spectrum of non-integer generalized dimensions, D_q. Therefore, the generalized dimensions, D_q , that characterize multiparticle production process can be determined from F_q-moments analyses using the following relations:

$$D_q = 1 - \frac{\alpha_q}{q-1}$$

or,

$$D_q = 1 - d_q \tag{9}$$

The mono-fractal structure of multiparticle spectra will show constant D_q values, which are associated with some collective phenomena (i.e., the formation of quark-gluon plasma) whereas the multifractal structures are characterized by decreasing values of the generalized dimensions, D_q, with increasing order of the moments, q. Decreasing behaviour of D_q with q are associated with a self-similar cascade process.

4. RESULTS AND DISCUSSIONS

Dependence of $\boldsymbol{ln < F_q >}$ on $\boldsymbol{lnM}$

In order to study the scaled factorial moment, the whole Cosθ-phase space has been divided into number of bins M = 2-30, and the scaled factorial moment $< F_q >$ for the order of moments q = 2 - 6 are calculated using Eqn. (4) for black and grey track producing particles respectively in ^{32}S-AgBr interactions at 200 AGeV. The values of $< F_q >$ as a function of lnM are plotted in Fig. 2 (a - b) in $Cos\theta$ - phase space for slow and fast target associated particles. It is evident from the graph that the values of $ln < F_q >$ increase with increasing lnM (i.e. decreasing bin size). It is important to record linear variation of $ln < F_q >$ with lnM with positive slope, which confirms the existence of intermittency in the emission pattern of target-associated protons (i.e. black and grey). The solid lines in Figs. 2 (a - b) represent the best-fitted line to the respective data points. The observation recorded about the behaviour of SFMs confirms the signature of intermittency in the emission spectrum of fast and slow target associated particles. Similar results have been reported by other workers [28-31].

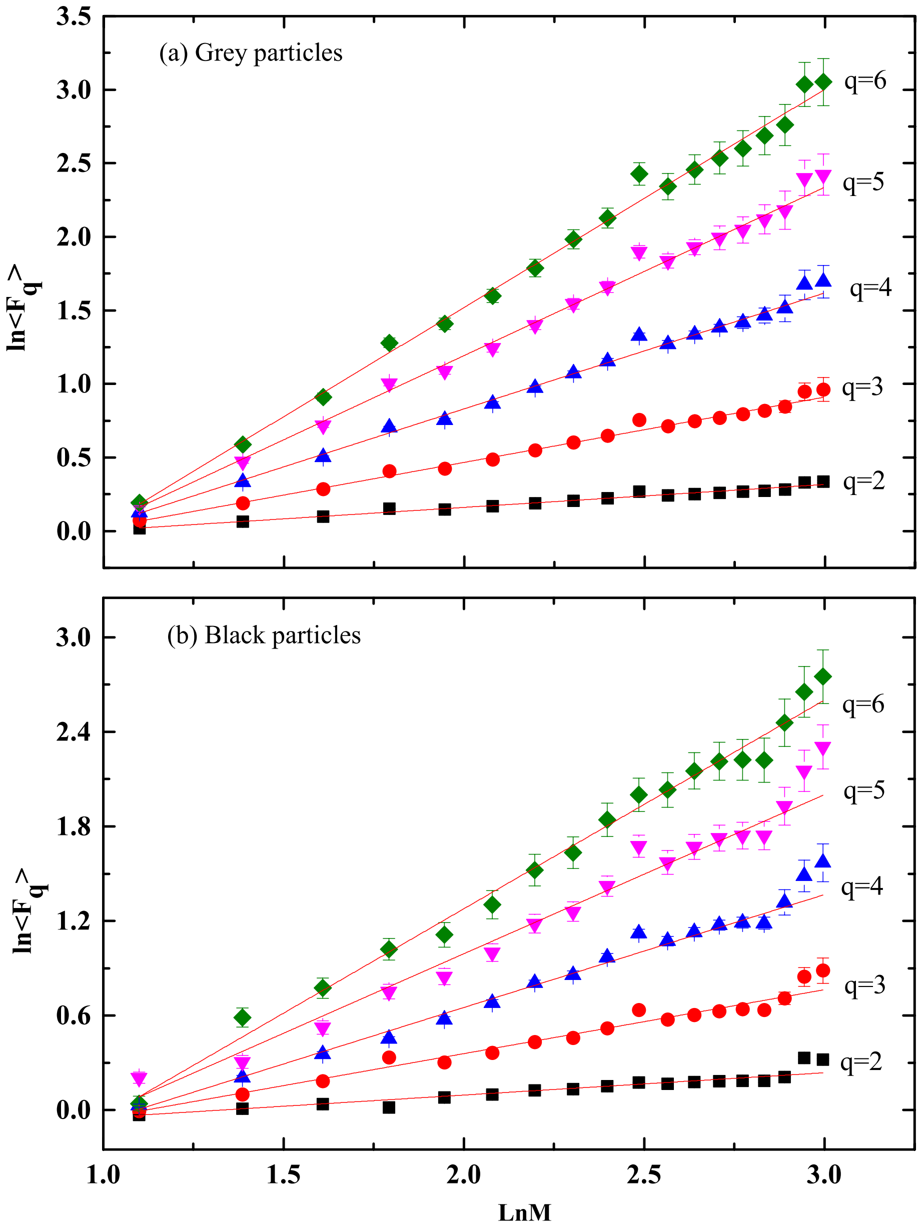

Fig. 2 (a and b): Variations of ln<F$_q$> as function of ln M for (a) grey particles and (b) black particles in Cosθ phase space in the interactions of ^{32}S-AgBr collisions at 200AGeV.

Dependence of α_q on q

The values of the intermittency index, α_q , which characterize the strength of intermittency effect have been obtained from the slopes of fitted solid lines in Fig. 2 (a and b) for various orders of factorial moments along with statistical errors for fast and slow target associated particles are given in Table 1 along with the values obtained by other workers for different projectiles at different energies. From the table it is observed that the parameter, α_q , increases with increasing order of moments for grey and black particles. Further, it is also observed that the values of α_q, are slightly larger for the grey particles in comparison to the black particles for each value of q. Thus it can be seen that the values of the intermittency index, α_q , are also higher for highly energetic particles. Similar results have been reported by other workers [29, 31].

Table 1: Values of intermittency index, α_q, obtained from least square fits of Eqn. (6) for the experimental data.

Data set/ Energy (A GeV)	Tracks	α_2	α_3	α_4	α_5	α_6	Ref.
^{32}S-AgBr 200	Grey	0.155 ± 0.004	0.443 ± 0.009	0.787 ± 0.012	1.144 ± 0.015	1.482 ± 0.018	**
^{32}S-AgBr 200	Black	0.141 ± 0.005	0.405 ± 0.017	0.716 ± 0.016	1.009 ± 0.039	1.324 ± 0.049	**
^{28}Si-Em 14.6	Grey	0.085 ± 0.004	0.184 ± 0.010	0.325 ± 0.017	0.513 ± 0.024	0.717 ± 0.035	[29]
^{28}Si-Em 14.6	Black	0.042 ± 0.002	0.099 ± 0.006	0.162 ± 0.011	0.295 ± 0.023	0.436 ± 0.031	[29]
^{16}O-AgBr 4.5	Grey	0.030 ± 0.002	0.101 ± 0.021	0.180 ± 0.047	----	----	[30]
^{16}O-AgBr 4.5	Black	0.009 ± 0.004	0.041 ± 0.017	0.094 ± 0.030	----	----	[30]
^{84}Kr-AgBr 0.95	Grey	0.110 ± 0.002	0.740 ± 0.010	1.720 ± 0.010	2.700 ± 0.010	----	[31]
^{84}Kr-AgBr 0.95	Black	0.110 ± 0.002	0.840 ± 0.040	2.28 ± 0.070	3.610 ± 0.100	----	[31]

** Corresponds to present work.

The Anomalous Fractal Dimensions, d_q

The anomalous fractal dimension [27, 32, 33], d_q, is used for the description of fractal objects, which can be evaluated by Eqn. (8). The formation of quark-gluon plasma in thermo-dynamical equilibrium may be responsible for the production of a phase transition to the hadron phase [34]. If the phase transition is of second order, the hadrons in final state would exhibit intermittent behaviour and the anomalous fractal dimension, d_q, would be independent of the order of moments. If on other hand the final state hadrons are produced as a result of the cascading process, the anomalous fractal dimensions d_q would increase linearly with q. To see the evidence of above observation, we have studied the variation of d_q with the order of the moments q which are shown in Fig. 3 for grey and black particles produced in the interactions of ^{32}S-AgBr collisions at 200 AGeV using F_q – moments. From the figures it may be noted that the anomalous fractal dimension, d_q, obtained, increases linearly with the increase of q, and it indicates multifractal geometry of the emission spectra of target fragments. Since the order independence of d_q is associated with the monofractal behaviour of multiparticle spectra whereas an increase will indicate multifractality. This analysis will be helpful in understanding the emission of target fragments, especially the emission of black particles. Similar trend has been reported by other workers for different projectiles and different energies [28-31].

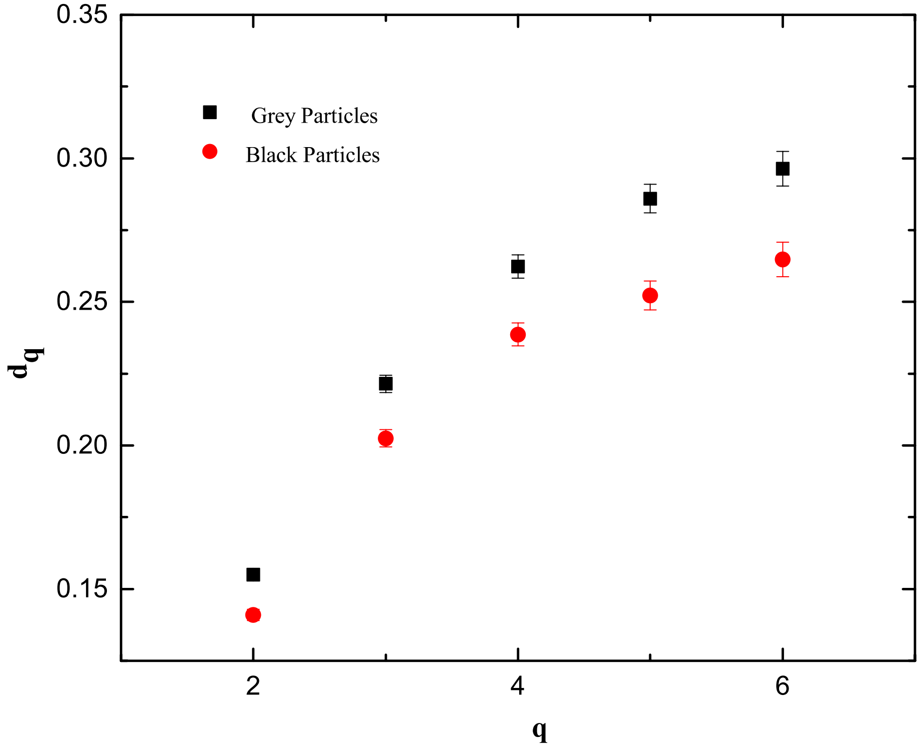

Fig. 3: Dependence of anomalous fractal dimension, d_q on q for grey and black particles in the interactions of ^{32}S-AgBr collisions at 200AGeV.

Variation of Generalized Fractal Dimension D_q with q

The power law behaviour of the scaled factorial moments (SFMs) of the multiplicity distribution on number of bins M reveals self-similar behaviour and also indicates the multifractal structures. The generalized dimensions, D_q, a parameter of fractality is calculated with the help of Eqn. (9) for F_q - moments in the interactions of ^{32}S- nucleus collisions at 200AGeV for grey and black particles respectively. The values of D_q, are plotted in Fig. 4 as a function of order of moments q in $Cos\theta$-phase space for F_q -moments. It is obvious from the figures that the values of D_q, decrease with the increasing order of the moments, q for both particles. The decreasing pattern in the values of, D_q, with the order of moments, q clearly gives an agreement with the multifractal cascade mechanism [35] and this behaviour also indicates that there is no existence of second-order phase transition. These results are similar as reported by other workers [29, 31, 36-37 and reference therein]. Therefore, the observed scaled factorial moment analysis reveal a self-similarity characteristic in multiparticle production.

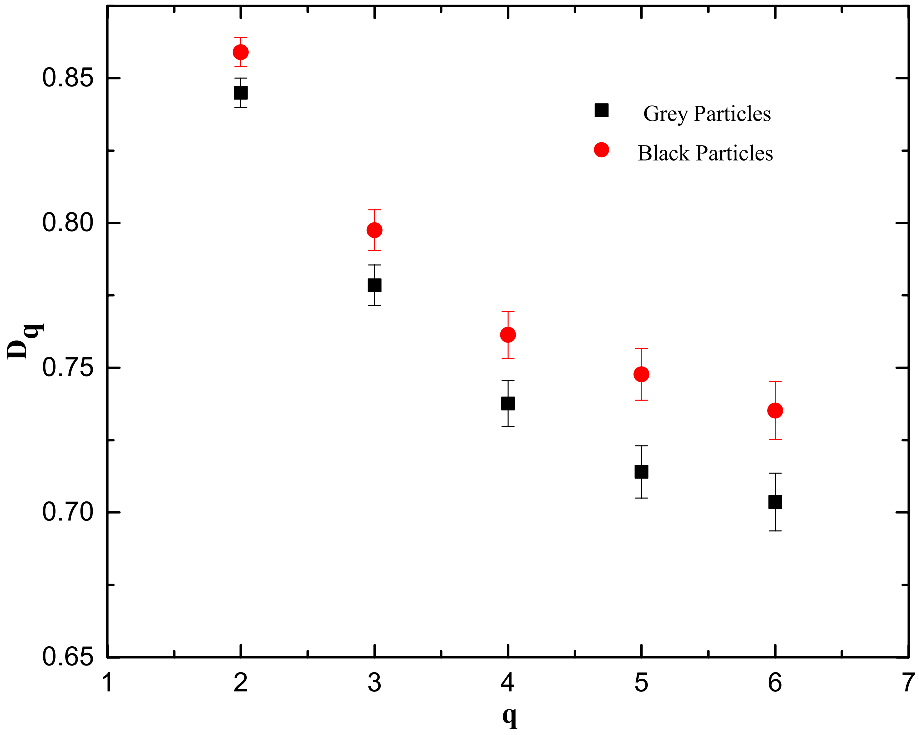

Fig. 4: Variation of generalized fractal dimension, D_q as a function of q for grey and black particles in the interactions of ^{32}S-AgBr collisions at 200AGeV.

5. CONCLUSIONS

The studies of “fractal behaviour of target fragments (i.e., grey and black particles) in the interactions of ^{32}S-AgBr collisions at 200AGeV are presented in this paper. The following conclusions may be drawn: The observed increasing trend in the values of scaled factorial moments, F_q with decreasing bin size clearly reflects the evaporation model and gives an evidence for an intermittency pattern of fluctuations in such heavy ion nucleus-nucleus collisions. The fractal behaviour of multiparticle production is observed for grey particles as well as black particles in the considered collisions of target fragmentation region. The anomalous fractal dimension, d_q, increases linearly with the increase of the order of the scaled factorial moments, thereby indicating multifractal geometry of the emission spectra of target fragments. Finally, the decreasing trend in the values of the generalized dimensions, D_q, with increasing order of moments, q, indicates the possibility that multiparticle production for grey and black particles is due to a self-similar cascade process.

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CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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