Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 3, 2568-2582
https://doi.org/10.28919/jmcs/5535
ISSN: 1927-5307

STABILITY OF TRIGINTIC FUNCTIONAL EQUATION IN MATRIX NORMED SPACES: FIXED POINT APPROACH

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Abstract. In this paper, we tend to find the general solution of trigintic functional equation and also prove the Hyers-Ulam stability of trigintic functional equation in matrix normed spaces through the fixed point method.

Keywords: Ulam-Hyers stability; trigintic functional inequality; fixed point; matrix normed space.

2010 AMS Subject Classification: 47H10.

1. INTRODUCTION

The stability problem for functional equations starts from the famous talk of Ulam [30] before the mathematics club of the university of Wisconsin in 1940. During which he discussed many of important unsolved problems. The universal Ulam stability problem: when is it true that by slightly changing the hypothesis of a theorem one can still assert that the thesis of the theorem remains true or approximately true? Within the following year, Hyers [10] gave the first declarative answer to the question of Ulam for additive functional equations on Banach spaces. Hyers result

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Received February 10, 2021

has since then seen many important generalizations, used to discuss the concept of approximate solution [1, 9, 24, 23].

Isac and Rassias [11] discussed generalization of the Hyers - Ulam stability for a vast field of mappings. Additionally, they mentioned some applications in non-linear analysis, particularly in fixed point theory. This methodology may also be applied to the cases of other functional equations [4, 5, 20, 25, 29, 31, 33, 34]. Also, the generalized Hyers – Ulam stability of functional equation and inequalities in matrix normed spaces has been studied by many mathematicians [13-16, 19, 32].

From previous few decades, the stability problems using different functional equations in many spaces have been investigated by many mathematicians [2, 3, 6, 7, 12, 17, 22, 26, 27, 30].

Recently, Ramdoss, Aruldass, Park and Paokanta [21] conferred the stability of trigintic functional equation in multi-Banach spaces through fixed point approach.

In this paper, we introduce the following functional equation

$$\begin{aligned}
& f(u+15v)-30f(u+14v)+435f(u+13v)-4060f(u+12v) \\
& +27405f(u+11v)-142506f(u+10v)+593775f(u+9v) \\
& -2035800f(u+8v)+5852925f(u+7v)-14307150f(u+6v) \\
& +30045015f(u+5v)-5467300f(u+4v)+86493225f(u+3v) \\
& -119759850f(u+2v)+145422675f(u+v)-1555117520f(u) \\
& +145422675f(u-v)-119759850f(u-2v)+86493225f(u-3v) \\
& -54627300f(u-4v)+30045015f(u-5v)-14307150f(u-6v) \\
& +5852925f(u-7v)-2035800f(u-8v)+593775f(u-9v) \\
& -142506f(u-10v)+27405f(u-11v)-4060f(u-12v) \\
& +435f(u-13v)-30f(u-14v)+f(u-15v)=30!\,f(v). \qquad (1.1)
\end{aligned}$$

where $30! = 2.652528598\times10^{32}$ is said to be trigintic functional equation since the function f(u) = c u^{30} is its solution.

In this paper, we discussed the general solution of the functional equation (1.1). Moreover, the stability of the functional equation (1.1) in matrix normed spaces with the help of fixed point method.

2. GENERAL SOLUTION OF TRIGINTIC FUNCTIONAL EQUATION IN (1)

In this section, we solve the trigintic functional equation in (1) in vector spaces.

Theorem 2.1 Let X and Y be vector spaces. If f: X $\rightarrow$ Y satisfies the function equation (1.1) for all u, v $\in$ X, then f is a trigintic mapping, that is, f (2u) = 2^{30}f(u) for all x $\in$ X.

Proof. Letting u = v = 0 in (1.1), we obtain that f(0) = 0. Replacing (u, v) with (u, u) and (u, – u) in (1.1), respectively, and subtracting two resulting equations, we get f(– u) = f(u). Hence f is an even mapping.

Replacing (u, v) by (15u, u) and (0, 2u) respectively in (1.1), and subtracting the two resulting equations, we have

$$\begin{aligned}
&30\,f(29u)-465\,f(28u)+4060\,f(27u)-26970\,f(26u)\\
&+142506\,f(25u)-597835\,f(24u)+2035800\,f(23u)\\
&-5825520\,f(22u)+14307150\,f(21u)-30187611\,f(20u)\\
&+54627300\,f(19u)-85899450\,f(18u)+11975985\,f(17u)\\
&-147458475\,f(16u)+155117520\,f(15u)-139569750\,f(14u)\\
&+119759850\,f(13u)-100800375\,f(12u)+546273000\,f(11u)\\
&+14307150\,f(9u)-60480225\,f(8u)+2035800\,f(7u)\\
&+85899450\,f(6u)+142506\,f(5u)-119787255\,f(4u)\\
&+4060\,f(3u)-1.326264299\times10^{32}\,f(2u)+30!\,f(u)=0, \qquad (2.1)
\end{aligned}$$

for all u $\in$ X. Refilling (u, v) by (14u, u) in (1.1), and increasing the out coming equation by 30, and subtracting the obtained result from (2.1), we have

$$\begin{aligned}
&435\,f(28u)-8990\,f(27u)+94830\,f(26u)-679644\,f(25u)\\
&+3677345\,f(24u)-15777450\,f(23u)+55248480\,f(22u)\\
&-161280600\,f(21u)+399026889\,f(20u)-846723150\,f(19u)\\
&+1552919550\,f(18u)-2475036900\,f(17u)+3445337025\,f(16u)\\
&-4207562730\,f(15u)+4513955850\,f(14u)-4242920400\,f(13u)\\
&+3491995125\,f(12u)-2540169450\,f(11u)+1638819000\,f(10u)\\
&-887043300\,f(9u)+368734275\,f(8u)-173551950\,f(7u)\\
&+146973450\,f(6u)-17670744\,f(5u)-115512075\,f(4u)\\
&-818090\,f(3u)-1.326264299\times10^{32}\,f(2u)+30!(31)\,f(u)=0, \qquad (2.2)
\end{aligned}$$

for all u $\in$ X. Refilling (u, v) by (13u, u) in (1.1), further multiplying the out coming equation by 435, and subtracting the obtained result from (2.2), we have

$$\begin{aligned}
&4060 f(27u) - 94395 f(26u) + 1086456 f(25u) - 8243830 f(24u)\\
&+46212660 f(23u) - 203043645 f(22u) + 724292400 f(21u)\\
&-2146995486 f(20u) + 5376887100 f(19u) - 11516661980 f(18u)\\
&+2.12878386\times 10^{10} f(17u) - 3.417921585\times 10^{10} f(16u)\\
&+4.788797202\times 10^{10} f(15u) - 5.874490778\times 10^{10} f(14u)\\
&+6.32332008\times 10^{10} f(13u) - 5.98573388\times 10^{10} f(12u)\\
&+4.95553653\times 10^{10} f(u) - 3.598573388\times 10^{10} f(u)\\
&+2.28758322\times 10^{10} f(u) - 1.270084726\times 10^{10} f(u)\\
&+6050058300 f(u) - 2399048925 f(u)\\
&+867902256 f(u) - 373804200 f(u) + 61172020 f(u)\\
&-1.326264299\times 10^{32} f(u) + 30!(466) f(u) = 0, \qquad (2.3)
\end{aligned}$$

for all u $\in$ X. Refilling (u, v) by (12u, u) in (1.1), further multiplying the out coming equation by 4060, and subtracting the obtained result from (2.3), we have

$$\begin{aligned}
&27405 f(26u) - 679644 f(25u) + 8239770 f(24u)\\
&-65051640 f(23u) + 375530715 f(22u) - 1686434100 f(21u)\\
&+6118352514 f(20u) - 18385988400 f(19u)\\
&+46570367030 f(18u) - 1.006949223\times 10^{11} f(17u)\\
&+1.876076222\times 10^{11} f(16u) - 3.032745215\times 10^{11} f(15u)\\
&+4.274800832\times 10^{11} f(14u) - 5.271828597\times 10^{11} f(13u)\\
&+5.700102627\times 10^{11} f(12u) - 5.408606952\times 10^{11} f(11u)\\
&+4.502392571\times 10^{11} f(10u) - 3.282866613\times 10^{11} f(9u)\\
&+2.090859907\times 10^{11} f(8u) - 1.159327026\times 10^{11} f(7u)\\
&+5.568798008\times 10^{10} f(6u) - 2.289497324\times 10^{10} f(5u)\\
&+7891543800 f(4u) - 2349558540 f(3u)\\
&-1.326264299\times 10^{32} f(2u) + 30!(4526) f(u) = 0, \qquad (2.4)
\end{aligned}$$

for all u $\in$ X. Refilling (u, v) by (11u, u) in (1.1), further multiplying the out coming equation by 27405, and subtracting the obtained result from (2.4), we have

$$
\begin{aligned}
&142506\,f(25u)-3681405\,f(24u)+46212660\,f(23u)-375503310\,f(22u)\\
&+2218942830\,f(21u)-10154051370\,f(20u)+1037405110600\,f(19u)\\
&-113829042600\,f(18u)+2.913925235\times10^{11}\,f(17u)\\
&-6.357760139\times10^{11}\,f(16u)+1.193786635\times10^{12}\,f(15u)\\
&-1.942866748\times10^{12}\,f(14u)+2.75483583\times10^{12}\,f(13u)\\
&-3.415298146\times10^{12}\,f(12u)+3.710134941\times10^{12}\,f(11u)\\
&-3.535069151\times10^{12}\,f(10u)+2.953732028\times10^{12}\,f(9u)\\
&-2.16126084\times10^{12}\,f(8u)+1.381128454\times10^{12}\,f(7u)\\
&-7.67695656\times10^{11}\,f(6u)+3.691924726\times10^{11}\,f(5u)\\
&-1.525078932\times10^{11}\,f(4u)+5.344236261\times10^{10}\,f(3u)\\
&-1.326264299\times10^{32}\,f(2u)+30!\,(31932)\,f(u)=0, \qquad (2.5)
\end{aligned}
$$

for all u $\in$ X. Refilling (u, v) by (10u, u) in (1.1), further multiplying the out coming equation by 142506, and subtracting the obtained result from (2.5), we have

$$
\begin{aligned}
&593775\,f(24u)-15777450\,f(23u)+203071050\,f(22u)\\
&-1686434100\,f(21u)+10153908670\,f(20u)-47211389550\,f(19u)\\
&+17628467220\,f(18u)-5.426844066\times10^{11}\,f(17u)\\
&+1.403078704\times10^{12}\,f(16u)-3.087808273\times10^{12}\,f(15u)\\
&+5.841851266\times10^{12}\,f(14u)-9.570967692\times10^{12}\,f(13u)\\
&+1.36511990\times10^{13}\,f(12u)-1.701346878\times10^{13}\,f(11u)\\
&+1.857010816\times10^{13}\,f(10u)-1.776987169\times10^{13}\,f(9u)\\
&+1.490523634\times10^{13}\,f(8u)-1.094467507\times10^{13}\,f(7u)\\
&+7.017022358\times10^{12}\,f(6u)-3.912402577\times10^{12}\,f(5u)\\
&+1.8863511\times10^{12}\,f(4u)-7.806965576\times10^{11}\,f(3u)\\
&-1.326264299\times10^{32}\,f(2u)+30!\,(174437)\,f(u)=0, \qquad (2.6)
\end{aligned}
$$

for all u $\in$ X. Refilling (u, v) by (9u, u) in (1.1), further multiplying the out coming equation by 593775, and subtracting the obtained result from (2.6), we have

$$\begin{aligned}
&2035800\, f(23u) - 55221075\, f(22u) + 724292400\, f(21u)\\
&-6118495206\, f(20u) + 37405110600\, f(19u)\\
&-176284078400\, f(18u) + 6.661227384\times10^{11}\, f(17u)\\
&-2.072241838\times10^{12}\, f(16u) + 5.407419719\times10^{12}\, f(15u)\\
&-1.199812752\times10^{13}\, f(14u) + 2.286535737\times10^{13}\, f(13u)\\
&-3.770631564\times10^{13}\, f(12u) + 5.409693615\times10^{13}\, f(11u)\\
&-6.777824069\times10^{13}\, f(10u) + 7.433503375\times10^{13}\, f(9u)\\
&-7.144311251\times10^{13}\, f(8u) + 6.016572986\times10^{13}\, f(7u)\\
&-4.434049291\times10^{13}\, f(6u) + 2.852394029\times10^{13}\, f(5u)\\
&-1.595388597\times10^{13}\, f(4u) + 7.71694216\times10^{12}\, f(3u)\\
&-1.326264299\times10^{32}\, f(2u) + 30!\,(768212)\, f(u) = 0, \qquad (2.7)
\end{aligned}$$

for all u $\in$ X. Refilling (u, v) by (8u, u) in (1.1), further multiplying the out coming equation by 2035800, and subtracting the obtained result from (2.7), we have

$$\begin{aligned}
&5852925\, f(22u) - 161280600\, f(21u) + 2146852794\, f(20u)\\
&-18385988400\, f(19u) + 113829636400\, f(18u)\\
&-5.426844066\times10^{11}\, f(17u) + 2.072239802\times10^{12}\, f(16u)\\
&-6.507964996\times10^{12}\, f(15u) + 1.712836845\times10^{13}\, f(14u)\\
&-3.830028417\times10^{13}\, f(13u) + 7.35039417\times10^{13}\, f(12u)\\
&-1.219859713\times10^{14}\, f(11u) + 1.760288619\times10^{14}\, f(10u)\\
&-2.217164481\times10^{14}\, f(9u) + 2.443451347\times10^{14}\, f(8u)\\
&-2.358857539\times10^{14}\, f(7u) + 1.994666708\times10^{14}\, f(6u)\\
&-1.475598527\times10^{14}\, f(5u) + 9.526463673\times10^{13}\, f(4u)\\
&-5.350449048\times10^{13}\, f(3u) - 1.326264299\times10^{32}\, f(2u)\\
&\qquad + 30!\,(2804012)\, f(u) = 0, \qquad (2.8)
\end{aligned}$$

for all u $\in$ X. Refilling (u, v) by (7u, u) in (1.1), further multiplying the out coming equation by 5852925, and subtracting the obtained result from (2.8), we have

$$\begin{aligned}
&14307150\ f(21u) - 399169581\ f(20u) + 65376887100\ f(19u) \\
&-46569773250\ f(18u) + 2.913925235\times 10^{11}\ f(17u) \\
&-1.40308074\times 10^{12}\ f(16u) + 5.407419719\times 10^{12}\ f(15u) \\
&-1.71283626\times 10^{13}\ f(14u) + 4.543839174\times 10^{13}\ f(13u) \\
&-1.023472777\times 10^{14}\ f(12u) + 1.977435186\times 10^{14}\ f(11u) \\
&-3.30209497\times 10^{14}\ f(10u) + 4.79228972\times 10^{14}\ f(9u) \\
&-6.068028812\times 10^{14}\ f(8u) + 6.720056324\times 10^{14}\ f(7u) \\
&-6.516838853\times 10^{14}\ f(6u) + 5.534093293\times 10^{14}\ f(5u) \\
&-4.111341216\times 10^{14}\ f(4u) + 2.670590763\times 10^{14}\ f(3u) \\
&-1.326264299\times 10^{32}\ f(2u) + 30!\,(8656937)\ f(u) = 0, \qquad (2.9)
\end{aligned}$$

for all u $\in$ X. Refilling (u, v) by (6u, u) in (1.1), further multiplying the out coming equation by 14307150, and subtracting the obtained result from (2.9), we have

$$\begin{aligned}
&30045015\ f(20u) - 846723150\ f(19u) + 11517255750\ f(18u) \\
&-1.006949223\times 10^{11}\ f(17u) + 6.357739781\times 10^{11}\ f(16u) \\
&-3.087808273\times 10^{12}\ f(15u) + 1.199813337\times 10^{13}\ f(14u) \\
&-3.830028417\times 10^{13}\ f(13u) + 1.023472634\times 10^{14}\ f(12u) \\
&-2.321150178\times 10^{14}\ f(11u) + 4.513514782\times 10^{14}\ f(10u) \\
&-7.58242586\times 10^{14}\ f(9u) + 1.106619686\times 10^{15}\ f(8u) \\
&-1.408584616\times 10^{15}\ f(7u) + 1.567663828\times 10^{15}\ f(6u) \\
&-1.527566783\times 10^{15}\ f(5u) + 1.304326871\times 10^{15}\ f(4u) \\
&-9.789076957\times 10^{14}\ f(3u) - 1.326264299\times 10^{32}\ f(2u) \\
&\qquad + 30!\,(22964087)\, f(u) = 0, \qquad (2.10)
\end{aligned}$$

for all u $\in$ X. Refilling (u, v) by (5u, u) in (1.1), further multiplying the out coming equation by 30045015, and subtracting the obtained result from (2.10), we have

$$\begin{aligned}
&54627300\ f(19u) - 1552325775\ f(18u) + 2.12878386\times 10^{10}\ f(17u) \\
&-1.87609658\times 10^{11}\ f(16u) + 1.193786635\times 10^{12}\ f(15u) \\
&-5.841845413\times 10^{12}\ f(14u) + 2.286535737\times 10^{13}\ f(13u) \\
&-7.350395601\times 10^{13}\ f(12u) + 1.977435186\times 10^{14}\ f(11u) \\
&-4.513514782\times 10^{14}\ f(10u) + 8.83036363\times 10^{14}\ f(9u) \\
&-1.492083626\times 10^{15}\ f(8u) + 2.189723856\times 10^{15}\ f(7u) \\
&-2.802386007\times 10^{15}\ f(6u) + 3.137223027\times 10^{15}\ f(5u)
\end{aligned}$$

$$-3.08273956\times10^{15}\ f\left(4u\right)+2.680444435\times10^{15}\ f\left(3u\right)$$
$$-1.326264299\times10^{32}\ f\left(2u\right)+30!\left(53009102\right)f\left(u\right)=0, \qquad (2.11)$$

for all u $\in$ X. Refilling (u, v) by (4u, u) in (1.1), further multiplying the out coming equation by 54627300, and subtracting the obtained result from (2.11), we have

$$86493225\ f\left(18u\right)-2475036900\ f\left(17u\right)+3.417718005\times10^{10}\ f\left(16u\right)$$
$$-3.032745215\times10^{11}\ f\left(15u\right)+1.942872601\times10^{12}\ f\left(14u\right)$$
$$-9.570967692\times10^{12}\ f\left(13u\right)+3.770630133\times10^{13}\ f\left(12u\right)$$
$$-1.219860259\times10^{14}\ f\left(11u\right)+3.302111358\times10^{14}\ f\left(10u\right)$$
$$-7.58265448\times10^{14}\ f\left(9u\right)+1.492280066\times10^{15}\ f\left(8u\right)$$
$$-2.536664555\times10^{15}\ f\left(7u\right)+3.747555965\times10^{15}\ f\left(6u\right)$$
$$-4.839261392\times10^{15}\ f\left(5u\right)+5.502121998\times10^{15}\ f\left(4u\right)$$
$$-5.583333149\times10^{15}\ f\left(3u\right)-1.326264299\times10^{32}\ f\left(2u\right)$$
$$+30!\left(107636402\right)f\left(u\right)=0, \qquad (2.12)$$

for all u $\in$ X. Refilling (u, v) by (3u, u) in (1.1), further multiplying the out coming equation by 86493225, and subtracting the obtained result from (2.12), we have

$$119759850\ f\left(17u\right)-3447372833\ f\left(16u\right)+4.788797202\times10^{10}\ f\left(15u\right)$$
$$-4.274742302\times10^{10}\ f\left(14u\right)+2.75483583\times10^{12}\ f\left(13u\right)$$
$$-1.365129984\times10^{13}\ f\left(12u\right)+5.409947635\times10^{13}\ f\left(11u\right)$$
$$-1.760648477\times10^{14}\ f\left(10u\right)+4.79557259\times10^{14}\ f\left(9u\right)$$
$$-1.108780523\times10^{15}\ f\left(8u\right)+2.200552599\times10^{15}\ f\left(7u\right)$$
$$-3.784879521\times10^{15}\ f\left(6u\right)+5.695237168\times10^{15}\ f\left(5u\right)$$
$$-7.582192512\times10^{15}\ f\left(4u\right)+9.070752951\times10^{15}\ f\left(3u\right)$$
$$-1.326264299\times10^{32}\ f\left(2u\right)+30!\left(194129627\right)f\left(u\right)=0, \qquad (2.13)$$

for all u $\in$ X. Refilling (u, v) by (2u, u) in (1.1), further multiplying the out coming equation by 119759850, and subtracting the obtained result from (2.13), we have

$$145422675\ f\left(16u\right)-4207562730\ f\left(15u\right)$$
$$+5.875076090\times10^{10}\ f\left(14u\right)-5.273026195\times10^{11}\ f\left(13u\right)$$
$$+3.41879014\times10^{12}\ f\left(12u\right)-1.706302412\times10^{13}\ f\left(11u\right)$$
$$+6.822847991\times10^{13}\ f\left(10u\right)-2.246701798\times10^{14}\ f\left(9u\right)$$

$$+6.21708118\times10^{14}\, f(8u) - 1.468744296\times10^{15}\, f(7u)$$
$$+3.001084836\times10^{15}\, f(6u) - 5.364123902\times10^{15}\, f(5u)$$
$$+8.473651298\times10^{15}\, f(4u) - 1.194323128\times10^{16}\, f(3u)$$
$$-1.326264299\times10^{32}\, f(2u) + 30!\,(313889477)\, f(u) = 0, \qquad (2.14)$$

for all u $\in$ X. Refilling (u, v) by (u, u) in (1.1), further multiplying the out coming equation by 145422675, and subtracting the obtained result from (2.14), we have

$$155117520\, f(15u) - 4653525530\, f(14u) + 6.74612125\times10^{10}\, f(13u)$$
$$-6.29777131\times10^{11}\, f(12u) + 4.250995636\times10^{12}\, f(11u)$$
$$-2.210517735\times10^{13}\, f(10u) + 9.210490544\times10^{13}\, f(9u)$$
$$-3.157882471\times10^{14}\, f(8u) + 9.078912107\times10^{14}\, f(7u)$$
$$-2.219289626\times10^{15}\, f(6u) + 4.660508218\times10^{15}\, f(5u)$$
$$-8.473651302\times10^{15}\, f(4u) + 1.341661456\times10^{16}\, f(3u)$$
$$-1.326264299\times10^{32}\, f(2u) + 30!\,(459312152)\, f(u) = 0, \qquad (2.15)$$

for all u $\in$ X. Replacing (u, v) by (0, u) in (1.1), further multiplying the out coming equation by 155117520, and subtracting the obtained result from (2.15), we have

$$-1.326264299\times10^{32}\, f(2u) + 30!\,(536870912)\, f(u) = 0, \qquad (2.16)$$

for all u $\in$ X. From (2.16), we get

$$f(2u) = 2^{30} f(u), \qquad (2.17)$$

for all u $\in$ X.

3. STABILITY OF TRIGINTIC FUNCTIONAL EQUATION IN MATRIX NORMED SPACES

Throughout this section, we are going to prove the Ulam - Hyers stability for the equation (1.1) in matrix normed spaces through fixed point technique.

Throughout this section, consider $(X, \|.\|_n)$ be a matrix normed spaces, $(Y, \|.\|_n)$ be a matrix Banach space and let n be a fixed non – negative integer.

Consider f: X $\to$ Y, set ψf: $X^2 \to$ Y and ψf_n: $M_n(X^2) \to M_n(Y)$, for all x, y $\in$ X and $u = [u_{ij}], v = [v_{ij}] \in M_n(X)$.

Theorem 3.1 Let t = ±1 be fixed and $\phi : X^2 \to [0, \infty)$ be a function such that there exists a $\lambda <$ 1 with

$$\phi(x,y) \le 2^{30t}\lambda\left(\frac{x}{2^t},\frac{y}{2^t}\right)\ \forall\, x,y \in X. \tag{3.1}$$

Let f: X $\to$ Y be a mapping satisfying

$$\left\|\psi f\left(\left[x_{ij}\right],\left[y_{ij}\right]\right)\right\| \le \sum_{i,j=1}^{n}\phi\left(x_{ij},y_{ij}\right), \tag{3.2}$$

for all $x=\left[x_{ij}\right], y=\left[y_{ij}\right]\in M_n(X)$. Then there exists a unique trigintic mapping T: X $\to$ Y such that

$$\left\|f\left(\left[x_{ij}\right]\right)-T_n\left(\left[x_{ij}\right]\right)\right\|_n \le \sum_{i,j=1}^{n}\frac{\lambda^{\frac{1-t}{2}}}{2^{30}(1-\lambda)}\phi^*\left(x_{ij}\right), \tag{3.3}$$

for all x = $[x_{ij}] \in M_n(X)$, where

$$\begin{aligned}\phi^*\left(x_{ij}\right)=\frac{1}{30!}[&\phi\left(0,2x_{ij}\right)+155117520\,\phi\left(0,x_{ij}\right)+\phi\left(15x_{ij},x_{ij}\right)\\
&+30\,\phi\left(14x_{ij},x_{ij}\right)+435\,\phi\left(13x_{ij},x_{ij}\right)+4060\,\phi\left(12x_{ij},x_{ij}\right)\\
&+27405\,\phi\left(11x_{ij},x_{ij}\right)+142506\,\phi\left(10x_{ij},x_{ij}\right)+593775\,\phi\left(9x_{ij},x_{ij}\right)\\
&+2035800\,\phi\left(8x_{ij},x_{ij}\right)+5852925\,\phi\left(7x_{ij},x_{ij}\right)+14307150\,\phi\left(6x_{ij},x_{ij}\right)\\
&+30045015\,\phi\left(5x_{ij},x_{ij}\right)+54627300\,\phi\left(4x_{ij},x_{ij}\right)\\
&+86493225\,\phi\left(3x_{ij},x_{ij}\right)+119759850\,\phi\left(2x_{ij},x_{ij}\right)+145422675\,\phi\left(x_{ij},x_{ij}\right)].\end{aligned}$$

Proof Putting n = 1 in (3.2), we get

$$\left\|\psi f(x,y)\right\| \le \phi(x,y). \tag{3.4}$$

By utilizing Theorem 2.1, we will get

$$\begin{aligned}\left\|-f(2x)+2^{30}f(x)\right\| \le \frac{1}{30!}\Big[&\phi(0,2x)+\phi(15x,x)+30\,\phi(14x,x)\\
&+435\,\phi(13x,x)+4060\,\phi(12x,x)+27405\,\phi(11x,x)\\
&+142506\,\phi(10x,x)+593775\,\phi(9x,x)+2035800\,\phi(8x,x)\\
&+5852925\,\phi(7x,x)+14307150\,\phi(6x,x)+30045015\,\phi(5x,x)\\
&+54627300\,\phi(4x,x)+86493225\,\phi(3x,x)+119759850\,\phi(2x,x)\\
&+145422675\,\phi(x,x)+155117520\,\phi(0,x)].\end{aligned}$$

Therefore,

$$\left\|f(2x)-2^{30}f(x)\right\| \le \phi^*(x), \tag{3.5}$$

for all $x \in X$. Hence

$$\left\| f(x) - \frac{1}{2^{30t}} f\left(2^t x\right) \right\| \leq \frac{\lambda^{\left(\frac{1-t}{2}\right)}}{2^{30}} \phi^*(x), \tag{3.6}$$

for all $x \in X$. Taking $\xi = \{f : X \to Y\}$ and therefore the generalized metric δ on ξ as follows:

$$\delta(f, f_1) = \inf\left\{\eta \in R_+ : \left\| f(x) - f_1(x) \right\| \leq \eta\phi^*(x), \text{for all } x \in X\right\}.$$

It is straightforward to ascertain that (ξ, δ) is a complete metric (see [18]). Set the mapping $\gamma : \xi \to \xi$ by

$$\gamma f(x) = \frac{1}{2^{30t}} f\left(2^t x\right), \text{for all } f \in \xi \text{ and } x \in X.$$

Suppose f, $f_1 \in \xi$ and m is an arbitrary constant with $\delta(f, f_1) = m$. Then $\left\| f(x) - f_1(x) \right\| \leq m\phi^*(x), \text{for all } x \in X.$ Using (3.1), we have

$$\left\| \gamma f(x) - \gamma f_1(x) \right\| = \left\| \frac{1}{2^{30t}} f\left(2^t x\right) - \frac{1}{2^{30t}} \left(2^t x\right) \right\| \leq \lambda T\phi^*(x), \text{ for all } x \in X.$$

Hence it holds that $\delta(\gamma f, \gamma f_1) \leq \lambda T$, that is, $\delta(\gamma f, \gamma f_1) \leq \lambda\delta(f, f_1)$, for all f, $f_1 \in \xi$.

By (3.6), we have $\delta(f, \gamma f_1) \leq \dfrac{\lambda^{\left(\frac{1-t}{2}\right)}}{2^{30}}$.

By Theorem 2.2 in [5], there exists a mapping g: $X \to Y$ that satisfying:

(a) g is a unique fixed point of δ, that is satisfied $g\left(2^t x\right) = 2^{30t} g(x)$, for all $x \in X$.

(b) $\delta\left(\gamma^k f, g\right) \to 0$ as $k \to \infty$. This suggests that

$$\lim_{k \to \infty} \frac{1}{2^{30kt}} f\left(2^{kt} x\right) = g(x), \text{ for all } x \in X.$$

(c) $\delta(f, g) \leq \dfrac{1}{1 - \lambda} \delta(f, \gamma f)$ implies

$$\left\| f(x) - g(x) \right\| \leq \frac{\lambda^{\frac{1-t}{2}}}{2^{30}(1 - \lambda)} \phi^*(x), \text{ for all } x \in X. \tag{3.7}$$

It follows from (3.1) and (3.2), that

$$\begin{aligned}\left\|\psi\, g(x,y)\right\| &= \lim_{k\to\infty}\frac{1}{2^{30kt}}\left\|\psi\, f\left(2^{kt}x, 2^{kt}y\right)\right\|\\ &\le \lim_{k\to\infty}\frac{1}{2^{30kt}}\phi\left(2^{kt}x, 2^{kt}y\right)\\ &\le \lim_{k\to\infty}\frac{2^{kt}\lambda^{t}}{2^{30kt}}\phi(x,y) = 0,\end{aligned}$$

for all x, y $\in$ X. Therefore, the mapping g: X $\to$ Y is trigintic mapping. From lemma 2.1 in [13] and 3.7, we get (3.3). Hence g: X $\to$ Y is a distinctive trigintic mapping satisfying (3.3).

Corollary 3.2 Let t = ± 1 be fixed and let α, β be non-negative real numbers with $\alpha \neq 30$. Let f: X $\to$ Y be a mapping satisfying

$$\left\|\psi f_n\left([x_{ij}],[y_{ij}]\right)\right\|_n \le \sum_{i,j=1}^{n}\beta\left(\left\|x_{ij}\right\|^{\alpha}+\left\|y_{ij}\right\|^{\alpha}\right), \tag{3.8}$$

for all $x=\left[x_{ij}\right], y=\left[y_{ij}\right]\in M_n(X)$. Then there exists a unique trigintic mapping g: X $\to$ Y such that

$$\left\| f_n\left([x_{ij}]\right)-g_n\left([x_{ij}]\right)\right\|_n \le \sum_{i,j=1}^{n}\frac{\beta_0}{\left|2^{30}-2^{\alpha}\right|}\left\|x_{ij}\right\|^{\alpha},$$

for all $x=\left[x_{ij}\right]\in M_n(X)$, where

$$\begin{aligned}\beta_0 = \frac{\beta}{30!}[&155117520+145422676(2^{\alpha})+119759850(3^{\alpha})\\ &+86493225(4^{\alpha})+54627300(5^{\alpha})+30045015(6^{\alpha})\\ &+14307150(7^{\alpha})+5852925(8^{\alpha})+2035800(9^{\alpha})\\ &+593775(10^{\alpha})+142506(11^{\alpha})+27405(12^{\alpha})\\ &+4060(13^{\alpha})+435(14^{\alpha})+30(15^{\alpha})+(16^{\alpha})].\end{aligned}$$

Proof The proof is similar to the proof of Theorem 3.1 by taking $\phi(x,y)=\beta\left(\|x\|^{\alpha}+\|y\|^{\alpha}\right)$ for all x, y $\in$ X. Then we can take $\lambda = 2^{t(\alpha-30)}$, and find the required result.

Corollary 3.2 Let t = ± 1 be fixed and let α, β be non-negative real numbers with $\alpha = a + b \neq 30$. Let f: X $\to$ Y be a mapping satisfying

$$\left\|\psi f_n\left([x_{ij}],[y_{ij}]\right)\right\|_n \le \sum_{i,j=1}^{n}\beta\left(\left\|x_{ij}\right\|^{a}.\left\|y_{ij}\right\|^{b}\right), \tag{3.9}$$

for all $x=\left[x_{ij}\right], y=\left[y_{ij}\right]\in M_n\left(X\right)$. Then there exists a unique trigintic mapping g: $X \to Y$ such that

$$\left\| f_n\left([x_{ij}]\right)-g_n\left([x_{ij}]\right)\right\|_n \le \sum_{i,j=1}^{n}\frac{\beta_0}{\left|2^{30}-2^{\alpha}\right|}\left\|x_{ij}\right\|^{\alpha},$$

for all $x=\left[x_{ij}\right]\in M_n\left(X\right)$, where

$$\begin{aligned}\beta_0 = \frac{\beta}{30!}[&155117520+145422676(2^a)+119759850(3^a)\\ &+86493225(4^a)+54627300(5^a)+30045015(6^a)\\ &+14307150(7^a)+5852925(8^a)+2035800(9^a)\\ &+593775(10^a)+142506(11^a)+27405(12^a)\\ &+4060(13^a)+435(14^a)+30(15^a)+(16^a)].\end{aligned}$$

Proof The proof is comparable to the proof of Theorem 3.1.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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