Available online at http://scik.org

J. Math. Comput. Sci. 11 (2021), No. 4, 4711-4719

https://doi.org/10.28919/jmcs/5662

ISSN: 1927-5307

A REMARK ON HESITANT FUZZY IDEALS IN ORDERED SEMIGROUPS

NAREUPANAT LEKKOKSUNG*

Division of Mathematics, Faculty of Engineering, Rajamangala University of Technology Isan
Khon Kaen Campus, Khon Kaen 40000, Thailand

Abstract. We investigate the concept of hesitant fuzzy sets. This concept is a generalized notion of fuzzy sets. The concept of fuzzy sets can be applied to the study of ordered semigroups. It is well-known that in regular ordered semigroups, any fuzzy bi-ideal and fuzzy quasi-ideal coincide. In this paper, we illustrate that this fact does not hold for hesitant fuzzy sets. Furthermore, we provide an example illustrating that in a regular ordered semigroup, there is a hesitant fuzzy bi-ideal which is not a hesitant fuzzy quasi-ideal. Moreover, we present a class of hesitant fuzzy sets in which hesitant fuzzy bi-ideals and hesitant fuzzy quasi-ideals coincide.

Keywords: hesitant fuzzy quasi-ideal; hesitant fuzzy bi-ideal; ordered semigroup.

2010 AMS Subject Classification: 20M12, 20M99, 08A72.

1. Introduction

The concept of fuzzy sets was first introduced by Zadeh as a generalization of vague sets (see [20]). Fuzzy sets are widely utilized in many branches of research, especially in decision making problems (see [5, 21]). Moreover, fuzzy sets can be applied to investigate algebraic properties of semigroups and their generalizations (see [4, 7, 8, 9, 11, 12, 18]).

---

*Corresponding author

E-mail address: nareupanat.le@rmuti.ac.th

Received March 8, 2021

There are many concepts extending the application of fuzzy sets; among them is the concept of hesitant fuzzy sets established by Torra. The notion was introduced for situations that the degree of membership is difficult to specify (see [15]). Therefore, hesitant fuzzy sets are suitable tools to deal with practical problems in decision making (see [16, 17]). Moreover, the concept of hesitant fuzzy sets can also be used to investigate properties of algebras. In 2016, Ahn, Lee and Jun extended the notions of ideals in ordered semigroups, relating it to the notions of hesitant fuzzy ideals. Based on their results, Ahn, Lee and Jun introduced the notions of hesitant fuzzy left, right, quasi- and bi-ideals. They also studied the interconnections between these kinds of hesitant fuzzy ideals in ordered semigroups (see [3]). Later on, many researchers conducted studies into algebraic systems in terms of hesitant fuzzy sets in various directions (see [1, 2, 6, 13, 14, 19]).

In the investigation of fuzzy ideals in ordered semigroups, every fuzzy quasi-ideal of an ordered semigroup is a fuzzy bi-ideal. The converse of this statement is true if an ordered semigroup is regular (see [10]). This assertion was explored and proven in terms of hesitant fuzzy sets by Ahn, Lee and Jun. They illustrated that every hesitant quasi-ideal of an ordered semigroup is a hesitant bi-ideal. Moreover, the authors illustrated that if an ordered semigroup is regular, then the concepts of hesitant quasi-ideal and hesitant bi-ideal coincide (see [3]).

In this paper, we illustrate using an example that in a regular ordered semigroup, a hesitant fuzzy bi-ideal may not always be a hesitant fuzzy quasi-ideal. Finally, we establish a particular class of hesitant fuzzy sets, the properties of which that every hesitant fuzzy bi-ideal is a hesitant fuzzy quasi-ideal in regular ordered semigroups.

2. Preliminaries

We provide brief concepts of ordered semigroups and hesitant fuzzy ordered semigroups in this section.

2.1. Ordered semigroups. The structure $\langle S;\circ,\leq\rangle$ consisting of a nonempty set S, a binary operation $\circ$ defined on S and a binary relation $\leq$ defined on S is called an *ordered semigroup* if

(1) $\langle S;\circ\rangle$ is a semigroup, that is, $\circ$ is associative,

(2) $\langle S;\leq\rangle$ is a partially ordered set,

(3) the binary operation $\circ$ is compatible with the order relation $\leq$, that is, for all $x, y \in S$ such that $x \leq y$, we have $x \circ z \leq y \circ z$ and $z \circ x \leq z \circ y$ for all $z \in S$.

Throughout this paper, we will denote $x \circ y$ by xy for any $x, y \in S$. We simplify ordered semigroup $\langle S; \circ, \leq \rangle$ as $\mathbf{S}$.

We observe that any semigroup can be regarded as an ordered semigroup by defining $\leq := \Delta$, where Δ is the equality relation on the underlying set of such semigroup.

Let $\mathbf{S}$ be an ordered semigroup, and $A, B \subseteq S$. We define the sets

$$(A] := \{x \in S : x \leq a \text{ where } a \in S\}$$

and

$$AB := \{ab : a \in A \text{ and } b \in B\}.$$

Let $\mathbf{S}$ be an ordered semigroup. A nonempty subset A of S such that $(A] \subseteq A$ is said to be

(1) a *left* (*resp., right*) *ideal* of $\mathbf{S}$ if $SA \subseteq A$ (resp., $AS \subseteq A$),

(2) a *quasi-ideal* of $\mathbf{S}$ if $(AS] \cap (SA] \subseteq A$,

(3) a *bi-ideal* of $\mathbf{S}$ if $AA \subseteq A$ and $ASA \subseteq A$.

We denote the set of all left ideals, right ideals, quasi-ideals, and bi-ideals of an ordered semigroup $\mathbf{S}$ by $\mathsf{L}(\mathbf{S})$, $\mathsf{R}(\mathbf{S})$, $\mathsf{Q}(\mathbf{S})$, and $\mathsf{B}(\mathbf{S})$, respectively. Thus, it is illustrated that $\mathsf{L}(\mathbf{S}) \subseteq \mathsf{Q}(\mathbf{S})$, $\mathsf{R}(\mathbf{S}) \subseteq \mathsf{Q}(\mathbf{S})$ and $\mathsf{Q}(\mathbf{S}) \subseteq \mathsf{B}(\mathbf{S})$.

In general, the sets $\mathsf{Q}(\mathbf{S})$ and $\mathsf{B}(\mathbf{S})$ may not be equal. But $\mathsf{Q}(\mathbf{S}) = \mathsf{B}(\mathbf{S})$ if $\mathbf{S}$ is a *regular ordered semigroup*. A regular ordered semigroup will mean that an ordered semigroup such that for any $a \in S$, we have $a \leq axa$ for some instances $x \in S$.

2.2. Hesitant fuzzy ordered semigroups.

Fuzzy sets and their generalizations can be used to investigate algebraic properties of ordered semigroups, namely, ideals in ordered semigroups can be studied in terms of both fuzzy ideals and hesitant fuzzy ideals. In this subsection, we reiterate some interesting results concerning fuzzy ideals and hesitant fuzzy ideals.

Let $[0,1]$ be a closed unit interval. We denote by $\mathrm{Sb}([0,1])$ as the set of all subsets of $[0,1]$.

Let X be a nonempty set. A mapping

(1) $f \colon X \to [0,1]$ is called a *fuzzy set* of X,

(2) $\mathscr{H}: X \to \mathsf{Sb}([0,1])$ is called a *hesitant fuzzy set* of X.

We denote the set of all fuzzy sets on X and hesitant fuzzy sets on X by $\mathsf{FS}(X)$ and $\mathsf{HFS}(X)$, respectively.

Remark 2.1. *The concept of hesitant fuzzy sets is a generalization of fuzzy sets in the sense that a fuzzy set can be regarded as a hesitant fuzzy set. In fact, there is a one-to-one function* $\varphi\colon \mathsf{FS}(X) \to \mathsf{HFS}(X)$ *defined by* $\varphi(f) := \mathscr{H}_f$ *for any* $f \in \mathsf{FS}(X)$, *where* $\mathscr{H}_f$ *is assigned by* $x \mapsto [0, f(x)]$ *for all* $x \in X$.

Let $\mathbf{S}$ be an ordered semigroup, and $x \in S$. We denote the set

$$\mathbf{S}_x := \{(u,v) \in S \times S : x \leq uv\}.$$

Let $\mathbf{S}$ be an ordered semigroup, and $f, g \in \mathsf{FS}(\mathbf{S})$. We define a binary relation $\subseteq$, and binary operations $\cap$, $\cup$ and $\circ$ on the set $\mathsf{FS}(\mathbf{S})$ as follows: $f \subseteq g$ if $f(x) \leq g(x)$ for all $x \in S$, $(f \cap g)(x) := \min\{f(x), g(x)\}$, $(f \cup g)(x) := \max\{f(x), g(x)\}$ and

$$(f \circ g)(x) := \begin{cases} \sup_{(u,v)\in \mathbf{S}_x}\{\min\{f(u), g(v)\}\} & \text{if } \mathbf{S}_x \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

for all $x \in S$.

Let $\mathbf{S}$ be an ordered semigroup. A fuzzy set f of S, such that for any $x, y \in S$, we have $f(x) \geq f(y)$ whenever $x \leq y$, is called

(1) a *fuzzy left* (*resp., right*) *ideal* of $\mathbf{S}$ if $f(xy) \geq f(y)$ (resp., $f(xy) \geq f(x)$) for all $x, y \in S$,

(2) a *fuzzy quasi-ideal* of $\mathbf{S}$ if $(\mathbf{1} \circ f) \cap (f \circ \mathbf{1}) \subseteq f$, where $\mathbf{1}(x) := 1$ for all $x \in S$,

(3) a *fuzzy bi-ideal* of $\mathbf{S}$ if $f(xy) \geq \min\{f(x), f(y)\}$ and $f(xyz) \geq \min\{f(x), f(z)\}$ for all $x, y, z \in S$.

We denote this by $\mathsf{FLS}(\mathbf{S})$, $\mathsf{FRS}(\mathbf{S})$, $\mathsf{FQS}(\mathbf{S})$, and $\mathsf{FBS}(\mathbf{S})$ the set of all fuzzy left ideals, fuzzy right ideals, fuzzy quasi-ideals and fuzzy bi-ideals of $\mathbf{S}$, respectively.

Then we have the following theorem:

Theorem 2.2 ([10])**.** *Let* $\mathbf{S}$ *be an ordered semigroup. Then:*

(1) $\mathsf{FLS}(\mathbf{S}) \subseteq \mathsf{FQS}(\mathbf{S})$,

(2) $\mathsf{FRS}(\mathbf{S}) \subseteq \mathsf{FQS}(\mathbf{S})$,

(3) $\mathsf{FQS}(\mathbf{S}) \subseteq \mathsf{FBS}(\mathbf{S})$,

(4) $\mathsf{FQS}(\mathbf{S}) = \mathsf{FBS}(\mathbf{S})$ *if* $\mathbf{S}$ *is regular.*

Let $\mathbf{S}$ be an ordered semigroup, $\mathscr{G}, \mathscr{H} \in \mathsf{HFS}(\mathbf{S})$, and $x, y, z \in S$. We denote the sets $\mathscr{H}(x)$, $\mathscr{H}(x) \cap \mathscr{H}(y)$, and $\mathscr{H}(x) \cap \mathscr{H}(y) \cap \mathscr{H}(z)$ by $\mathscr{H}_x$, $\mathscr{H}_x^y$, and $\mathscr{H}_x^y[z]$, respectively. Moreover, we write $\mathscr{G} \sqsubseteq \mathscr{H}$ if $\mathscr{G}_x \subseteq \mathscr{H}_x$ for all $x \in S$. Then, we define new hesitant fuzzy sets $\mathscr{G} \sqcup \mathscr{H}$ and $\mathscr{G} \sqcap \mathscr{H}$ by $(\mathscr{G} \sqcup \mathscr{H})(x) := \mathscr{G}_x \cup \mathscr{H}_x$ and $(\mathscr{G} \sqcap \mathscr{H})(x) := \mathscr{G}_x \cap \mathscr{H}_x$ for all $x \in S$. Another possibility to obtain a new hesitant fuzzy set can be done as follows. Let $\mathscr{G}, \mathscr{H} \in \mathsf{HFS}(\mathbf{S})$. We define a binary operation $\tilde{\circ}$ on $\mathsf{HFS}(\mathbf{S})$ by

$$(\mathscr{G} \tilde{\circ} \mathscr{H})_x := \begin{cases} \bigcup_{(u,v) \in \mathbf{S}_x} \{\mathscr{G}_u \cap \mathscr{H}_v\} & \text{if } \mathbf{S}_x \neq \emptyset, \\ \emptyset & \text{otherwise,} \end{cases}$$

for all $x \in S$. An element $[\chi_S] \in \mathsf{HFS}(\mathbf{S})$ is defined by $[\chi_S]_x := [0,1]$ for all $x \in S$.

Let $\mathbf{S}$ be an ordered semigroup. A hesitant fuzzy set $\mathscr{H}$ of $\mathbf{S}$ satisfying $x \leq y$ implies that $\mathscr{H}_x \supseteq \mathscr{H}_y$ for all $x, y \in S$ which is said to be

(1) a *hesitant fuzzy left* (*resp., right*) *ideal* of $\mathbf{S}$ if $\mathscr{H}_{xy} \supseteq \mathscr{H}_y$ (resp., $\mathscr{H}_{xy} \supseteq \mathscr{H}_x$) for all $x, y \in S$,

(2) a *hesitant fuzzy quasi-ideal* of $\mathbf{S}$ if $([\chi_S] \tilde{\circ} \mathscr{H}) \sqcap (\mathscr{H} \tilde{\circ} [\chi_S]) \sqsubseteq \mathscr{H}$,

(3) a *hesitant fuzzy bi-ideal* of $\mathbf{S}$ if $\mathscr{H}_{xy} \supseteq \mathscr{H}_x^y$ and $\mathscr{H}_{xyz} \supseteq \mathscr{H}_x^z$ for all $x, y, z \in S$.

We denote the set of all hesitant fuzzy left ideals, hesitant fuzzy right ideals, hesitant fuzzy quasi-ideals and hesitant fuzzy bi-ideals of $\mathbf{S}$ as $\mathsf{HFLS}(\mathbf{S})$, $\mathsf{HFRS}(\mathbf{S})$, $\mathsf{HFQS}(\mathbf{S})$, and $\mathsf{HFBS}(\mathbf{S})$, respectively. More information about hesitant fuzzy ideals in ordered semigroups can be found in [3].

The following theorem was proven by Ahn, Lee and Jun in 2016.

Theorem 2.3 ([3])**.** *Let* $\mathbf{S}$ *be an ordered semigroup. Then:*

(1) $\mathsf{HFLS}(\mathbf{S}) \subseteq \mathsf{HFQS}(\mathbf{S})$,

(2) $\mathsf{HFRS}(\mathbf{S}) \subseteq \mathsf{HFQS}(\mathbf{S})$,

(3) $\mathsf{HFQS}(\mathbf{S}) \subseteq \mathsf{HFBS}(\mathbf{S})$,

(4) $\mathsf{HFQS}(\mathbf{S}) = \mathsf{HFBS}(\mathbf{S})$ *if* $\mathbf{S}$ *is regular.*

3. RESULTS

Firstly, we give an example which illustrates that Theorem 2.3(4) is not true.

Example 3.1. *Let $S = \{a,b,c\}$. Define a binary operation $\circ$ on S by the following multiplication table.*

$\circ$	a	b	c
a	a	a	a
b	a	b	a
c	a	a	c

Let $\leq := \{(a,c)\} \cup \Delta_S$ be a binary relation on S, where $\Delta_S := \{(x,x) : x \in S\}$. We can see that $\mathbf{S} := \langle S; \circ, \leq \rangle$ is a regular ordered semigroup. Define $\mathscr{H} : S \to \mathrm{Sb}([0,1])$ a hesitant fuzzy set on S by

$$\mathscr{H}_x := \begin{cases} [0.2, 0.5] & \text{if } x = a, \\ [0.5, 0.6] & \text{if } x = b, \\ [0.3, 0.4] & \text{if } x = c, \end{cases}$$

for any $x \in S$. We show that $\mathscr{H}$ is a hesitant fuzzy bi-ideal of $\mathbf{S}$.

Let us observe that $\mathscr{H}_a \supsetneq \mathscr{H}_x$ for any $x \in S \setminus \{a\}$, and $ax = a = ay$ for all $x, y \in S$. The former observation illustrates that for any $x, y \in S$, we have $x \leq y$, which implies that $\mathscr{H}_x \supseteq \mathscr{H}_y$. Next, we show that for any $x, y \in S$, we have $\mathscr{H}_{xy} \supseteq \mathscr{H}_x^y$. By the idempotency, commutativity of the binary operation $\circ$ on S, we consider the following nontrivial case: $\mathscr{H}_{bc} = [0.2, 0.5] \supsetneq \emptyset = \mathscr{H}_b^c$. Lastly, we show that for any $x, y, z \in S$, we have $\mathscr{H}_{xyz} \supseteq \mathscr{H}_x^z$. By the idempotency, commutativity of the binary operation $\circ$ on S, we consider the following nontrivial case: $\mathscr{H}_{bbc} = [0.2, 0.5] \supsetneq \emptyset = \mathscr{H}_b^c$. Altogether, we obtain that $\mathscr{H}$ is a hesitant fuzzy bi-ideal of $\mathbf{S}$.

It can be seen that $\mathbf{S}_a = (S \times S) \setminus \{(b,b)\}$. Then, we can carefully calculate that $(\mathscr{H} \tilde{\circ} [\chi_S])_a = \bigcup_{(u,v) \in \mathbf{S}_a} (\mathscr{H}_u \cap [\chi_S]_v) = [0.2, 0.6]$ and $([\chi_S] \tilde{\circ} \mathscr{H})_a = \bigcup_{(u,v) \in \mathbf{S}_a} ([\chi_S]_u \cap \mathscr{H}_v) = [0.2, 0.6]$. Then we have that $(\mathscr{H} \tilde{\circ} [\chi_S])_a \cap ([\chi_S] \tilde{\circ} \mathscr{H})_a = [0.2, 0.6] \supsetneq [0.2, 0.5] = \mathscr{H}_a$. Therefore, $\mathscr{H}$ is not a hesitant fuzzy quasi-ideal of $\mathbf{S}$.

Now, the question becomes when does a hesitant fuzzy bi-ideal a hesitant fuzzy quasi-ideal of an ordered semigroup? Before we address this question, we introduce a notion which is important for this answer.

Let **S** be an ordered semigroup, and $\mathscr{H} \in \mathsf{HFS}(\mathbf{S})$. We call $\mathscr{H}$ a *hesitant fuzzy set of* **S** *with chain-valued* if either $\mathscr{H}_x \subseteq \mathscr{H}_y$, or $\mathscr{H}_x \supseteq \mathscr{H}_y$ for any $x, y \in S$. We denote the set of all hesitant fuzzy sets of **S** with chain-valued by $\text{c-}\mathsf{HFS}(\mathbf{S})$.

Our main theorem shows that Theorem 2.3(4) is true only in a particular class of hesitant fuzzy sets of **S**.

Theorem 3.2. *Let* **S** *be a regular ordered semigroup, and let* $\mathscr{H} \in \text{c-}\mathsf{HFS}(\mathbf{S})$. *In this case, the following statements are equivalent.*

(1) $\mathscr{H}$ *is a hesitant fuzzy quasi-ideal of* **S**.

(2) $\mathscr{H}$ *is a hesitant fuzzy bi-ideal of* **S**.

Proof. $(1) \Rightarrow (2)$. The proof of this direction can be found in [3, Theorem 3.7].

$(2) \Rightarrow (1)$. Assume that $\mathscr{H}$ is a hesitant fuzzy bi-ideal of **S**. Let $x \in S$. Clearly,

$$((\mathscr{H}\tilde{\circ}[\chi_S]) \sqcap ([\chi_S]\tilde{\circ}\mathscr{H}))_x = (\mathscr{H}\tilde{\circ}[\chi_S])_x \cap ([\chi_S]\tilde{\circ}\mathscr{H})_x.$$

Since **S** is regular, it is clear that $\mathbf{S}_x \neq \emptyset$. As such, let us consider

$$(\mathscr{H}\tilde{\circ}[\chi_S])_x = \bigcup_{(z,w)\in\mathbf{S}_x} \{\mathscr{H}_z \cap [\chi_S]_w\}.$$

Obviously, if $(\mathscr{H}\tilde{\circ}[\chi_S])_x \subseteq \mathscr{H}_x$, then

$$\mathscr{H}_x \supseteq (\mathscr{H}\tilde{\circ}[\chi_S])_x \supseteq (\mathscr{H}\tilde{\circ}[\chi_S])_x \cap ([\chi_S]\tilde{\circ}\mathscr{H})_x = ((\mathscr{H}\tilde{\circ}[\chi_S]) \sqcap ([\chi_S]\tilde{\circ}\mathscr{H}))_x.$$

Thus, we can conclude that $(\mathscr{H}\tilde{\circ}[\chi_S])_x \supsetneq \mathscr{H}_x$. Therefore, $\mathscr{H}_z \cap [\chi_S]_w \supsetneq \mathscr{H}_x$ for some $(z,w) \in \mathbf{S}_x$. That is,

$$\mathscr{H}_z \supsetneq \mathscr{H}_x \tag{1}$$

and $x \leq zw$. Following on, for any given $(u,v) \in \mathbf{S}_x$, we can produce $x \leq uv$. By the regularity of **S** and $x \in S$, there exists $s \in S$ such that $x \leq xsx$. Now, we have $x \leq xsx$, $x \leq zw$ and $x \leq uv$. Since $\leq$ is compatible with $\circ$, we obtain $x \leq z(wsu)v$. By the ideality of $\mathscr{H}$, we have that

$\mathscr{H}_x \supseteq \mathscr{H}_{z(wsu)v} \supseteq \mathscr{H}_z^v$. This implies that $\mathscr{H}_z^v = \mathscr{H}_v$, otherwise we would have $\mathscr{H}_x \supseteq \mathscr{H}_z$, which contradicts (1). Hence, $\mathscr{H}_x \supseteq \mathscr{H}_v = [\chi_S]_u \cap \mathscr{H}_v$. By the arbitrariness of $(u,v) \in \mathbf{S}_x$, we have that $\mathscr{H}_x \supseteq ([\chi_S]\tilde{\circ}\mathscr{H})_x$. Therefore,

$$\mathscr{H}_x \supseteq ([\chi_S]\tilde{\circ}\mathscr{H})_x \supseteq (\mathscr{H}\tilde{\circ}[\chi_S])_x \cap ([\chi_S]\tilde{\circ}\mathscr{H})_x = ((\mathscr{H}\tilde{\circ}[\chi_S]) \sqcap ([\chi_S]\tilde{\circ}\mathscr{H}))_x.$$

This completes the proof. □

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] M.Y. Abbasi, A.F. Talee, S.A. Khan, K. Hila, A Hesitant Fuzzy Set Approach to Ideal Theory in Γ-Semigroups, Adv. Fuzzy Syst. 2018 (2018), Article ID 5738024.

[2] M.Y. Abbasi, A.F. Talee, X.Y. Xie, S.A. Khan, Hesitant fuzzy ideal extension in po-semigroups, TWMS J. Appl. Eng. Math. 8 (2018), 509–521.

[3] S.S. Ahn, K.J. Lee, Y.B. Jun, Ideal theory in ordered semigroups based on hesitant fuzzy sets, Honam Math. J. 38 (2016), 783–794.

[4] R. Chinram, S. Saelee, Fuzzy ideals and fuzzy filters of ordered ternary semigroups, J. Math. Res. 2 (2010), 93–97.

[5] D.H. Hong, C.H. Choi, Multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets Syst. 114 (2000), 103–113.

[6] Y.B. Jun, S.Z. Song, G. Muhiuddin, Hesitant fuzzy semigroups with a frontier, J. Intell. Fuzzy Syst. 30 (2016), 1613–1618.

[7] A. Kanlaya, A. Iampan, Coincidences of different types of fuzzy ideals in ordered Γ-semigroups, Korean J. Math. 22 (2014), 367–381.

[8] N. Kehayopulu, M. Tsingelis, Fuzzy bi-ideals in ordered semigroups, Inform. Sci. 171 (2005), 13–28.

[9] N. Kehayopulu, M. Tsingelis, Fuzzy interior ideals in ordered semigroups, Lobachevskii J. Math. 21 (2006), 65–71.

[10] N. Kehayopulu, M. Tsingelis, Fuzzy ideals in ordered semigroups, Quasigroups Related Syst. 15 (2007), 279–289.

[11] N. Lekkoksung, P. Jampachon, Characterizations of regular ordered ternary semigroups in term of fuzzy subsets, Southeast Asian J. Sci. 2 (2013), 141–153.

[12] N. Lekkoksung, P. Jampachon, On regular ordered ternary semigroups, Far East J. Math. Sci. 92 (2014), 67–84.

[13] A.F. Talee, M.Y. Abbasi, S.A. Khan, Hesitant fuzzy ideals in semigroups with two frontiers, J. Basic Appl. Eng. Res. 4 (2017), 385–388.

[14] A.F. Talee, M.Y. Abbasi, S.A. Khan, Hesitant fuzzy sets approach to ideal theory in ternary semigroups, Int. J. Appl. Math. 31 (2018), 527–539.

[15] V. Torra, Hesitant fuzzy sets, Int. J. Intell. Syst. 25 (2010), 529–539.

[16] R.M. Rodriguez, L. Martinez, F. Herrera, Hesitant fuzzy linguistic term sets for decision making, IEEE Trans. Fuzzy Syst. 20 (2011), 109–119.

[17] M. Xia, Z. Xu, Hesitant fuzzy information aggregation in decision making, Internat. J. Approx. Reason. 52 (2011), 395–407.

[18] X.Y. Xie, F. Yan, Fuzzy ideals extenstions of ordered semigroups, Lobachevskii J. Math. 19 (2005), 29–40.

[19] P. Yiarayong, Applications of hesitant fuzzy sets to ternary semigroups, Heliyon, 6 (2020), e03668.

[20] L.A. Zadeh, Fuzzy set, Inf. Control. 8 (1965), 338–353.

[21] Z. Zhang, Trapezoidal interval type-2 fuzzy aggregation operators and their application to multiple attribute group decision making, Neural. Comput. Appl. 29 (2018), 1039–1054.