Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 3, 3151-3154
https://doi.org/10.28919/jmcs/5684
ISSN: 1927-5307

GENERALIZED PRODUCTS OF $\Delta-$SYNCHRONIZED FUZZY AUTOMATA

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Abstract. The main purpose of this paper is to the study the properties of generalized products of $\Delta-$synchronized fuzzy automata.

Keywords: $\Delta-$synchronized fuzzy automata; generalized direct product (GDP); generalized restricted direct product (GRDP).

2010 AMS Subject Classification: 03D05, 20M35, 18B20, 68Q45, 68Q70, 94A45.

1. INTRODUCTION

Fuzzy sets was first introduced by Zadeh in 1965[4]. The notion of the automaton was first fuzzified by Wee [3]. $\Delta-$ synchronized fuzzy automata was introduced by V. Karthikeyan *et.al* in [1]. In this paper, we prove that the generalized direct product and generalized restricted direct product of $\Delta-$ synchronized fuzzy automata(for short, $\Delta-$ SFA) is $\Delta-$synchronized fuzzy automata.

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Received March 12, 2021

2. PRELIMINARIES

2.1. Fuzzy automata (FA) [2]. A finite fuzzy automaton is $F = (T, I, \zeta)$ where

$T-$ set of states$\{s_1, s_2, ...s_n\}$

$I-$ alphabets

$\zeta-$ function from $T \times I \times T \to [0,1]$

$\zeta(s_r, b, s_s) = \tau \; 0 \leq \tau \leq 1$

2.2. $\Delta-$synchronized fuzzy automata [1]. Let $F = (T, I, \zeta)$ be a fuzzy automaton. We say that the fuzzy automaton is $\Delta-$synchronized fuzzy automaton at the state q_t if there exist a real number $\Delta-$ with $0 \leq \Delta \leq 1$ and a word $w \in A^*$ that takes each state s_i of S into s_k such that $\zeta(s_r, w, s_k) \geq \Delta$.

3. GENERALIZED DIRECT PRODUCTS OF $\Delta-$SYNCHRONIZED FUZZY AUTOMATA

3.1. Definition. Let $F_i = (T_i, I_i, \zeta_i), i = 1,2...n$ be FA. Then the generalized direct product is $F = \prod_{i=1}^n F_i = (\prod_{i=1}^n T_i, \prod_{i=1}^n I_i, \prod_{i=1}^n \zeta_i)$ and is defined by $\prod_{i=1}^n \zeta_i : \prod_{i=1}^n T_i \times \prod_{i=1}^n I_i \times \prod_{i=1}^n T_i \to [0,1]$.

$\prod_{i=1}^n \zeta_i((s_i, s_j.....s_z), (b_1, b_2, ...b_n), (s'_i, s'_j...s'_z))$

$= \vee\{\zeta_1(s_i, b_1, s'_i) \wedge \zeta_2(s_j, b_2, s'_j) \wedge .... \wedge \zeta_n(s_z, b_n, s'_z)\}$

$= \vee\{\wedge\{\zeta_1(s_i, b_1, s'_i), \zeta_2(s_j, b_2, s'_j), ..., \zeta_n(s_z, b_n, s'_z)\}$

$\forall (s_i, s_j, ..., s_z), (s'_i, s'_j, ...s'_z) \in \prod_{i=1}^n S_i, \; (b_1, b_2, ...bn) \in \prod_{i=1}^n I_i.$

3.2. Definition. Let $F_i = (T_i, I_i, \zeta_i), i = 1,2...n$ be FA. Then the generalized restricted direct product is $F = \cap_{i=1}^n F_i = (\cup_{i=1}^n T_i, I, \cup_{i=1}^n \zeta_i)$. Define $\cup_{i=1}^n \zeta_i : (\cup_{i=1}^n T_i \times I \times \cup_{i=1}^n \zeta_i) \to [0,1]$.

$\cup_{i=1}^n \zeta_i((s_i, s_j.....s_z), b, (s'_i, s'_j...s'_z)) = \vee\{\zeta_1(s_i, b, s'_i) \wedge \zeta_2(s_j, b, s'_j) \wedge .... \wedge \zeta_n(s_z, b, s'_z)\}$

$= \vee\{\wedge\{\zeta_1(s_i, b, s'_i), \zeta_2(s_j, b, s'_j), ..., \zeta_n(s_z, b, s'_z)\}$

$\forall (s_i, s_j, ..., s_z), (s'_i, s'_j, ...s'_z) \in \prod_{i=1}^n S_i, \; b \in \prod_{i=1}^n I_i.$

Theorem 3.3. Let $F_i = (T_i, I, \zeta_i), i = 1,2,...,n$ be FA. Let $\prod_{i=1}^n F_i$ is the GDP of FA F_i. Then $\forall (s_i, s_j, ...s_z), (s'_i, s'_j...s'_z) \in \prod_{i=1}^n T_i$, and $\forall (y_1, y_2, ..., y_n) \in \prod_{i=1}^n I_i^*$

$(\prod_{i=1}^n \zeta_{i^*})((s_i, s_j, ...s_z), (y_1, y_2, ...y_n), (s'_i, s'_j...s'_z)) = \zeta_{1^*}(s_i, y_1, s'_i) \wedge \zeta_{2^*}(s_j, y_2, s'_j) \wedge ..... \wedge \zeta_{n^*}(s_j, y_2, s'_j).$

Proof. Let $y_i \in I_i^*, y_i \neq \lambda, i = 1,2,...,n$ and $|y_1| = |y_2| = ... = |y_n| = m$. Then the result is true for $m = 1$. Suppose the result is true for all $y_i \in I^*, i = 1,2,...,n$ and $|y_i| = m-1, i = 1,2,..,n, m > 1$.

Let $y_i = a_i u_i, i = 1,2,...n$ where $a_i \in I_i$ and $u_i \in I_i^*, i = 1,2,...n$. Then

$\prod_{i=1}^n \zeta_i^*((s_i,s_j,..s_z),(a_1u_1,a_2u_2,..a_nu_n),(s_i',s_j',..s_z')) = \prod_{i=1}^n \zeta_i^*((s_i,s_j,..s_z),(a_1u_1),(s_i',s_j',..s_z')) \wedge$

$(\prod_{i=1}^n \zeta_i^*((s_i,s_j,..s_z),(a_2u_2),(s_i',s_j',..s_z')) \wedge .. \wedge \prod_{i=1}^n \zeta_i^*((s_i,s_j,..s_z)(a_nu_n)(s_i',s_j',..s_z'))$

$$= \{\vee\{\prod_{i=1}^n \zeta_i((s_i,s_j,...,s_z),a_1,(s_a,s_b....,s_e) \wedge \prod_{i=1}^n \zeta_i^*((s_a,s_b....,s_e),u_1,(s_i',s_j',..,s_z'))|(s_a,s_b...,s_e) \in \prod_{i=1}^n T_i\}\}\wedge$$

$$\{\vee\{\prod_{i=1}^n \zeta_i((s_i,s_j,...,s_z),a_2,(s_a',s_b',..,s_e')) \wedge (\prod_{i=1}^n \zeta_i)^*((s_a',s_b',..,s_e'),u_2,(s_i',s_j',..,s_z'))|(s_a',s_b',..,s_e') \in \prod_{i=1}^n T_i\}\}$$

$$\wedge,..\wedge\{\prod_{i=1}^n \zeta_i((s_i,s_j,...,s_z),a_n,(s_a',s_b',...,s_e')) \wedge \{\prod_{i=1}^n \zeta_i^*((s_a',s_b',...,s_e'),u_n,(s_i',s_j',...,s_z'))|(s_a',s_b',..,s_e') \in \prod_{i=1}^n T_i\}\}.$$

$= \zeta_1^*(s_i,a_1u_1,s_i') \wedge \zeta_2^*(s_j,a_2u_2,s_j') \wedge .... \wedge \zeta_n^*(s_z,a_nu_n,s_z')$

$= \zeta_1^*(s_i,y_1,s_i') \wedge \zeta_2^*(s_j,y_2,s_j') \wedge .... \wedge \zeta_n^*(s_z,y_n,s_z')$

$= \prod_{i=1}^n \zeta_i^*(s_i,s_j,...s_z),y_i,s_i',s_j',...s_z')$

Theorem 3.4. Let $F_i = (T_i,I,\zeta_i), i = 1,2,...,n$ be FA. Let $\cap_{i=1}^n F_i$ is the GRDP of fuzzy automata F_i. Then $\forall(s_i,s_j,...s_z),(s_i',s_j',...s_z') \in \cap_{i=1}^n T_i,$ and $\forall y \in I^*$

$\cap_{i=1}^n \zeta_i^*((s_i,s_j,....s_z),y,(s_i',s_j',...s_z')) = \zeta_{1^*}(s_i,y,s_i') \wedge \zeta_{2^*}(s_j,y,s_j') \wedge ..... \wedge \zeta_{n^*}(s_z,y,s_z').$

Proof. The proof of the result is follows by induction on $|y| = m$.

If $m = 1$ then the result is trivial. Suppose the result is true for all $y \in I^*$. Let $y = bv$, where $b \in I, v \in I^*$ and $|v| = m-1, m > 1$. Then

$\cap_{i=1}^n \zeta_i^*((s_i,s_j,...s_z),y,(s_i',s_j',...s_z')) = \cap_{i=1}^n \zeta_i^*((s_i,s_j,...s_z),bv,(s_i',s_j',...s_z'))$

$= \vee\{\cap_{i=1}^n \zeta_i((s_i,s_j,...s_z),b,(s_a,s_b,....s_e)) \wedge \cap_{i=1}^n \zeta_i^*((s_a,s_b,....s_e),v,(s_i',s_j',...s_z')|(s_a,s_b,....s_e) \in \cap_{i=1}^n T_i)\}$

$= \vee\{\zeta_1(s_i,b,s_a) \wedge \zeta_2(s_j,b,s_b) \wedge ... \wedge \zeta_n(s_z,b,s_e) \wedge \zeta_1^*(s_a,v,s_i') \wedge \zeta_2^*(s_b,v,s_j') \wedge ..... \wedge \zeta_n^*(s_e,v,s_z')|(s_a,s_b,....s_e)$

$\in \cap_{i=1}^n T_i\}$

$= \zeta_1^*(s_i,bv,s_i') \wedge \zeta_2^*(s_j,bv,s_j') \wedge ...... \wedge \zeta_n^*(s_n,bv,s_n')$

$= \zeta_1^*(s_i,y,s_i') \wedge \zeta_2^*(s_j,y,s_j') \wedge .... \wedge \zeta_n^*(s_z,y,s_z').$

Theorem 3.5. The GDP of Δ−synchronized FA is Δ− synchronized FA.

Proof. Let $F_i = (T_i,I_i,\zeta_i),\ i = 1,2$ be Δ−synchronized FA. Then the GDP of F_i is given by $\prod_{i=1}^n F_i = (\prod_{i=1}^n T_i, \prod_{i=1}^n I_i, \prod_{i=1}^n \zeta_i)$.

Define $\prod_{i=1}^n \zeta_i : \prod_{i=1}^n T_i \times \prod_{i=1}^n I_i \times \prod_{i=1}^n T_i \to [0,1]$.

Since $F_i, i = 1,2,...n$ are Δ−synchronized FA then there exists Δ−synchronized words $w_i \in I^*, i = 1,2,...n$ and the states $s_k \in T_1,\ s_l \in T_2, ..., S_e \in T_n$ such that

$\zeta_1^*(s_i, w_1, s_k) > 0$ $\zeta_2^*(s_j, w_2, s_l) > 0$

Now, $(\prod_{i=1}^{n} \zeta_i^*)((s_i, s_j, ...s_z), w, (s_k, s_l, ...s_e)) > 0$

$\Leftrightarrow \prod_{i=1}^{n} \zeta_i^*((s_i, s_j, ...s_z), w_1 w_2 ... w_n, (s_k, s_l, ...s_e)) > 0$

$\Leftrightarrow \zeta_1^*(s_i, w_1, s_k) \wedge \zeta_2^*(s_j, w_2, s_l) > 0 \wedge \zeta_n^*(s_z, w_n, s_z') > 0.$

Hence $\prod_{i=1}^{n} F_i$ is $\Delta-$synchronized FA.

Theorem 3.6. The GRDP of $\Delta-$ synchronized FA is $\Delta-$synchronized FA.

Proof. Let $F_i = (T_i, I, \zeta_i),\ i = 1, 2, ...n$ be $\Delta-$synchronized FA. Then the GRDP of F_i is given by $\cap_{i=1}^{n} F_i = (\cap_{i=1}^{n} T_i, I, \prod_{i=1}^{n} \zeta_i)$.

Define $\cap_{i=1}^{n} \zeta_i : \cap_{i=1}^{n} T_i \times I \times \cap_{i=1}^{n} T_i \to [0, 1]$. Since $F_i, i = 1, 2, ..n$ are $\Delta-$synchronized FA then there exists $\Delta-$synchronized words $w \in I^*$, and the states $s_k \in T_1$, $s_l \in T_2, ...s_e \in T_n$ such that

$\zeta_1^*(s_i, w, s_k) > 0, \zeta_2^*(s_j, w, s_l) > 0, ..., \zeta_n^*(s_z, w, s_e) > 0$

Now, $(\cap_{i=1}^{n} \zeta_i^*)((s_i, s_j, ...s_z), w, (s_k, s_l, ..., s_e)) > 0 \Leftrightarrow (\cap_{i=1}^{n} \zeta_i^*)((s_i, s_j, ..s_z), w, (s_k, s_l, ...s_e)) > 0$

$= (\zeta_1^*(s_i, w, s_k) \wedge \zeta_2^*(s_j, w, s_l) \wedge ... \wedge \zeta_n^*(s_z, w, s_e)) > 0$

Hence $\cap_{i=1}^{n} F_i$ is $\Delta-$synchronized FA.

4. Conclusion

In this paper, using the concept of generalized products, we prove that generalized direct product and generalized restricted direct product of $\Delta-$synchronized FA is $\Delta-$synchronized FA.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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