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COMMON FIXED POINT UNDER JUNGCK CONTRACTIVE CONDITION IN A DIGITAL METRIC SPACE

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Abstract: Digital topology is concerned with the topological characteristics of digital image pictures or objects. A peculiar arrangement of non-negative numbers configures digital images. Digital image processing is a technique of dismantling the picture into its fundamental components and analyzing its various features with respect to component parts. In analyzing the fundamental segments of image pictures, the connected segments are separated out to ascertain the relationship of adjacency. During this process of tracking, coding and thinning, it is kept in mind that the connectedness peculiarity of the object remains unchanged.

The features of the component subsets and their relationships can be detailed when the image is decomposed into its constituents. Some of the characteristics of these constituent points or subsets are depending on their positions. Thus, the primary topological features of digital images like connectedness, adjacency, etc. can be the basic clues for their processing.

Various kinds of contraction mappings and related fixed-point theorems can be applied in the field of science and technology, including mathematics, game theory, computer science, engineering, environmental science, etc. Fix point theorems are applied in computational techniques in engineering and science to explore the areas of parallel and

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distributed computation, simulation, modeling and image processing-digital images. In image processing fixed point theorems are applied to get digital contraction which would be a mathematical basis of contour filling, border following algorithm and thinning of a digital image.

To broaden the applicability of contraction principle and associated fixed point theorem in image processing, we wish to explore some of them as significant applicable tools for digital image processing.

Keywords: digital image; digital metric space; common fixed point; digital contractive map.

2010 AMS Subject Classification: 47H10, 47H09.

1. INTRODUCTION

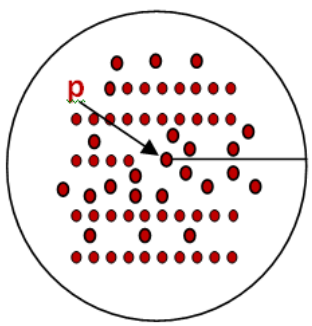
In nonlinear analysis contraction principal plays a vital role. The wider applications of contraction mapping in the field of functional analysis and topology are notable. Contraction principle in a complete metric space (the idea of the metric space was propounded by M. Ferchet [1]) assures the subsistence of fixed points and their uniqueness as well. It was first introduced by Stephen Banach [3] in 1922. Banach fixed point theorem possesses a fixed point as a solution of a self-mapping using the convergence of the Cauchy sequences. Banach fixed point theorem extended the classical consequence of “a continuous map on a closed unit ball” bestowed by L.E.J. Brouwer [2] in 1912. J. Schauder [4] has given another extension for the same.

Number of authors has contributed to generalize fixed point theorem, thereafter, among them few notables are A. Tychonoff [5], S. Lefschetz [6], S. Kakutani [7], M. Edelstein [9], R. Kannan [10], etc. Some advances have further been made by the authors of references [32], [33], [34] to generalize the idea of contraction mapping using different type of contractions on general and digital metric spaces.

In 1942, K. Menger [8] used distribution function to introduce probabilistic metric space instead of general metric space. Similarly, some more spaces like d-complete topological spaces, F-complete metric spaces, G -metric spaces were further introduced by many authors later, as referred in [14], [18] and [25], respectively. To enrich the notion of fixed-point results based on contraction maps, probabilistic metric space in lieu of formal metric has been wielded by some workers of the field, a particular example of the same has been shown in ref. [35].

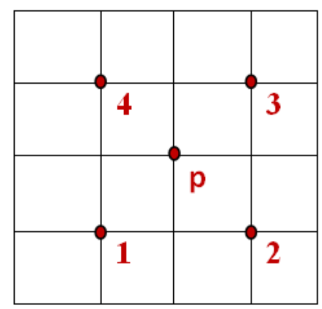
Various spaces with their fixed-point results are studied under the theory of topological fixed points. The geometrical situations which are dealt under the topology are not depending only on definite shape of the objects. The notion of general topology, in general, encompasses infinitely many points in a small arbitrary neighborhood of the certain point, whereas in digital topology some finite number of such points is considered in the neighborhood [11]. This could well be reconciled by the diagram shown below [32].

In image processing, general topology is not appropriate, as it assumes spaces which carry infinite number of points, even in its smallest neighborhood.



General Topology

$\boldsymbol{N(p) = \infty}$



Digital Topology

$\boldsymbol{N(p) = 4}$

Hence, digital topology is taken in to account which is the topological properties of 2-dimensional (2D) and 3-dimensional (3D) digital images. It furnishes a solid mathematical basis for image thinning, object counting, contour filling and other image processing techniques. The 2D image array elements are denoted by pixels, whereas the same for 3D are known as voxels. In 2D-plane or 3D-space, every such pixel or voxel is related with the lattice points which are the points having integral coordinates. A pixel or voxel associated lattice point assumes the value 0 which stands for a black point or 1 which stands for a white point.

Digital topology was accorded as a tool of digital image processing by A. Rosenfield [11] [12]. Then after, the idea of digital fundamental group for a 3-dimentional image thinning was introduced by Kong [13]. Boxer [15] [17] [20] [21] [22] [23] studied various digital continuous mappings and forwarded the approach of digital topology. The concept of digital continuous function and features of fixed point in digital images was first proposed by Rosenfield [12].

Application of Lefchetz fixed point theorem for digital images was studied and fostered by Ege and Karaca [26] [27]. Digital metric space was defined by Ege and Karaca [28]. Also, they have given the proof of the Banach contraction theorem for digital images. Han [30] further improved the Banach fixed point theorem for digital metric spaces. Wielding a digital contraction defined on two mappings, a common coincidence point or a common fixed point has been procured by the authors in [32] [36].

Our interest in this paper is to further enrich the idea of digital topology by making use of some more contractions and their fixed common points in processing the digital images.

2. PRELIMINARIES

Prior to deal with the said purpose, it is obligatory to start with some basic points.

Consider any subset K of set of lattice points Z^n, where n is a positive integer in Euclidean space of dimensions, n and let the adjacency relation for elements of K is p. Then, the pair (K,p) will be a digital image [11] [12]. Thus, a digital image is a graph (K,p) for some positive integer n and adjacency relation p on K.

Definition 1: Consider positive integers p,n; $1 \leq p \leq n$ and two definite points r,s such that, $r = (r_{1,}r_{2,} \dots \dots \dots . r_{\mathrm{n}}), s = (s_{1,}s_{2,} \dots \dots \dots . s_{\mathrm{n}}) \in Z^n$, r,s are p-adjacent [17] if for at most p, we have

$$|r_i - s_i| = 1, \text{ for all } i \text{ and } |r_j - s_j| \neq 1,\ r_j = s_j \text{ for all other indices } j.$$

Therefore, from above, we get

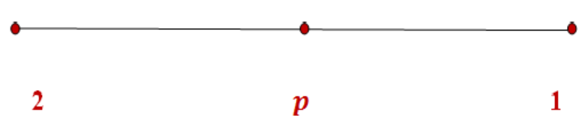
For $r \in Z^n$, number of points $s \in Z^n$ adjacent to r can be depicted by $k(p,n)$, where $k(p,n)$ is not depending on r. In short, we use $k = k(p,n)$

Now, let us visualize this as follows:

(i) When $r \in Z$, that is, $n = 1$, so p will assume the value $p = 1$. Therefore, we may write

$$k(p,n) = k(1,1) = 2$$

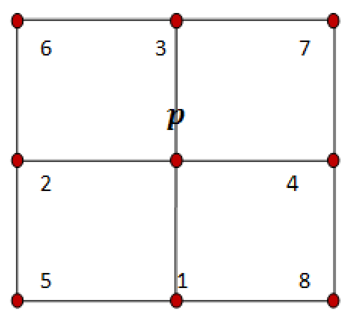
As the two points in this case are $r-1$ and $r+1$, 1-adjacent to $r \in Z$.



(ii) When, $r \in Z^2$, that is, $n=2$, so p will assume the values $p=1,2$.

If $p=2$. Then, 2-adjacent to $r=(r_{1,}r_2)$ will be $(r_1 \pm 1, r_2), (r_1, r_2 \pm 1), (r_1 \pm 1, r_2 \pm 1)$.

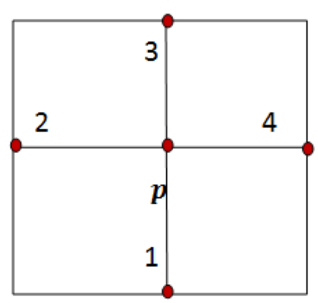
So, there are 8 points 2-adjacent to r, such that $k=k(2,2)=8$.



2-adjacency

If $p=1$. Then, 1-adjacent to $r=(r_{1,}r_2)$ will be $(r_1 \pm 1, r_2), (r_1, r_2 \pm 1)$

So, there are 4 points 2-adjacent to r such that $k=k(1,2)=4$.



1-adjacency

(iii)When, $r \in Z^3$, that is, $n=3$, so p will assume the value $p=1,2,3$.

If $p=3$. Then, 3-adjacent to $r=(r_{1,}r_2, r_3)$ will be

$$(r_1 \pm 1, r_2, r_3), (r_1, r_2 \pm 1, r_3), (r_1, r_2, r_3 \pm 1),$$

$$(r_1 \pm 1, r_2 \pm 1, r_3), (r_1 \pm 1, r_2, r_3 \pm 1), (r_1, r_2 \pm 1, r_3 \pm 1),$$

$$(r_1 \pm 1, r_2 \pm 1, r_3 \pm 1)$$

So, there are 26 points 3-adjacent to r such that $k = k(3,3) = 26$.

If $p = 2$. Then, 2-adjacent to $r = (r_1, r_2, r_3)$ will be

$$(r_1 \pm 1, r_2, r_3), (r_1, r_2 \pm 1, r_3), (r_1, r_2, r_3 \pm 1),$$

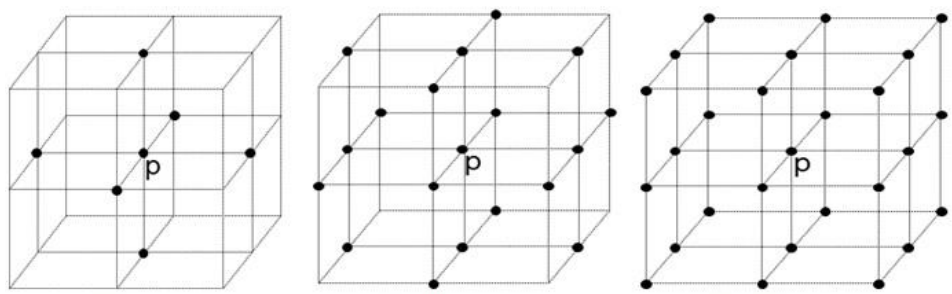
$$(r_1 \pm 1, r_2 \pm 1, r_3), (r_1 \pm 1, r_2, r_3 \pm 1), (r_1, r_2 \pm 1, r_3 \pm 1),$$

So, there are 18 points 2-adjacent to r such that $k = k(2,3) = 18$.

If $p = 1$. Then, 1-adjacent to $r = (r_1, r_2, r_3)$ will be

$$(r_1 \pm 1, r_2, r_3), (r_1, r_2 \pm 1, r_3), (r_1, r_2, r_3 \pm 1)$$

So, there are 6 points 1-adjacent to r such that $k = k(1,3) = 6$.



6-adjacency 18-adjacency 26-adjacency

The generalization to n-D digital image can be shown by the following rule [19].

$$k(p,n) = \sum_{i=n-p}^{n-1} 2^{n-i}\binom{n}{i}, 1 \le p \le n \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad \textbf{(1)}$$

Where, $\binom{n}{i} = \frac{n!}{i!(n-i)!}$

Consider a set K, subset of Z^n such that $K \neq \emptyset$ with $1 \le p \le n$ and $k = k(p,n)$. Then, we say $k(p,n)$ is a digital image having p-adjacency [11]. Also, the pair (K,p) is known as n-D digital image [11] [12].

Definition 2: [16] A point of Z^n, p-adjacent to r is called p-neighbour of $r \in Z^n$ if for $p \in \{2,4,8,6,18,26\}$ and $n \in \{1,2,3\}$, we have

$N_p(\mathrm{r}) = \{\mathrm{s}: \mathrm{s}$ is p-adjacent to $r\}$

$N_p(\mathrm{r})$ is known as p-neighbourhood to p-neighbour of r. r and s are p-neighbours, if s is p-adjacent to r.

Definition 3: [15] [16] A digital interval $[x,y]_z$ is defined for $x, y \in Z$ and $x < y$ such that

$[x, y]_z = \{z \in Z : x \leq z \leq y\}$.

Definition 4: [16] The p-connected digital image $K \in Z^n$ is defined for each point, $x, y \in K$ if there exist a set of points $\{x_0, x_1, x_2, \dots \dots \dots, x_m\}$ so that $x = x_0$ and $y = x_m$ & x_i and x_{i+1} are p-neighbours for all $i = 0,1,2, \dots \dots \dots, m - 1$.

Definition 5: If (K, p_0) and (L, p_1) are the digital images of Z^{n_0} and Z^{n_1}, respectively and $G: K \to L$ is a mapping. Then

(i)G is called (p_0, p_1)-continuous [17] if p_0-connected subsets F of K are mapped to p_1-connected to L.

(ii)G is (p_0, p_1)-continuous iff the images of p_0-adjacent of K are p_1-adjacent in L or, the images of p_0-adjacent of K are coincident. That is, x_0, x_1 are p_0-adjacent of K then,$G(x_0)$ and $G(x_1)$ are p_1-adjacent in L or, $G(x_0) = G(x_1)$.

(iii) **Isomorphism:** [32]G is (p_0, p_1)-isomorphism if the following conditions are satisfied:

$(a)G$ is (p_0, p_1)-continuous

$(b)G$ is a bijection function

$(c)G^{-1}$ is (p_0, p_1)-continuous.

Then, we say $K \cong_{(p_0, p_1)} \mathrm{L}$.

Definition 6: [17] A digital p-path from $x \to y$ in K is a $(2, p)$-continuous mapping $\mathrm{G}: [0, m]_z \to K$ such that $G(0) = x$ and $G(\mathrm{m}) = y$. The set K is called p-path connected in (K, p) if for each two points there is a p-path.

A simple closed p-curve of $m \geq 4$ points [23] in (K, p) is a sequence $\{G(0), G(1), G(2), \dots \dots \dots, G(\mathrm{m} - 1)\}$ of p-path images $\mathrm{G}: [0, m]_z \to K$ so that $G(\mathrm{i})$ and $G(\mathrm{j})$ are p-adjacent iff $j = i \pm mod\ m$.

Fixed point theorem for digital images:

Consider $G: (K, p) \to (K, p)$ is a (p, p)-continuous map. A digital image(K, p) retains the fixed point [26] if there exists $\alpha \in \mathrm{K}$ such that

$$G(\alpha) = \alpha, \forall (p, p)\text{-continuous function } G: K \to K$$

In digital isomorphism, the above characteristic of the fixed point is protected.

Now, consider the digital metric space (K, d, p) having p-adjacency with d as an Euclidean metric on Z^n.

Definition 7: [26] [27] [28] Consider a sequence $< \alpha_n >$ on a digital metric space (K, d, p). The sequence $< \alpha_n >$ be a Cauchy sequence, if $\forall \varepsilon > 0 \, \exists \, c \in N$, we have

$$d(\alpha_m, \alpha_n) < \varepsilon, \ \forall \, m, n > c$$

Theorem 1: [30] For any Cauchy sequence $< \alpha_n >$ on a digital metric space (K, d, p), we have

$$\alpha_m = \alpha_n \ , \forall m, n > c \in N$$

Definition 8: [30] A Cauchy sequence $< \alpha_n >$ on a digital metric space (K, d, p) is said to converge on $\alpha_0 \in$ K, if $\forall \, \varepsilon > 0$ there exists $c \in N$, such that

$$d(\alpha_n, \alpha_0) < \varepsilon \, \forall \, n > c$$

Definition 9: [30] Let (K, d, p) is a digital metric space. It is said to be complete if a Cauchy sequence $< \alpha_n >$ on the digital metric space(K, d, p) is convergent to a point $\alpha_0 \in$ K.

Theorem 2: [30] Every digital metric space(K, d, p) is a complete digital metric space.

Definition 10: [30] A self-map $G: (K, d, p) \to (K, d, p)$ on a digital metric space(K, d, p) is known as a digital contraction if for $\beta \in [0,1)$, we have

$$d(Gx, Gy) \leq \beta d(x, y), \ \forall x, y \in K.$$

Where, β is known as the constant of digital contraction.

Statement 1: [30] Every digital contraction map $G: (K, d, p) \to (K, d, p)$ on a digital metric space(K, d, p) have digital continuity.

Now, we define digital contraction for two mappings.

Theorem 3: [28] Banach Contraction Principle

Consider a digital metric space(K, d, p) on Euclidean metric Z^n and $G: K \to K$ be a digital contraction. Then, a unique fixed point $a_0 \in K$ subsists, which results $G(a_0) = a_0$.

Theorem 4: [32] Consider a digital metric space (K, d, p) with coefficient $k \geq 1$ defined for the mappings $f, g: D \to K$ such that

(i) $d(f\mathrm{x}_x, fy) \geq h.min\{d(gx, gy), d(fy, gy), \frac{1}{2k}[d(gx, fy) + d(gy, fx)]\} \ \forall x, y \in Khk < 1.$

(ii) Either $f(D)$ or g(D) is complete.
Then, f and g have a coincidence point and hence, a fixed point.

This theorem can be followed with less efforts as claimed in reference [32].

3. MAIN RESULTS

Our motive is to ascertain the applicability of contraction principal in digital topology particularly by establishing some common fixed point on Jungck contractive condition in digital metric space to widen the use of fixed-point theorems in digital topology.

Definition 11: A Cauchy sequence is convergent in a digital metric space if it possesses a convergent subsequence in it.

Definition 12: On a complete digital metric space (K, d, p), the digital version of Jungck type contraction for two commuting and self-mappings f and g with coefficient, $\alpha \in (0,1)$, if f is continuous and $g(K) \subset f(K)$, is given by

$$d(g(x), g(y)) \leq \alpha d(f(x), f(y)), \text{ for each } x, y \in K.$$

Statement 2: Consider a sequence $\{y_n\}$ in a complete digital metric space (K, d, p) with $\alpha \in (0,1)$ such that, $(y_{n+1}, y_n) \leq \alpha\, d(y_n, y_{n-1})$ $\forall n$, then $\{y_n\}$ is a convergent sequence.

Theorem 5: Consider two commuting and self-mappings f and g on a complete digital metric space (K, d, p) with coefficient $\alpha \in (0,1)$ such that

$(i) f$ is continuous

$(ii) g(K) \subset f(K)$

(iii) $d(g(x), g(y)) \leq \alpha d(f(x), f(y))$, for each $x, y \in K$.

Then, f and g have a common fixed point in K, which is unique.

Proof: Consider $x_0 \in K$ is any arbitrary point, then from (ii) we obtain, x_1 such that,

$$g(x_0) = f(x_1) = y_0.$$

Now, we may obtain $x_2 \in L$ corresponding to this x_1 which further gives

$$g(x_1) = f(x_2)$$

By proceeding in same manner, we may construct a sequence of the form $\{y_n\}$ such that

$y_{2n} = g(x_{2n}) = f(x_{2n+1})$, for every $n \geq 0$.

Now, substituting $x = x_{2n}$ and $y = x_{2n-1}$ in $d(g(x), g(y)) \leq \alpha d(f(x), f(y))$, we have

$$d(g(x_{2n}), g(x_{2n-1})) \leq \alpha d(f(x_{2n}), f(x_{2n-1}))$$

$$\Rightarrow d(g(x_{2n}), g(x_{2n-1})) \leq \alpha d(g(x_{2n-1}), g(x_{2n-2}))$$

$$\Rightarrow d(y_{2n}, y_{2n-1}) \leq \alpha d(y_{2n-1}, y_{2n-2})$$

Therefore, $\{y_{2n}\}$ is a convergent sequence (Statement 2). Hence, $\{y_{2n}\}$ is a Cauchy sequence, as every convergent sequence in a metric space is a Cauchy sequence. Due to completeness of the metric space, we may have $p \in f(L)$ such that

$$y_{2n+1} = g(x_{2n+1}) = f(x_{2n+2}), \text{ converges to } p \text{ as } n \to \infty.$$

Henceforth, we obtain, $v \in L$ such that, $f(v) = p$. As the sequence $\{y_n\}$ is a digital Cauchy sequence having a convergent subsequence $\{y_{2n+1}\}$ so, the sequence $\{y_n\}$ is also convergent. This shows the convergence of the sequence, $\{y_{2n}\}$, as it is a subsequence of a convergent sequence $\{y_n\}$. Therefore, $\{g(x_{2n})\}$ and $\{f(x_{2n+1})\}$ converges to p as $n \to \infty$.

Now, substituting $x = v$ and $y = x_{2n+1}$ in (iii), we obtain

$$d(g(v), g(x_{2n+1})) \leq \alpha d(f(v), f(x_{2n+1}))$$

This implies that,

$$d(g(v), p) \leq \alpha d(f(v), p) \text{ as } n \to \infty.$$

$$\Rightarrow d(g(v), p) \leq \alpha d(g(v), p)$$

$$\Rightarrow d(g(v), p) \leq 0.$$

Therefore, by metric property, we have

$$g(v) = p \Rightarrow g(v) = f(v) = p.$$

This is a common digital fixed point termed as a *common d-fixed point* in a digital metric space under Jungck contractive condition on a digital metric. The uniqueness of the common d-fixed point is itself yielded with the help of condition (iii).

This is the clear depiction of a unique common fixed point in a digital metric space for any two mappings under Jungck type contractive condition.

4. CONCLUSION

Although, fixed point theorems have variety of uses but, in the field of science and technology it is not less than a boon. In processing of an image, fixed point theorems can be utilized to reduce the size and enhance the quality of the image by making use of the contraction principle and its application. It is found that the quality of a compressed image is not so good every time even, it could not have the information same as the source image. Also, the memory of the data is sometimes very large. This can be another kind of headache for the storage of the data. Nevertheless, compressions of digital image sources are the need of the hour. So, by making use of the contraction principle and common fixed point, as shown in the main result, we can reinforce the knowledge of digital topology especially, in digital image processing to have a compressed image without much redundancy.

5. ACKNOWLEDGEMENT

A number of researchers have paid their attention in obtaining and establishing some results under certain contractive conditions, the work of some of them are detailed by us while introducing the concept. This doesn't mean that the works done by others in this regard are not valuable and admissible. It is just to avoid the unnecessary comprehensiveness of the matter and to save the time of the reader as well. Still, we are thankful to all of them.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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