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ALMOST BI-HYPERIDEALS AND THEIR FUZZIFICATION OF SEMIHYPERGROUPS

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Abstract. In this paper, we introduce the concept of almost bi-hyperideals of semihypergroups which is a generalization of bi-hyperideals, and we give some properties of them. Moreover, we consider the connections between almost bi-hyperideals and their fuzzification of semihypergroups.

Keywords: bi-hyperideal; almost bi-hyperideal; fuzzy almost bi-hyperideal; semihypergroup.

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1. INTRODUCTION

The concepts of left, right, two-sided almost ideals of semigroups were introduced by Grosek and Satko [7] in 1980. They studied the characterization of these ideals when a semigroup contains no proper left, right, two-sided ideals. Later in 1981, Bogdanovic [1] introduced the notion of almost bi-ideals in semigroups as a generalization of bi-ideals. The concept of fuzzy subsets was first introduced by Zadeh [18] as a function from a nonempty set X to the unit interval $[0,1]$. The fuzzy subset theory is a generalization of traditional mathematics set theory. In 2018, Wattanatripop, Chinram and Changphas [17] introduced the notion of fuzzy almost

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bi-ideals in semigroups and discussed some relationships between almost bi-ideals and fuzzy almost bi-ideals of semigroups. Then, Simuen, et al. [14] investigated some properties of fuzzy almost bi-Γ-ideals of Γ-semigroups.

Algebraic hyperstructure was introduced in 1934, by Marty [10], as the 8^{th} Congress of Scandinavian Mathematicians. In a classical algebraic structure, composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a nonempty set. There are many authors expanded the concept of hyperstructuree, see, e.g., [3], [4], [6], [11], [12], [16]. In this work, the authors focus on semihypergroups. Semihypergroups are studied by many authors, for instance, [2], [5], [8], [9], [13]. In 2020, Suebsung, Kaewnoi and Chinram [15] studied the concept of almost hyperideals in semihypergroups and gave some interesting properties.

In this paper, we introduce the concept of almost bi-hyperideals of semihypergroups as a generalization of bi-hyperideals and investigate some properties of them. Then, we discuss the connections between almost bi-hyperideals and fuzzy almost bi-hyperideals of semihypergroups.

2. Preliminaries

Let H be a nonempty set. A *hyperoperation* on H is a mapping $\circ : H \times H \to \mathscr{P}^*(H)$, where $\mathscr{P}^*(H)$ denotes the set of all nonempty subsets of H. Then, the structure $(H,\circ)$ is called a *hypergroupoid*. If $A,B \in \mathscr{P}^*(H)$ and $x \in H$, then we denote

$$A \circ B = \bigcup_{a\in A, b\in B} a\circ b,\ A\circ x = A\circ\{x\} \text{ and } x\circ B = \{x\}\circ B.$$

A hypergroupoid $(H,\circ)$ is called a *semihypergroup* if for every $x,y,z \in H$, $(x\circ y)\circ z = x\circ(y\circ z)$, which means that

$$\bigcup_{u\in x\circ y} u\circ z = \bigcup_{v\in y\circ z} x\circ v.$$

A nonempty subset A of a semihypergroup $(S,\circ)$ is called a *subsemihypergroup* of S if $A\circ A \subseteq A$. A subsemihypergroup B of a semihypergroup $(S,\circ)$ is called a *bi-hyperideal* of S if $B\circ S\circ B \subseteq B$. For more convenient, we write S instead of a semihypergroup $(S,\circ)$ and AB instead of $A\circ B$, for any nonempty subsets A and B of S.

A *fuzzy subset* [18] of a nonempty set X is a mapping $f : X \to [0,1]$. Let f and g be any two fuzzy subsets of a nonempty set X. Then, $f \subseteq g$ if and only if $f(x) \leq g(x)$ for all $x \in X$. The *intersection* and the *union* of two fuzzy subsets f and g of a nonempty set X, denoted by $f \cap g$ and $f \cup g$, respectively, are defined by letting $x \in X$,

$$(f \cap g)(x) = \min\{f(x), g(x)\},$$

$$(f \cup g)(x) = \max\{f(x), g(x)\}.$$

Let X be a nonempty set. For a fuzzy subset f of X, the *support* of f is defined by $supp(f) := \{x \in X \mid f(x) \neq 0\}$. Let A be a nonempty subset of X. The *characteristic mapping* χ_A of A is a fuzzy subset of X defined by

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise}, \end{cases}$$

for all $x \in X$.

For any element s of X and $t \in (0,1]$, a *fuzzy point* s_t of X defined by

$$s_t(x) = \begin{cases} t & \text{if } x = s, \\ 0 & \text{otherwise}, \end{cases}$$

for all $x \in X$.

Lemma 2.1. *Let A and B be nonempty subsets of a nonempty set X and let f and g be fuzzy subsets of X. Then the following statements hold:*

(i) $\chi_{A \cap B} = \chi_A \cap \chi_B$;

(ii) $A \subseteq B$ *if and only if* $\chi_A \subseteq \chi_B$;

(iii) $supp(\chi_A) = A$;

(iv) *if* $f \subseteq g$, *then* $supp(f) \subseteq supp(g)$.

Proof. The proof is straightforward. □

Let f and g be fuzzy subsets of a semihypergroup S. A *product* $f \circ g$ is defined by

$$(f \circ g)(x) = \begin{cases} \sup\limits_{x \in yz} \{\min\{f(y), g(z)\}\} & \text{if } \exists\, y, z \in S \text{ such that } x \in yz, \\ 0 & \text{otherwise}, \end{cases}$$

for all $x \in S$.

Lemma 2.2. *If A and B are subsets of a semihypergroup S, then $\chi_A \circ \chi_B = \chi_{AB}$.*

Proof. Let $x \in S$. If $\chi_{AB}(x) = 0$, then $x \notin AB$. This means that $x \notin ab$ for all $a \in A$ and $b \in B$. Thus, $(\chi_A \circ \chi_B)(x) = 0$. That is, $\chi_{AB}(x) = (\chi_A \circ \chi_B)(x)$. If $\chi_{AB}(x) = 1$, then $x \in AB$. This implies that $x \in ab$ for some $a \in A$ and $b \in B$. Hence, $(\chi_A \circ \chi_B)(x) = \sup\limits_{x \in ab}\{\min\{\chi_A(a), \chi_B(b)\}\} = 1$. So, $\chi_{AB}(x) = (\chi_A \circ \chi_B)(x)$. Therefore, $\chi_A \circ \chi_B = \chi_{AB}$. □

3. Almost Bi-Hyperideals

In this section, we introduce the concept of almost bi-hyperideals of semihypergroups and give some of its properties.

Definition 3.1. A nonempty subset B of a semihypergroup S is called an *almost bi-hyperideal* of S if $BxB \cap B \neq \emptyset$ for all $x \in S$.

Example 3.2. Let $S = \{a, b, c\}$. Define a hyperoperation $\cdot$ on S by the following table:

$\cdot$	a	b	c
a	$\{a\}$	$\{b,c\}$	$\{b,c\}$
b	$\{b,c\}$	$\{b,c\}$	$\{b,c\}$
c	$\{c\}$	$\{c\}$	$\{c\}$

Then, $(S, \cdot)$ is a semihypergroup. Let $B = \{b, c\}$. Hence,

$$BaB \cap B = \{b,c\} \neq \emptyset,$$

$$BbB \cap B = \{b,c\} \neq \emptyset,$$

$$BcB \cap B = \{b,c\} \neq \emptyset.$$

Therefore, B is an almost bi-hyperideal of S.

Proposition 3.3. *Every bi-hyperideal of a semihypergroup S is an almost bi-hyperideal.*

Proof. Let B be a bi-hyperideal of a semihypergroup S. Then, $BSB \subseteq B$. It follows that for any $x \in S$, $BxB \subseteq BSB \subseteq B$. That is, $BxB \cap B = BxB \neq \emptyset$ for all $x \in S$. Hence, B is an almost bi-hyperideal of S. □

In general, an almost bi-hyperideal of a semihypergroup need not to be a bi-hyperideal as the following example.

Example 3.4. Consider $S = \{a,b,c\}$ together with the hyperoperation $\cdot$ on S defined in Example 3.2. Let $B = \{a,b\}$. By routine computations, B is an almost bi-hyperideal of S, but B is not a bi-hyperideal of S because $BSB = S \nsubseteq B$.

Next, we discuss some properties of almost bi-hyperideals of semihypergroups.

Theorem 3.5. *Let B be an almost bi-hyperideal of a semihypergroup S. If A is any subset of S containing B, then A is also an almost bi-hyperideal of S.*

Proof. Assume that A is a subset of S containing B. Since B is an almost bi-hyperideal of S and $B \subseteq A$, we have that $BxB \cap B \neq \emptyset$ and $BxB \cap B \subseteq AxA \cap A$ for all $x \in S$. It turns out that $AxA \cap A \neq \emptyset$ for all $x \in S$. Hence, A is an almost bi-hyperideal of S. □

Corollary 3.6. *The union of any two almost bi-hyperideals of a semihypergroup S is also an almost bi-hyperideal of S.*

Proof. Let A and B be any two almost bi-hyperideals of a semihypergroup S. Since $A \subseteq A \cup B$ and by Theorem 3.5, we get that $A \cup B$ is an almost bi-hyperideal of S. □

Example 3.7. Let $S = \{a,b,c\}$. Then the hyperoperation $\cdot$ on S defined by the following:

$\cdot$	a	b	c
a	$\{a\}$	$\{b,c\}$	$\{c\}$
b	$\{b,c\}$	$\{b,c\}$	$\{c\}$
c	$\{b,c\}$	$\{b,c\}$	$\{c\}$

Hence, $(S,\cdot)$ is a semihypergroup. Let $B_1 = \{a,b\}$ and $B_2 = \{a,c\}$. By routine calculations, B_1 and B_2 are almost bi-hyperideals of S. However, $B = B_1 \cap B_2 = \{a\}$ is not an almost bi-hyperideal of S because $BbB \cap B = \emptyset$.

By Example 3.7, we have that the intersection of any two almost bi-hyperideals of a semihypergroup S need not to be an almost bi-hyperideal of S.

Theorem 3.8. *Let S be a semihypergroup. Then S contains a proper almost bi-hyperideal if and only if there exists an element x of S such that $S \setminus \{x\}$ is an almost bi-hyperideal of S.*

Proof. Assume that S contains a proper almost bi-hyperideal. Let A be a proper almost bi-hyperideal of S. Then, there exists $x \in S$ such that $x \notin A$. So, $A \subseteq S \setminus \{x\}$. By Theorem 3.5, $S \setminus \{x\}$ is an almost bi-hyperideal of S. Conversely, consider $S \setminus \{x\}$ for some $x \in S$. Then, $S \setminus \{x\}$ is a proper subset of S. By assumption, we have that $S \setminus \{x\}$ is an almost bi-hyperideal of S. Therefore, S contains a proper almost bi-hyperideal. □

Theorem 3.9. *Let S be a semihypergroup and $|S| > 1$. Then S has no proper almost bi-hyperideals if and only if for every $x \in S$ there exists $a \in S$ such that $(S \setminus \{x\})a(S \setminus \{x\}) = \{x\}$.*

Proof. Assume that S has no proper almost bi-hyperideals. Let $x \in S$. Then, $S \setminus \{x\}$ is not an almost bi-hyperideal of S. Thus, there exists $a \in S$ such that

$$[(S \setminus \{x\})a(S \setminus \{x\})] \cap (S \setminus \{x\}) = \emptyset.$$

We obtain that

$$(S \setminus \{x\})a(S \setminus \{x\}) \subseteq S \setminus (S \setminus \{x\}) = \{x\}.$$

This implies that $(S \setminus \{x\})a(S \setminus \{x\}) = \{x\}$.

Conversely, suppose that S contains a proper almost bi-hyperideal B. Let $x \in S \setminus B$. By assumption, there exists $a \in S$ such that $(S \setminus \{x\})a(S \setminus \{x\}) = \{x\}$. Since $B \subseteq S \setminus \{x\}$ and by Theorem 3.5, we get that $S \setminus \{x\}$ is an almost bi-hyperideal of S. It follows that

$$\emptyset = \{x\} \cap (S \setminus \{x\}) = [(S \setminus \{x\})a(S \setminus \{x\})] \cap (S \setminus \{x\}) \neq \emptyset.$$

This is a contradiction. Therefore, S has no proper almost bi-hyperideals. □

4. Fuzzy Almost Bi-Hyperideals

In this section, we introduce the concept of fuzzy almost bi-hyperideals of semihypergroups, and we study the connections between almost bi-hyperideals and their fuzzification of semihypergroups.

Definition 4.1. Let f be a fuzzy subset of a semihypergroup S such that $f \neq 0$. Then f is called a *fuzzy almost bi-hyperideal* of S if for every fuzzy point s_t of S, $(f \circ s_t \circ f) \cap f \neq 0$.

Theorem 4.2. *Let f be a fuzzy almost bi-hyperideal of a semihypergroup S. If g is a fuzzy subset of S such that $f \subseteq g$, then g is a fuzzy almost bi-hyperideal of S.*

Proof. Assume that g is a fuzzy subset of S such that $f \subseteq g$. By assumption, $(f \circ s_t \circ f) \cap f \subseteq (g \circ s_t \circ g) \cap g$ and $(f \circ s_t \circ f) \cap f \neq 0$ for all fuzzy points s_t of S. It follows that $(g \circ s_t \circ g) \cap g \neq 0$ for all fuzzy points s_t of S. Hence, g is a fuzzy almost bi-hyperideal of S. □

Corollary 4.3. *Let f and g be fuzzy almost bi-hyperideals of a semihypergroup S. Then $f \cup g$ is also a fuzzy almost bi-hyperideal of S.*

Proof. It follows from Theorem 4.2. □

Example 4.4. Consider $S = \{a, b, c\}$ together with the hyperoperation $\cdot$ on S defined in Example 3.7. Let f and g be fuzzy subsets of S defined by

$$f(a) = 0,\ f(b) = 0,\ f(c) = 0.3$$

and

$$g(a) = 0,\ g(b) = 0.7,\ g(c) = 0.$$

It not difficult to show that

$$[(f \circ s_t \circ f) \cap f](c) \neq 0 \quad \text{and} \quad [(g \circ s_t \circ g) \cap g](b) \neq 0$$

for all fuzzy points s_t of S. Then, f and g are fuzzy almost bi-hyperideals of S. Moreover, for a fuzzy point a_t of S, $f \cap g$ is not a fuzzy almost bi-hyperideal of S because $[((f \cap g) \circ a_t \circ (f \cap g)) \cap (f \cap g)](x) = 0$ for all $x \in S$.

Theorem 4.5. *Let B be a nonempty subset of a semihypergroup S. Then B is an almost bi-hyperideal of S if and only if χ_B is a fuzzy almost bi-hyperideal of S.*

Proof. Assume that B is an almost bi-hyperideal of S. Then, $BsB \cap B \neq \emptyset$ for all $s \in S$. So, there exists $x \in S$ such that $x \in BsB$ and $x \in B$. Thus, $x \in b_1 s b_2$ for some $b_1, b_2 \in B$. It follows that

$$(\chi_B \circ s_t \circ \chi_B)(x) = \sup_{x \in b_1 s b_2} \{\min\{\chi_B(b_1), s_t(s), \chi_B(b_2)\}\} \neq 0 \text{ and } \chi_B(x) = 1.$$

This implies that $(\chi_B \circ s_t \circ \chi_B) \cap \chi_B \neq 0$ for all fuzzy points s_t of S. Hence, χ_B is a fuzzy almost bi-hyperideal of S.

Conversely, assume that χ_B is a fuzzy almost bi-hyperideal of S. Let $s \in S$. Then, $(\chi_B \circ s_t \circ \chi_B) \cap \chi_B \neq 0$. Thus, there exists $x \in S$ such that $(\chi_B \circ s_t \circ \chi_B)(x) \neq 0$ and $\chi_B(x) \neq 0$. Then, there exist $b_1, b_2 \in B$ such that $x \in b_1 s b_2$ and $x \in B$. This means that $x \in BsB$ and $x \in B$. That is, $BsB \cap B \neq \emptyset$. Therefore, B is an almost bi-hyperideal of S. □

Theorem 4.6. *Let f be a fuzzy subset of a semihypergroup S. Then f is a fuzzy almost bi-hyperideal of S if and only if $supp(f)$ is an almost bi-hyperideal of S.*

Proof. Assume that f is a fuzzy almost bi-hyperideal of S. Let $s \in A$ and s_t be a fuzzy point of S. Then, $(f \circ s_t \circ f) \cap f \neq 0$. Thus, there exists $x \in S$ such that $[(f \circ s_t \circ f) \cap f](x) \neq 0$. That is, $(f \circ s_t \circ f)(x) \neq 0$ and $f(x) \neq 0$. We obtain that there exist $y_1, y_2 \in S$ such that $x \in y_1 s y_2$,

$$0 \neq (f \circ s_t \circ f)(x) = \sup_{x \in y_1 s y_2} \{\min\{f(y_1), s_t(s), f(y_2)\}\}.$$

So, $f(y_1) \neq 0$ and $f(y_2) \neq 0$. Hence, $x, y_1, y_2 \in supp(f)$. It follows that $(\chi_{supp(f)} \circ s_t \circ \chi_{supp(f)})(x) \neq 0$ and $\chi_{supp(f)}(x) \neq 0$. Also, $(\chi_{supp(f)} \circ s_t \circ \chi_{supp(f)}) \cap \chi_{supp(f)} \neq 0$. This means that $\chi_{supp(f)}$ is a fuzzy almost bi-hyperideal of S. By Theorem 4.5, $supp(f)$ is an almost bi-hyperideal of S.

Conversely, assume that $supp(f)$ is an almost bi-hyperideal of S. By Theorem 4.5, $\chi_{supp(f)}$ is a fuzzy almost bi-hyperideal of S. Let s_t be any fuzzy point of S. Then, $(\chi_{supp(f)} \circ s_t \circ \chi_{supp(f)}) \cap \chi_{supp(f)} \neq 0$. Thus, there exists $x \in S$ such that $[(\chi_{supp(f)} \circ s_t \circ \chi_{supp(f)})](x) \neq 0$ and $\chi_{supp(f)}(x) \neq 0$. That is, there exist $y_1, y_2 \in S$ such that $x \in y_1 s y_2$,

$$0 \neq (\chi_{supp(f)} \circ s_t \circ \chi_{supp(f)})(x) = \sup_{x \in y_1 s y_2} \{\min\{\chi_{supp(f)}(y_1), s_t(s), \chi_{supp(f)}(y_2)\}\},$$

which implies that, $\chi_{supp(f)}(y_1) \neq 0$ and $\chi_{supp(f)}(y_2) \neq 0$. It turns out that $f(y_1) \neq 0, f(y_2) \neq 0$ and $f(x) \neq 0$. Hence, $(f \circ s_t \circ f) \cap f \neq 0$. Consequently, f is a fuzzy almost bi-hyperideal of S. □

An almost bi-hyperideal M of a semihypergroup S is *minimal* if for any almost bi-hyperideal A of S such that $A \subseteq M$ implies that $A = M$.

Definition 4.7. Let S be a semihypergroup. A fuzzy almost bi-hyperideal f of S is called *minimal* if for any fuzzy almost bi-hyperideal g of S such that $g \subseteq f$, we get $supp(f) = supp(g)$.

Next, we investigate the minimality of fuzzy almost bi-hyperideals of semihypergroups.

Theorem 4.8. *Let B be a nonempty subset of a semihypergroup S. Then B is a minimal almost bi-hyperideal of S if and only if χ_B is a minimal fuzzy almost bi-hyperideal of S.*

Proof. Assume that B is a minimal almost bi-hyperideal of S. By Theorem 4.5, χ_B is a fuzzy almost bi-hyperideal of S. Let g be a fuzzy almost bi-hyperideal of S such that $g \subseteq \chi_B$. By Lemma 2.1, $supp(g) \subseteq supp(\chi_B) = B$. By Theorem 4.6, $supp(g)$ is an almost bi-hyperideal of S. By the minimality of B, we have $supp(g) = B = supp(\chi_B)$. Therefore, χ_B is a minmal fuzzy almost bi-hyperideal of S.

Conversely, assume that χ_B is a minmal fuzzy almost bi-hyperideal of S. Then, B is an almost bi-hyperideal of S. Let A be any almost bi-hyperideal of S such that $A \subseteq B$. By Theorem 4.5, χ_A is a fuzzy almost bi-hyperideal of S such that $\chi_A \subseteq \chi_B$. Since χ_B is minimal, we get that $supp(\chi_A) = supp(\chi_B)$. We obtain that $A = supp(\chi_A) = supp(\chi_B) = B$ by Lemma 2.1. Consequently, B is a minimal almost bi-hyperideal of S. □

Corollary 4.9. *Let S be a semihypergroup. Then S has no proper almost bi-hyperideals if and only if for every fuzzy almost bi-hyperideal f of S, $supp(f) = S$.*

Proof. Assume that S has no proper almost bi-hyperideals. Let f be a fuzzy almost bi-hyperideal of S. By Theorem 4.6, $supp(f)$ is an almost bi-hyperideal of S. By assumption, we have that $supp(f) = S$. Conversely, let B be any almost bi-hyperideal of S. Then, χ_B is a fuzzy almost bi-hyperideal of S by Theorem 4.5. It follows that $B = supp(\chi_B) = S$. This shows that S has no proper almost bi-hyperideals. □

Let S be a semihypergroup. An almost bi-hyperideal P of S is *prime* if for any almost bi-hyperideals A and B of S such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. An almost bi-hyperideal P of S is *semiprime* if for any almost bi-hyperideal A of S such that $AA \subseteq P$ implies that $A \subseteq P$. An almost bi-hyperideal P of S is *strongly prime* if for any almost bi-hyperideals A and B of S such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 4.10. A fuzzy almost bi-hyperideal h of a semihypergroup S is called a *fuzzy prime almost bi-hyperideal* of S if for any two fuzzy almost bi-hyperideals f and g of S,

$$f \circ g \subseteq h \text{ implies that } f \subseteq h \text{ or } g \subseteq h.$$

Definition 4.11. A fuzzy almost bi-hyperideal h of a semihypergroup S is called a *fuzzy semiprime almost bi-hyperideal* of S if for any fuzzy almost bi-hyperideal f of S,

$$f \circ f \subseteq h \text{ implies that } f \subseteq h.$$

Definition 4.12. A fuzzy almost bi-hyperideal h of a semihypergroup S is called a *fuzzy strongly prime almost bi-hyperideal* of S if for any two fuzzy almost bi-hyperideals f and g of S,

$$(f \circ g) \cap (g \circ f) \subseteq h \text{ implies that } f \subseteq h \text{ or } g \subseteq h.$$

We note that every fuzzy strongly prime almost bi-hyperideal of a semihypergroup is a fuzzy prime almost bi-hyperideal, and every fuzzy prime almost bi-hyperideal of a semihypergroup is a fuzzy semiprime almost bi-hyperideal, but the converse is not true in general.

Finally, we study the relationships between prime (resp., semiprime, strongly prime) almost bi-hyperideals and their fuzzification of semihypergroups.

Theorem 4.13. *Let P be a nonempty subset of a semihypergroup S. Then P is a prime almost bi-hyperideal of S if and only if χ_P is a fuzzy prime almost bi-hyperideal of S.*

Proof. Assume that P is a prime almost bi-hyperideal of S. By Theorem 4.5, χ_P is a fuzzy almost hyperideal of S. Let f and g be fuzzy almost bi-hyperideals of S such that $f \circ g \subseteq \chi_P$. Suppose that $f \nsubseteq \chi_P$ and $g \nsubseteq \chi_P$. Then, there exist $x, y \in S$ such that $f(x) \neq 0$ and $g(y) \neq 0$, but $\chi_P(x) = 0$ and $\chi_P(y) = 0$. It follows that $x \notin P$ and $y \notin P$. Since $f(x) \neq 0$ and $g(y) \neq 0$, we get that $x \in supp(f)$ and $y \in supp(g)$. Thus, $supp(f) \nsubseteq P$ and $supp(g) \nsubseteq P$. By Theorem 4.6, we have that $supp(f)$ and $supp(g)$ are almost bi-hyperideals of S. Since P is prime, $supp(f)supp(g) \nsubseteq P$. Then, there exists $t \in ab$ for some $a \in supp(f)$ and $b \in supp(g)$ such that $t \notin P$. So, $\chi_P(t) = 0$, and then $(f \circ g)(t) = 0$ because $f \circ g \subseteq \chi_P$. Since $a \in supp(f)$ and $b \in supp(g)$, we have that $f(a) \neq 0$ and $g(b) \neq 0$. Hence, $\min\{f(a), g(b)\} \neq 0$, which implies that $(f \circ g)(t) = \sup_{t \in ab}\{\min\{f(a), g(b)\}\} \neq 0$. This is a contradiction to the fact that $(f \circ g)(t) = 0$. Therefore, $f \subseteq \chi_P$ or $g \subseteq \chi_P$. Consequently, χ_P is a fuzzy prime almost bi-hyperideal of S.

Conversely, assume that χ_P is a fuzzy prime almost bi-hyperideal of S. By Theorem 4.5, P is an almost bi-hyperideal of S. Let A and B be any two almost bi-hyperideals of S such that $AB \subseteq P$. By Lemma 2.1 and Lemma 2.2, we get that $\chi_A \circ \chi_B = \chi_{AB} \subseteq \chi_P$. Again by Theorem 4.5, χ_A and χ_B are fuzzy almost bi-hyperideals of S. By hypothesis, $\chi_A \subseteq \chi_P$ or $\chi_B \subseteq \chi_P$. That is, $A \subseteq P$ or $B \subseteq P$. Hence, P is a prime almost bi-hyperideal of S. □

The proof of the following theorem is similar to Theorem 4.13.

Theorem 4.14. *Let P be a nonempty subset of a semihypergroup S. Then P is a semiprime almost bi-hyperideal of S if and only if χ_P is a fuzzy semiprime almost bi-hyperideal of S.*

Theorem 4.15. *Let P be a nonempty subset of a semihypergroup S. Then P is a strongly prime almost bi-hyperideal of S if and only if χ_P is a fuzzy strongly prime almost bi-hyperideal of S.*

Proof. Assume that P is a strongly prime almost bi-hyperideal of S. By Theorem 4.5, χ_P is a fuzzy almost bi-hyperideal of S. Let f and g be any two fuzzy almost bi-hyperideals of S such that $(f \circ g) \cap (g \circ f) \subseteq \chi_P$. Suppose that $f \nsubseteq \chi_P$ and $g \nsubseteq \chi_P$. Then, there exist $x, y \in S$ such that $f(x) \neq 0$ and $g(y) \neq 0$, but $\chi_P(x) = 0$ and $\chi_P(y) = 0$. So, $x \in supp(f)$ and $y \in supp(g)$ such that $x \notin P$ and $y \notin P$. It follows that $supp(f) \nsubseteq P$ and $supp(g) \nsubseteq P$. By Theorem 4.6 and the hypothesis, we have that $[supp(f)supp(g)] \cap [supp(g)supp(f)] \nsubseteq P$. Hence, there exists $t \in [supp(f)supp(g)] \cap [supp(g)supp(f)]$ such that $t \notin P$. Also, $\chi_P(t) = 0$, and then $[(f \circ g) \cap (g \circ f)](t) = 0$ because $(f \circ g) \cap (g \circ f) \subseteq \chi_P$. Since $t \in supp(f)supp(g)$ and $t \in supp(g)supp(f)$, we have that $t \in a_1b_1$ and $t \in b_2a_2$ for some $a_1, a_2 \in supp(f)$ and $b_1, b_2 \in supp(g)$. It turns out that

$$(f \circ g)(t) = \sup_{t \in a_1b_1} \{\min\{f(a_1), g(b_1)\}\} \neq 0 \text{ and } (g \circ f)(t) = \sup_{t \in b_2a_2} \{\min\{g(b_2), f(a_2)\}\} \neq 0.$$

This implies that $\min\{(f \circ g)(t), (g \circ f)(t)\} \neq 0$, that is, $[(f \circ g) \cap (g \circ f)](t) \neq 0$. This is a contradiction with the fact that $[(f \circ g) \cap (g \circ f)](t) = 0$. Hence, $f \subseteq \chi_P$ or $g \subseteq \chi_P$. Therefore, χ_P is a fuzzy strongly prime almost bi-hyperideal of S.

Conversely, assume that χ_P is a fuzzy strongly prime almost bi-hyperideal of S. Then, P is an almost bi-hyperideal of S by Theorem 4.5. Let A and B be any two almost bi-hyperideals of S such that $(AB) \cap (BA) \subseteq P$. By Theorem 4.5, χ_A and χ_B are fuzzy almost bi-hyperideals of S.

By Lemma 2.2, $\chi_{AB} = \chi_A \circ \chi_B$ and $\chi_{BA} = \chi_B \circ \chi_A$. By Lemma 2.1, we have that $(\chi_A \circ \chi_B) \cap (\chi_B \circ \chi_A) = \chi_{AB} \cap \chi_{BA} = \chi_{(AB)\cap(BA)} \subseteq \chi_P$. By assumption, we get that $\chi_A \subseteq \chi_P$ or $\chi_B \subseteq \chi_P$ implies that $A \subseteq P$ or $B \subseteq P$. Consequently, P is a strongly prime almost bi-hyperideal of S. □

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] S. Bogdanovic, Semigroups in which some bi-ideal is a group, Rev. Res. Fac. Sci. Univ. Novi Sad. 11 (1981), 261-266.

[2] T. Changphas, B. Davvaz, Properties of hyperideals in ordered semihypergroups, Italian J. Pure Appl. Math. 33 (2014), 425-432.

[3] P. Corsini, Prolegomena of hypergroup theory (2^{ed}), Aviani Editore, Tricesimo, 1993.

[4] P. Corsini, V. Leoreanu, Applications of hyperstructure theory, Kluwer Academic Publishers, Dordrecht, 2003.

[5] B. Davvaz, Some result on congruences on semihypergroups, Bull. Malays. Math. Sci. Soc. 23 (2000), 53-58.

[6] B. Davvaz, V. Leoreanu-Fotea, Hyperring theory and applications, International Academic Press, Palm Harbor, 2007.

[7] O. Grosek, L. Satko, A new notion in the theory of semigroups, Semigroup Forum. 20 (1980), 233-240.

[8] K. Hila, B. Davvaz, J. Dine, Study on the structure of Γ-semihypergroups, Commun. Algebra. 40 (2012), 2932-2948.

[9] K. Hila, B. Davvaz, K. Naka, On quasi-hyperideals in semihypergroups, Commun. Algebra. 39 (2011), 4183-4194.

[10] F. Marty, Sur une generalization de la notion de group, Proceeding of 8^{th} Congress des Mathematician Scandinave. (1934), 45-49.

[11] W. Nakkhasen, B. Pibaljommee, Hyperideals in EL-semihyperrings, Thai J. Math. Spec. Iss., Ann. Meet. Math. 2017 (2018), 133–143.

[12] B. Pibaljommee, W. Nakkhasen, Connections of (m,n)-bi-quasi hyperideals in semihyperrings, Thai J. Math. Spec. Iss., Ann. Meet. Math. 2019 (2020), 39–48.

[13] M.D. Salvo, D. Freni, G.L. Faro, Fully simple semihypergroups, J. Algebra. 399 (2014), 358-377.

[14] A. Simuen, S. Abdullah, W. Yonthanthum, R. Chinram, Almost bi-Γ-ideals and fuzzy almost bi-Γ-ideals of Γ-semigroups, Eur. J. Pure Appl. Math. 13 (2020), 620-630.

[15] S. Suebsung, T. Kaewnoi, R. Chinram, A not on almost hyperideals in semihypergroups, Int. J. Math. Computer Sci. 15 (2020), 127-133.

[16] T. Vougiouklis, Hyperstructures and their representations, Hadronic Press, Nonanhum, 1994.

[17] K. Wattanatripop, R. Chinram, T. Changphas, Fuzzy almost bi-ideals in semigroups, Int. J. Math. Computer Sci. 13 (2018), 51-58.

[18] L.A. Zadeh, Fuzzy sets, Inform. Control. 8 (1965), 338-353.