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COST OPTIMIZATION OF SINGLE SERVER RETRIAL QUEUEING MODEL WITH BERNOULLI SCHEDULE WORKING VACATION, VACATION INTERRUPTION AND BALKING

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Abstract: In present paper an M/M/1 retrial queueing model with working vacation interruption using Bernoulli schedule and Balking is analyzed under classical retrial policy. When sever is occupied in regular busy period customer enters in system with probability b and during working vacation of server, customer joins the orbit with probability v . Whenever no customers are in the system after vacation completion then either server returns to normal free state with probability q or goes for multiple working vacation with probability $1-q$. Server provides the service at lower rate during working vacation than normal busy period. In this paper steady state probabilities have been obtained using probability generating function technique. Some important performance measures of this model are also evaluated numerically and some results are shown graphically using MATLAB software.

Keywords: queueing; balking; working vacation; interruption; Bernoulli schedule.

2010 Subject Classification: 60K25, 60K30.

1. INTRODUCTION

Retrial queues have wide applications in field of communication networking, computer systems, call centres and telephone switching systems etc. In retrial queues arriving customers, on finding the busy server, instead of joining the queue in front of server, join the virtual room called orbit and

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retry for their request after some random period of time. Otherwise, arrival will get service immediately, if server is free at the time of arrival. Retrial queues have been studied extensively in literature. Yang, Templeton [23] and Falin [8] analyzed the retrial queues. Gomez-Corral [9] studied stochastic analysis of retrial queues with general retrial times. Artalejo and Corral [3] did the pioneer work on retrial queueing system.

The impatient customer behaviour is a challenge in modeling queueing system. The pioneer work had been done by Haight [7] to study queueing model with balking. Multi server queue with fixed probability of balking is presented by Al-Seedy et al. [19]. Choudhury and Medhi [1] analyzed M/M/C queueing model with balking and reneging. Ammar et al. [22] studied busy period of single server queueing model with balking and reneging. Yue et al. [6] investigate M/M/1/N system with dynamic balking probability. Vijaya Laxmi et al. [16] analyzed finite buffer queueing model with working vacation and impatient behaviour of customers. Kumar and Sharma [17] present multi-server finite capacity queueing model with impatient behaviour of customers. Kumar and Sharma [18] also extended their work to the case of infinite capacity. Some researchers studied queueing model with dynamic probability of balking.

A special class of vacation queueing models where system don't stop working completely rather provides service at relatively slow rate is working vacation. The vacation models with Bernoulli schedule where server goes on vacation or remains in system with probability $1-q$ or q respectively provides a control on congestion of system. Many researchers worked on working vacation queueing models. In this respect an important work on GI/G/1 queueing model with Bernoulli schedule vacation was done by Keilson and Servi [10]. Servi and Finn [12,13] did pioneer work on working vacation queueing model. Takagi [5] also studied single server queueing model with Bernoulli vacation. Later on a general working vacation queueing model was analyzed by Banik et al. [2]. Arivudainambi et al. [4] analyzed single server retrial queue with working vacation.

This paper analyses M/M/1 retrial queue with working vacation and vacation interruption with different levels of customer impatience at the time of arrival in busy state of server in normal as well in working vacation period. Bernoulli schedule of working vacation at vacation completion instant provides an option that server may go on multiple working vacation with probability $1-q$ or may return to normal state with probability q , when there is no customer in the system i.e. a generalization of single and multiple working vacation. The concept of vacation interruption is also important for effective utilization of server where vacation is interrupted on service completion

instant if customers are present in the system. Li and Tian [11] studied queuing model with vacation interruption. Later on Zhang and Hou [15] analyzed M/G/1 queueing model with working vacation and vacation interruption. A pioneer work on M/G/1 retrial queueing model with Bernoulli schedule working vacation and vacation interruption has been given by Gao et al. [20,21]. GI/M/1 queues with vacation interruption under controlled Bernoulli schedule was analyzed by Tao [14].

In this paper, we have considered classical retrial queue with Bernoulli Schedule of working vacation interruption along with impatient customer behaviour in both normal and vacation state using the method of probability generating functions. The various sections of the paper are described as below:

The model description of the paper is given in section 2. The steady state equations are described in Section 3. Section 4 describes some system performance measures and normalization condition. Section 5 illustrates the graphical results of the model and finally conclusion is given in section 6.

2. MODEL DESCRIPTION

In present paper, M/M/1 queueing model with impatient behaviour of customers, Bernoulli working vacation and vacation interruption under classical retrial policy is considered. Description of model of present paper is given as follows:

- (1) Customer arrives in the system with arrival rate λ which follows Poisson distribution. On arrival customer decides to join the system or balk depending on the state of server. If server is not busy then arriving customer gets service immediately otherwise if the server is busy in normal working state then arrival either joins the orbit with probability b or balk with probability $1-b$. On the other hand if server is busy in working vacation state then arriving customer joins the orbit with probability v or balk the system with probability $1-v$
- (2) Customers get service using first come first served (FCFS) basis. In normal busy state service time is assumed to follow exponential distribution with mean $1/\mu$.
- (3) Customers in the orbit retry for service with retrial rate ξ , which follows Poisson distribution. If retrial customers find free server then request is accepted immediately otherwise retrial customers waits in the orbit for his turn.

(4) When system is empty then server goes for working vacation. Vacation period is exponentially distributed with parameter ϕ . In working vacation state customer gets service with lower service rate $\theta (< \mu)$ which is exponentially distributed. On completion of vacation if server finds customer in the system then server goes to normal busy period otherwise if there is no customer in the system then either server goes to regular free state in normal service period with probability q or continues vacation with probability $1-q$, thereby giving rise to a generalization of single or multiple working vacations.

Let $N(t)$ is number of customers in the orbit (a free pool) at a given time t and $H(t)$ denotes the state of server at a given time t . The possible values of the server states $H(t)$ can be:

$$H(t) = \begin{cases} 0, \text{ the server is free in normal service state} \\ 1, \text{ the server is busy in normal service state} \\ 2, \text{ the server is free in working vacation state} \\ 3, \text{ the server is busy in working vacation state} \end{cases}$$

Then, $\{N(t), H(t)\}$ is a Markov process having state space

$$S = \{(n, h), n \geq 0, h = 0, 1, 3\} \cup \{(0, 2)\}$$

In present model, we consider Bernoulli Schedule working vacation and vacation interruption policy. So, the states $\{(n, 2), n \geq 1\}$ doesn't exist.

3. STEADY STATE EQUATIONS

The balance equations governing the present model are:

$$\lambda p_{00} = q\phi p_{02} \quad (1)$$

$$(\lambda + n\xi)p_{n0} = \mu p_{n1} + \theta p_{n3} \quad n \geq 1 \quad (2)$$

$$(\lambda b + \mu)p_{01} = \xi p_{10} + \phi p_{03} + \lambda p_{00} \quad (3)$$

$$(\lambda b + \mu)p_{n1} = (n + 1)\xi p_{n+10} + \phi p_{n3} + \lambda p_{n0} + \lambda b p_{n-11} \quad n \geq 1 \quad (4)$$

$$(\lambda + \phi q)p_{02} = \theta p_{03} + \mu p_{01} \quad (5)$$

$$(\lambda v + \theta + \phi)p_{03} = \lambda p_{02} \quad (6)$$

$$(\lambda v + \theta + \phi)p_{n3} = \lambda v p_{n-13} \quad n \geq 1 \quad (7)$$

Probability generating functions are:

$$F_0(z) = \sum_{n=0}^{\infty} p_{n0} z^n \quad (8)$$

$$F_1(z) = \sum_{n=0}^{\infty} p_{n1} z^n \quad (9)$$

$$F_3(z) = \sum_{n=0}^{\infty} p_{n3} z^n \quad (10)$$

On multiplying equation (2) by z^n and summing over $n=1$ to ∞ and adding equation (1)

$$\lambda F_0(z) + z \xi F_0'(z) - \mu F_1(z) - \theta F_3(z) = \frac{-\lambda^2}{q\varphi} p_{00} \quad (11)$$

Similarly using equations(3) and (4) we get

$$(\lambda b + \mu - \lambda b z) F_1(z) - \xi F_0'(z) - \lambda F_0(z) - \varphi F_3(z) = 0 \quad (12)$$

From equations(6) and (7) we have

$$(\lambda v + \theta + \varphi - \lambda v z) F_3(z) = \frac{\lambda^2}{q\varphi} p_{00} \quad (13)$$

Substituting value of $F_1(z)$ from equation (11) in (12) and using equation (13)

$$\begin{aligned} & (1-z)\xi(\mu - \lambda b z)F_0'(z) - \lambda^2 b(1-z)F_0(z) \\ & = \lambda(\mu + \lambda b - \lambda b z)p_{02} - \frac{\lambda\varphi\mu + \lambda\theta(\mu + \lambda b - \lambda b z)}{\lambda v + \theta + \varphi - \lambda v z} p_{02} \\ F_0'(z) - \frac{\lambda^2 b}{\xi(\mu - \lambda b z)} F_0(z) & = \frac{\lambda(\mu + \lambda b - \lambda b z)}{(1-z)\xi(\mu - \lambda b z)} p_{02} - \frac{\lambda\varphi\mu + \lambda\theta(\mu + \lambda b - \lambda b z)}{(\lambda v + \theta + \varphi - \lambda v z)(1-z)\xi(\mu - \lambda b z)} p_{02} \end{aligned} \quad (14)$$

Solving differential equation (14) we get

$$F_0(z) = -(\mu - \lambda b z)^{\frac{-\lambda}{\xi}} \left(\varphi \mu I_1(z) + \theta I_2(z) - I_3(z) \right) \frac{\lambda^2 p_{00}}{q\varphi\xi} \quad (15)$$

where

$$I_1(z) = \int_0^z (1-x)^{-1} (\mu - \lambda bx)^{\frac{\lambda}{\xi}-1} (\lambda v + \theta + \varphi - \lambda vx)^{-1} dx \quad (16)$$

$$I_2(z) = \int_0^z (1-x)^{-1} (\mu - \lambda bx)^{\frac{\lambda}{\xi}-1} (\lambda v + \theta + \varphi - \lambda vx)^{-1} (\mu + \lambda b - \lambda bx) dx \quad (17)$$

$$I_3(z) = \int_0^z (1-x)^{-1} (\mu - \lambda bx)^{\frac{\lambda}{\xi}-1} (\mu + \lambda b - \lambda bx) dx \quad (18)$$

Eliminating $F'_0(z)$ from equation (11) and equation (12), we get

$$F_1(z) = \frac{(\theta + \varphi z)F_3(z) - \lambda(1-z)F_0(z) - \frac{\lambda^2}{q\varphi} p_{00}}{(1-z)(\lambda bz - \mu)} \quad (19)$$

From equation (13)

$$F_3(z) = \frac{\lambda^2}{q\varphi(\lambda v + \theta + \varphi - \lambda vz)} p_{00} \quad (20)$$

From equation (15), (19) and (20) we observe that $F_0(z)$, $F_1(z)$ and $F_3(z)$ are all expressed in terms of p_{00} whose values can be obtained from normalization condition.

4. SYSTEM PERFORMANCE MEASURES

From equation (20), we have

$$\begin{aligned} F_3(1) &= \lim_{z \rightarrow 1} F_3(z) \\ &= \frac{\lambda^2}{q\varphi(\theta + \varphi)} p_{00} \end{aligned} \quad (21)$$

On differentiating equation (20) we get

$$F'_3(z) = \frac{\lambda^3 v}{q\varphi(\lambda v + \theta + \varphi - \lambda vz)^2} p_{00} \quad (22)$$

$$F'_3(1) = \lim_{z \rightarrow 1} F'_3(z)$$

$$= \frac{\lambda^3 \nu}{q\varphi(\theta + \varphi)^2} p_{00} \quad (23)$$

Taking limit in equation (15) we get

$$\begin{aligned} F_0(1) &= \lim_{z \rightarrow 1} F_0(z) \\ &= -(\mu - \lambda b)^{\frac{-\lambda}{\xi}} I p_{00} \end{aligned} \quad (24)$$

where

$$I = \frac{\lambda^2}{q\varphi\xi} (\phi\mu I_1(1) + \theta I_2(1) - I_3(1))$$

From equation (11)

$$F_0'(z) = \frac{1}{z\xi} \left(\mu F_1(z) + \theta F_3(z) - \lambda F_0(z) - \frac{\lambda^2}{q\varphi} p_{00} \right) \quad (25)$$

$$F_0'(1) = \frac{1}{\xi} \left(\mu F_1(1) + \theta F_3(1) - \lambda F_0(1) - \frac{\lambda^2}{q\varphi} p_{00} \right) \quad (26)$$

From equation (19) we get

$$\begin{aligned} F_1(1) &= \lim_{z \rightarrow 1} F_1(z) \\ &= \lim_{z \rightarrow 1} \frac{(\theta + \varphi z)F_3(z) - \lambda(1-z)F_0(z) - \frac{\lambda^2}{q\varphi} p_{00}}{(1-z)(\lambda bz - \mu)} \quad \left(\frac{0}{0} \right) \text{ form} \end{aligned}$$

Using L-Hospital rule

$$F_1(1) = \frac{\lambda F_0(1) + \varphi F_3(1) + (\theta + \varphi)F_3'(1)}{(\mu - \lambda b)} \quad (27)$$

On differentiating equation (19) and using L-Hospital's rule twice we obtain

$$F_1'(1) = \frac{(\mu - \lambda b)[(\theta + \varphi)F_3''(1) + 2\varphi F_3'(1) + 2\lambda F_0'(1)] + 2\lambda b[(\theta + \varphi)F_3'(1) + \varphi F_3(1) + \lambda F_0(1)]}{2(\mu - \lambda b)^2} \quad (28)$$

Normalization condition is

$$p_{02} + F_0(1) + F_1(1) + F_3(1) = 1$$

implies

$$\frac{\lambda}{q\varphi} p_{00} \left(1 + (\mu - \lambda b)^{\frac{-\lambda}{\xi}} I(1) + \frac{\lambda(\theta + \lambda v)}{(\theta + \varphi)(\mu - \lambda b)} + \lambda(\mu - \lambda b)^{\frac{-\lambda}{\xi}-1} I(1) + \frac{\lambda}{\theta + \varphi} \right) = 1$$

$$p_{00} = \frac{q\varphi}{\lambda} \left(1 + (\mu - \lambda b)^{\frac{-\lambda}{\xi}} (1 + \lambda(\mu - \lambda b)^{-1}) I(1) + \frac{\lambda(\theta + \lambda v) + \lambda(\mu - \lambda b)}{(\theta + \varphi)(\mu - \lambda b)} \right)^{-1}$$

Let expected (average) number of customers in the orbit is denoted by $E[L_k]$, where states of server takes values $k=0, 1, 2, 3$.

So, from equation (26)

$$\begin{aligned} E[L_0] &= \lim_{z \rightarrow 1} F'_1(z) \\ &= \frac{1}{\xi} \left(\mu F_1(1) + \theta F_3(1) - \lambda F_0(1) - \frac{\lambda^2}{q\varphi} p_{00} \right) \end{aligned} \quad (29)$$

Similarly,

$$\begin{aligned} E[L_1] &= \lim_{z \rightarrow 1} F'_1(z) \\ &= \frac{(\mu - \lambda b)[(\theta + \varphi)F_3''(1) + 2\varphi F_3'(1) + 2\lambda F_0'(1)] + 2\lambda b[(\theta + \varphi)F_3'(1) + \varphi F_3(1) + \lambda F_0(1)]}{2(\mu - \lambda b)^2} \end{aligned} \quad (30)$$

$$\begin{aligned} E[L_3] &= \lim_{z \rightarrow 1} F'_3(z) \\ &= \frac{\lambda^3 v}{q\varphi(\theta + \varphi)^2} p_{00} \end{aligned} \quad (31)$$

Expected orbit length is given by

$$\begin{aligned} E[L_q] &= E[L_0] + E[L_1] + E[L_3] \\ &= F'_0(1) + F'_1(1) + F'_3(1) \end{aligned} \quad (32)$$

Expected length of system is given by

$$E[L_s] = E[L_q] + F_1(1) + F_3(1) \quad (33)$$

The probability of server being in busy state is:

$$Pr_B = F_1(1) + F_3(1) \quad (34)$$

The probability of server being in free state is:

$$\begin{aligned} Pr_F &= F_0(1) + p_{02} \\ &= 1 - Pr_B \end{aligned} \quad (35)$$

The probability of server being in regular service state is

$$Pr_N = F_0(1) + F_1(1) \quad (36)$$

The probability of server being in working vacation state is

$$\begin{aligned} Pr_V &= F_3(1) + p_{02} \\ &= 1 - Pr_N \end{aligned} \quad (37)$$

5. GRAPHICAL RESULTS

In this section, we illustrate the effect of various parameters like ξ , θ , μ on expected orbit length $E[L_q]$ along with effect of ξ on probabilities of server being in busy and working vacation state. Further we have optimized the cost with respect to θ using parabolic method.

In below graphs, we have fixed the parameters $\lambda = 0.9$, $\mu=1.8$, $\xi=0.9$, $\theta=0.3$, $\varphi=0.4$, $b=0.7$, $v=0.5$ unless they are used as a variables in the graph.

a) SENSITIVITY ANALYSIS

Figure 1, figure 2, and figure 3 show how expected orbit length $E[L_q]$ varies with change in ξ , θ , μ for two set of values of v and b as shown in the graphs.

Figure 1 reveals that as ξ increases $E[L_q]$ decreases. It is because of the reason as ξ increases mean retrial time decreases due to which retrial and primary customers compete for access to the server, resulting in decrease in expected queue length.

Figure 2 shows that $E[L_q]$ varies inversely with θ , as expected. Since as θ increases the mean service time in vacation decreases resulting in decrease in expected queue length. This decrease is not so obvious for large values of θ because we have considered that as soon as service of a customer is completed in working vacation period server resumes the normal working state, if any customer is waiting in the system.

Figure 3 shows that $E[L_q]$ changes inversely with μ . This is due to the fact that as μ increases mean service time in normal working state decreases. This results in increase in $E[L_q]$

Figure 4 and figure 5 depict that with increase in ξ probability of server being in busy state and in working vacation state increases. This is due to the reason that mean retrial time decrease as ξ increases.

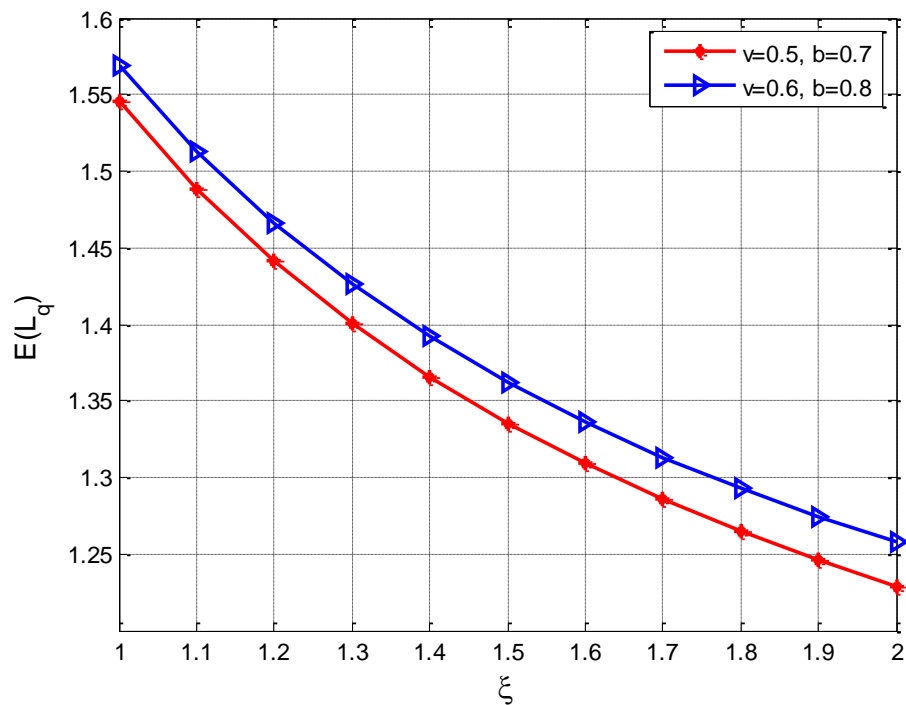


Figure 1. The effect of ξ on $E[L_q]$ for two different set of values of b & v

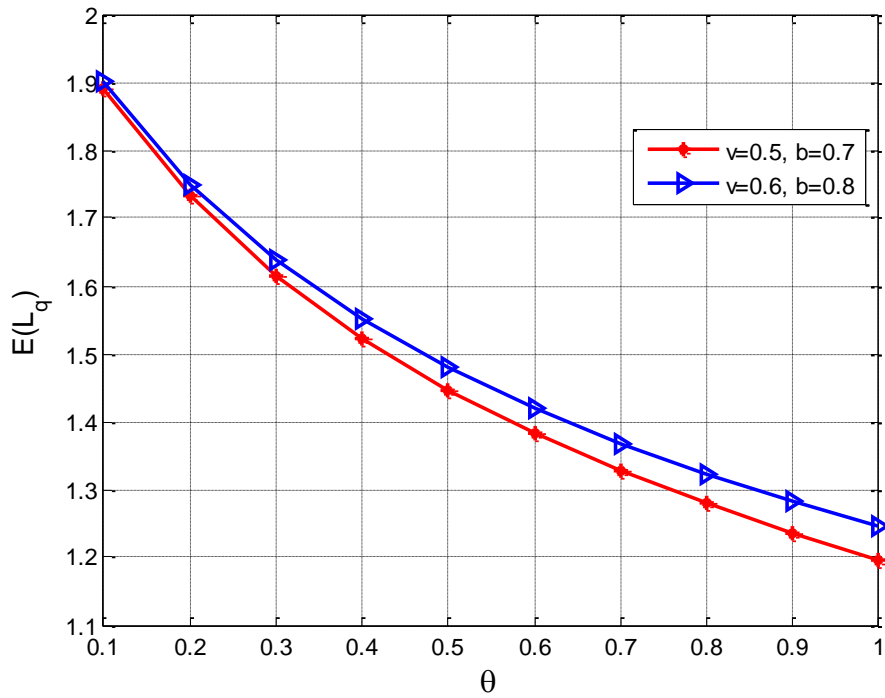


Figure 2. The effect of θ on $E[L_q]$ for two different set of values of b & v

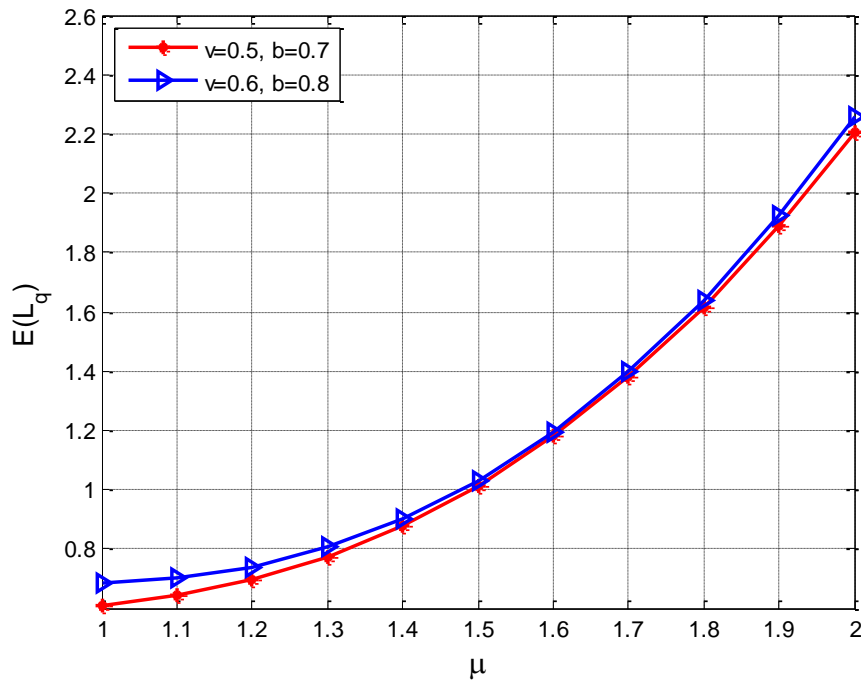


Figure 3. The effect of μ on $E[L_q]$ for two different set of values of b & v

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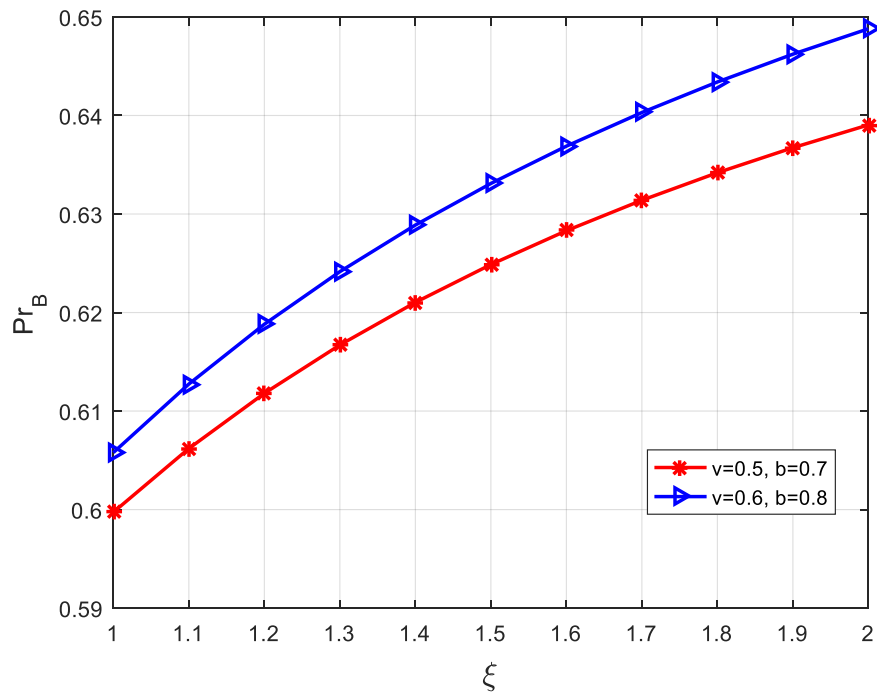


Figure 4. The effect of ξ on Pr_B for different set of values of b and v

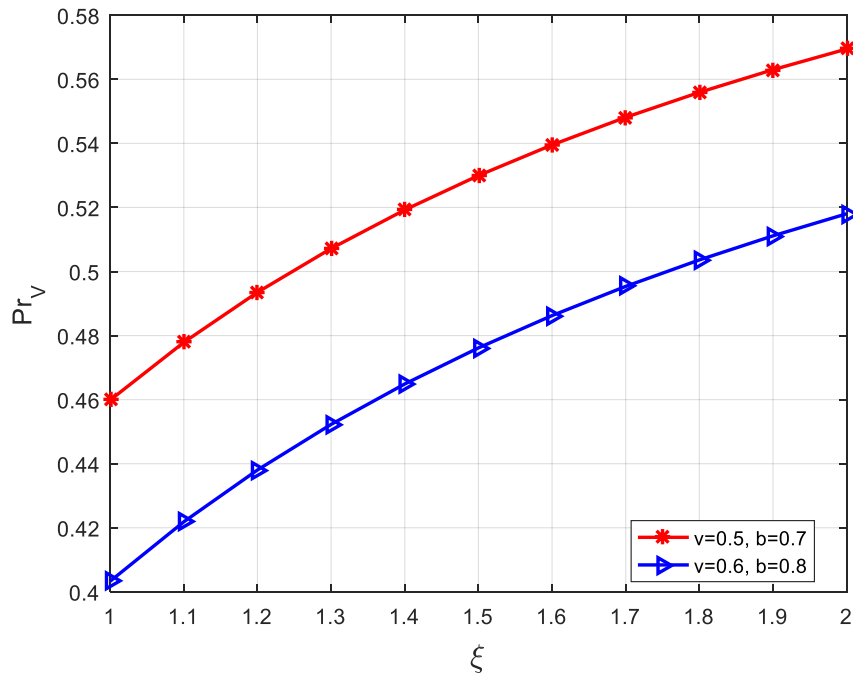


Figure 5. The effect of ξ on Pr_V for different set of values of b and v

b) COST ANALYSIS

In this subsection we formulate an operating cost function and minimize this function with respect to θ to find the optimal value of θ . For this we define some cost elements as

C_{Lq} = Cost per unit time for each customer present in the orbit.

C_{μ} = Cost per unit time for service in normal state.

C_{φ} = Cost per unit time invacation period.

C_{θ} = Cost per unit time for service in working vacation state.

The corresponding cost function per unit time is defined as

$$F(\theta) = E[Lq]C_{Lq} + \mu C_{\mu} + \theta C_{\theta} + \varphi C_{\varphi}$$

In order to find the optimal cost $F(x)$ and corresponding value of x we take $C_{Lq}=26$, $C_{\mu}=42$, $C_{\theta}=29$, $C_{\varphi}=13$ in parabolic method. This method works by generating quadratic function through calculated points in every iterations to which the function $F(x)$ can be approximated. The point at which $F(x)$ is optimum in three point pattern $\{x_1, x_2, x_3\}$ is given by

$$x_L = \frac{0.5(F(x_1)(x_2^2 - x_3^2) + F(x_2)(x_3^2 - x_1^2) + F(x_3)(x_1^2 - x_2^2))}{F(x_1)(x_2 - x_3) + F(x_2)(x_3 - x_1) + F(x_3)(x_1 - x_2)}$$

The new value obtained here replaces one of the three points to improve the current 3 point pattern. This process is used iteratively till optimum value is obtained up to desire degree of accuracy.

Table 1 shows that optimum value $F(\theta) = 131.446$ corresponding to $\theta = 0.2676$ with permissible error of 10^{-3} , which agrees with results of Figure 6.

Table 1. The parabolic method to optimize the cost function

x_1	x_2	x_3	$F(x_1)$	$F(x_2)$	$F(x_3)$	x_L
0.1	0.2	0.4	132.8311	131.6328	131.9832	0.2809
0.2	0.2809	0.4	131.6328	131.4524	131.9832	0.2738
0.2	0.2738	0.2809	131.6328	131.4474	131.4524	0.2685
0.2	0.2685	0.2738	131.6328	131.446	131.4474	0.2679
0.2	0.2679	0.2685	131.6328	131.446	131.446	0.2676
0.2	0.2676	0.2679	131.6328	131.446	131.446	0.2676

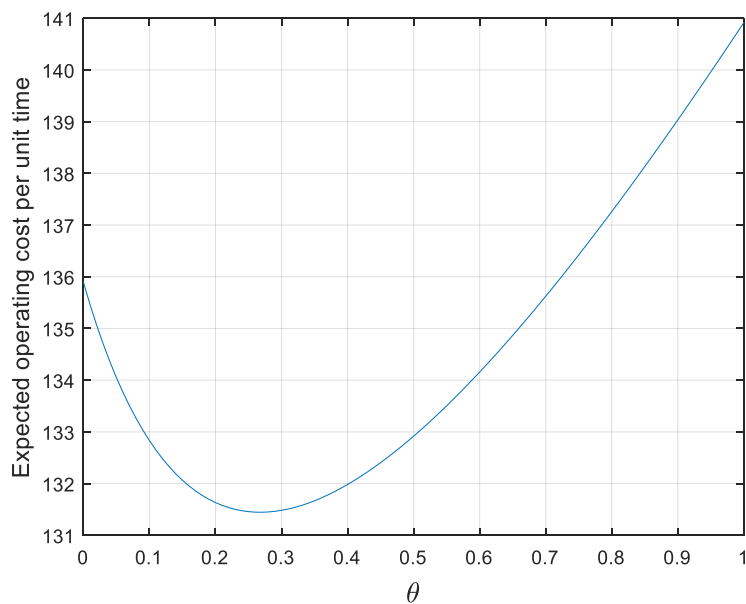


Figure 6. The effect of θ on Expected operating cost per unit time

6. CONCLUSION

The objective of this paper is to analyze the different balking probabilities of customers in busy normal and working vacation states under vacation interruption policy. The explicit formula have been obtained for expected queue length and probabilities of various server states by using probability generating function method. The numerical results obtained show that the model has practical applicability in several real world situations. The numerical results have been interpreted using MATLAB software. Further we have obtained the optimum value of cost with respect to lower service rate in working vacation period using Parabolic Method. This model has applications in various network systems and telecommunications.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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