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ESTIMATION OF COEFFICIENT BOUNDS FOR THE SUBCLASSES OF ANALYTIC FUNCTIONS ASSOCIATED WITH CHEBYSHEV POLYNOMIAL

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Abstract. The most essential and the needed concept used by the theory of complex function is the Quasi subordination. The subordination along with the majorization concepts are getting collaborated with the help of this Quasi subordination concept. In this article, a novel subclass consisting of univalent analytic functions are investigated, analysed and reviewed. The Chebyshev polynomials that is associated with the open unit disk is defined. Further the estimates are found in these classes having the coefficients of functions by utilizing the benefits of Chebyshev polynomials. Then the Fekete-Szegö inequalities are obtained which provides the results representing the associated new classes thereby briefly making the quasi-subordination to involve along with majorization results.

Keywords: univalent function; subordination; quasi-subordination; majorization; chebyshev polynomials; coefficient estimates; Fekete-Szegö inequality.

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1. INTRODUCTION AND PRELIMINARY RESULTS

Let us denote $\mathscr{A}$ as the class of functions which is in the form

(1) $$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

that seems to be analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and is again normalized by the conditions $f(0) = 0 = f'(0) - 1$. Also let us consider $\mathscr{S}$ as the subclass of $\mathscr{A}$ that consists of univalent functions in $\mathbb{U}$.

Let as assume $\omega(z)$ as the class of analytic functions in the form

$$\omega(z) = \omega_1 z + \omega_2 z^2 + \omega_3 z^3 + \cdots,$$

which satisfies with a suitable condition $|\omega(z)| < 1$ in $\mathbb{U}$, that is mentioned to be the class which is comprised of the Schwarz functions. By Recalling the subordination principle between the analytic functions, $f(z)$ and $g(z)$ are analytic in $\mathbb{U}$. This could be mentioned that the function $f(z)$ is considered as the subordinate to $g(z)$, Schwarz function $\omega(z)$ gets existed, then $f(z) = g(\omega(z))$, $(z \in \mathbb{U})$. This make the functions are subordinate. This subordination condition is denoted by $f \prec g$ (or) $f(z) \prec g(z)$, $(z \in \mathbb{U})$. Particularly the function $g(z)$ is univalent in $\mathbb{U}$, then the above mentioned subordination is much equivalent to the conditions $f(0) = g(0)$, $f(\mathbb{U}) \subset g(\mathbb{U})$.

According to Ma-Minda [21], the following subordination principle is stated as:

Let us consider $\mathscr{S}^*(\phi)$ as the deliberated class representing the starlike function $f \in \mathscr{S}$ for which $\frac{zf'(z)}{f(z)} \prec \phi(z)$, $(z \in \mathbb{U})$ and $\mathscr{C}(\phi)$ be assumed as the class representing the convex function $f \in \mathscr{S}$ by which $1 + \frac{zf''(z)}{f'(z)} \prec \phi(z)$, $(z \in \mathbb{U})$. Ma-Minda [21] analysed and reviewed these classes.

Quasi-subordination is the extension of subordination which is introduced and studied by Robertson in [29, 30]. The function $f(z)$ is Quasi-subordinate onto the function $g(z)$ in $\mathbb{U}$ whenever if arises the Schwarz function $\omega(z)$ along with an functions which is analytic given as $\varphi(z)$ by satisfying the condition $|\varphi(z)| < 1$ as $f(z) = \varphi(z) g(\omega(z))$ in $\mathbb{U}$, which is also represented by $f \prec_q g$, where

(2) $$\varphi(z) = d_0 + d_1 z + d_2 z^2 + \cdots \quad \text{and} \quad |d_n| \leq 1.$$

By setting the value $\varphi(z) \equiv 1$ then the subordination reduction is typically done by quasi-subordination. And by setting $\omega(z) = z$, then we get $f(z) = \varphi(z)g(z)$ and also consider that $f(z)$ which is majorized by $g(z)$ and is mentioned as $f(z) \ll g(z)$ in $\mathbb{U}$. Hence the quasi-subordination might be considered as the generalization by representing the notion of the subordination in addition with the majorization which emphasize the reputation of majorization. Quasi-subordination were discussed in [19, 28, 3, 6, 17, 11, 16, 27]. Many authors have recently examined different aspects of meromorphic families, including multivalent and univalent meromorphics, as well as corresponding functions that include both linear and non linear operators (for example, see [22, 4, 14, 13, 26, 15, 20, 31, 25, 32, 7]).

The major role considered in the numerical analysis, applied mathematics and the approximation theory might play a crucial role in case of the chebyshev polynomials. Considerably it is applied mostly in mathematical applications. Hence an extensive review might need to undergo on the chebyshev polynomials. Considering the chebyshev polynomials, it is of various kinds (specifically four). Of the four kinds, the orthogonal polynomials based research is mostly preferred by most of the researchers. Considering the short antiquity of the Chebyshev polynomials which is of the first kind ie: $T_n(t)$, the second kind $U_n(t)$ and its applications where anybody might refer the articles [23, 8, 10, 1, 5, 2, 12].

The Chebyshev polynomials are represented by $T_n(x)$ and $U_n(x)$ for all $x \in [-1,1]$ respectively are well defined by,

$$T_n(x) = \cos n\theta, \quad x \in [-1,1],$$

and second kind,

$$U_n(x) = \frac{\sin(n+1)\theta}{\sin\theta}, \quad x \in [-1,1],$$

where n denotes the polynomial degree and $x = \cos\theta$.

Let us define the function

$$\Phi(z,t) = \frac{1}{1-2tz+z^2},$$

where $t = \cos\theta$, $\theta \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$, and $z \in \mathbb{U}$.

$\Phi(z,t)$ can be written as

$$\begin{aligned}
\Phi(z,t) &= \frac{1}{1-2tz+z^2} \\
&= 1 + \sum_{n=1}^{\infty} \frac{\sin(n+1)\theta}{\sin\theta} z^n \\
&= 1 + 2\cos\theta z + (3\cos^2\theta - \sin^2\theta) z^2 + \cdots \\
&= 1 + U_1(t)z + U_2(t)z^2 + \cdots, \qquad z \in \mathbb{U}, \quad t \in \left(\frac{1}{2}, 1\right],
\end{aligned}$$

where

$$U_{n-1} = \frac{\sin(n\cos^{-1} t)}{\sqrt{1-t^2}}, \qquad n \in \mathbb{N},$$

are the Chebyshev polynomials of second kind.

Furthermore, we know that

$$U_n(t) = 2tU_{n-1}(t) - U_{n-2}(t)$$

and

$$U_1(t) = 2t, \qquad U_2(t) = 4t^2 - 1, \qquad U_3(t) = 8t^3 - 4t, \qquad U_4(t) = 16t^4 - 12t^2 + 1, \cdots.$$

The generating function of the first kind of Chebyshev polynomial $T_n(t)$, $t \in [-1,1]$, is given by

$$\sum_{n=0}^{\infty} T_n(t) z^n = \frac{1-tz}{1-2tz+z^2}, \qquad (z \in \mathbb{U}).$$

The first kind of Chebyshev polynomial $T_n(t)$ in addition with the second kind of Chebyshev polynomial $U_n(t)$ might gets related by considering the following relationship:

$$\begin{aligned}
\frac{dT_n(t)}{dt} &= nU_{n-1}(t); \\
T_n(t) &= U_n(t) - tU_{n-1}(t); \\
2T_n(t) &= U_n(t) - U_{n-2}(t).
\end{aligned}$$

Definition 1.1. *A function $f \in \mathscr{S}$ is said to be in the class $\mathscr{H}_q(\alpha,\beta,\Phi)$, $\alpha,\beta \in \mathbb{R}$ if it satisfies the following condition*

$$(1-\beta)\left[(1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right)\right] + \beta\left[\frac{zf'(z) + \alpha z^2 f''(z)}{(1-\alpha)f(z) + \alpha z f'(z)}\right] - 1 \prec_q \Phi(z,t) - 1, \ (z \in \mathbb{U}).$$

Definition 1.2. *A function $f \in \mathscr{S}$ is said to be in the class $\mathscr{S}_q(\alpha, \Phi)$, $0 \leq \alpha \leq 1$ if*

$$\left(f'(z)\right)^{\alpha}\left(\frac{zf'(z)}{f(z)}\right)^{1-\alpha} - 1 \prec_q \Phi(z,t) - 1, \quad (z \in \mathbb{U}).$$

Definition 1.3. *A function $f \in \mathscr{S}$ is said to be in the class $\mathscr{L}_q(\alpha, \Phi)$, $0 \leq \alpha \leq 1$ if*

$$\left(f'(z)\right)^{\alpha}\left(1 + \frac{zf''(z)}{f'(z)}\right)^{1-\alpha} - 1 \prec_q \Phi(z,t) - 1, \quad (z \in \mathbb{U}).$$

Our current research proves that the usage of the Chebyshev polynomials might gets expanded to provide the basic coefficients of analytic functions in $\mathscr{H}_q(\alpha, \beta, \Phi(z,t))$, $\mathscr{S}_q(\alpha, \Phi(z,t))$, $\mathscr{L}_q(\alpha, \Phi(z,t))$. We also have to find the Fekete-Szegö estimation for the above defined class associated with quasi-subordination and majorization.

The lemma that is following in regards to the coefficients of functions in $\omega(z)$ might needed for proving our main results.

Lemma 1.1. [18] *Let the Schwarz function $\omega(z)$ be analytic function in $\mathbb{U}$ with $\omega(0) = 0$, $|\omega(z)| < 1$ and*

$$\omega(z) = \omega_1 z + \omega_2 z^2 + \omega_3 z^3 + \cdots . \tag{3}$$

Then

$$\left|\omega_2 - \mu\omega_1^2\right| \leq \max\{1, |\mu|\},$$

for any complex number μ. The result is sharp for the function $\omega(z) = z$ or $\omega(z) = z^2$.

Lemma 1.2. [9] *Let us consider the Schwarz function $\omega(z)$ as the analytic function in $\mathbb{U}$ with $\omega(0) = 0$, $|\omega(z)| < 1$ and given by (3) then*

$$|\omega_n| \leq \begin{cases} 1, & \text{for } n = 1 \\ 1 - |\omega_1|^2, & \text{for } n \geq 2. \end{cases}$$

The result is sharp for the function $\omega(z) = z$ or $\omega(z) = z^n$.

Lemma 1.3. [18] *If $\varphi(z)$ given in* (2) *be analytic function in $\mathbb{U}$ with $|\varphi(z)| < 1$. Then $|d_0| \leq 1$ and $|d_n| \leq 1 - |d_0|^2 \leq 1$ for $n > 0$.*

2. Fekete-Szegö Coefficient Estimates Associated with Quasi-Subordination

Theorem 2.1. *Let α,β be positive real numbers, let $\varphi(z)$ is given in (2) and if $f(z)$ given by (1) belongs to $\mathscr{H}_q(\alpha,\beta,\Phi)$, then*

$$|a_2| \leq \frac{2t}{1+\alpha}.$$

and for some $\mu \in \mathbb{C}$

$$|a_3-\mu a_2^2| \leq \frac{t}{(1+2\alpha)} \max\left\{1, \left|\frac{4\mu(1+2\alpha)t-2(1+3\alpha-\alpha\beta+\alpha^2\beta)t}{(1+\alpha)^2} - \frac{4t^2-1}{2t}\right|\right\}.$$

Proof. If $f \in \mathscr{H}_q(\alpha,\beta,\Phi)$, then by the Definition 1.1 we have

$$\begin{aligned}
&(1-\beta)\left[(1-\alpha)\frac{zf'(z)}{f(z)}+\alpha\left(1+\frac{zf''(z)}{f'(z)}\right)\right]+\beta\left[\frac{zf'(z)+\alpha z^2 f''(z)}{(1-\alpha)f(z)+\alpha z f'(z)}\right]-1\\
&\qquad = \varphi(z)\left[\Phi(\omega(z),t)-1\right]\\
&1+(1+\alpha)a_2 z+\left[2(1+2\alpha)a_3-(1+3\alpha-\alpha\beta+\alpha^2\beta)a_2^2\right]z^2+\cdots\\
&\qquad = \left[d_0U_1(t)\omega_1\right]z+\left[d_0U_1(t)\omega_2+d_0U_2(t)\omega_1^2+d_1U_1(t)\omega_1\right]z^2+\cdots
\end{aligned}$$

Equating the coefficient of z and z^2 we have the following coefficients

$$a_2 = \frac{d_0U_1(t)\omega_1}{1+\alpha} \tag{4}$$

and

$$a_3 = \frac{U_1(t)}{2(1+2\alpha)}\left(d_1\omega_1+d_0\left[\omega_2+\left(\frac{U_2(t)}{U_1(t)}+\frac{(1+3\alpha-\alpha\beta+\alpha^2\beta)d_0U_1(t)}{(1+\alpha)^2}\right)\omega_1^2\right]\right). \tag{5}$$

Since μ is a complex number, from (4) and (5), we get

$$\begin{aligned}
a_3-\mu a_2^2 &= \frac{U_1(t)}{2(1+2\alpha)}\\
&\times\left(d_1\omega_1+d_0\omega_2-\left[\frac{2\mu(1+2\alpha)-(1+3\alpha-\alpha\beta+\alpha^2\beta)}{(1+\alpha)^2}U_1(t)d_0^2-\frac{U_2(t)}{U_1(t)}d_0\right]\omega_1^2\right).
\end{aligned} \tag{6}$$

Since $\varphi(z)$ given in (2) is analytic and bounded in $\mathbb{U}$, Hence applying the result in [24, p. 172], for some y $(|y|\leq 1)$. We have

$$|d_0| \leq 1 \qquad \text{and} \qquad d_1=(1-d_0^2)y. \tag{7}$$

Substituting the value of d_1 in (6), which yields

$$\begin{aligned} a_3-\mu a_2^2 &= \frac{U_1(t)}{2(1+2\alpha)}\left(y\omega_1+d_0\omega_2+\frac{U_2(t)}{U_1(t)}d_0\omega_1^2\right.\\ &\left.-\left(\left[\frac{2\mu(1+2\alpha)-(1+3\alpha-\alpha\beta+\alpha^2\beta)}{(1+\alpha)^2}U_1(t)\right]\omega_1^2+y\omega_1\right)d_0^2\right). \end{aligned} \tag{8}$$

If $d_0=0$, the equation (8) becomes

$$a_3-\mu a_2^2=\frac{U_1(t)y\omega_1}{2(1+2\alpha)}$$

Using Lemma 1.2 and Lemma 1.3

$$|a_3-\mu a_2^2|\leq\frac{U_1(t)}{2(1+2\alpha)}$$

If $d_0\neq 0$, from equation (8), let

$$\begin{aligned} F(d_0) &= y\omega_1+\left(\omega_2+\frac{U_2(t)}{U_1(t)}\omega_1^2\right)d_0\\ &-\left(\frac{2\mu(1+2\alpha)-(1+3\alpha-\alpha\beta+\alpha^2\beta)}{(1+\alpha)^2}U_1(t)\omega_1^2+y\omega_1\right)d_0^2 \end{aligned}$$

which is a polynomial in d_0 and hence it is analytic in $|d_0|\leq 1$, and maximum $|F(d_0)|$ is attained at $d_0=e^{i\theta},(0\leq\theta<2\pi)$. Therefore we obtained that

$$\max_{0\leq\theta<2\pi}|F(e^{i\theta})|=|F(1)|$$

and

$$|a_3-\mu a_2^2|=\frac{U_1(t)}{2(1+2\alpha)}\left|\omega_2-\left[\frac{2\mu(1+2\alpha)-(1+3\alpha-\alpha\beta+\alpha^2\beta)}{(1+\alpha)^2}U_1(t)-\frac{U_2(t)}{U_1(t)}\right]\omega_1^2\right|.$$

By Lemma 1.1

$$|a_3-\mu a_2^2|\leq\frac{U_1(t)}{2(1+2\alpha)}\max\left\{1,\left|\frac{2\mu(1+2\alpha)-(1+3\alpha-\alpha\beta+\alpha^2\beta)}{(1+\alpha)^2}U_1(t)-\frac{U_2(t)}{U_1(t)}\right|\right\}$$

Which completes the proof. □

By Applying the same technique as in Theorem 2.1 for the classes $\mathscr{S}_q(\alpha,\Phi)$ and $\mathscr{L}_q(\alpha,\Phi)$, the following Theorems are obtained.

Theorem 2.2. *Let $\varphi(z)$ be given in (2) and if $f(z)$ given by (1) belongs to $\mathscr{S}_q(\alpha,\Phi)$, then*

$$|a_2| \leq \frac{2t}{1+\alpha}.$$

and for some $\mu \in \mathbb{C}$

$$|a_3-\mu a_2^2| \leq \frac{t}{2+\alpha}\max\left\{1,\left|\frac{2\mu t-(1-\alpha)(2+\alpha)t}{(1+\alpha)^2}-\frac{4t^2-1}{2t}\right|\right\},\ 0\leq\alpha\leq 1.$$

Theorem 2.3. *Let $\varphi(z)$ be given in (2) and if $f(z)$ given by (1) belongs to $\mathscr{L}_q(\alpha,\Phi)$, then*

$$|a_2| \leq t.$$

and for some $\mu \in \mathbb{C}$

$$|a_3-\mu a_2^2| \leq \frac{2t}{3(2-\alpha)}\max\left\{1,\left|3\mu(2-\alpha)t-4(1-\alpha)t-\frac{4t^2-1}{2t}\right|\right\},\ 0\leq\alpha\leq 1..$$

3. Fekete-Szegö Coefficient Estimates Associated with Majorization

Theorem 3.1. *Let α,β be positive real numbers, let $\varphi(z)$ is given in (2) and if $f(z)$ given by (1) satisfies the condition*

(9)
$$(1-\beta)\left[(1-\alpha)\frac{zf'(z)}{f(z)}+\alpha\left(1+\frac{zf''(z)}{f'(z)}\right)\right]+\beta\left[\frac{zf'(z)+\alpha z^2f''(z)}{(1-\alpha)f(z)+\alpha zf'(z)}\right]-1\ll\Phi(z,t)-1,$$

then

$$|a_2| \leq \frac{2t}{1+\alpha}.$$

and for some $\mu \in \mathbb{C}$

$$|a_3-\mu a_2^2| \leq \frac{t}{(1+2\alpha)}\max\left\{1,\left|\frac{4\mu(1+2\alpha)t-2(1+3\alpha-\alpha\beta+\alpha^2\beta)t}{(1+\alpha)^2}-\frac{4t^2-1}{2t}\right|\right\}.$$

Proof. If $f(z)$ in (9), then by the definition of majorization we have

$$\begin{aligned}(1-\beta)&\left[(1-\alpha)\frac{zf'(z)}{f(z)}+\alpha\left(1+\frac{zf''(z)}{f'(z)}\right)\right]+\beta\left[\frac{zf'(z)+\alpha z^2f''(z)}{(1-\alpha)f(z)+\alpha zf'(z)}\right]-1\\&=\varphi(z)\left[\Phi(z,t)-1\right]\end{aligned}$$

$$\begin{aligned}1+(1+\alpha)a_2z+&\left[2(1+2\alpha)a_3-(1+3\alpha-\alpha\beta+\alpha^2\beta)a_2^2\right]z^2+\cdots\\&=\left[d_0U_1(t)\right]z+\left[d_0U_2(t)+d_1U_1(t)\right]z^2+\cdots\end{aligned}$$

Equating the coefficient of z and z^2 we have the following coefficients

$$a_2 = \frac{d_0 U_1(t)}{1+\alpha} \tag{10}$$

and

$$a_3 = \frac{U_1(t)}{2(1+2\alpha)}\left(d_1 + d_0\left[\frac{U_2(t)}{U_1(t)} + \frac{(1+3\alpha-\alpha\beta+\alpha^2\beta)d_0U_1(t)}{(1+\alpha)^2}\right]\right). \tag{11}$$

Since μ is a complex number, from (10) and (11), we get

$$a_3 - \mu a_2^2 = \frac{U_1(t)}{2(1+2\alpha)}\left(d_1 + \frac{U_2(t)}{U_1(t)}d_0 - \frac{2\mu(1+2\alpha)-(1+3\alpha-\alpha\beta+\alpha^2\beta)}{(1+\alpha)^2}U_1(t)d_0^2\right). \tag{12}$$

Substituting the value of d_1 from (7) in (12), which implies that

$$a_3 - \mu a_2^2 = \frac{U_1(t)}{2(1+2\alpha)}\left(y + \frac{U_2(t)}{U_1(t)}d_0 - \left[\frac{2\mu(1+2\alpha)-(1+3\alpha-\alpha\beta+\alpha^2\beta)}{(1+\alpha)^2}U_1(t)+y\right]d_0^2\right).$$

If $d_0 = 0$, the equation (12) becomes

$$|a_3 - \mu a_2^2| \leq \frac{U_1(t)}{2(1+2\alpha)}. \tag{13}$$

If $d_0 \neq 0$, let

$$H(d_0) = y + \frac{U_2(t)}{U_1(t)}d_0 - \left[\frac{2\mu(1+2\alpha)-(1+3\alpha-\alpha\beta+\alpha^2\beta)}{(1+\alpha)^2}U_1(t)+y\right]d_0^2.$$

which is a polynomial in d_0 and hence it is analytic in $|d_0| \leq 1$, and maximum $|F(d_0)|$ is attained at $d_0 = e^{i\theta}, (0 \leq \theta < 2\pi)$. Therefore we obtained that

$$\max_{0\leq\theta<2\pi} |H(e^{i\theta})| = |H(1)|$$

and consequently

$$|a_3 - \mu a_2^2| \leq \frac{U_1(t)}{2(1+2\alpha)} \max\left\{1, \left|\frac{2\mu(1+2\alpha)-(1+3\alpha-\alpha\beta+\alpha^2\beta)}{(1+\alpha)^2}U_1(t) - \frac{U_2(t)}{U_1(t)}\right|\right\}. \tag{14}$$

From equation (13) and (14) we get the required result. □

By Implementing the same method as in Theorem 3.1 for the classes $\mathscr{S}_q(\alpha, \Phi(z,t))$ and $\mathscr{L}_q(\alpha, \Phi(z,t))$, we get the following Theorems.

Theorem 3.2. *Let $\varphi(z)$ be given in (2) and if $f(z)$ given by (1) satisfies the condition*

$$\left(f'(z)\right)^{\alpha}\left(\frac{zf'(z)}{f(z)}\right)^{1-\alpha}-1\ll\Phi(z,t)-1,\ 0\leq\alpha\leq 1,$$

then

$$|a_2|\leq\frac{2t}{1+\alpha}.$$

and for some $\mu\in\mathbb{C}$

$$|a_3-\mu a_2^2|\leq\frac{t}{2+\alpha}\max\left\{1,\left|\frac{2\mu t-(1-\alpha)(2+\alpha)t}{(1+\alpha)^2}-\frac{4t^2-1}{2t}\right|\right\}.$$

Theorem 3.3. *Let $\varphi(z)$ be given in (2) and if $f(z)$ given by (1) satisfies the condition*

$$\left(f'(z)\right)^{\alpha}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\alpha}-1\ll\Phi(z,t)-10\leq\alpha\leq 1,$$

then

$$|a_2|\leq t.$$

and for some $\mu\in\mathbb{C}$

$$|a_3-\mu a_2^2|\leq\frac{2t}{3(2-\alpha)}\max\left\{1,\left|3\mu(2-\alpha)t-4(1-\alpha)t-\frac{4t^2-1}{2t}\right|\right\}.$$

4. CONCLUDING REMARK

From the above researches lot of various mind blowing consequences that generates the results that gets asserted in the theorems proven above might gets derived by particularly specialising the parameters that provides the results.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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