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ON DISJUNCTIVE DOMINATION NUMBER OF CORONA RELATED GRAPHS

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Abstract. A disjunctive dominating set in a graph G is a set $S \subset V(G)$ such that every vertex $v \in V(G) \setminus S$ is either adjacent to a vertex in S or has at least 2 vertices in S at a distance 2 from it in G. The disjunctive domination number of G, denoted by $\gamma_2^d(G)$, is the minimum cardinality of a disjunctive dominating set in G. In this paper we investigate disjunctive domination number of some corona related graphs.

Keywords: domination; disjunctive domination number; corona of graphs.

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1. INTRODUCTION

Domination in graphs is one of the best tools for understanding situations that can be modeled by networks. A dominating set can be interpreted as a set of processors from which information can be passed on to all the other processors in the network. Hence the determination of the domination parameters is a significant problem. There are many variations of domination. A vertex can exert influence on, or dominate, all vertices not only in its immediate neighborhood, but in a specified distance not too far from it. Such a situation is handled in distance domination introduced by Meir and Moon [11] in 1975. For an integer $k \geq 1$, a distance k-dominating set

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of a connected graph G is a set S of vertices of G such that every vertex of $V(G)$ is at distance at most k from some vertex of S. The distance k-domination number $\gamma_k(G)$ of G is the minimum cardinality of a distance k-dominating set of G. A similar situation also occurs in exponential domination proposed by Dankelmann et al. [2] in 2009.

Motivated by distance domination and exponential domination, Goddard et al. [5] introduced disjunctive domination. A subset S of vertices of G is called a disjunctive dominating set if every vertex in $V \setminus S$ is either adjacent to a vertex in S or has at least 2 vertices in S at a distance 2 from it in G. The disjunctive domination number of G, denoted by $\gamma_2^d(G)$, is the minimum cardinality of a disjunctive dominating set in G. Some properties of disjunctive domination are studied in [8].

The corona of two graphs is defined in [7] and some results on the corona of two graphs are given in [3]. Domination in corona of two graphs is studied in [4] and disjunctive domination in corona of two graphs is studied in [5]. Two variants of corona of graphs are defined in [10] and [6]. Exponential domination of these corona graphs are studied in [1]. In this paper we study disjunctive domination in neighborhood and edge corona of graphs.

1.1. Terminology and Notation.

From the definition of disjunctive domination we note that if S is a disjunctive dominating set and $v \in V$ is not dominated by S, then it has at least 2 vertices in S at a distance 2 from it. In this case we say that $v \in V$ is disjunctively dominated by S.

A universal vertex is a vertex that is adjacent to every other vertex in the graph.

A subdivision of an edge $e = uv$ of a graph G is the replacement of the edge e by a path (u, w, v). The graph obtained from a graph G by subdividing each edge of G exactly once is called the subdivision graph of G and is denoted by $S(G)$.

For all standard terminology and notation we follow [7]. The terms related to the theory of domination in graphs are used as in the sense of Haynes et al. [9]. The graph G considered in this paper is connected and nontrivial and H is an arbitrary graph, unless otherwise specified.

2. Disjunctive Domination in Neighborhood Corona of Graphs

Definition 2.1. [6] *Let G and H be two graphs on n and m vertices respectively. Then the neighborhood corona, $G\star H$ is the graph obtained by taking n copies of H and for each i, making all vertices in the i^{th} copy of H adjacent with the neighbors of $v_i \in G, i = 1,2,...,n$.*

Notation: H_v denotes the copy of H in $G\star H$ corresponding to $v \in G$.

In neighborhood corona $G\star H$ if we consider $H = K_1$ then $G\star H$ becomes a splitting graph. The splitting graph was introduced by Sampathkumar and Walikar [12].

Observation 2.2. *Let $v \in G$ and v' be any vertex in the copy of H corresponding to v. Then, for any $u \neq v$ in G, $d(u,v) = d(u,v')$ in $G\star H$. This follows directly from the definition of neighborhood corona of graphs.*

Theorem 2.3. *If S is a disjunctive dominating set of neighborhood corona of any graph, then for any $v \in S$ there exists $u \in S$ such that $d(u,v) \leq 2$.*

Proof. Let $G_1 = G\star H$, S is a disjunctive dominating set of G_1 and $v \in S$. Choose vertex $v_1 \in G_1$ such that $d(v,v_1) = 2$. Such a vertex always exists in G_1 because if $v \in G$, v_1 can be any vertex in the copy of H corresponding to v and if $v \in H$ then v_1 can be chosen as the vertex on G corresponding to this $v \in H$. Then for the domination or disjunctive domination of v_1 there must be another vertex u in S such that $d(u,v) \leq 2$. □

Observation 2.4. *For any two graphs G and H , $\gamma_2^d(G\star H) \geq 2$*

Theorem 2.5. *If radius of G is less than or equal to 2, then for any graph H, $\gamma_2^d(G\star H) = 2$. In particular, if G has a universal vertex, then for any graph H, $\gamma_2^d(G\star H) = 2$.*

Proof. Radius of $G\star H$ is also 2. Hence $\gamma_2^d(G\star H) > 1$. Let $u \in C(G)$, where $C(G)$ is the center of G. Then $S = \{u,u'\}$, where u' is any vertex in the copy of u, is a disjunctive dominating set of $G\star H$. So $\gamma_2^d(G\star H) = 2$. □

Theorem 2.6. *For any two graphs G and H,*

$$\gamma_2(G) \leq \gamma_2^d(G\star H) \leq 2\gamma_2(G)$$

where $\gamma_2(G)$ is the distance -2 domination number of G.

Proof. Let S be a γ_2^d-set of $G\star H$. Let $S' = (S\cap V(G))\cup\{v\in V(G) : S\cap H_v \neq \phi\}$. Then S' is a distance-2 dominating set of G. Hence $\gamma_2(G) \leq \gamma_2^d(G\star H)$. Now let S be a distance-2 dominating set of G and let S' be a set of vertices formed by taking exactly one vertex from each $H_v, v\in S$. Then $S\cup S'$ is a disjunctive dominating set of $G\star H$. Hence $\gamma_2^d(G\star H) \leq 2\gamma_2(G)$. □

The bounds given in the above theorem are sharp. For example the lower bound is achieved by the family of graphs $\mathscr{G}$ given in figure 1. The upper bound is achieved by the family of graphs $G = S(K_{1,n})$ obtained from $K_{1,n}$ by subdividing each edge once. The case when $n = 2$ and $H = K_1$ is illustrated in figure 2.

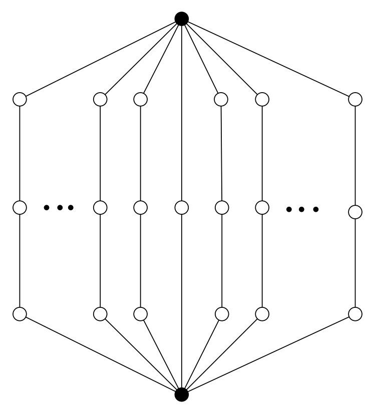
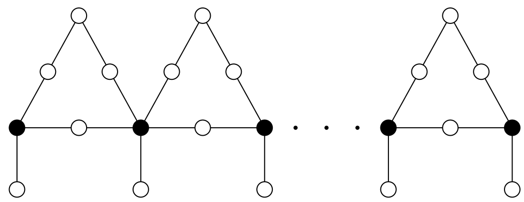


FIGURE 1. A family of graphs $\mathscr{G}$ for which $\gamma_2(\mathscr{G}) = \gamma_2^d(\mathscr{G}\star H)$

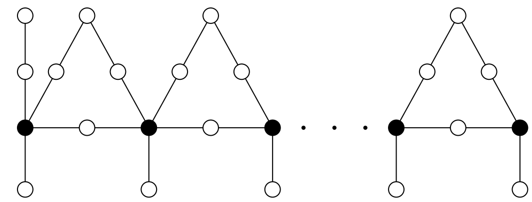
FIGURE 2. A graph G and $G\star K_1$ for which $\gamma_2^d(G\star K_1) = 2\gamma_2(G)$

Observation 2.7. *Corresponding to each positive integer $k \geq 2$, there exists a graph G for which $\gamma_2(G) = k$ and $\gamma_2^d(G\star H) = k+i$ where $0 \leq i \leq k$. This is illustrated in figure 3. If there are $(k-1)$ copies of C_6 in each of the graphs G_0, G_1,..., G_k as shown in figure 3, then $\gamma_2(G_0) = \gamma_2(G_1) = ... = \gamma_2(G_k) = k$ and $\gamma_2^d(G_i\star H) = k+i$ for $0 \leq i \leq k$.*



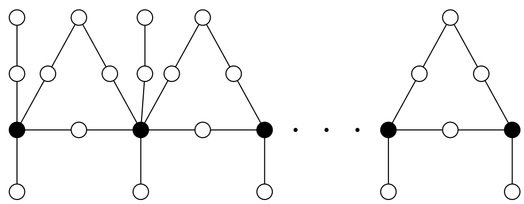
(A) Graph G_0 with $\gamma_2(G_0) = \gamma_2^d(G_0 \star H) = k$

(There are (k-1) copies of C_6, $k \geq 2$)



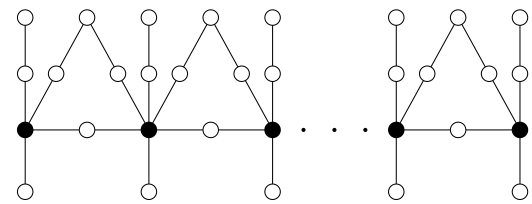
(B) Graph G_1 with

$\gamma_2(G_1) = k$, $\gamma_2^d(G_1 \star H) = k+1$



(C) Graph G_2 with

$\gamma_2(G_2) = k$, $\gamma_2^d(G_2 \star H) = k+2$



(D) Graph G_k with

$\gamma_2(G_k) = k$, $\gamma_2^d(G_k \star H) = 2k$

FIGURE 3. Graphs $G_0, G_1, G_2, ..., G_k$ with $\gamma_2(G_0) = \gamma_2(G_1) = ... = \gamma_2(G_k) = k$ and $\gamma_2^d(G_i \star H) = k+i$ for $0 \leq i \leq k$. In each graph, there are $k-1$ copies of C_6 where $k \geq 2$.

Theorem 2.8. *For any two graphs G and H,*

$$\gamma_2^d(G \star H) \leq 2\gamma_2^d(G).$$

Equality is attained if and only if G has a universal vertex.

Proof. Let S be a γ_2^d-set of G. It can be observed that all vertices in $G \star H$, except the vertices in the copy H_v corresponding to $v \in S$ are dominated or disjunctively dominated by S. Let S' be a set formed by taking exactly one vertex from H_v corresponding to each $v \in S$. Then $S \cup S'$ is a disjunctive dominating set of $G \star H$ and $|S \cup S'| = 2|S|$. Hence $\gamma_2^d(G \star H) \leq 2\gamma_2^d(G)$.

If G has a universal vertex, it follows from theorem 2.5 that

$$\gamma_2^d(G \star H) = 2\gamma_2^d(G) = 2.$$

If G has no universal vertex then, every γ_2^d-set of G must contain at least two vertices. Let S be a γ_2^d-set of G and $u \in S$. Then there exist at least one vertex $v \in S$ such that $d(u,v) \leq 4$.

Case (i) $d(u,v) \leq 2$. Let S be a γ_2^d-set of G and S' be a set formed by taking exactly one vertex from each copy of a vertex in $S \setminus \{u,v\}$. Then $S \cup S'$ia a disjunctive dominating set of $G \star H$ and $|S \cup S'| = 2\gamma_2^d(G) - 2$. Hence $\gamma_2^d(G \star H) < 2\gamma_2^d(G)$.

Case (ii) $3 \leq d(u,v) \leq 4$. Let w be a vertex on the uv-path of G such that $d(u,w)$ and $d(v,w)$ are both less than or equal to 2. If S and S' are sets as in *case.1*, then $S \cup S' \cup \{w\}$ ia a disjunctive dominating set of $G \star H$ of cardinality $2\gamma_2^d(G) - 1$. Hence $\gamma_2^d(G \star H) < 2\gamma_2^d(G)$. □

Remark 2.9. *There are graphs for which* $\gamma_2^d(G \star H) \leq \gamma_2^d(G)$, $\gamma_2^d(G \star H) = \gamma_2^d(G)$ *and* $\gamma_2^d(G \star H) \geq \gamma_2^d(G)$. *Following are some examples for this.*

(1) Disjunctive domination number of Petersen graph is 2 *as realized by any pair of vertices. Disjunctive domination number of neighborhood corona of Petersen graph and any graph H is also* 2. *Thus in this case* $\gamma_2^d(G) = \gamma_2^d(G \star H)$.

(2) Let $G = Q_4$, *the hypercube of dimension* 4 *and H be any other graph. The set* $\{0000, 1111\}$ *of its vertices is a disjunctive dominating set of G. Hence* $\gamma_2^d(G) = 2$. *But no two vertices in* $\gamma_2^d(G \star H)$ *disjunctively dominate all the vertices in it, but the set* $\{0000, 0011, 1111\}$ *is one of its disjunctive dominating set. Hence* $\gamma_2^d(G \star H) = 3$. *In this case,* $\gamma_2^d(G) < \gamma_2^d(G \star H)$.

(3) Let G be a graph obtained by subdividing each edge once of $K_{1,n}$ *where* $n > 2$. *Then* $\gamma_2^d(G) = n > 2$. *But* $\gamma_2^d(G \star H) = 2$ *because the centre vertex together with one vertex in its copy is a disjunctive dominating set of* $G \star H$. *Hence in this case,* $\gamma_2^d(G \star H) < \gamma_2^d(G)$.

Theorem 2.10. *For any two graphs G and H,* $\gamma_2^d(G \star H) \leq \gamma_2^d(G)$ *if G has a* γ_2^d*-set in which corresponding to every* $u \in S$ *there exists* $v \in S$ *such that* $d(u,v) \leq 2$.

Proof. Let S be a γ_2^d-set of G. It can be observed that all vertices in $G \star H$, except the vertices in the copy H_v corresponding to $v \in S$ are dominated or disjunctively dominated by S. All the vertices in H_v are at a distance 2 from v. These vertices are dominated or disjunctively dominated by S if there is another vertex $u \in S$ such that $d(u,v) \leq 2$. Hence if G has such a γ_2^d-set, then it is a disjunctive dominating set of $G \star H$ as well. Thus, $\gamma_2^d(G \star H) \leq \gamma_2^d(G)$. □

Theorem 2.11. *For any graph H and for any positive integer n,*

$$\gamma_2^d(P_n \star H) = 2 \; if \; n = 1,2.$$

and if $n \geq 3$,

$$\gamma_2^d(P_n \star H) = \begin{cases} 2\lceil \frac{n}{5} \rceil - 1 & if\, n \equiv 1,2 \; (mod\; 5) \\ \\ 2\lceil \frac{n}{5} \rceil & otherwise \end{cases}$$

Proof. Let v_i be vertices of P_n and H_{v_i} be the copy of H corresponding to $v_i \in P_n$, where $i \in \{1,2,...,n\}$.

case (i) $n \equiv 0\; (mod\; 5)$. Let $n = 5k$. The set $\{v_2, v_4, v_8, v_{10}, v_{14}, ..., v_i, v_{i+2}, v_{i+6}, ..., v_{5k}\}$ is a disjunctive dominating set if k is even and $\{v_2, v_4, v_8, v_{10}, v_{14}, ..., v_{5k-1}\}$ is a disjunctive dominating set if k is odd. Number of vertices in these sets are $2k$. Hence $\gamma_2^d(P_n \star H) \leq 2k$. The reverse inequality can be seen as follows. Let S be a disjunctive dominating set. In order to dominate or disjunctively dominate the vertex v_1, the set S must contain a vertex from the set $\{v_1, v_2\} \cup H_{v_2}$ or two vertices from $H_{v_1} \cup \{v_3\} \cup H_{v_3}$ respectively. If S dominates the vertex v_1 , then either the vertex v_2 or vertices in H_{v_1} or Hv_2 is at a distance 2 from this vertex. In order to dominate or disjunctively dominate this vertex or vertices another vertex is required in its first or second neighborhood. If S disjunctively dominates v_1, then it must contain at least 2 vertices from the set $H_{v_1} \cup \{v_3\} \cup H_{v_3}$. Thus in order to dominate or disjunctively dominate vertices v_1, v_2 and their copies of H at least two vertices are required from their first or second neighborhood. But these vertices can dominate or disjunctively dominate at the most vertices up to v_5 and their copies. Thus S must contain at least 2 vertices from the first five vertices on P_n or their copies. A similar argument shows that at least 2 vertices are required from every set of five consecutive vertices on P_n and their copies.

Hence $\gamma_2^d(P_n \star H) \geq 2k$.

Thus when $n = 5k$, $\gamma_2^d(P_n \star H) = 2k = 2\lceil \frac{n}{5} \rceil$.

case (ii) $n \equiv 1,2\; (mod\; 5)$. Let $n = 5k+1$ *or* $5k+2$. As in case (i) it can be seen that every disjunctive dominating set must contain at least 2 vertices to dominate or disjunctively dominate any set of 5 consecutive vertices and their copies. Thus $2k$ vertices are needed to dominate or disjunctively dominate any set of $5k$ consecutive vertices and their copies. In order to dominate or disjunctively dominate the remaining one or two vertices and their copies one more vertex is required. Hence $\gamma_2^d(P_n \star H) \geq 2k+1$. On the other hand the set $\{v_2, v_4, v_8, v_{10}, v_{14}, ..., v_{5k}, v_{5k+1}\}$ is a disjunctive dominating set of cardinality $2k+1$. Thus $\gamma_2^d(P_n \star H) \leq 2k+1$.

Thus when $n = 5k+1$ *or* $5k+2$, $\gamma_2^d(P_n \star H) = 2k+1 = 2\lceil \frac{n}{5} \rceil - 1$.

case (iii) $n \equiv 3,4$ $(mod\ 5)$. Let $n = 5k+3$ *or* $5k+4$. As in the above cases it can be proved that $2k+2$ vertices are necessary and sufficient to dominate or disjunctively dominate all the vertices in $P_n \star H$, giving the result $\gamma_2^d(P_n \star H) = 2k+2 = 2\lceil \frac{n}{5} \rceil$.

By summing up the cases (i), (ii) and (iii) we get the theorem. ☐

Theorem 2.12. *For any positive integer $n \geq 3$ and for any graph H,*

$$\gamma_2^d(C_n \star H) = \begin{cases} 2\lceil \frac{n}{6} \rceil - 1 & \text{if } n \equiv 1,2\ (mod\ 6) \\ \\ 2\lceil \frac{n}{6} \rceil & \text{otherwise} \end{cases}$$

Proof. Let $\{v_1, v_2, v_3, ..., v_n\}$ be the vertices of C_n and H_{v_i} be the copy of H corresponding to $v_i \in C_n$, where $i \in \{1, 2, ..., n\}$.

case (i) $n \equiv 0$ $(mod\ 6)$. Let $n = 6k$. The set $\{v_1, v_3, v_7, v_9, v_{11}, v_{15}, ..., v_i, v_{i+2}, v_{i+6}, ..., v_{n-3}\}$ is a disjunctive dominating set of order $2k$. Hence $\gamma_2^d(C_n \star H) \leq 2k$. The reverse inequality can be seen as follows. Let S be a disjunctive dominating set. Consider an arbitrary vertex v_i on C_n. Then S must contain v_i or one of its neighbors to dominate it or two vertices in its second neighborhood to dominate it disjunctively. If v_i is dominated by a vertex in S, then there is at least one vertex in the neighborhood of v_i or copy of v_i or its neighbor which is at a distance 2 from this vertex in S. S must contain at least one more vertex from its first or second neighborhood for the disjunctive domination of this vertex . But these two vertices in S together can dominate or disjunctively dominate at most 6 consecutive vertices on C_n and their copies. Thus S must contain at least 2 vertices from every set of 6 consecutive vertices on C_n and their copies. Hence $\gamma_2^d(C_n \star H) \geq 2k$.

Thus when $n = 6k$, $\gamma_2^d(C_n \star H) = 2k = 2\lceil \frac{n}{6} \rceil$.

case (ii) $n \equiv 1,2 \ (mod\ 6)$. Let $n = 6k+1$ *or* $6k+2$. As in case (i) it can be seen that every disjunctive dominating set must contain at least 2 vertices to dominate or disjunctively dominate any set of 6 consecutive vertices and their copies. Thus $2k$ vertices are needed to dominate or disjunctively dominate any set of $6k$ consecutive vertices and their copies. In order to dominate or disjunctively dominate the remaining one or two vertices and their copies one more vertex is required. Hence $\gamma_2^d(C_n \star H) \geq 2k+1$. On the other hand the set $\{v_1, v_3, v_7, v_9, v_{11}, v_{15}, \ldots, v_i, v_{i+2}, v_{i+6}, \ldots, v_{6k-3}, v_{6k+1}\}$ is a disjunctive dominating set of cardinality $2k+1$. Thus $\gamma_2^d(C_n \star H) \leq 2k+1$.

Thus when $n = 6k+1$ *or* $6k+2$, $\gamma_2^d(C_n \star H) = 2k+1 = 2\lceil \frac{n}{6} \rceil - 1$.

case (iii) $n \equiv 3,4,5 \ (mod\ 6)$. Let $n = 6k+3$ *or* $6k+4$ *or* $6k+5$. As in the above cases it can be proved that $2k+2$ vertices are necessary and sufficient to dominate or disjunctively dominate all the vertices in $C_n \star H$, giving the result $\gamma_2^d(C_n \star H) = 2k+2 = 2\lceil \frac{n}{6} \rceil$.

By summing up the cases (i), (ii) and (iii) we get the theorem. □

Theorem 2.13. *$\gamma_2^d(G \star H) = 2$ for $G \cong K_n,\ K_{1,n},\ W_{1,n}$ where n is a positive integer greater than 3 for $W_{1,n}$.*

Proof. Since these graphs have a universal vertex it follows from theorem 2.5. □

Theorem 2.14. *For all positive integers m,n, $\gamma_2^d(K_{m,n} \star H) = 2$.*

Proof. Let $U = \{u_1, u_2, \ldots, u_m\}$ and $V = \{v_1, .v_2, \ldots, v_n\}$ be the partition of the vertex set of $K_{m,n}$. Any one vertex in U dominates all the vertices in V and the vertices in their copies. Similarly an arbitrary vertex in V dominates all the vertices in U and the vertices in their copies. Thus a γ_2^d-set contains exactly two vertices. Hence, $\gamma_2^d(K_{m,n} \star H) = 2$. □

3. DISJUNCTIVE DOMINATION IN EDGE CORONA OF GRAPHS

Definition 3.1. [10] *Let G and H be two graphs on disjoint sets of n_1 and n_2 vertices, m_1 and m_2 edges respectively. The edge corona $G \diamond H$ of G and H is the graph obtained by taking one copy of G and m_1 copies of H and then joining two end-vertices of the i^{th} edge of G to every vertex in the i^th copy of H.*

Theorem 3.2. *For any nontrivial graph G and a graph H,*

$$\gamma_2^d(G) \le \gamma_2^d(G \diamond H).$$

Proof. Let S be a γ_2^d-set of $G \diamond H$. Let H_e denote the copy of H corresponding to an edge $e \in E(G)$. Let the set S' be formed such that it contains one of the incident vertices of each edge $e \in E(G)$ for which $|S \cap H_e| = 1$ and both the incident vertices if $|S \cap H_e| \ge 2$. Let $D = (S \cap V(G)) \cup S'$. Then $D \subset V(G)$. S is a disjunctive dominating set of $G \diamond H$ and $d(w,D) \le d(w,S)$ for any vertex $w \in V(G)$ shows that D is a disjunctive dominating set of G. Hence $\gamma_2^d(G) \le \gamma_2^d(G \diamond H)$. □

Note. It may be noted that $\gamma_2^d(G \diamond H)$ can be much larger than $\gamma_2^d(G)$. For example if G is the friendship graph F_n, which is constructed by joining n copies of C_3 with a common vertex, then $\gamma_2^d(G) = 1$ whereas $\gamma_2^d(G \diamond H) = n$. The case when $n = 4$ is illustrated in figure 4.

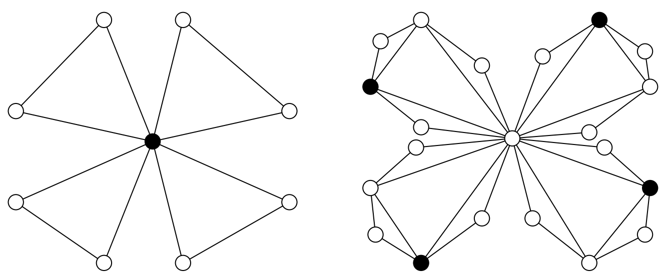


FIGURE 4. $\gamma_2^d(F_4) = 1$ but $\gamma_2^d(F_4 \diamond K_1) = 4$

Theorem 3.3. *For every positive integer $n > 1$, $\gamma_2^d(P_n \diamond H) = \lceil \frac{n}{3} \rceil$*

Proof. Let $v_1, v_2, v_3, \ldots v_n$ be the vertices and $e_1, e_2, e_3, \ldots, e_{n-1}$ be the edges of P_n. Let $H_1, H_2, \ldots H_{n-1}$ be the copies of H corresponding to the edges of P_n.

case (i) $n \equiv 0$ $(mod\ 3)$. Let $n = 3k$. It is obvious that the set $\{v_2, v_5, v_8, \ldots v_{n-1}\}$ is a disjunctive dominating set of cardinality k.

Hence $\gamma_2^d(P_n \diamond H) \le k$.

For the reverse inequality let S be a disjunctive dominating $P_n \diamond H$. In order to dominate or disjunctively dominate the vertex v_1, the set S must contain at least one vertex from $\{v_1, v_2\} \cup H_1$ or two vertices from $\{v_3\} \cup H_2$ respectively. Thus S must contain at least one vertex from first three vertices or from the copies of H corresponding to the edges between these vertices. If v_1 is dominated by a vertex in S, this vertex can dominate at most vertices up to v_3. Hence to dominate or disjunctively dominate vertices in H_3, S must contain at least one more vertex from $\{v_3, v_4, v_5\} \cup H_5$. If v_1 is disjunctively dominated by two vertices in S, then those two vertices can dominate or disjunctively dominate vertices upto v_4. Hence to dominate or disjunctively dominate H_4, another vertex is required in its first or second neighborhood. It can be observed that at least one vertex is required from every set of three consecutive vertices or from the copies of H corresponding to the edges between these vertices. Hence, $\gamma_2^d(P_n \diamond H) \geq k$

Thus when $n = 3k$, $\gamma_2^d(P_n \diamond H) = k = \lceil \frac{n}{3} \rceil$.

case (ii) $n \equiv 1,2$ $(mod\ 3)$. Let $n = 3k+1$ *or* $3k+2$. In this case $\{v_2, v_5, v_8, ...v_{3k-1}, v_{3k+1}\}$ is a disjunctive dominating set of cardinality $k+1$.

Hence $\gamma_2^d(P_n \diamond H) \leq k+1$.

The reverse inequality can be seen as follows. Let S be an arbitrary disjunctive dominating set. As before it can be seen that at least one vertex is required from every set of three consecutive vertices or from the copies of H corresponding to the edges between these vertices. Hence, $\gamma_2^d(P_n \diamond H) \geq k+1$

Thus when $n = 3k+1$ *or* $3k+2$, $\gamma_2^d(P_n \diamond H) = k+1 = \lceil \frac{n}{3} \rceil$. By summing up the cases (i) and (ii) we get the theorem. □

Theorem 3.4. *For every positive integer $n > 3$,*

$$\gamma_2^d(C_n \diamond H) = \lceil \frac{n}{3} \rceil$$

.

Proof. The proof is similar to the proof of $\gamma_2^d(P_n \diamond H)$. □

Theorem 3.5. *For every positive integer $n \geq 3$,*

$$\gamma_2^d(K_n \diamond H) = 2.$$

Proof. Let u be an arbitrary vertex in K_n. It dominates all the vertices in K_n and the copies of H corresponding to the edges incident with u. Let e be an edge which is not incident with u and let H_e be the copy of H corresponding to e. Vertices in H_e are at a distance 2 from u. Let $v \neq u$ be any other vertex in K_n. Then H_e is dominated or disjunctively dominated by $\{u,v\}$, i.e, it is a γ_2^d-set of $K_n \diamond H$. Hence, $\gamma_2^d(K_n \diamond H) = 2$. □

Theorem 3.6. *For $m,n \geq 2$,*

$$\gamma_2^d(K_{m,n} \diamond H) = 2.$$

Proof. Any two vertices in $K_{m,n}$ dominates or disjunctively dominates all the vertices in $K_{m,n}$ as well as the vertices in the copies of H corresponding to its edges. It is also obvious that a single vertex cannot dominate all the vertices. Hence, $\gamma_2^d(K_{m,n} \diamond H) = 2$. □

Theorem 3.7. *If $W_{1,n}$ is the wheel graph on $n+1$ vertices, then*

$$\gamma_2^d(W_{1,n} \diamond H) = \lceil \frac{n}{4} \rceil + 1$$

Proof. Let u be the center of the wheel. It dominates all the vertices in $W_{1,n}$ and the vertices in all the copies of H corresponding to the edges of $W_{1,n}$ incident at u. Let V' denote the vertices in the copies of H corresponding to the edges not incident at the center u. All the vertices in V' are at a distance 2 from u. For the disjunctive domination of these vertices, each vertex in V' needs at least one more vertex at a distance 2 from it. Since at least $\lceil \frac{n}{4} \rceil + 1$ vertices are required for this $\gamma_2^d(W_{1,n} \diamond H) \geq \lceil \frac{n}{4} \rceil + 1$. Since $S = \{v_1, v_5, v_9, ..., v_{4k+1}\}$ of the vertices on the rim of the wheel is such a set, we get $\{u\} \cup S$ is a disjunctive dominating set of $W_{1,n} \diamond H$. Hence $\gamma_2^d(W_{1,n} \diamond H) \leq \lceil \frac{n}{4} \rceil + 1$. Thus,

$$\gamma_2^d(W_{1,n} \diamond H) = \lceil \frac{n}{4} \rceil + 1$$

□

Theorem 3.8. $\gamma_2^d(G \diamond H) = 1$ *if and only if* $G = K_{1,n}$.

Proof. Let $G = K_{1,n}$. The center vertex of $K_{1,n}$ dominates all the vertices in $K_{1,n} \diamond H$. Hence, $\gamma_2^d(K_{1,n} \diamond H) = 1$. Conversely let $\gamma_2^d(G \diamond H) = 1$. This is possible if and only if all the edges of G are incident at a single vertex, i.e, $G = K_{1,n}$. □

4. Conclusion

In this paper we have made an attempt to study the properties of disjunctive domination of neighborhood and edge corona of graphs. We also found the disjunctive domination number of neighborhood and edge corona of some standard classes of graphs. Such studies are significant because if a larger network can be broken into smaller networks, the solution of smaller ones can be used to find that of larger one. It will be interesting to study the impact of other graph operations on disjunctive domination.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] B. Atay, A. Aytac, On exponential domination and graph operations, Int. J. Nonlinear Anal. Appl. 8 (2017), 243-250.

[2] P. Dankelmann, D. Day, D. Erwin, S. Mukwembi, H. Swart, Domination with exponential decay, Discrete Math. 309 (2009), 5877–5883.

[3] F. Harary, R. Frucht, On the corona of two graphs, Aequationes Math. 4 (1970), 322-325.

[4] C.E. Go, S.R. Canoy, Domination in the corona and join of graphs, Int. Math. Forum, 6 (2011), 763-771.

[5] W. Goddard, M.A. Henning, C.A. McPillan, The disjunctive domination number of a graph, Quaest. Math. 37 (2014), 547–561.

[6] I. Gopalapillai, The spectrum of neighborhood corona of graphs, Kragujevac J. Math. 35(3) (2011), 493–500.

[7] F. Harary, Graph Theory, Addison-Wesley, Reading, MA, 1969.

[8] M.A. Henning, S.A. Marcon, Domination versus disjunctive domination in graphs, Quaest. Math. 39 (2016), 261–273.

[9] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, Fundamentals of domination in graphs. Marcel Dekker Inc., New York, 1998.

[10] Y. Hou, W.-C. Shiu, The spectrum of the edge corona of two graphs, Electron. J. Linear Algebra, 20 (2010), 586-594.

[11] A. Meir, J. Moon, Relations between packing and covering numbers of a tree, Pac. J. Math. 61 (1975), 225–233.

[12] E. Sampathkumar, H.B. Walikar, On the splitting graph of a graph, J. Karnatak Univ. Sci. 35/36 (1980-1981), 13-16.