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A COMPREHENSIVE STUDY OF VEHICLE ROUTING PROBLEM WITH TIME WINDOWS USING AN IMPROVED MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS

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Abstract: The vehicle routing problem with time windows (VRPTW) which has both capacity and time constraints is an extension of vehicle routing problem (VRP). The problem is solved by optimizing routes for the vehicles so as to meet all given constraints as well as to minimize travelling distance and number of vehicles. This paper proposes to analyze a multi objective evolutionary algorithm (MOEA) that incorporates Various heuristics for local exploitation in the evolutionary search and the concept of pareto's optimality to solve multi objective optimization in VRPTW. In this paper we model VRPTW as a modified version specialized for a multi objective context, using Evolutionary Algorithm to get a set of pareto optimal solutions considering three objectives, the number of vehicles, the total travel distance and the total delivery time at the same time. This new approach is validated with very good results and the comparison is performed on a standard benchmark problems showing that the algorithm outperforms highly competitive results compared with previously published studies.

Keywords: vehicle routing problem with time windows; multi-objective optimization; evolutionary optimization algorithms; pareto optimal set.

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1. INTRODUCTION

The Vehicle Routing Problem with Time Windows (VRPTW) was first introduced by Solomon in 1987 [19]. It involves the routing of a set of vehicles with limited capacity from a central depot to a set of geographically dispersed customers with known demands and predefined time windows. In other words, for every customer, goods must be distributed in certain time window with known demands along with least cost and distance. The vehicles can not arrive earlier or later than the time [18]. In case if the vehicle arrives earlier then the earliest arrival time and waiting time will occur. Each customer should also consider the service period for loading or unloading the goods for each route. VRP Using time windows aims to reduce three objectives namely the number of vehicles, the total travelling time and the total delivery time. With the time windows, the total routing and scheduling cost consist not only the total travel distance and time costs, but also include the cost of waiting time due to a vehicle arrives too early at a customer location.

The VRPTW is known to be complex comparing with the travelling salesman problem (TSP) as it admits servicing customers with time windows using multiple vehicles [5]. The optimal solutions can be found for VRPTW using exact methods, the computational time required to solve VRPTW is prohibited for large problem [7]. Therefore heuristic methods are often applied for solving optimal solution in a reasonable amount of time.

Mathematical Model for Vehicle Routing Problem with Time Windows

In VRPTW, the objective which comes into consideration along with minimization of the total travel time is to minimize the number of vehicles servicing the customers [7]. So it is a bio-objective optimization problem which we can extend to a Multi-Objective problem by connecting other real –life parameters. In this circumstance, there is an associated time window with each customer to service.

Here, the vehicles have to wait, if they arrive too early for the service and if they arrive after the closure of the time windows, the service cannot be rendered.

There are three constraints to consider for solving VRPTW as follows.

i. First for most all the customers request that service must be visited.

ii. Each Vehicle should begin and end at the same depot.

iii. Each vehicle should arrive within the particular arrival time. It should not come too early and should not be late.

The Mathematical model for VRPTW is presented as below [19]

Inputs:

b_j^k: The time at which the vehicle k starts its service at node j,

b_j^k=max $\{e_j, b_j^k + t_{ij}\}$

c_{ij}: Travel distance between node i and node j

d_i: The demand of the customer at node i,i $\in$N

e_i: The earliest start time of the vehicle for node i

l_i : The latest start time of the vehicle for node i

N: The total number of service nodes

Q: The capacity of Vehicle

V: The fleet of vehicles

t_{ij}: the travel time of edge (i,j)which also includes the service time at node i

Decision variable:

$$X_{ijk} = \begin{cases} 1, \text{ if vehicle k goes from node i to j} \\ 0 \text{ otherwise} \end{cases}$$

Model: Equations

$$min \sum_{k\in v} \sum_{i\in N} \sum_{j\in N} c_{ij} x_{ij}^k \quad (1)$$

such that

$$\sum_{k\in v} \sum_{i\in N} x_{ij}^k = 1 , \forall j \in N \setminus \{0\} \quad (2)$$

$$\sum_{k\in v} \sum_{i\in N} x_{ij}^k = 1 , \forall i \in N\{0\} \quad (3)$$

$$\sum_{i\in N} d_i \sum_{j\in N} x_{ij}^k \leq Q \, , \forall k \in V \tag{4}$$

$$\sum_{i\in N} x_{ih}^k - \sum_{j\in N} x_{hj}^k = 0 \, , \forall h \in N \setminus \{0\} \, , \forall k \in V \tag{5}$$

$$\sum_{j\in N\setminus\{0\}} x_{0j}^k = 1 \, , \forall k \in V \tag{6}$$

$$\sum_{i\in N\setminus\{0\}} x_{i0}^k = 1 \, , \forall k \in V \tag{7}$$

$$x_{ij}^k\left(b_i^k + t_{ij}\right) \leq b_j^k , \forall i \in N, \forall j \in N, \forall k \in V \tag{8}$$

$$e_i \leq b_i^k \leq l_i , \forall i \in N, \forall k \in V \tag{9}$$

$$x_{ij}^k \in \{0,1\} \, , \forall i \in N, \forall k \in V \tag{10}$$

Equation (1) shows the minimization of the total distance of travel. Equations (1) & (3) describe that each node being serviced only once by one vehicle. Equation (4) models that the total demand for a particular route of a vehicle should not exceed its capacity. It is stated in equation (5) that the vehicle which has entered the node must leave the same. Equations (6) & (7) express that each vehicle must return to depot after servicing the customers. In equation (8) it is denoted that vehicle k cannot arrive at j before $b_i^k + t_{ij}$if it is to travel from node i to node j .Equation (9) which states that start of every vehicle must be within the time windows and equation (10) is a integrality constraint.

The rest of this paper is organized as follows. The literature review on multi-objective approach is explained in the Second Section and the new multi- objective approach is introduced for the optimization of three objective functions namely Number of vehicles, total travelling time and total delivery time. In Section 3, the proposed EA for solving the VRPTW as a Multi-Objective problem is explained. Section 4 presents the Multi-Objective performs metrics which is relevant to this study. Then the improved MOEA for VRPTW has been introduced in Section 5. Section 6 presents the experimental design and set-up in addition with NSGA-II. The comparison of proposed MOEA with previous published studies is described in Section 7. Finally Section 8 provides some conclusions and ideas for further research in this field.

2. MULTI OBJECTIVE OPTIMIZATION PROBLEM

Multi-objective optimization which is known to be a multi-criteria optimization is the process of simultaneously optimizing two or more conflicting objectives related to certain constraints. As multi-objective optimization problems (MOOP), the solution to the bi-objective GVRPTW is a non – dominated solution received from an optimal pareto- front [6]. In multi-objective optimization problem, if it is well formed then there is no way to have a single solution that minimizes each objective simultaneously to its fullest. In each case, the solution is eagerly expected to the extent that if it is optimized further, then the next other objective(s) will deplore as a result [12]. After finding a solution, the solution is compared to other such solutions to know how much better this solution as quantification and this is the goal when setting up and solving a multi objective optimization problem [13]. There are two goals in a multi-objective optimization.

- **i.** To find a set of solution as close as possible to the desired pareto-optimal front, in terms of both proximity and diversity.
- **ii.** To find a set of solutions as diverse as possible.

In general, Mops include many incommensurable and contrariety objectives [19,7,12]. Without loss of generality Mop can be formulated as follows.

$e(y)=(e_{1,}(y),e_2(y), \ldots, e_e(y))\geq 0$,where

$y=(y_1, \ldots,y_m)\in R^m$is in the decision position and

$z=(z_1, \ldots, z_n) \in R^n$ is in the objective position.

The set of decision vectors that satisfies the constraints and indicates feasible set Y.

The vector function $f:y\rightarrow R^n$ denotes the feasible region.

$Z=\{f(y)/y\in Y\}$. The conflicting objectives give a set of alternative solutions. These solutions are optimal in the way that no solution is able to dominate those solutions in an overall aspect. Because no objective can be amended or reformed without degrading at least one of the others. Such solutions are called as the non dominated solutions or the pareto optimal solutions. This brings the research on the solutions to MOP_S more effective in the field of intelligent optimization.

3. EVOLUTIONARY ALGORITHMS

Evolutionary Algorithms (EA) which is known to be stochastic search algorithms inspired by Darwin's theory of evolution are the optimization algorithms that search for optimal solutions by evolving a multiset of candidate solutions based on mechanisms of natural solution and genetics. The exact methods can be used to obtain optimal solutions for small instances of the VRPTW. But for larger instances [9] ,the computation time requirements increase considerably . For this reason heuristic methods are usually employed for optimal solutions in small instances of VRPTW. In particular, EA automatically generates the entire population of solutions that fulfill the whole range of trade –offs.

3.1. Evolutionary Optimization Algorithms (EOA)

Evolutionary optimization algorithms (EOA) use a population based approach in its search procedure which means more than one solution participate in an iteration and produce a new population of solutions in each iteration. EOA are very popular due to their following reasons.

(i) Any derivative information is not required in EOA .

(ii) They are relatively simple to implement.

(iii) EOA are so flexible and also have a wide-spread applicability.

Braysy and Gendreau [3,4] provide an excellent survey of them utilizing a number of heuristics and give a comparison of the obtained results. Braysy et al .[2] have focused on evolutionary approaches. There are many studies those have considered the bi-objective optimization of the VRPTW, use an EA to minimize the two objectives namely the number of vehicles and the travel distance. Tan et al. [20] have used the dominance rank scheme for assigning fitness to individuals and have designed a problem specific crossover operator and multi-mode mutation operator. Further they have also considered three local search heuristics. The work proposed here is a number of a route and travel distance, but also the delivery time. Here we try to analyze the results for comparison with those from previous algorithms. Finally we introduce improved comparisons with the popular NSGA-II algorithm [8] by using multi-objective performance metrics.

3.2. Problem Definitions for VRPTW Objectives

To define three objective functions for VRPTW, any number of relevant objective functions f_i to optimize can be specified. We concentrate in this paper on the key objectives, namely the number of routs or vehicles(f_1), the total travel distance (f_2), by summing the route distances and the total travel time (f_3) by summing the arrival time back at the depot have computed as follows,

$f_1(R) = |R| = k$, $\quad f_2(R) = \sum_{k=1}^{k} D\kappa$, $\quad f_3(R) = \sum_{\kappa=1}^{\kappa} T\kappa$ (11)

Here $R = \{r_1, \ldots, r_k\}$,

$D_k = \sum_{i=0}^{N\kappa} d\mathrm{u}(i,k)\ u(i+1,k)$ and

$T_k = \sum_{i=0}^{\mathrm{Nk}} (t u(i,k) u(i+1,k) + w\big(u(i+1,k)\big) + S u(i+1,k))$

Subject to the demands associated with customers serviced in each route r_k not exceeding the vehicle capacity that is, $G_k = \sum_{i \epsilon V_K^*} g_i \leq Q$; for all k = 1, 2, . . ., k (12)

Where $V_k^* = \{u(1,k), \ldots, u(N_k, k)\}$ and no arrival times after the customers constraints that is,

$b_{u(i,k)} \leq a(u(i,k)) \leq e_{u(i,k)}$, for all $k = 1, 2, \ldots, k$, $1 \leq i \leq \mathcal{N}_K$ (13)

Here $a(u(i,k)) = \ell\big(u(i-1,k)\big) + t_{u(i-1,k)u(i,k)}$

As the minimization of three objectives is not usually possible, a set of non-dominated solutions is needed to obtain each solution is better that the others on at least one objective.

4. Multi-Objective Performance Metrics

Multi-Objective problems have to compare whole set of solutions, whereas, the single objective problems have to compare the best solutions from the various approaches studied. So that, the definition and application of Multi-Objective performance metrics is crucial.

We have two existing metrics. These are particularly applicable to the problem. The coverage metric $\mathcal{M}_c$[19] is used to compare the number of solutions in set $\mathcal{B}$ that are covered by solutions in $\mathcal{A}$(ie dominated to equal to) to the cardinality of $\mathcal{B}$. Formally this ratio maps the ordered pair $(\mathcal{A}, \mathcal{B})$ to the interval [0,1] as the general coverage metric[15].

$$\mathcal{M}_c\,(\mathcal{A}, \mathcal{B}) = \frac{|\{b \in \mathcal{B} : \exists a \in A, a \leq b\}|}{|\mathcal{B}|}$$

The value $\mathcal{M}_c\,(\mathcal{A}, \mathcal{B})$ =1 indicates that all solutions in $\mathcal{B}$ are covered by solutions in $\mathcal{A}$ and $\mathcal{M}_c\,(\mathcal{A}, \mathcal{B})$ = 0 means that no one solution in $\mathcal{B}$ is covered by those in$\mathcal{A}$. Since $\mathcal{M}_c\,(\mathcal{A}, \mathcal{B})$ is not necessarily equal to 1- $\mathcal{M}_c\,(\mathcal{B}, \mathcal{A})$ and both$\mathcal{M}_c\,(\mathcal{A}, \mathcal{B})$ and $\mathcal{M}_c\,(\mathcal{B}, \mathcal{A})$ do not necessarily to be computed.

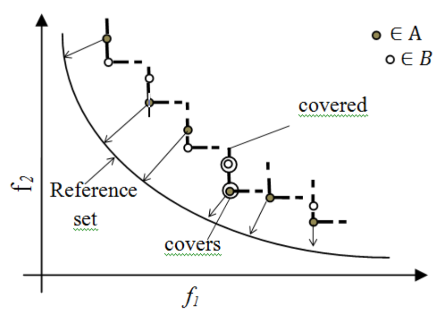

Fig.1. Representation of $\mathcal{M}_c$ and $\mathcal{M}_\mathcal{D}$

This idea makes the algorithm with the best performance to provide solutions with the largest coverage of the solutions from others. Figure 1 presents a simple Example with $\mathcal{M}_c\,(\mathcal{A}, \mathcal{B})$ =3/5 which is better than $\mathcal{M}_c\,(\mathcal{B}, \mathcal{A})$=2/6 and also it is noted that the average distance from points in set $\mathcal{A}$ to the reference set smaller than that of the points in $\mathcal{B}$. Thus the algorithm giving solutions $\mathcal{A}$ is deemed better than that producing solution $\mathcal{B}$.

5. AN IMPROVED MULTI-OBJECTIVE EA FOR VRPTW

The proposed multi-objective Evolutionary algorithm (MOEA) consists of selection, Cross-over and mutation and uses a simple list-based encoding as in most EAs. Here the implementation of a similarity measure to preserve population diversity is the main novel characteristic approach. This Characteristic is based on the Jaccard’s similarity coefficient in which the measurement shows how similar two sets are as the ratio between the numbers of elements. This metric is adapted to VRPTW for the BiEA[10] in which each solution is treated as set of arcs and used it for calculating the average similarity between solutions in the population.

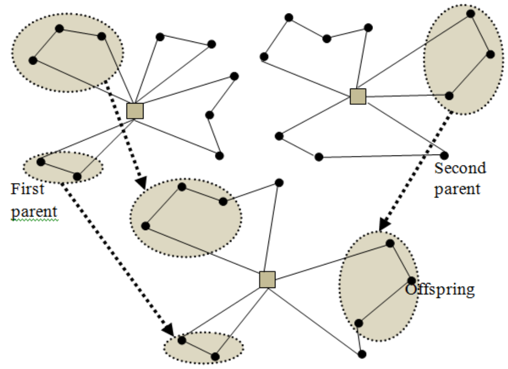

Fig. 2: The recombination process

Here MOEA differs from the BiEA in improved mutation stage and also mainly in dealing with more than two objectives.

5.1 Fitness Assignment

The assignment of fitness to solutions is done by using a dominance depth criteria [8] by which individuals are gathered into non-dominated fronts and the relative depth of the front confines the fitness.

5.2 Recombination

Next a tournament selection is used to choose the first parent according to fitness in recombination stage as usual. But at the same time, the second parent is chosen on the basis of lowest similarity measure. After that, the designing of recombination is structured to preserve routes from both parents. The two example parents in recombination is shown in fig 2. Both routes on the left are selected from the first parent to be copied into the offspring . Then the route only on the right can be copied from the second parent.

5.3 Mutation

Mutation is applied with a probability μ after on offspring has been generated. This includes three basic functions.

i. Select Route () which selects a route stochastically according to the largest ratio between the distance and the number of customers ie., the selection is as the routes with a larger travel distance and fewer customers.

ii. Select customer () which stochastically selects one customer from a particular route according to the longest average length of its inbound and outbound arcs.

iii. Insert customers () which tries to insert a set of customers into a specific route where the lowest travel distance is obtained. If there is no specified route, it tests all existing routes.

In addition, the three functions above given are used by the mutation operators. The three operators are:

- *Reallocation*-It takes a number of customers from a given route and allocates them to other existing routes. The process of this reallocation is illustrated in Fig.3
- *Exchange*–which swaps a sequence of customers between two routes, if it is possible, as illustrated in Fig.4
- *Reposition*-This uses select customer and insert customers respectively in order to select one customer from a specific route and reinserts it into the same route. This is illustrated in Fig.5.

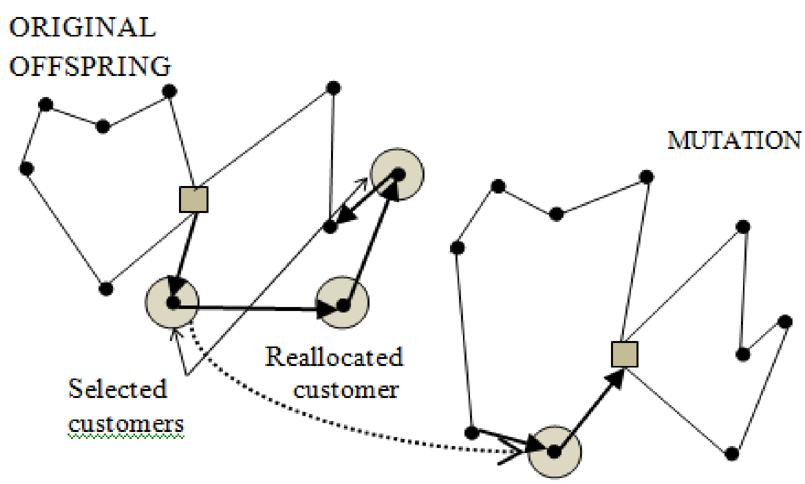

Fig. 3: The reallocation mutation operator

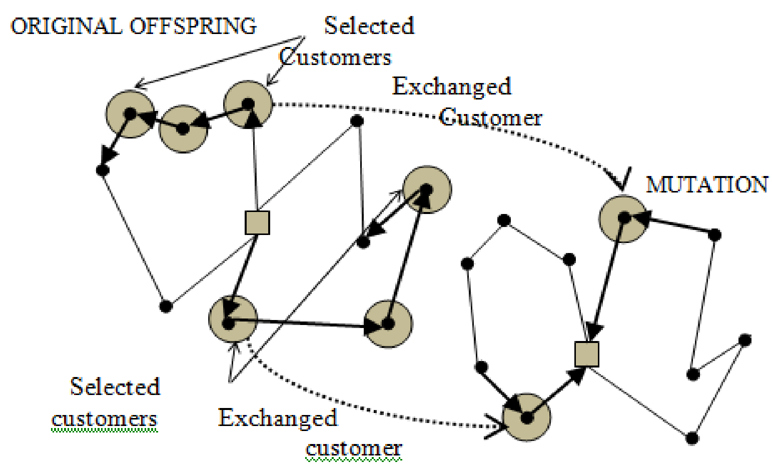

Fig. 4: The Exchange mutation operator.

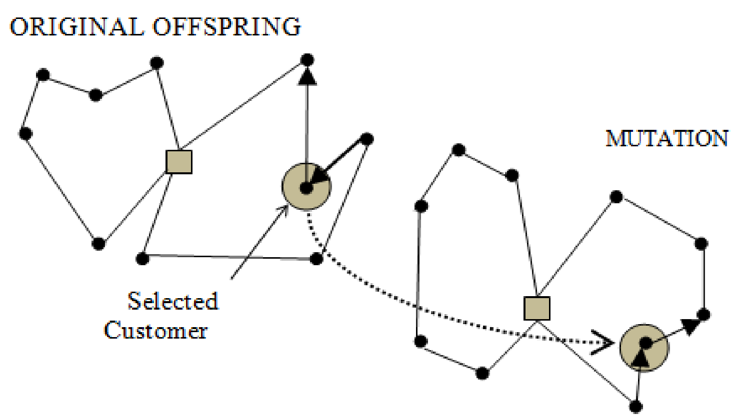

Fig. 5: The Reposition Mutation operator.

The Process of Mutation Is as Follows

At first, two routes are chosen using select Route().If two routes are same, the reallocation operation is performed or else, the exchange operator is executed immediately. Then select Route () takes plane to select another route and then the reposition operator is carried out. Finally, as the completion of the process, the parent and offspring populations are combined together, fitness levels are assigned and then the individuals with highest fitness animate the next new generation. The similarity is computed for the solutions in the front after the population size is attained in the last selected front. So that the least similar are chosen as desired. Likewise, the whole process for parent selection and offspring generation is repeated for a fixed number of generations.

6. EXPERIMENTAL STUDY FOR COMPARISONS

This study describes the VRPTW benchmark instances used for testing the proposed MOEA algorithm and also tests how this improved MOEA performance is well determined comparing with previous approaches. Finally evaluates the performance of MOEA as a direct fully multi-objective comparison with NISGA-II[8].

6.1 Solomon's Benchmark Instances

The standard benchmark set of Solomon is used to carry out controlled experiments [19]. This includes 56 instances of size N=100 and are categorized as: C1 and C2 where customers are situated in geographical clusters, R1and R2 where customers are distributed randomly and RC1 and RC2 in which the customers have a mixed of random locations and customers. Also, the instances in sets C1,R1and RC1have time constraint acts as a short scheduling horizon which together with the vehicle capacity constraint and permits customers few in number to be serviced by the same vehicle. Whereas, the instances in sets C2,R2and RC2have constraints with large vehicle capacities and allows many customers to be serviced by the same vehicle [19].

A recent analysis by Tan et al.[20] found that categories C1and C2 both have positively correlating objectives. That is the travel cost of a solution will increase with the number of vehicles. But the majority to the instances in four categories R1,R2,RC1 and RC2 were found to have conflicting objectives.

6.2 Experimental Set-Up

This algorithm was implemented in Java and tested on a computer cluster. The evolutionary parameters of our algorithm were set to the following suitable values that have proved to work well in preliminary testing.

Population size (pop size) =100

Number of generation (num Gen) =500

Tournament size (T size) =2

Crossover rate (γ) =1.0

Mutation rate (μ) =0.1

Each version of the algorithm was run 30 items with different random number of 30 items with different random number of seeds in order to provide reliable statistics for each bench mark instance.

6.3 Non-Dominated Sorting GA (NSGA –II)

The Elitist Non-dominated sorting (NSGA-II) is one of the effectively used Evolutionary multi-objective optimization (EMO) which tries to find multiple pareto-optimal solutions in MOP and also NSGA –II has the following the significant.

- It uses an elitist principle
- It has a descriptive diversity preserving mechanism
- It emphasizes or gives importance to non- dominated solutions.

For any generation t, the offspring population named as Q_t is created at first by using the parent population (say P_t) and the usual generator operators. Then the two populations are merged together so as to form a new population (say, R_t) which of size 2n.The new population R_t is assorted into various non-domination classes. Now, the new generated population is filled by points of different non-dominated fronts, one at a time. The process of filling begins with the first non-domination front and continues with the second non-domination front, and so on. As the overall population size of R_t is 2N, All fronts cannot be accommodated in N slots available for the new population. Fronts those who all could not be accommodated are deleted from the class. Once the last allowed front has to be considered, more points may exist in the front than the remaining slots in the new generated population. This schematic is illustrated in Fig. 6.

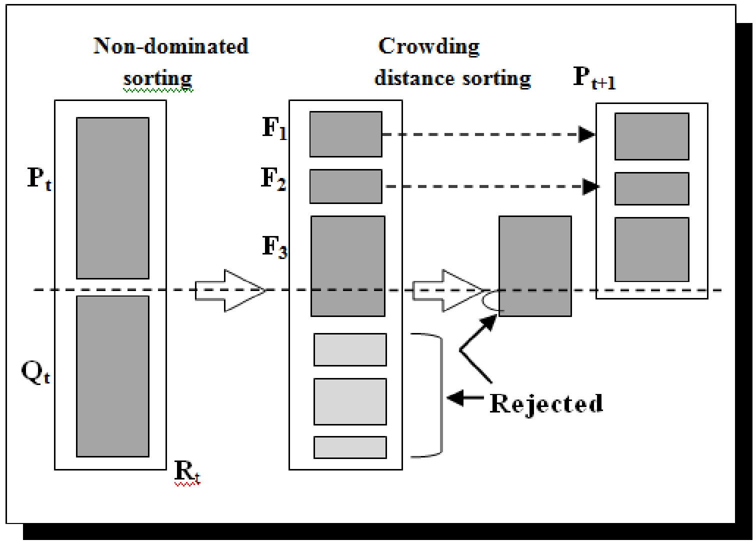

Fig. 6: Schematic of the NSGA-II procedure

7. RESULTS ANALYSIS

7.1. Comparisons with Previous Studies

Many previous studies have already considered the VRPTW as a multi - objective problem. But they have failed to present their results in a multi - objective manner. Since they have only shown averages from their best results, this study compares the averages of our best results with those previous studies. There after performs the proper multi - objective comparisons with NSGA - II. The following table gives the average number of routes as well as travel distances from those from many previous studies and from MOEA. For each problem instance, the entire solutions in the resulting non-dominated set are taken from all repetitions and the average is computed for each objective. After that they are averaged over each category. Noting that, in each algorithm, and category, the average number of routes (upper figure) and the average travel distance (lower figure) are shown in the table. The total average number of routes and travel distance for all 56 instances are presented in the last column (Accumulated). At last, the percentage difference between the results obtained from MOEA are shown in the bottom of the table. Since the categories C1 and C2 do not have conflicting objectives, MOEA has achieved similar results to the previous studies. The lowest number of routes, as well as the accumulations for other categories was received by BiEA [2]. But for MOEA, the solutions was found with lower travel distance for categories R2, RC1, RC2 and also accumulated. In the table it is shown that the solutions for Tan et al. [20] the category R_1 has the lowest travel distance, where results for MOEA are 0.33% which is higher. Similarly, the shortest distances for categories R2 and RC2 were obtained and by ombuki et al. [16], where the results for MOEA are 2.61%, 3.46% and 0.16% which are higher, but at the same time have smaller numbers of routes. In addition to that, the travel distances from MOEA for category RC1are clearly the shortest. The above shown result proves that the performance of MOEA is comparable to the algorithms which are previously published.

7.2. Multi-Objective Performance Comparisons with NSGA –II

As simple averages are often misleading, the performance of our MOEA is evaluated in a multi-objective approach, and the results are analyzed by using multi-objective coverage and convergence performance metrics for comparison of non-dominated solution from MOEA with those from NSGA –II [12]. The implementation of NSGA –II used the same solution representation with the same crossover and mutation operators as MOEA . The difference by comparison lies in the way that MOEA selects one parent according to the similarity, whereas NSGA-II uses fitness as the only criteria to select both parents. MOEA considers similarity for the identification of those solutions are taken to the next generation, while NSGA –II utilizes the crowding distance which does not involve any routing information.

For computation, the hyper volume metric M_H needs an appropriate reference points Z to be set. The reference point for every instance was set at $Z=(N, D^{max})$. Because, each instance has an obvious maximal solution, with largest number of routes that is equal to the number of customers N, and longest travel distance D^{max} which is equal to twice the sum of the distance for all customers from the depot.

There is a set of 30 M_H values for each problem instance from the 30 runs performed, with each value calculated by using the non- dominated sets obtained by MOEA and NSGA-II starting from the same initial populations. In table 2, It is shown that the hyper volume metric averages for each category and the numbers of instances with specific improvements over the other approaches. To see the difference among the categories, there is a small difference between two algorithms for categories C1& C2.The MOEA shows special improvement over NSGA-II in most problem instance for categories R1& RC1. On the other hand, the MOEA has significant improvement in some problem instances for categories R2&RC2.It is clear that there are no problem instances at all for which NSGA-II has significant performance better than the MOEA.

Author	C1	C2	R1	R2	RC1	RC2	Accumulated
Ten et al. [7]	10.00	3.00	12.92	3.55	12.38	4.25	441.00
	828.91	590.81	1187.35	951.74	1355.37	1068.26	56293.06
Jung and Moon[23]	10.00	3.00	13.25	5.36	13.00	6.25	486.00
	828.38	589.86	1179.95	878.41	1343.65	1004.21	54779.02
Ombuki et al. [8]	10.00	3.00	13.17	4.55	13.00	5.63	471.00
	828.48	590.60	1204.48	893.03	1384.95	1025.31	55740.33
Le Bouthillier and Crainic [27]	10.00	3.00	12.08	2.73	11.50	3.25	407.00
	828.38	589.86	1209.19	960.95	1386.38	1133.30	57412.37
BiEA [9]	10.00	3.00	12.50	3.18	12.38	4.00	430.00
	830.64	589.86	1191.22	926.97	1349.81	1080.11	56125.35
Homberger and Gehring [29]	10.00	3.00	11.91	2.73	11.50	3.25	405.00
	828.38	589.38	1212.73	955.03	1386.44	1108.52	57192.00
Pisinger and Ropke [42]	10.00	3.00	11.92	2.73	11.50	3.25	405.00
	828.38	589.86	1212.39	957.72	1385.78	1123.49	57332.00
Garcia-Najera and Bullinaria[35]	10.00	3.00	12.50	3.18	12.38	4.00	430.00
	830.64	589.86	1191.22	926.97	1349.81	1080.11	56125.35
MOEA	10.00	3.00	13.08	4.00	12.63	5.38	459.00
% difference number of routes	0.00	0.00	9.85	46.52	9.78	65.38	13.33
% difference travel distance	0.00	0.08	0.62	2.22	0.34	3.23	1.09

Table 1: Travel distance and number of routes, averaged over categories, for the best solutions found in previous studies and by MOEA.

Algorithm	C1	C2	R1	R2	RC1	RC2
NSGA-II	0.77	0.87	0.66	0.79	0.69	0.81
	(0)	(0)	(0)	(0)	(0)	(0)
MOEA	0.81	0.74	0.71	0.76	0.75	0.81
	(1)	(0)	(10)	(3)	(8)	(2)

Table 2: Hyper volume metric values, averaged over instance categories, for solutions obtained with NSGA-II and MOEA. The number of instances for which the result is significantly better than the other approach are shown in brackets.

Algorithm	**C1**	**C2**	**R1**	**R2**	**RC1**	**RC2**
NSGA-II	0.81	0.87	0.14	0.37	0.13	0.35
	(0)	(2)	(0)	(4)	(0)	(0)
	25.06	4.88	51.56	49.81	72.21	67.17
	(0)	(0)	(0)	(0)	(0)	(0)
MOEA	0.93	0.88	0.78	0.41	0.80	0.45
	(4)	(2)	(12)	(5)	(8)	(7)
	12.20	5.07	27.89	50.67	33.91	65.86
	(3)	(0)	(12)	(0)	(8)	(1)

Table 3: Coverage (upper figure) and convergence (lower figure), averaged over instance categories, for solutions obtained with NSGA-II and MOEA. The numbers of instances which are significantly better than the other approach are shown in brackets.

Next, The coverage values M_C (MOEA, NSGA-II) and (NSGA-II, MOEA)were computed for all pairs of runs (i,j=1, 2, . . ., 30) which means 900 M_C values each. The following table gives the averages of $M_C(x,y)$ values over all the instances within each problem category and there was a significant improvement for the numbers of instances over the other approach.

The improvement in each instance is explained in the figure. Six plots show the mean population diversity and also variance on the vertical axis for each instance category and a function for first 500 generations in horizontal axis. Observing this it is very clear that the MOEA definitely preserves a higher diversity for the categories R1, R2, RC1 and RC2 which are the improvements over NSGA-II. Whereas the non-multi –objective categories C1 and C2were not. Further the different behavior for R1and RC1 to that of R2 and RC2 is also stable with that distinction in the hyper volume results and the increased numbers of solutions in MOEA pareto approximations compared to NSGA-II. Overall then, It can be concluded that the MOEA diversities present a significantly gentle drop in all categories except 2. This leads that MOEA performs a wider exploration in the search space and significantly better than NSGA-II.

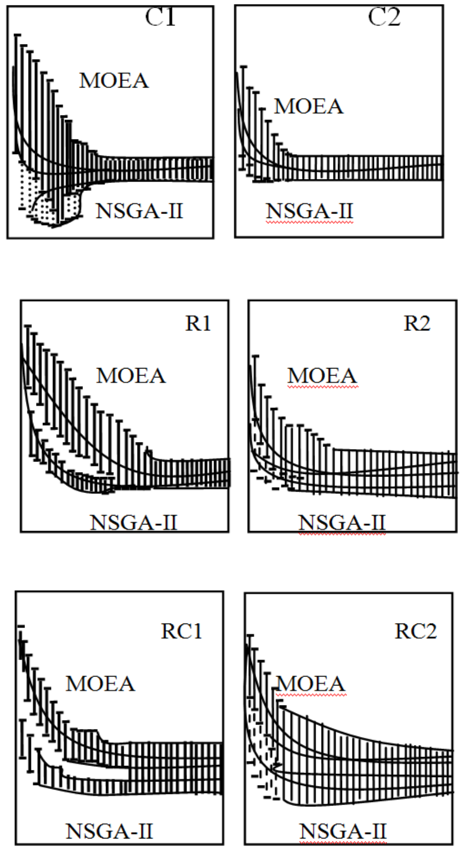

Fig.7. Average population diversity for each instance category as a function of generation, for MOEA and NSGA-II.

7.3. Comparisons with Tri-Objective Performance

In tri-objective performance, the application of our MOEA is illustrated to other objectives and for optimizing more than two objectives at the same time. Here the number of routes (R), travel distance (D) and delivery time (T) were first to be minimized in pairs, giving three objective settings RD, RT and DT and then all three of them were minimized together (RDT). Using the coverage performance metric M_C for each setting and instance, the outcome set of non-dominated solution after each of the 30 repetitions was recorded. The compared results provide the following observations. For categories C1 and C_2, we have mixed results with all settings having a high coverage of each other. The contrarily coverage for RD, DT and RDT by RD is low (14%),and the coverage for RD, DT and RDT by RT is closed to zero. The settings

DT and RDT are most interesting cases due to their coverage of RD and RT is much higher. Between them, the coverage of DT by RDT is significantly larger than the coverage of RDT by DT. These results are evidently indicated that setting MOEA minimizes all three objectives does definitely leads to better non-dominated solutions. The above said comparison results are presented in the Table 4.

Obj.	**Covers**	**C1**	**C2**	**R1**	**R2**	**RC1**	**RC2**
	RT	0.87	0.64	0.04	0.01	0.05	0.02
		(6)	(3)	(6)	(1)	(4)	(2)
RD	DT	0.82	0.72	0.08	0.11	0.14	0.08
		(1)	(0)	(1)	(4)	(0)	(0)
	RDT	0.82	0.72	0.08	0.11	0.13	0.08
		(1)	(1)	(1)	(3)	(0)	(1)
	RD	0.68	0.55	0.01	0	0.01	0.01
		(0)	(1)	(2)	(0)	(2)	(0)
RT	DT	0.68	0.63	0.03	0.04	0.06	0.02
		(0)	(0)	(0)	(0)	(0)	(0)
	RDT	0.68	0.62	0.04	0.03	0.07	0.03
		(0)	(0)	(0)	(0)	(0)	(0)
	RD	0.91	0.9	0.31	0.14	0.36	0.21
		(2)	(3)	(11)	(5)	(8)	(6)
DT	RT	0.97	0.92	0.49	0.42	0.46	0.48
		(5)	(4)	(12)	(11)	(8)	(8)
	RDT	0.91	0.88	0.43	0.4	0.42	0.42
		(2)	(2)	(4)	(4)	(2)	(3)
	RD	0.89	0.91	0.32	0.16	0.38	0.23
		(3)	(3)	(11)	(6)	(8)	(7)
RDT	RT	0.97	0.93	0.52	0.44	0.49	0.44
		(6)	(3)	(12)	(11)	(8)	(8)
	DT	0.89	0.89	0.44	0.43	0.46	0.41
		(2)	(2)	(4)	(4)	(2)	(3)

CONCLUSION

The paper has proposed and analyzed the performance of an improved multi-objective Evolutionary Algorithm (MOEA) to solve vehicle Routing Problem with Time Windows (VRPTW). MOEA can minimize simultaneously for any number of objectives and also has been tested on the number of routes, travel distance and delivery time. In this paper a crucial difference of the MOEA proposed to most EA_S is that in addition using fitness to select good parents, it also uses the similarity measure to maintain population diversity. The first from the two parents is selected on the basis of fitness and the second on the basis of similarity. This is the key improvement in the introduction for a similarity measure between solutions which is primarily used to choose the second parent for the recombination process.

An improved MOEA has been tested using a popular bench mark set of 56 VRPTW problem instances, nearly half of which have conflicting objectives. The experimental results have made to explore the effect that similarity measure had on population diversity and solution quality. Comparing three methods to select second parent including fitness, it is demonstrated that the similarity measure has a high importance which had in maintaining diversity and arriving for better solutions. This is the best approach and this would work best on problems with conflicting objectives. Later, it was confirmed that not offering much improvement on others. The proposed MOEA was evaluated by using two multi-objective performance metrics namely hyper volume and coverage. This shows the significantly better results of MOEA than the popular NSGA-II for both bi-objective and tri-objective performances.

Therefore, it is concluded that the proposed MOEA performance comparison against the well known evolutionary multi-objective optimizer NSGA-II has shown that the solutions obtained by MOEA are much better for almost all instance categories. There is also a scope to explore the extension of the MOEA for optimizing even more objectives such as route balance and this scope will pursue the comparison of our results with other multi-criterion optimization methods and also further multi-objective performance metrics.

Conflict of Interests

The authors declare there is no conflict of interests.

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