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Q_8 DIFFERENCE CORDIAL LABELING

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Abstract. Let Q_8 be a quaternion group. Let $G = (V, E)$ be a graph. Let $f : V(G) \to Q_8$. For each edge xy assign the label 0 when $|o(f(x)) - o(f(y))| = 0$ and 1 otherwise. The function f is called Q_8 cordial difference labeling of G if $|v_f(x) - v_f(y)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x), v_f(y)$ denote the total number of vertices labeled with x, y in Q_8 and $e_f(0), e_f(1)$ denote the total number of edges labeled with 0,1 respectively. A graph G which admits a group Q_8 difference cordial labeling is called Q_8 difference cordial graph. In this paper, we prove the existence of this labeling to the graphs viz., path, ladder related graphs and snake related graphs.

Keywords: group Q_8 cordial; cordial labeling; quaternion group labeling.

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1. Introduction

The concept of graph labeling was introduced by Rosa [4] in 1967. The cordial labeling of graph was introduced by Cahit [2]. For standard terminology and notation related to graph

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theory we follow Balakrishnan and Ranganathan [1]. Lourdusamy et. al., introduced the concept of S_3 remainder cordial labeling [3]. In this paper, we discussed the concept of group Q_8 difference cordial labeling.

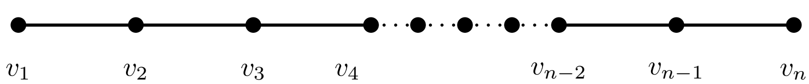
2. MAIN RESULTS

Definition 2.1. *Consider the quaternion group Q_8. Let the elements of Q_8 be $\pm 1, \pm i, \pm j, \pm k$. Now $o(1) = 1, o(-1) = 2, o(\pm i) = o(\pm j) = o(\pm k) = 4$.*

Definition 2.2. *Let $G = (V, E)$ be a graph. Let $f : V(G) \to Q_8$. For each edge xy assign the label 0 when $|o(f(x)) - o(f(y))| = 0$ and 1 otherwise. The function f is called Q_8 difference cordial lebeling of G if $|v_f(x) - v_f(y)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x), v_f(y)$ denote the total number of vertices labeled with x, y in Q_8 and $e_f(0), e_f(1)$ denote the total number of edges labeled with 0,1 respectively. A graph G which admits a group Q_8 difference cordial labeling is called Q_8 difference cordial graph.*

Theorem 2.3. *The path P_n is group Q_8- difference cordial graph.*

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of P_n.



Let the vertex label $f : V(P_n) \to Q_8$ be defined as follows: for $n = 8s$ and $s \geq 1$,

$$f(v_\beta) = \begin{cases} i, & \beta = 8s-7 \\ -i, & \beta = 8s-6 \\ 1, & \beta = 8s-5 \\ j, & \beta = 8s-4 \\ -j, & \beta = 8s-3 \\ k, & \beta = 8s-2 \\ -1, & \beta = 8s-1 \\ -k, & \beta = 8s \end{cases}$$

Now we see that $|v_f(x) - v_f(y)| \leq 1$. This implies that

$$|e_f(0) - e_f(1)| = \begin{cases} 0, & \text{if } n \text{ is odd} \\ 1, & \text{if } n \text{ is even} \end{cases}$$

Hence the path P_n is group Q_8- difference cordial graph. □

Theorem 2.4. *Let G be the comb graph $P_n \odot K_1$. Then G is group Q_8-difference cordial graph.*

Proof. Let the vertex set be $V(G) = \{v_\gamma^\beta / 1 \leq \gamma \leq n,\ \beta = 1,2\}$. Let the edge set be $E(G) = \{v_\gamma^1 v_\gamma^2 / 1 \leq \gamma \leq n\} \cup \{v_\gamma^1 v_{\gamma+1}^1 / 1 \leq \gamma \leq n-1\}$. Define $f : V(G) \to Q_8$ as follows.

$$f(v_\gamma^1) = \begin{cases} 1, & \gamma \equiv 1\ (mod\ 4) \\ -i, & \gamma \equiv 2\ (mod\ 4) \\ -j, & \gamma \equiv 3\ (mod\ 4) \\ -k, & \gamma \equiv 0\ (mod\ 4) \end{cases}$$

and

$$f(v_\gamma^2) = \begin{cases} i, & \gamma \equiv 1\ (mod\ 4) \\ j, & \gamma \equiv 2\ (mod\ 4) \\ k, & \gamma \equiv 3\ (mod\ 4) \\ -1, & \gamma \equiv 0\ (mod\ 4) \end{cases}$$

Clearly we see that $|v_f(x) - v_f(y)| \leq 1$.Let $s = \left\lceil \frac{n}{2} \right\rceil$. Then

$$e_f(1) = \begin{cases} n, & s \text{ is odd} \\ n-1, & s \text{ is even} \end{cases}$$

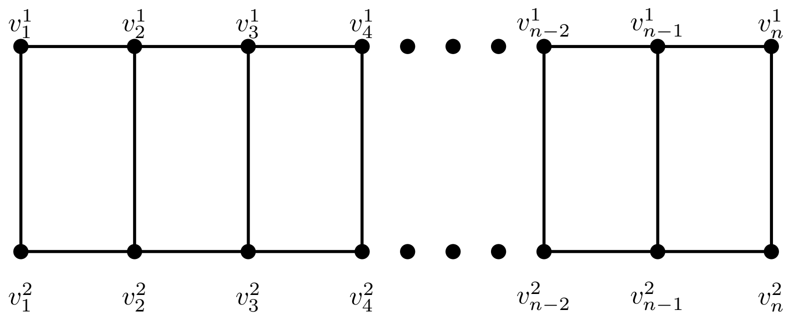
and

$$e_f(0) = \begin{cases} n-1, & s \text{ is odd} \\ n, & s \text{ is even} \end{cases}$$

It is obvious that $|e_f(0) - e_f(1)| \leq 1$. Therefore comb $P_n \odot K_1$ is group Q_8-difference cordial graph. □

Theorem 2.5. *Let G be the ladder graph L_n. Then G is group Q_8-difference cordial graph.*

Proof. Let $V(L_n) = \{v_1^1,\ v_2^1,\ v_3^1,\dots,v_n^1,\ v_1^2,\ v_2^2,\ v_3^2,\dots,v_n^2\}$ and $E(L_n) = \{v_\gamma^1 v_\gamma^2/1 \le \gamma \le n\} \cup \{v_\gamma^1 v_{\gamma+1}^1, v_\gamma^2 v_{\gamma+1}^2/1 \le \gamma \le n-1\}$.



Define a map $f : V(L_n) \to Q_8$ as follows: for $n = 4s, s \ge 1$,

$$f(v_\gamma^1) = \begin{cases} 1,\ \gamma = 1,5,9,\dots,4s-3, \\ -i,\ \gamma = 2,6,10,\dots,4s-2, \\ -j,\ \gamma = 3,7,11,\dots,4s-1, \\ k,\ \gamma = 4,8,12,\dots,4s. \end{cases}$$

and

$$f(v_\gamma^2) = \begin{cases} i,\ \gamma = 1,5,9,\dots,4s-3, \\ j,\ \gamma = 2,6,10,\dots,4s-2, \\ -1,\ \gamma = 3,7,11,\dots,4s-1, \\ -k,\ \gamma = 4,8,12,\dots,4s. \end{cases}$$

We can verify that $|v_f(x) - v_f(y)| \le 1$. This implies that

$$e_f(0) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor - 1 + n, & \text{if } n \text{ is odd} \\ \left(\frac{n}{2} - 1\right) + n, & \text{if } n \text{ is even} \end{cases}$$

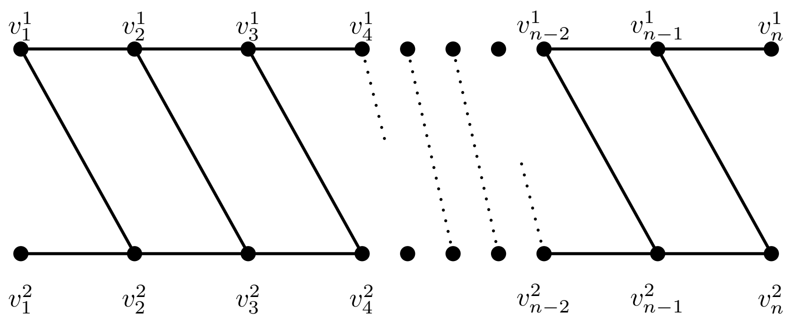
and

$$e_f(1) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor + n, & \text{if } n \text{ is odd} \\ \left(\frac{n}{2} - 1\right) + n, & \text{if } n \text{ is even} \end{cases}$$

Thus the ladder L_n is group Q_8-difference cordial graph. □

Theorem 2.6. *Let G be the slanting ladder graph SL_n. Then G is group Q_8-difference cordial graph.*

Proof. Let $V(SL_n) = \{v_\gamma^\beta / 1 \le \gamma \le n,\ \beta = 1,2\}$ and $E(SL_n) = \{v_\gamma^1 v_{\gamma+1}^1, v_\gamma^2 v_{\gamma+1}^2, v_\gamma^1 v_{\gamma+1}^2 / 1 \le \gamma \le n-1\}$.



Define $f : V(SL_n) \to Q_8$ by $f(v_{4s-3}^1) = i, f(v_{4s-2}^1) = 1, f(v_{4s-1}^1) = -i, f(v_{4s}^1) = -1, f(v_{4s-3}^2) = j, f(v_{4s-2}^2) = -j, f(v_{4s-1}^2) = k, f(v_{4s}^2) = -k$. It is obvious that $|v_f(x) - v_f(y)| \le 1$. It is observed as

$$e_f(0) = \begin{cases} \lfloor \frac{n}{2} \rfloor - 1 + n, & \text{if } n \text{ is odd} \\ \left(\frac{n}{2}\right) - 1 + n, & \text{if } n \text{ is even} \end{cases}$$

and

$$e_f(1) = \begin{cases} \lfloor \frac{n}{2} \rfloor - 1 + n, & \text{if } n \text{ is odd} \\ \left(\frac{n}{2}\right) - 2 + n, & \text{if } n \text{ is even} \end{cases}$$

Hence the slanting ladder SL_n is group Q_8- difference cordial graph. □

Theorem 2.7. *Let G be the triangular ladder graph TL_n. Then G is group Q_8-difference cordial graph.*

Proof. Let $V(TL_n) = \{v_\gamma^\beta / 1 \leq \gamma \leq n,\ \beta = 1,2\}$ and $E(TL_n) = \{v_\gamma^1 v_{\gamma+1}^1, v_\gamma^1 v_{\gamma+1}^2, v_\gamma^2 v_{\gamma+1}^2 / 1 \leq \gamma \leq n-1\} \cup \{v_\gamma^1 v_\gamma^2 / 1 \leq \gamma \leq n\}$. Define a function $f : V(TL_n) \to Q_8$ as follows.

$$f(v_\gamma^1) = \begin{cases} 1, & \gamma \equiv 1\ (mod\ 4) \\ i, & \gamma \equiv 2\ (mod\ 4) \\ -1, & \gamma \equiv 3\ (mod\ 4) \\ -i, & \gamma \equiv 0\ (mod\ 4) \end{cases}$$

and

$$f(v_\gamma^2) = \begin{cases} j, & \gamma \equiv 1\ (mod\ 4) \\ -j, & \gamma \equiv 2\ (mod\ 4) \\ k, & \gamma \equiv 3\ (mod\ 4) \\ -k, & \gamma \equiv 0\ (mod\ 4) \end{cases}$$

Clearly, $|v_f(x) - v_f(y)| \leq 1$. This implies that $e_f(0) = 2n-2$ and $e_f(1) = 2n-1$. Therefore triangular ladder TL_n is group Q_8-difference cordial graph. □

Theorem 2.8. *Let G be the braid graph B_n. Then G is group Q_8-difference cordial graph.*

Proof. Let $V(B_n) = \{v_\gamma^\beta / 1 \leq \gamma \leq n,\ \beta = 1,2\}$ and $E(B_n) = \{v_\gamma^1 v_{\gamma+1}^1, v_\gamma^2 v_{\gamma+1}^2,\ v_\gamma^1 v_{\gamma+1}^2, / 1 \leq \gamma \leq n-1\} \cup \{v_\gamma^2 v_{\gamma+2}^1 / 1 \leq \gamma \leq n-2\}$. Define a map $f : V(B_n) \to Q_8$ as follows.

$$f(v_\gamma^1) = \begin{cases} 1, & \gamma \equiv 1\ (mod\ 4) \\ i, & \gamma \equiv 2\ (mod\ 4) \\ -1, & \gamma \equiv 3\ (mod\ 4) \\ -i, & \gamma \equiv 0\ (mod\ 4) \end{cases}$$

and

$$f(v_\gamma^2) = \begin{cases} j, & \gamma \equiv 1\ (mod\ 4) \\ -j, & \gamma \equiv 2\ (mod\ 4) \\ k, & \gamma \equiv 3\ (mod\ 4) \\ -k, & \gamma \equiv 0\ (mod\ 4) \end{cases}$$

It follows that $|v_f(x) - v_f(y)| \leq 1$. This implies that $e_f(0) = 2n-3$ and $e_f(1) = 2n-2$. Hence braid graph B_n is group Q_8-difference cordial graph. □

Theorem 2.9. *Let G be the open triangular ladder graph OTL_n. Then G is group Q_8-difference cordial graph.*

Proof. Let $V(OTL_n) = \{v_\gamma^\beta / 1 \leq \gamma \leq n,\ \beta = 1,2\}$. Let $E(OTL_n) = \{v_\gamma^1 v_{\gamma+1}^1, v_\gamma^2 v_{\gamma+1}^2, v_\gamma^1 v_{\gamma+1}^2 / 1 \leq \gamma \leq n-1\} \cup \{v_\gamma^1 v_\gamma^2 / 2 \leq \gamma \leq n-1\}$. Define $f : V(OTL_n) \rightarrow Q_8$ as follows.

$$f(v_\gamma^1) = \begin{cases} 1, & \gamma \equiv 1\ (mod\ 4) \\ i, & \gamma \equiv 2\ (mod\ 4) \\ -1, & \gamma \equiv 3\ (mod\ 4) \\ -i, & \gamma \equiv 0\ (mod\ 4) \end{cases}$$

and

$$f(v_\gamma^2) = \begin{cases} j, & \gamma \equiv 1\ (mod\ 4) \\ -j, & \gamma \equiv 2\ (mod\ 4) \\ k, & \gamma \equiv 3\ (mod\ 4) \\ -k, & \gamma \equiv 0\ (mod\ 4) \end{cases}$$

Clearly, $|v_f(x) - v_f(y)| \leq 1$. It is observed as

$$e_f(0) = \begin{cases} 2n-2, & \text{if } n \text{ is odd} \\ 2n-3, & \text{if } n \text{ is even} \end{cases}$$

and

$$e_f(1) = \begin{cases} 2n-3, & \text{if } n \text{ is odd} \\ 2n-2, & \text{if } n \text{ is even} \end{cases}$$

It can be easily verified that the open triangular ladder OTL_n is group Q_8-difference cordial graph. □

Theorem 2.10. *Let G be the open diagonal ladder graph $ODTL_n$. Then G is group Q_8-difference cordial graph.*

Proof. Let $V(ODTL_n)=\{v_\gamma^\beta/1\leq\gamma\leq n,\ \beta=1,2\}$. Let $E(ODTL_n)=\{v_\gamma^1 v_{\gamma+1}^1, v_\gamma^2 v_{\gamma+1}^2, v_\gamma^1 v_{\gamma+1}^2, v_{\gamma+1}^1 v_\gamma^2/1\leq\gamma\leq n-1\}\cup\{v_\gamma^1 v_\gamma^2/2\leq\gamma\leq n\}$. Define function $f:V(ODTL_n)\to Q_8$ as follows.

$$f(v_\gamma^1)=\begin{cases}1, & \gamma\equiv 1\ (mod\ 4)\\ i, & \gamma\equiv 2\ (mod\ 4)\\ -1, & \gamma\equiv 3\ (mod\ 4)\\ -i, & \gamma\equiv 0\ (mod\ 4)\end{cases}$$

and

$$f(v_\gamma^2)=\begin{cases}j, & \gamma\equiv 1\ (mod\ 4)\\ -j, & \gamma\equiv 2\ (mod\ 4)\\ k, & \gamma\equiv 3\ (mod\ 4)\\ -k, & \gamma\equiv 0\ (mod\ 4)\end{cases}$$

It is easy to show that $|v_f(x)-v_f(y)|\leq 1$. This implies that

$$e_f(0)=\begin{cases}5\left\lfloor\frac{n}{2}\right\rfloor, & \text{if } n \text{ is odd}\\ 5\left\lfloor\frac{n}{2}\right\rfloor-3, & \text{if } n \text{ is even}\end{cases}$$

and

$$e_f(1)=\begin{cases}5\left\lfloor\frac{n}{2}\right\rfloor-1, & \text{if } n \text{ is odd}\\ 5\left\lfloor\frac{n}{2}\right\rfloor-3, & \text{if } n \text{ is even}\end{cases}$$

Hence the open diagonal ladder $ODTL_n$ is group Q_8-difference cordial graph. □

Theorem 2.11. *Let G be the alternate double triangular snake graph $DA(TS_n)$. Then G is group Q_8-difference cordial graph.*

Proof. Let the vertex set be $V(DA(TS_n))=\{v_\gamma^\beta/1\leq\gamma\leq n,\ \beta=1,2,3\}$. Let the edge set be $E(DA(TS_n))=\{v_\gamma^2 v_{\gamma+1}^2/1\leq\gamma\leq n-1\}\cup\{v_\gamma^2 v^1_{\lceil\frac{\gamma}{2}\rceil},\ v_\gamma^2 v^3_{\lceil\frac{\gamma}{2}\rceil}/1\leq\gamma\leq n\}$. Define a function $f:V(DA(TS_n))\to Q_8$ as follows:

For $n=4s$ and $s\geq 1$,

$$f(v_\gamma^1)=\begin{cases}i, & \gamma \text{ is odd}\\ -i, & \gamma \text{ is even}\end{cases}$$

$$f(v^2_\gamma)=\begin{cases} j, & \gamma=4s-3,\\ -j, & \gamma=4s-2,\\ 1, & \gamma=4s-1,\\ -k, & \gamma=4s.\end{cases}$$

and

$$f(v^3_\gamma)=\begin{cases} -1, & \gamma \text{ is odd}\\ k, & \gamma \text{ is even}\end{cases}$$

It can be easily verified that $|v_f(x)-v_f(y)|\leq 1$. Therefore

$$e_f(0)=\begin{cases} \lfloor \frac{n}{2}\rfloor \times 3, & \text{if } n \text{ is odd}\\ n+2\left(\lceil \frac{n}{4}\rceil -1\right), & \text{if } n \text{ is even}\end{cases}$$

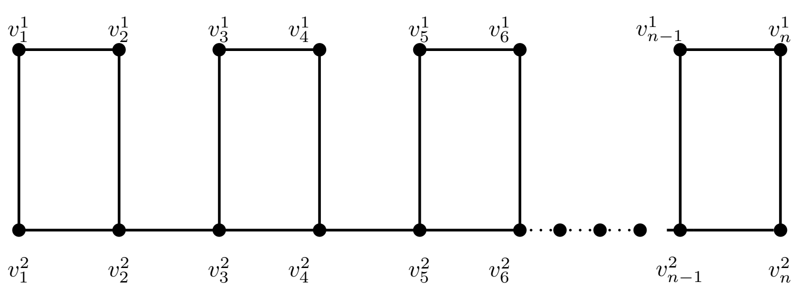
and

$$e_f(1)=\begin{cases} \lfloor \frac{n}{2}\rfloor \times 3, & \text{if } n \text{ is odd}\\ \left(\lfloor \frac{n}{4}\rfloor \times 2\right)+n, & \text{if } n \text{ is even}\end{cases}$$

Thus the alternate double triangular snake $DA(TS_n)$ is group Q_8-difference cordial graph. □

Theorem 2.12. *Let G be the alternate quadrilateral snake graph $A(QS_n)$. Then G is group Q_8-difference cordial graph.*

Proof. Let $V(A(QS_n))=\{v^1_1,\ v^1_2,\ v^1_3,\dots,v^1_n,\ v^2_1,\ v^2_2,\ v^2_3,\dots,v^2_n\}$ be the vertex set. Let the edge set be $E(A(QS_n))=\{v^1_\gamma v^2_\gamma/1\leq\gamma\leq n\}\cup\{v^2_\gamma v^2_{\gamma+1},/1\leq\gamma\leq n-1\}\cup\{v^1_\gamma v^1_{\gamma+1}/1\leq\gamma\leq n-1 \text{ and } \gamma \text{ is odd}\}$.



Define a map $f : V(A(QS_n)) \to Q_8$ as follows.

$$f(v_\gamma^1) = \begin{cases} i, & \gamma \equiv 1 \ (mod\ 4) \\ -i, & \gamma \equiv 2 \ (mod\ 4) \\ -j, & \gamma \equiv 3 \ (mod\ 4) \\ -k, & \gamma \equiv 0 \ (mod\ 4) \end{cases}$$

and

$$f(v_\gamma^2) = \begin{cases} 1, & \gamma \equiv 1 \ (mod\ 4) \\ j, & \gamma \equiv 2 \ (mod\ 4) \\ k, & \gamma \equiv 3 \ (mod\ 4) \\ -1, & \gamma \equiv 0 \ (mod\ 4) \end{cases}$$

We can show that $|v_f(x) - v_f(y)| \leq 1$. This implies that

$$e_f(0) = \begin{cases} \left\lfloor \frac{n-2}{4} \right\rfloor + n, & \text{if } n \text{ is odd} \\ n + \left\lfloor \frac{n}{4} \right\rfloor, & \text{if } n \text{ is even} \end{cases}$$

and

$$e_f(1) = \begin{cases} \left(n + \left\lfloor \frac{n}{4} \right\rfloor\right) - 1 & \text{if } n \text{ is odd} \\ \left(n + \left\lceil \frac{n}{4} \right\rceil\right) - 1, & \text{if } n \text{ is even} \end{cases}$$

Hence the alternate quadrilateral snake graph $A(QS)$ is group Q_8-difference cordial graph. □

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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