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CERTAIN PROMINENCE OF PARTIALLY $Q\alpha$-COMPACT FUZZY SETS

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Abstract: In this paper, we define notions of partially $Q\alpha$-cover, shortly $pQ\alpha$-cover and partially $Q\alpha$-compact, shortly $pQ\alpha$-compact fuzzy sets. Also, we established some theorems about our notion in fuzzy topological spaces, fuzzy subspaces, fuzzy T_1-spaces and fuzzy T_2-spaces (Hausdorff spaces) and various prominence in fuzzy topological spaces. Finally, we shown that our notion does not satisfies "good extension" property.

Keywords: fuzzy set; fuzzy topological spaces; $pQ\alpha$-cover; $pQ\alpha$-compact.

2010 AMS Subject Classification: 03E72.

1. INTRODUCTION

The opinion of fuzzy set and operations of fuzzy sets were first presented in 1965 by Zadeh [35], reciting for the first time fuzziness mathematically. Chang[4] the notion of fuzzy compactness was first introduced by using open cover and Wong[32] develop the theory of fuzzy topological spaces by the sense of Chang[4] and discussed various covering properties of these spaces. Lowen introduced fuzzy compactness, strong fuzzy compactness and ultra fuzzy compactness

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[16, 18] and also generalized Lowen's compactness by Chadwick [3]. Also Lowen discussed compact product fuzzy topological spaces and compactness in fuzzy real line [17, 19]. In the fuzzifying topological spaces, Ying [33, 34] introduced the notion of fuzzifying compactness. Guojun[9] introduced N-compactness and Dongsheng[5] generalize the N-compactness, Shi [28] represented a new definition of fuzzy compactness in L-fuzzy topological spaces and Li et al. [14] introduced the notion of degree of fuzzy compactness in L-fuzzy topological spaces. Hong [10] introduced RS-compactness and several properties presented on RS-compactness by Noiri [25]. Also Kudri and Warner [13] and Shafei et al. [26] studied characterizations of RS-compactness. Georgiou and Papadopoulos [7] and Warner [31] discussed about fuzzy compactness. Moreover, Gantner et al. [6] introduced α -compactness and Benchalli et al. [2] established relation between α -compact fuzzy topological spaces and ordinary topological spaces. Zhongfu [36] introduced $Q\alpha$ -compactness and fuzzy strong Q-compactness with the concept of Q-neighborhoods convergence of nets. Ghosh [8] introduced q-compactness, Ali et al. [1] introduced Q-compact fuzzy sets in different form with the concept of q-coincident and Talukder et al. [29] defined $Q\alpha$ -compact fuzzy set in another notion. Warner and McLean [30] presented a good definition of compactness and kudri [12] gave the extension of this notion and studied its properties. Mahbub et al. [20, 21] introduced two notions of compactness and Q-compactness in intuitionistic fuzzy topological spaces. Recently, Mattam and Gopalan [22] presented fuzzy soft relation based on cartesian product of fuzzy soft sets and the notion of fuzzy soft equivalence was introduced. Shakila and Selvi [27] introduced fuzzy soft paraopen sets and paraclosed sets in fuzzy soft topological spaces and discussed their some basic properties. The purpose about this paper, we present and discussion the notion of $pQ\alpha$ -compact fuzzy sets and obtain various prominence of the notion. We search that this notion has various apparent results.

2. PRELIMINARIES

To this segment, we remember several essential definitions which are requirement for the subsequent segment. These are needed in our discussion and can be set up to the papers referred to.

Definition 2.1[35]: Suppose W is a non-empty set and $I = [0, 1]$ is the unit closed interval. A fuzzy set in W is a function $a : W \rightarrow I$ which assigns to each component $p \in W$. $a(p)$ indicates a grade or the degree of membership of p . I^W is indicated by all fuzzy sets in W . Any member of I^W may also is mentioned as fuzzy subset of W .

Definition 2.2[35]: A fuzzy set is mentioned as empty fuzzy set iff its grade of membership is identically 0 . It is represented by 0_W .

Definition 2.3[35]: A fuzzy set is mentioned as whole fuzzy set iff its grade of membership is identically 1. It is identified by 1_W .

Definition 2.4[35]: Suppose a and b are two fuzzy sets in W . We prescribe

1) $a = b$ iff $a(p) = b(p)$ for every $p \in W$

2) $a \subseteq b$ iff $a(p) \leq b(p)$ for every $p \in W$

3) $\sigma = a \cup b$ iff $\sigma(p) = (a \cup b)(p) = \max[a(p), b(p)]$ for every $p \in W$

4) $\eta = a \cap b$ iff $\eta(p) = (a \cap b)(p) = \min[a(p), b(p)]$ for every $p \in W$

5) $\gamma = a^c$ iff $\gamma(p) = 1 - a(p)$ for every $p \in W$ and we declare that a^c is complement of a .

Note: Two fuzzy sets a and b are disjoint iff $a \cap b = 0$.

De-Morgan's laws 2.5[35]: De-Morgan's Laws effective for fuzzy sets in W i.e. when a and b are any fuzzy sets in W , then

1) $1 - (a \cup b) = (1 - a) \cap (1 - b)$

2) $1 - (a \cap b) = (1 - a) \cup (1 - b)$

For every fuzzy set in a in W , $a \cap (1 - a)$ not mandatory be 0 (zero) and $a \cup (1 - a)$ not mandatory be 1(one).

Distributive laws 2.6[35]: Distributive laws stay effective for fuzzy sets in W i.e. when a , b and d are fuzzy sets in W , so

1) $a \cup (b \cap d) = (a \cup b) \cap (a \cup d)$

2) $a \cap (b \cup d) = (a \cap b) \cup (a \cap d)$.

Definition 2.7[23]: Suppose a is a fuzzy set in W , so the set { $p \in W$: $a(p) > 0$ } is mentioned as the support of a and is denoted by a_0 .

Definition 2.8[15]: Suppose W is a non-empty set and G is a collection of subsets of W . Then G is mentioned as topology on W if

1) ϕ , $W \in G$

2) if $P_i \in G$ for each $i \in J$, then $\bigcup_{i \in J} P_i \in G$

3) if P , $Q \in G \Rightarrow P \cap Q \in G$

The pair (W, G) is mentioned as topological space and any part $U \in G$ is mentioned as open set in the topology G .

Definition 2.9[15]: A subset M of a topological space (W, G) is compact iff every open cover of M has a finite subcover.

Definition 2.10[4]: Suppose W is a non-empty set and $g \subseteq I^W$ i.e. g is a family of fuzzy sets in W . Then g is mentioned as fuzzy topology on W if

1) 0 , $1 \in g$

2) if $a_i \in g$ for each $i \in J$, then $\bigcup_{i \in J} a_i \in g$

3) if a , $b \in g$, then $a \cap b \in g$

The pair (W, g) is mentioned as fuzzy topological space and shortly, fts. All members in g are mentioned as g -open fuzzy set. A fuzzy set is mentioned as g -closed iff its complements is g -open. When there is no confusion is likely to arise, we shall name a g -open (g -closed) fuzzy set simply an open (closed) fuzzy set.

Definition 2.11[23]: Suppose (W, g) is an fts and $M \subseteq W$. Then the gathering $g_M = \{ a \mid M : a \in g \}$ is also fuzzy topology on M, mentioned as the subspace fuzzy topology on M and the pair (M, g_M) is mentioned as fuzzy subspace of (W, g).

Definition 2.12[11]: Suppose (W, g) is an fts. Then (W, g) is mentioned as fuzzy T_1-space if and only if for every p, $q \in W$, $p \neq q$, there exist a, $b \in g$ such that $a(p) > 0$, $a(q) = 0$ and $b(p) = 0$, $b(q) > 0$.

Definition 2.13[11]: Suppose (W, g) is an fts. Then (W, g) is called fuzzy T_2 -space (Hausdorff space) if and only if for every p, $q \in W$, $p \neq q$, there exist a, $b \in g$ such that $a(p) > 0$, $b(q) > 0$ and $a \cap b = 0$.

Definition 2.14[16]: Suppose (W, G) is a topological space. A function $h : W \to \mathbf{R}$ (with usual topology) is mentioned lower semi-continuous (l. s. c.) if for each $u \in \mathbf{R}$, the set $h^{-1}(u, \infty) \in g$. In a topology G on a set W, suppose $\omega(G)$ is the set of all l. s. c. functions from (W, G) to I (with usual topology); so $\omega(G) = \{ a \in I^W : a^{-1}(u, 1] \in G , u \in I_1 \}$. It is shows that $\omega(G)$ is a fuzzy topology on W.

Let P be a property of topological spaces and FP be its fuzzy topology analogue. Then FP is called a ‘good extension’ of P “iff the statement (W, G) has P iff $(W, \omega (G))$ has FP” holds good for every topological space (W, G). Thus characteristic functions are l . s. c.

3. MAIN OUTCOMES

In this segment, we introduced the notion of $PQ\alpha$ -compact fuzzy sets and set up certain apparent prominence of $PQ\alpha$ -compact fuzzy sets.

Definition 3.1: Suppose (W, g) is an fts, σ is a fuzzy set in W and $0 < \alpha \leq 1$. Consider $E = \{ a_i : i \in J \} \subseteq I^W$ is a collection of fuzzy sets. Then $E = \{ a_i : i \in J \}$ is mentioned as partially $Q\alpha$ -cover, shortly $PQ\alpha$ -cover of σ iff $\sigma(p) + a_i(p) \geq \alpha$ for every $p \in \sigma_0 (\sigma_0 \neq W)$

and for some $a_i \in E$. When every a_i is open, then E is mentioned as an open $PQ\alpha$-cover of σ in (W, g). A sub collection of $PQ\alpha$-cover of σ in W which is also a $PQ\alpha$-cover of σ is mentioned as $PQ\alpha$-subcover of σ.

When $\sigma(p) \neq 0$ for every $p \in W$ i.e. $\sigma_0 = W$, then $PQ\alpha$-cover and $Q\alpha$-cover in [29] will be same and also for Q-cover in [1], when $\sigma_0 = W$ and $\alpha = 1$.

Example 3.2: Let $W = \{u, v, w\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Suppose a_1, $a_2 \in I^W$ defined with $a_1(u) = 0.05$, $a_1(v) = 0.02$, $a_1(w) = 0.01$ and $a_2(u) = 0.01$, $a_2(v) = 0.04$, $a_2(w) = 0.03$. Again, consider $\sigma \in I^W$ with $\sigma(u) = 0.04$, $\sigma(v) = 0$, $\sigma(w) = 0.06$, so $\sigma_0 = \{u, w\}$. Choose $\alpha = 0.08$. Thus we observe that $\sigma(u) + a_1(u) > \alpha$, $\sigma(w) + a_2(w) > \alpha$, where $u, w \in \sigma_0$. Hence $\{ a_1, a_2 \}$ is a $PQ\alpha$-cover of σ.

Example 3.3: Consider $W = \{u, v, w\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Suppose a_1, a_2, $a_3 \in I^X$ defined by $a_1(u) = 1$, $a_1(v) = 0$, $a_1(w) = 0.03$; $a_2(u) = 0$, $a_2(v) = 1$, $a_2(w) = 1$ and $a_3(u) = 0$, $a_3(v) = 0$, $a_3(w) = 0.03$. Put $g = \{ 0, a_1, a_2, a_3, 1 \}$, so (W, g) is an fts. Freshly, let $\sigma \in I^X$ be defined by $\sigma(u) = 0.02$, $\sigma(v) = 0$, $\sigma(w) = 0.03$, so $\sigma_0 = \{u, w\}$. Select $\alpha = 0.07$. Thus we see that $\sigma(u) + a_1(u) > \alpha$, $\sigma(w) + a_2(w) > \alpha$, where $u, w \in \sigma_0$. Hence $\{ a_1, a_2 \}$ is an open $PQ\alpha$-cover of σ in (W, g).

Definition 3.4: Suppose (W, g) is an fts and $0 < \alpha \leq 1$. A fuzzy set σ in W is mentioned as $PQ\alpha$-compact if and only if every open $PQ\alpha$-cover of σ has a finite $PQ\alpha$-subcover i.e. there exist a_{i_1}, a_{i_2}, ….. , $a_{i_n} \in \{a_i\}$ such that $\sigma(p) + a_{i_k}(p) \geq \alpha$ for every $p \in \sigma_0$. When $\sigma \subseteq \eta$ and $\eta \in I^W$, then η is also $PQ\alpha$-compact i.e. all super sets of $PQ\alpha$-compact fuzzy set is also $PQ\alpha$-compact.

The ideas of some theorems are taken from [15, 24].

Theorem 3.5: Suppose (W, g) is an fts, $M \subset W$, σ is a fuzzy set in W, where $\sigma_0 \subset M$ and $0 < \alpha \leq 1$. Then σ is $PQ\alpha$-compact in (W, g) if and only if σ is $PQ\alpha$-compact in (M, g_M).

Proof: Assume σ is $PQ\alpha$-compact in (W, g). Consider $\{ a_i : i \in J \}$ is an open $PQ\alpha$-cover of σ in (M, g_M). So there exist $b_i \in g$ such that $a_i = b_i \mid M \subseteq b_i$. Thus $\sigma(p) + a_i(p) \geq \alpha$ for each $p \in \sigma_0$ and consequently $\sigma(p) + b_i(p) \geq \alpha$ for each $p \in \sigma_0$. Hence $\{ b_i : i \in J \}$ is an open $PQ\alpha$-cover of σ in (W, g). But σ is $PQ\alpha$-compact in (W, g), so σ has finite $PQ\alpha$-subcover i.e. there exist $b_{i_k} \in \{b_i\}$ ($k \in J_n$) such that $\sigma(p) + b_{i_k}(p) \geq \alpha$ for each $p \in \sigma_0$. But, then $\sigma(p) + (b_{i_k} \mid M)(p) \geq \alpha$ for each $p \in \sigma_0$ and therefore $\sigma(p) + a_{i_k}(p) \geq \alpha$ for each $p \in \sigma_0$. Thus $\{a_i\}$ holds a finite $PQ\alpha$-subcover $\{ a_{i_1}, a_{i_2}, \ldots.., a_{i_n} \}$ and thus σ is $PQ\alpha$-compact in (M, g_M).

Conversely, assume σ is $PQ\alpha$-compact in (M, g_M). Permit $\{ b_i : i \in J \}$ is an open $PQ\alpha$-cover of σ in (W, g). Put $a_i = b_i \mid M$, so $\sigma(p) + b_i(p) \geq \alpha$ for each $p \in \sigma_0$ and hence $\sigma(p) + (b_i \mid M)(p) \geq \alpha$ for each $p \in \sigma_0$ which implies that $\sigma(p) + a_i(p) \geq \alpha$ for each $p \in \sigma_0$. But $a_i \in g_M$, so $\{ a_i : i \in J \}$ is an open $PQ\alpha$-cover of σ in (M, g_M). Since σ is $PQ\alpha$-compact in (M, g_M), then there exist $a_{i_k} \in \{a_i\}$ ($k \in J_n$) such that $\sigma(p) + a_{i_k}(p) \geq \alpha$ for each $p \in \sigma_0$. Thus we have $\sigma(p) + (b_{i_k} \mid M)(p) \geq \alpha$ for each $p \in \sigma_0$ and consequently $\sigma(p) + b_{i_k}(p) \geq \alpha$ for each $p \in \sigma_0$. Thus $\{b_i\}$ contains a finite $PQ\alpha$-subcover $\{ b_{i_1}, b_{i_2}, \ldots.., b_{i_n} \}$ and therefore σ is $PQ\alpha$-compact in (W, g).

Corollary 3.6: Consider (V, g^*) is a fuzzy subspace of (W, g) and $M \subset V \subset W$. Suppose σ is a fuzzy set in M with $\sigma_0 \subset M$. So σ is $PQ\alpha$-compact in (W, g) if and only if σ is $PQ\alpha$-compact in (V, g^*).

Proof: Assume g_M and g_M^* are the subspace fuzzy topologies on M. So by preceding theorem (3.5), σ is $PQ\alpha$-compact in (W, g) or (V, g^*) if and only if σ is $PQ\alpha$-compact in (M, g_M) or (M, g_M^*). However $g_M = g_M^*$.

Theorem 3.7: Suppose (W, g) is an fts and σ is a $PQ\alpha$-compact fuzzy set in W. When $\eta \subseteq \sigma$ and $\eta \in g^c$, then η is also $PQ\alpha$-compact in (W, g).

Proof: Suppose $\{ a_i : i \in J \}$ is an open $PQ\alpha$-cover of η. At the moment $\{a_i\} \cup \eta^c$ is an open $PQ\alpha$ -cover of σ. But $\eta(p) + a_i(p) \geq \alpha$ for every $p \in \eta_0$, so $\sigma(p) + \max(a_i(p), \eta^c(p)) \geq \alpha$ for each $p \in \sigma_0$. Therefore $\eta(p) + a_i(p) \leq \sigma(p) + a_i(p) \geq \alpha$ for each $p \in \eta_0, \sigma_0$. As σ is $PQ\alpha$-compact in (W, g), so every open $PQ\alpha$-cover of σ has a finite $PQ\alpha$-subcover i.e. there exist a finite subset $J_n \subset J$ such that $\{ a_{i_k} : k \in J_n \} \cup \eta^c$ is an open $PQ\alpha$-cover of σ. So $\{ a_{i_k} : k \in J_n \}$ is a finite subfamily of $\{ a_i : i \in J \}$ and hence is an open $PQ\alpha$-cover of η i.e. $\{ a_{i_k} : k \in J_n \}$ is a finite $PQ\alpha$-subcover of η. Thus η is $PQ\alpha$-compact in (W, g). `

Theorem 3.8: Suppose (W, g) is an fts and σ, η are $PQ\alpha$-compact fuzzy sets in W. So $\sigma \cap \eta$ $(\sigma \cap \eta \neq 0)$ is also $PQ\alpha$-compact in (W, g).

Proof: Suppose $E = \{ a_i : i \in J \}$ is an open $PQ\alpha$-cover of $\sigma \cap \eta$. So E is open $PQ\alpha$-cover of both σ and η respectively. As σ is $PQ\alpha$-compact in (W, g), so every open $PQ\alpha$-cover of σ has a finite $PQ\alpha$-subcover i.e. there exist, say $a_{i_k} \in E$ $(k \in J_n)$ such that $\sigma(p) + a_{i_k}(p) \geq \alpha$ for all $p \in \sigma_0$. Again, η is $PQ\alpha$-compact in (W, g), then every open $PQ\alpha$-cover of η has a finite $PQ\alpha$-subcover i.e. there exist, say $a_{i_r} \in E$ $(r \in J_n)$ such that $\eta(p) + a_{i_r}(p) \geq \alpha$ for each $p \in \eta_0$. Hence $\{ a_{i_k}, a_{i_r} \}$ is a finite $PQ\alpha$-subcover of E. Therefore $\sigma \cap \eta$ is $PQ\alpha$-compact in (W, g).

Theorem 3.9: Suppose σ and η are $PQ\alpha$-compact fuzzy sets in an fts (W, g). Then $\sigma \cup \eta$ $((\sigma \cup \eta)_0 \neq W)$ is also $PQ\alpha$-compact in (W, g).

Proof: As $\sigma \subseteq \sigma \cup \eta$, $\eta \subseteq \sigma \cup \eta$ and σ & η are $PQ\alpha$-compacts in (W, g), so it is clearly shows that $\sigma \cup \eta$ is also $PQ\alpha$-compact in (W, g).

Theorem 3.10: Suppose (W, g) is an fts and σ is a fuzzy set in W. If each family of closed fuzzy sets in (W, g) which has empty intersection has a finite subfamily with empty intersection, on that situation σ is $PQ\alpha$-compact. Conversely is not necessarily true in general.

Proof: Suppose $\{ a_i : i \in J \}$ is an open $PQ\alpha$-cover of σ i.e. $\sigma(p) + a_i(p) \geq \alpha$ for all $p \in \sigma_0$. For first condition from the theorem, we have $\bigcap_{i \in J} a_i^c = 0_W$. So we can write $\bigcup_{i \in J} a_i = 1_W$.

Again, from the second condition of the theorem, we have $\bigcap_{k \in J_n} a_{i_k}^c = 0_W$ implies that $\bigcup_{k \in J_n} a_{i_k} = 1_W$ and hence $\sigma(p) + a_{i_k}(p) \geq \alpha$ for all $p \in \sigma_0$. Thus, it is clear that $\{ a_{i_k} : k \in J_n \}$ is a finite $PQ\alpha$-subcover of $\{ a_i : i \in J \}$. Therefore σ is $PQ\alpha$-compact.

Now, for the converse, we think about the following example.

Consider $W = \{u, v, w\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Suppose a, $b \in I^W$ defined by $a(u) = 0.04$, $a(v) = 0.02$, $a(w) = 0.03$ and $b(u) = 0.05$, $b(v) = 0.04$, $b(w) = 0.04$. Put $g = \{ 0, a, b, 1 \}$, so (W, g) is an fts. Again, let $\sigma \in I^W$ be defined with $\sigma(u) = 0.03$, $\sigma(v) = 0.02$, $\sigma(w) = 0$, so $\sigma_0 = \{u, v\}$. Take $\alpha = 0.06$. Clearly σ is $PQ\alpha$-compact in (W, g).

Again, closed fuzzy sets are $a^c(u) = 0.06$, $a^c(v) = 0.08$, $a^c(w) = 0.07$ and $b^c(u) = 0.05$, $b^c(v) = 0.06$, $b^c(w) = 0.06$. Thus we see that $a^c \cap b^c \neq 0$. Hence it is not necessarily true in general for the converse.

We give an example which follows that $PQ\alpha$-compact fuzzy sets in an fts not mandatory to closed.

Example 3.11: Let $W = \{u, v, w\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Let a , $b \in I^W$ be defined by $a(u) = 0.02$, $a(v) = 0.03$, $a(w) = 0.01$ and $b(u) = 0.04$, $b(v) = 0.05$, $b(w) = 0.03$. Choose $g = \{ 0 , a , b , 1 \}$, so we observe that (W, g) is an fts. Furthermore, let $\sigma \in I^W$ be defined with $\sigma(u) = 0.07$, $\sigma(v) = 0$, $\sigma(w) = 0.08$, then $\sigma_0 = \{u, w\}$. Take $\alpha = 0.09$. Clearly σ is $PQ\alpha$ -compact in (W, g). On the other hand, σ is not closed in (W, g), since the complement of σ i.e. σ^c is not open in (W, g).

Consider the following example which shows that the subsets of $PQ\alpha$ -compact fuzzy set in an fts not mandatory to $PQ\alpha$ -compact.

Example 3.12: Consider $W = \{u, v, w\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Consider a_1 , $a_2 \in I^W$ defined by $a_1(u) = 0.03$, $a_1(v) = 0.01$, $a_1(w) = 0.02$ and $a_2(u) = 0.04$, $a_2(v) = 0.02$, $a_2(w) = 0.03$. Choose $g = \{ 0 , a_1 , a_2 , 1 \}$, then (W, g) is an fts. Again, let σ , $\eta \in I^W$ be defined by $\sigma(u) = 0.05$, $\sigma(v) = 0.04$, $\sigma(w) = 0$ and $\eta(u) = 0.03$, $\eta(v) = 0.02$, $\eta(w) = 0$, where $\sigma_0 = \{u, v\}$, $\eta_0 = \{u, v\}$. Hence we see that $\eta \subset \sigma$. Take $\alpha = 0.06$. Clearly σ is $PQ\alpha$ -compact in (W, g). On the other hand, $\eta(v) + a_k(v) < \alpha$ for $v \in \eta_0$ and $k = 1, 2$. So η is not $PQ\alpha$ -compact in (W, g).

Theorem 3.13 Suppose (W, g) is a fuzzy T_1-space and σ is a fuzzy set in W where $\sigma_0 \subset W$ and $0 < \alpha \leq 1$. If σ is a $PQ\alpha$ -compact in (W, g) and $p \notin \sigma_0$, then there exist a , $b \in g$ such that $a(p) > 0$ and $\sigma_0 \subseteq b^{-1}(0, 1]$. But conversely is not necessarily true in general.

Proof: Suppose $q \in \sigma_0$. Then it is clear that $p \neq q$. Since (W, g) is fuzzy T_1-space, then there exist a_q , $b_q \in g$ such that $a_q(p) > 0$, $a_q(q) = 0$ and $b_q(p) = 0$, $b_q(q) > 0$. Since σ is $PQ\alpha$ -compact in (W, g), so $\sigma(q) + b_q(q) \geq \alpha$ for all $q \in \sigma_0$ i.e. $\{ b_q : q \in \sigma_0 \}$ is an open $PQ\alpha$ -cover of σ , then $\{ b_q : q \in \sigma_0 \}$ has a finite $PQ\alpha$ -subcover i.e. there exist b_{q_1} , b_{q_2} , ……. , $b_{q_n} \in \{b_q\}$ such that $\sigma(q) + b_{q_k}(q) \geq \alpha$ for each $q \in \sigma_0$ when $\sigma(q) > 0$ and some

$b_{q_k} \in \{b_q\}$. Now, consider $b = b_{q_1} \cup b_{q_2} \cup \ldots.. \cup b_{q_n}$ and $a = a_{q_1} \cap a_{q_2} \cap \ldots\ldots \cap a_{q_n}$. Hence b and a are open fuzzy sets i.e. b , $a \in g$. Moreover, $\sigma_0 \subseteq b^{-1}(0,1]$ and $a(p)>0$, as $a_{q_k}(p)>0$ for each k.

For the converse, we put the following example.

Consider $W = \{u, v\}$, $I = [0,1]$ and $0<\alpha\leq 1$. Suppose a_1 , a_2 , $a_3 \in I^W$ defined by $a_1(u) = 0.02$, $a_1(v) = 0$; $a_2(u) = 0$, $a_2(v) = 0.02$ and $a_3(u) = 0.02$, $a_3(v) = 0.02$. Put $g = \{ 0 , a_1 , a_2 , a_3 , 1 \}$, so (W, g) is a fuzzy T_1-space. Moreover, let $\sigma \in I^W$ be defined with $\sigma(u) = 0$, $\sigma(v) = 0.06$, then $\sigma_0 = \{v\}$ and $u \notin \sigma_0$. Here a_1 , $a_2 \in g$ where $a_1(u)>0$ and $a_2^{-1}(0,1] = \{v\}$. Therefore $\sigma_0 \subseteq a_2^{-1}(0,1]$. Furthermore, choose $\alpha = 0.09$. So we observe that σ is not $PQ\alpha$-compact in (W, g), as $\sigma(v) + a_k(v) < \alpha$ for $k = 1, 2, 3$. Hence the converse of the theorem is not necessarily true in general.

Theorem 3.14: Suppose (W, g) is a fuzzy T_1-space, σ and η are fuzzy sets set in W where σ_0, $\eta_0 \subset W$ and $0<\alpha\leq 1$. If σ and η are disjoint $PQ\alpha$-compact fuzzy sets in (W, g), then there exist a , $b \in g$ such that $\sigma_0 \subseteq a^{-1}(0,1]$ and $\eta_0 \subseteq b^{-1}(0,1]$. But conversely is not true in general.

Proof: Suppose $q \in \sigma_0$. So $q \notin \eta_0$, as σ and η are disjoint and it is clear that $p \neq q$. As η is $PQ\alpha$-compact in (W, g), so by theorem (3.13), there exist a_q , $b_q \in g$ such that $a_q(q)>0$ and $\eta_0 \subseteq b_q^{-1}(0,1]$. But σ is $PQ\alpha$-compact in (W, g), so $\sigma(q) + a_q(q) \geq \alpha$ for all $q \in \sigma_0$ i.e. $\{ a_q : q \in \sigma_0 \}$ is an open $PQ\alpha$-cover of σ and thus $\{ a_q : q \in \sigma_0 \}$ has a finite $PQ\alpha$-subcover i.e. there exist a_{q_1} , a_{q_2} , ……. , $a_{q_n} \in \{a_q\}$ such that $\sigma(q) + a_{q_k}(q) \geq \alpha$ for each $q \in \sigma_0$ when $\sigma(q) > 0$ and some $a_{q_k} \in \{a_q\}$. Furthermore, as η is $PQ\alpha$-compact in (W, g), so we have $\eta(p) + b_{q_k}(p) \geq \alpha$ for each $p \in \eta_0$, when $\eta(p) > 0$ and some $b_{q_k} \in \{b_q\}$, as $\eta_0 \subseteq {b_{q_k}}^{-1}(0,1]$ for each k.

Now, let $a = a_{q_1} \cup a_{q_2} \cup \ldots.. \cup a_{q_n}$ and $b = b_{q_1} \cap b_{q_2} \cap \ldots\ldots \cap b_{q_n}$. Hence a and b are open fuzzy sets i.e. a , $b \in g$. Thus we see that $\sigma_0 \subseteq a^{-1}(0,1]$ and $\eta_0 \subseteq b^{-1}(0,1]$.

For the converse, let $W = \{u, v\}$, $I = [0,1]$ and $0<\alpha\leq 1$. Consider a_1 , a_2 , $a_3 \in I^W$ defined by $a_1(u) = 0.02$, $a_1(v) = 0$; $a_2(u) = 0$, $a_2(v) = 0.03$, and $a_3(u) = 0.02$, $a_3(v) = 0.03$. Substitute $g = \{ 0 , a_1 , a_2 , a_3 , 1\}$, so (W, g) is a fuzzy T_1-space. Again, let σ , $\eta \in I^W$ be defined with $\sigma(u) = 0.04$, $\sigma(v) = 0$; $\eta(u) = 0$, $\eta(v) = 0.03$, then $\sigma_0 = \{u\}$, $\eta_0 = \{v\}$. Hence a_1 , $a_2 \in g$ where ${a_1}^{-1}(0,1] = \{u\}$ and ${a_2}^{-1}(0,1] = \{v\}$. Hence we observe that, $\sigma_0 \subseteq {a_1}^{-1}(0,1]$ and $\eta_0 \subseteq {a_2}^{-1}(0,1]$, where σ and η are disjoint. Choose $\alpha = 0.09$. But σ and η are not $PQ\alpha$ -compacts in (W, g), since $\sigma(u) + a_k(u) < \alpha$ for $u \in \sigma_0$ and $\eta(v) + a_k(v) < \alpha$ for $v \in \eta_0$, where $k = 1$, 2 , 3. Hence the converse of the theorem is not true in general.

The following example which shows that, $PQ\alpha$ -compact fuzzy sets in fuzzy T_1 -space not mandatory to closed.

Example 3.15: Consider the fuzzy T_1-space (W, g) in the example of the theorem (3.14). Again, suppose $\sigma \in I^W$ is defined with $\sigma(u) = 0$, $\sigma(v) = 0.04$, where $\sigma_0 = \{v\}$. Choose $\alpha = 0.06$. Clearly σ is $PQ\alpha$ -compact in fuzzy T_1-space (W, g). On the other hand σ is not closed, as the complement of σ i.e. σ^c is not open in (W, g).

Theorem 3.16: Suppose (W, g) is a fuzzy T_2 -space (Hausdorff space) and σ is a fuzzy set in W where $\sigma_0 \subset W$ and $0<\alpha\leq 1$. If σ is a $PQ\alpha$ -compact in (W, g) and $p \notin \sigma_0$, so there exist a , $b \in g$ such that $a(p) > 0$, $\sigma_0 \subseteq b^{-1}(0,1]$ and $a \cap b = 0$. But converse of the theorem is not true in general.

Proof: Allow $q \in \sigma_0$. So clearly $p \neq q$. Since (W, g) is fuzzy T_2-space, so there exist a_q, $b_q \in g$ such that $a_q(p) > 0$, $b_q(q) > 0$ and $a_q \cap b_q = 0$. As σ is $PQ\alpha$-compact in (W, g), so $\sigma(q) + b_q(q) \geq \alpha$ for each $q \in \sigma_0$ i.e. { b_q : $q \in \sigma_0$ } is an open $PQ\alpha$-cover of σ in (W, g) and hence { b_q : $q \in \sigma_0$ } has a finite $PQ\alpha$-subcover i.e. there exist b_{q_1}, b_{q_2}, ……., $b_{q_n} \in \{b_q\}$ such that $\sigma(q) + b_{q_k}(q) \geq \alpha$ for each $q \in \sigma_0$ when $\sigma(q) > 0$ and some $b_{q_k} \in \{b_q\}$. Now, let $b = b_{q_1} \cup b_{q_2} \cup \ldots.. \cup b_{q_n}$ and $a = a_{q_1} \cap a_{q_2} \cap \ldots\ldots \cap a_{q_n}$. Hence b, $a \in g$ i.e. b and a are open fuzzy sets. Moreover, $\sigma_0 \subseteq b^{-1}(0, 1]$ and $a(p) > 0$, since $a_{q_k}(p) > 0$ for every k.

Lastly, we will show that $a \cap b = 0$. As $a_{q_k} \cap b_{q_k} = 0$ implies $a \cap b_{q_k} = 0$, from distributive law, it follows that $a \cap b = a \cap (b_{q_1} \cup b_{q_2} \cup \ldots.. \cup b_{q_n}) = 0$.

For the converse, we give the following example.

Consider $W = \{u, v,\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Suppose a_1, a_2, $a_3 \in I^W$ defined by $a_1(u) = 0.02$, $a_1(v) = 0$; $a_2(u) = 0$, $a_2(v) = 0.03$ and $a_3(u) = 0.02$, $a_3(v) = 0.03$. Choose $g = \{ 0, a_1, a_2, a_3, 1\}$, so (W, g) is a fuzzy T_2-space. Again, let $\sigma \in I^W$ be defined with $\sigma(u) = 0$, $\sigma(v) = 0.04$, where $\sigma_0 = \{v\}$ and $u \notin \sigma_0$. But a_1, $a_2 \in g$ where $a_1(u) > 0$, $a_2^{-1}(0, 1] = \{v\}$ and $a_1 \cap a_2 = 0$. Therefore $\sigma_0 \subseteq a_2^{-1}(0, 1]$. Furthermore, choose $\alpha = 0.09$. So we observe that σ is not $PQ\alpha$-compact in (W, g), as $\sigma(v) + a_k(v) < \alpha$ for $k = 1, 2, 3$. Hence the converse of the theorem is not true in general.

Note: The proof of $a \cap b = 0$ is same in [1].

Theorem 3.17: Suppose (W, g) is a fuzzy T_2-space (Hausdorff space) and σ, η are fuzzy sets in W where σ_0, $\eta_0 \subset W$ and $0 < \alpha \leq 1$. If σ and η are disjoint $PQ\alpha$-compact fuzzy sets in (W, g), so there exist a, $b \in g$ such that $\sigma_0 \subseteq a^{-1}(0, 1]$, $\eta_0 \subseteq b^{-1}(0, 1]$ and $a \cap b = 0$. But conversely is not true in general.

Proof: Suppose $q \in \sigma_0$. So $q \notin \eta_0$, since σ and η are disjoint and it is clear that $p \neq q$. But η is $PQ\alpha$ -compact in (W, g), so by theorem (3.16), there exist a_q , $b_q \in g$ such that $a_q(q) > 0$ and $\eta_0 \subseteq b_q^{-1}(0, 1]$. But σ is $PQ\alpha$ -compact in (W, g), so $\sigma(q) + a_q(q) \geq \alpha$ for all $q \in \sigma_0$ i.e. $\{ a_q : q \in \sigma_0 \}$ is an open $PQ\alpha$ -cover of σ and thus $\{ a_q : q \in \sigma_0 \}$ has a finite $PQ\alpha$ -subcover i.e. there exist a_{q_1} , a_{q_2} , ……. , $a_{q_n} \in \{a_q\}$ such that $\sigma(q) + a_{q_k}(q) \geq \alpha$ for each $q \in \sigma_0$ when $\sigma(q) > 0$ and some $a_{q_k} \in \{a_q\}$. Furthermore, as η is $PQ\alpha$ -compact in (W, g), so we have $\eta(p) + b_{q_k}(p) \geq \alpha$ for each $p \in \eta_0$, when $\eta(p) > 0$ and some $b_{q_k} \in \{b_q\}$ also $\eta_0 \subseteq b_{q_k}^{-1}(0, 1]$ for each k.

Now, let $a = a_{q_1} \cup a_{q_2} \cup$ ….. $\cup\, a_{q_n}$ and $b = b_{q_1} \cap b_{q_2} \cap$ …… $\cap\, b_{q_n}$. Thus it is clear that $\sigma_0 \subseteq a^{-1}(0, 1]$ and $\eta_0 \subseteq b^{-1}(0, 1]$. Consequently a and b are open fuzzy sets i.e. a , $b \in g$.

To sum up, we will show that $a \cap b = 0$. But $a_{q_k} \cap b_{q_k} = 0$ make out $a_{q_k} \cap b = 0$, by distributive law, we observe that $a \cap b = (a_{q_1} \cup a_{q_2} \cup$ ….. $\cup\, a_{q_n}) \cap b = 0$.

For the converse, consider the fuzzy T_2 -space (W, g) in the theorem (3.16) . Again, let σ, $\eta \in I^W$ be defined with $\sigma(u) = 0.03$, $\sigma(v) = 0$; $\eta(u) = 0$, $\eta(v) = 0.01$, where $\sigma_0 = \{u\}$, $\eta_0 = \{v\}$. But a_1 , $a_2 \in g$ where $a_1^{-1}(0, 1] = \{u\}$, $a_2^{-1}(0, 1] = \{v\}$ and $a_1 \cap a_2 = 0$. Hence we observe that, $\sigma_0 \subseteq a_1^{-1}(0, 1]$ and $\eta_0 \subseteq a_2^{-1}(0, 1]$, where σ and η are disjoint. Choose $\alpha = 0.06$. Then σ and η are not $PQ\alpha$ -compacts in (W, g), since $\sigma(u) + a_k(u) < \alpha$ for $u \in \sigma_0$ and $\eta(v) + a_k(v) < \alpha$ for $v \in \eta_0$, where $k = 1, 2, 3$. Hence the converse of the theorem is not true in general.

Note: The proof of $a \cap b = 0$ is same in[1].

The raising example shows that $PQ\alpha$ -compact fuzzy sets in fuzzy T_2 -space not mandatory to closed.

Example 3.18: Consider the fuzzy T_2-space in the theorem (3.16). Again, let $\sigma \in I^W$ be given by $\sigma(u) = 0$, $\sigma(v) = 0.04$ where $\sigma_0 = \{v\}$. Take $\alpha = 0.07$. It is clearly shows that σ is $PQ\alpha$-compact in (W, g), but σ is not closed, since its complement σ^c is not open in (W, g).

We give the following precedent which shows that the "good extension" property does not true for $PQ\alpha$-compact fuzzy set.

Precedent 3.19: Consider $W = \{u, v, w\}$ and $G = \{ \phi , \{v\} , \{w\} , \{v, w\} , W \}$. Then (W, G) is a topological space. Again, let a_1 , a_2 , $a_3 \in I^W$ defined by $a_1(u) = 0$, $a_1(v) = 1$, $a_1(w) = 0$; $a_2(u) = 0$, $a_2(v) = 0$, $a_2(w) = 0.06$ and $a_3(u) = 0$, $a_3(v) = 1$, $a_3(w) = 0.06$. Then $\omega(G) = \{ 0 , a_1 , a_2 , a_3 , 1\}$ and $(W, \omega (G))$ is an fts. Furthermore, let $\sigma \in I^W$ be defined with $\sigma(u) = 0$, $\sigma(v) = 0.02$, $\sigma(w) = 0.01$, then $\sigma_0 = \{v, w\}$. It is clearly shows that σ_0 is compact in (W, G) . Select $\alpha = 0.08$. Thus we notice that σ is not $PQ\alpha$ -compact in $(W, \omega (G))$, as there do not exist $a_k \in \omega(G)$ ($k = 1$, 2 , 3) such that $\sigma(w) + a_k(w) \geq \alpha$ where $w \in \sigma_0$. Again, let $\eta \in I^W$ defined by $\eta(u) = 0.09$, $\eta(v) = 0.02$, $\eta(w) = 0$ where $\eta_0 = \{u,v\}$. It is clear that η is $PQ\alpha$-compact in $(W, \omega (G))$, but $\eta_0 = \{u,v\}$ is not compact in (W, G). It is notice that the "good extension property" does not true good for $PQ\alpha$-compact fuzzy sets.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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