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EFFECT OF POROSITY ON THE REFLECTION AND REFRACTION COEFFICIENTS IN A LIQUID SATURATED POROUS HALF-SPACE BENEATH LIQUID LAYER

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Abstract: The reflection and refraction of Rayleigh wave has been contemplated in a liquid saturated porous layer beneath uniform liquid layer. In this paper, analytical expression for reflection and refraction coefficients of three types of wave's fast dilatational wave, slow dilatational wave and SV-wave are procured. It can be found that the reflection and refraction coefficient dependency on porosity and angle of incidence are well. For the numerical interpretation of the results a specific model is chosen. The discussions are done with the help of graphs drawn for reflection and refraction coefficient with angle of incidence.

Keywords: wave propagation; reflection and refraction; SV wave; porous media.

2010 AMS Subject Classification: 74J15, 74L05, 74L10.

1. INTRODUCTION

The earth is mostly covered with water and beneath this there are large deposits of porous materials. These crude materials are highly needed for making daily use products. This makes very important to study this topic for various fields of science and technology. The porosity of

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rock also affects velocity, which can be useful. Some extra properties like porosity, lithology and permeability are required for knowing wave propagation through solid/porous rocks (Domenico, [1]). The porosity is affected in sedimentary rocks by their maximum depth of burial. Velocity variations can affect seismic data in a variety of ways, producing, for example false anticlines which are not there (Christensen and Szymanski, [2]). The relation between velocity and density may be found for particular rock types. Porosity is, in-turn, also involved through the effect on elasticity and density. Love waves propagation in a fluid-saturated porous layer has been discussed by Konczak [3]. Deresiewicz [4] used the theory of Biot [5] for the wave propagation in a statistically isotropic fluid-saturated porous medium to study the propagation of Love waves in a porous layer resting on an elastic, homogeneous and isotropic semi-infinite space. Deresiewicz and Rice [6] explored the various aspects of the presence of boundaries in liquid-saturated porous solids on the propagation of plane harmonic seismic waves. Gogna [7] considered the surface wave propagation in a homogeneous anisotropic layer lying over a homogeneous isotropic elastic half-space and under a uniform layer of liquid. Sharma et al. [8] have discussed the surface wave propagation in a liquid-saturated porous solid layer, overlying an impervious, transversely isotropic, elastic, solid half-space and under a uniform layer of liquid. Chaudhary et al. [9, 10] have discussed the reflection and transmission of plane SH-wave in the self reinforced, viscoelastic and monoclinic media. Chattopadhyay [11] have discussed the reflection and refraction in triclinic crystalline media.

Present problem deals with the reflection and refraction of Rayleigh wave in a liquid-saturated porous elastic solid beneath uniform thickness of layer of homogeneous liquid. In this problem the effect of porosity has been encountered for the reflection and refraction of different types of waves in liquid-saturated porous medium. The expressions for reflection and refraction have been obtained and results are shown in figures with the graphs plotted for reflection and refraction coefficient against angle of incidence. The scenario of this problem is more relevant to the study of oceanic model.

2. FORMULATION OF THE PROBLEM

The uniform thickness h of liquid layer resting on half-space of liquid-saturated porous elastic medium has been taken in account. The interface of these two medium is taken at $z=0$ and upper boundary at $z=-h$ as manifested in fig.1.

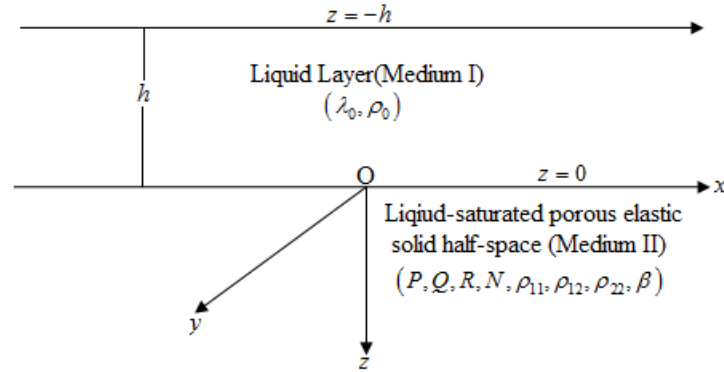


Fig.1: Physical model of problem.

For liquid layer (medium I), the displacement potential ϕ_0 is governed by

$$\frac{\partial^2 \phi_0}{\partial x^2} + \frac{\partial^2 \phi_0}{\partial z^2} = \frac{1}{\alpha^2} \frac{\partial^2 \phi_0}{\partial t^2} \quad (1a)$$

where, $\alpha (= \sqrt{\lambda_0 / \rho_0})$, ρ_0 and λ_0 are the velocity of the dilatational wave, density and elastic constant of the liquid layer respectively.

The components of displacement u_0, w_0 and pressure p are related by

$$u_0 = \frac{\partial \phi_0}{\partial x}, w_0 = \frac{\partial \phi_0}{\partial z},$$

and $p = -\sigma_z = -\lambda_0 \nabla^2 \phi_0$ (1b)

Assuming $\phi_0 = \bar{\phi}_0(z) e^{ik(x-ct)}$ and putting in (1) and solving, we get

$$\bar{\phi}_0(z) = A_0 e^{kz\sqrt{1-c^2/\alpha^2}} + B_0 e^{-kz\sqrt{1-c^2/\alpha^2}}$$

where, A_0 and B_0 are variable constants.

Therefore, we have solution of liquid layer in terms of displacement potential as

$$\phi_0 = \left\{ A_0 e^{kz\sqrt{1-c^2/\alpha^2}} + B_0 e^{-kz\sqrt{1-c^2/\alpha^2}} \right\} e^{ik(x-ct)} \quad (1c)$$

The dynamics of a physical field for the liquid-saturated porous solid (medium II) are given by Biot [5] in the absence of dissipation as

$$\text{and } \left. \begin{aligned} N\nabla^2\bar{u} + \text{grad}[(D+N)\text{div}\bar{u} + Q\text{div}\bar{U}] &= \frac{\partial^2}{\partial t^2}(\rho_{11}\bar{u} + \rho_{12}\bar{U}), \\ \text{grad}(Q\text{div}\bar{u} + R\text{div}\bar{U}) &= \frac{\partial^2}{\partial t^2}(\rho_{12}\bar{u} + \rho_{22}\bar{U}), \end{aligned} \right\} \quad (2)$$

where, \bar{u} is displacements in solids and \bar{U} is displacements in liquid parts of porous medium; D, N, Q and R are elastic Moduli and nonnegative ; ρ_{11}, ρ_{12} and ρ_{22} are dynamical coefficients.

Interference of Moduli with relations between the stress in the solid, σ_{ij} and that in the liquid, σ , on the one hand, and the strain in the solid,

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

and the dilatations, $e = \text{div } \bar{u}$ and $\varepsilon = \text{div } \bar{U}$, on the other:

$$\left. \begin{aligned} \sigma_{ij} &= (De + Q\varepsilon)\delta_{ij} + 2N\varepsilon_{ij} \\ \sigma &= Qe + R\varepsilon \end{aligned} \right\} \quad (3)$$

where, δ_{ij} is kronecker delta.

If $\bar{\rho}_s$ and $\bar{\rho}_f$ are the mass of the solid and liquid per unit volume of the aggregate respectively, ρ_s and ρ_f represent their respective mass densities and β , the porosity of aggregate, then we have

$$\text{and } \left. \begin{aligned} \bar{\rho}_s &= \rho_{11} + \rho_{12} = (1 - \beta)\rho_s \\ \bar{\rho}_f &= \rho_{12} + \rho_{22} = \beta\rho_f \end{aligned} \right\} \quad (4)$$

The following relations are also satisfied by following Biot [5]

$$\left. \begin{aligned} \rho_{11} > 0, \rho_{12} \leq 0, \rho_{22} \geq 0, \rho_{11}\rho_{22} - \rho_{12}^2 > 0, \\ PR - Q^2 > 0 \text{ and } \rho_{11}R + \rho_{22}P - 2\rho_{12}Q > 0, \end{aligned} \right\} \quad (5)$$

where, $P = D + 2N$.

Taking a Helmholtz resolution for two displacement vectors as

$$\left. \begin{aligned} \bar{u} &= \text{grad}\phi + \text{curl}H \\ \bar{U} &= \text{grad}\psi + \text{curl}G \end{aligned} \right\} \quad (6)$$

Intersection of above (6) in (2), we get a couple of equations that are satisfied identically by setting

$$\left. \begin{aligned} P\nabla^2\phi + Q\nabla^2\psi &= \frac{\partial^2}{\partial t^2}(\rho_{11}\phi + \rho_{12}\psi) \\ Q\nabla^2\phi + R\nabla^2\psi &= \frac{\partial^2}{\partial t^2}(\rho_{12}\phi + \rho_{22}\psi) \end{aligned} \right\} \quad (7)$$

and

$$\left. \begin{aligned} N\nabla^2H &= \frac{\partial^2}{\partial t^2}(\rho_{11}H + \rho_{12}G) \\ 0 &= \frac{\partial^2}{\partial t^2}(\rho_{12}H + \rho_{22}G) \end{aligned} \right\} \quad (8)$$

Assuming

$$\left. \begin{aligned} \phi &= \bar{\phi}(z)e^{ik(x-ct)} \\ \psi &= \bar{\psi}(z)e^{ik(x-ct)} \end{aligned} \right\} \quad (9)$$

where, c is the phase velocity and k is the wave number.

Putting these values of (9) in (7), we get

$$\left. \begin{aligned} P(\bar{\phi}'' - k^2\bar{\phi}) + Q(\bar{\psi}'' - k^2\bar{\psi}) &= -k^2c^2(\rho_{11}\bar{\phi} + \rho_{12}\bar{\psi}) \\ Q(\bar{\phi}'' - k^2\bar{\phi}) + R(\bar{\psi}'' - k^2\bar{\psi}) &= -k^2c^2(\rho_{11}\bar{\phi} + \rho_{12}\bar{\psi}) \end{aligned} \right\} \quad (10)$$

where, prime denotes the differentiation with respect to 'z'. Eliminating $\bar{\psi}''$ from (10), we get

$$\bar{\psi} = \frac{1}{\rho_{22}Q - \rho_{12}R} \left[\frac{A}{k^2c^2} \bar{\phi}'' + \left(\rho_{11}R - \rho_{12}Q - \frac{A}{c^2} \right) \bar{\phi} \right] \quad (11)$$

Substituting this value of $\bar{\psi}$ in (10) will give us

$$A\bar{\phi}^{IV} + (Bc^2 - 2A)k^2\bar{\phi}'' + k^2(A - Bc^2 + Cc^4)\bar{\phi} = 0 \quad (12)$$

where,

$$A = PR - Q^2, \quad C = (\rho_{11}\rho_{22} - \rho_{12}^2)$$

and (13)

$$B = (\rho_{11}R - 2\rho_{12}Q + \rho_{22}P)$$

Having A, B, C being nonnegative, the solution of (12) is taken as

$$\bar{\phi} = \bar{\phi}_1 + \bar{\phi}_2$$

With

$$\begin{aligned} \bar{\phi}_1 &= A_1 e^{kz\sqrt{1-c^2/\alpha_1^2}} + A_2 e^{-kz\sqrt{1-c^2/\alpha_1^2}} \\ \bar{\phi}_2 &= A_3 e^{kz\sqrt{1-c^2/\alpha_2^2}} + A_4 e^{-kz\sqrt{1-c^2/\alpha_2^2}} \end{aligned} \quad (14)$$

where, A_1, A_2, A_3, A_4 are arbitrary constants and

$$\frac{1}{\alpha_1^2} = \frac{B - (B^2 - 4AC)^{1/2}}{2A}, \quad \frac{1}{\alpha_2^2} = \frac{B + (B^2 - 4AC)^{1/2}}{2A} \quad (15)$$

Thus we obtain

$$\phi = (\bar{\phi}_1 + \bar{\phi}_2) e^{ik(x-ct)} = \phi_1 + \phi_2 \text{ (say)} \quad (16)$$

where,

$$\begin{aligned} \left\{ \nabla^2 + \left(\frac{kc}{\alpha_1} \right)^2 \right\} \phi_1 &= 0 \\ \left\{ \nabla^2 + \left(\frac{kc}{\alpha_2} \right)^2 \right\} \phi_2 &= 0 \end{aligned} \quad (17)$$

The solution of (16) and (17) corresponds to unbounded solution for the two dilatational waves. The wave corresponding to ϕ_1 being the faster one, called fast P (or P_f) wave propagation with the phase velocity α_1 and that corresponding to ϕ_2 being the slower one, called 'slow' P (or P_s) wave propagating with the phase velocity α_2 .

From equations (11) and (16), we get

$$\psi = \mu_1 \phi_1 + \mu_2 \phi_2 \quad (18)$$

where,

$$\text{and } \left. \begin{aligned} \mu_1 &= \frac{\rho_{11}R - \rho_{12}Q - \frac{A}{\alpha_1^2}}{\rho_{22}Q - \rho_{12}R} \\ \mu_2 &= \frac{\rho_{11}R - \rho_{12}Q - \frac{A}{\alpha_2^2}}{\rho_{22}Q - \rho_{12}R} \end{aligned} \right\} \quad (19)$$

similarly, the assumption

$$\left. \begin{aligned} H &= \bar{H}(z) e^{ik(x-ct)} \\ G &= \bar{G}(z) e^{ik(x-ct)} \end{aligned} \right\}$$

when inserted in equation (8), we get

$$\text{and } \left. \begin{aligned} \bar{G} &= -\frac{\rho_{12}}{\rho_{22}} \bar{H} \\ \nabla^2 \bar{H} &= \frac{1}{\alpha_3^2} \frac{\partial^2 \bar{H}}{\partial t^2} \end{aligned} \right\} \quad (20)$$

$$\text{where, } \alpha_3^2 = \frac{N\rho_{22}}{C}$$

i.e., in an unbounded medium α_3 is the phase velocity of only shear wave.

The displacements $\bar{u} = (u, 0, w)$ in the solid and $\bar{U} = (U, 0, W)$ in liquid in xz -plane are given as follows:

$$\left. \begin{aligned} u &= \frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_2}{\partial x} + \frac{\partial \psi_1}{\partial z}, \\ w &= \frac{\partial \phi_1}{\partial z} + \frac{\partial \phi_2}{\partial z} - \frac{\partial \psi_1}{\partial x}, \\ U &= \mu_1 \frac{\partial \phi_1}{\partial x} + \mu_2 \frac{\partial \phi_2}{\partial x} - \frac{\rho_{12}}{\rho_{22}} \frac{\partial \psi_1}{\partial z}, \\ W &= \mu_1 \frac{\partial \phi_1}{\partial z} + \mu_2 \frac{\partial \phi_2}{\partial z} + \frac{\rho_{12}}{\rho_{22}} \frac{\partial \psi_1}{\partial x}, \end{aligned} \right\} \quad (21)$$

$$\text{Where, } \psi_1 = \left(-\bar{H} \right)_y$$

Stress component will be given by

$$\sigma_z = (P + Q\mu_1)\nabla^2\phi_1 + (P + Q\mu_2)\nabla^2\phi_2 - 2N\left(\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_2}{\partial x^2} + \frac{\partial^2\psi_1}{\partial x\partial z}\right),$$

and

$$\tau_{zx} = N\left(2\frac{\partial^2\phi_1}{\partial x\partial z} + 2\frac{\partial^2\phi_2}{\partial x\partial z} + \frac{\partial^2\psi_1}{\partial z^2} - \frac{\partial^2\psi_1}{\partial x^2}\right), \quad (22a)$$

$$\sigma = (Q + R\mu_1)\nabla^2\phi_1 + (Q + R\mu_2)\nabla^2\phi_2$$

As the displacement components are proportional to $e^{ik(x-ct)}$, we therefore take the expressions for the potentials ϕ_1, ϕ_2 and ψ_1 as

$$\phi_1 = \left(A_1 e^{kz\sqrt{1-c^2/\alpha_1^2}} + A_2 e^{-kz\sqrt{1-c^2/\alpha_1^2}}\right) e^{ik(x-ct)}$$

$$\phi_2 = \left(A_3 e^{kz\sqrt{1-c^2/\alpha_2^2}} + A_4 e^{-kz\sqrt{1-c^2/\alpha_2^2}}\right) e^{ik(x-ct)} \quad (22b)$$

$$\psi_1 = \left(A_5 e^{kz\sqrt{1-c^2/\alpha_3^2}} + A_6 e^{-kz\sqrt{1-c^2/\alpha_3^2}}\right) e^{ik(x-ct)}$$

where, A_1, A_2, A_3, A_4, A_5 and A_6 are all arbitrary constants.

3. BOUNDARY CONDITIONS

The free surface $z = -h$ of the liquid layer the stress component vanish i.e.

$$-(P)_1 = (\sigma_z) = \lambda_0 \nabla^2 \phi = 0 \quad (23)$$

At $z=0$ i.e. the interface of liquid layer and fluid saturated porous solid, according to Deresiewicz and Skalak [12] for open pore boundary conditions are as follows:

$$\left. \begin{aligned} \left(\frac{\sigma}{\beta}\right)_{II} &= (\sigma_z)_I \\ (\sigma_z + \sigma)_{II} &= (\sigma_z)_I \\ (1-\beta)\left(\dot{u}\right)_{II} + \beta\left(\dot{U}\right)_{II} &= \left(\dot{u}_0\right)_I \\ (1-\beta)\left(\dot{w}\right)_{II} + \beta\left(\dot{W}\right)_{II} &= \left(\dot{w}_0\right)_I \\ (\sigma_{xz})_{II} &= 0 \\ \left(\dot{w}\right)_{II} - \left(\dot{W}\right)_{II} &= 0 \end{aligned} \right\} \quad (24)$$

Using the above boundary condition (23) and (24) in (1), (21) and (22a) we obtain seven homogeneous equations in $A_0, B_0, A_1, A_2, A_3, A_4, A_5$ and A_6 and the coefficients of these taken respectively as $C_1, C_2, C_3, C_4, C_5, C_6, C_7$ and C_8 irrespective of boundary conditions and equations, where $C_1, C_2, C_3, C_4, C_5, C_6, C_7$ and C_8 are

$$C_1 = \left\{ e^{-kh} \sqrt{1 - \frac{c^2}{\alpha^2}}, \frac{\lambda_0 c^2}{\alpha^2}, \frac{\lambda_0 c^2}{\alpha^2}, \sqrt{1 - \frac{c^2}{\alpha^2}}, 1, 0, 0 \right\}$$

$$C_2 = \left\{ e^{kh} \sqrt{1 - \frac{c^2}{\alpha^2}}, \frac{\lambda_0 c^2}{\alpha^2}, \frac{\lambda_0 c^2}{\alpha^2}, -\sqrt{1 - \frac{c^2}{\alpha^2}}, 1, 0, 0 \right\}$$

$$C_3 = \left\{ 0, -\frac{(Q+R\mu_1)c^2}{\beta\alpha_1^2}, -\left[\{P+Q+\mu_1(Q+R)\} \frac{c^2}{\alpha_1^2} + 2Nk^2 \right], \sqrt{1 - \frac{c^2}{\alpha_1^2}} \{1 - \beta(1 - \mu_1)\}, \right. \\ \left. -\{1 - \beta(1 - \mu_1)\}, -2iN\sqrt{1 - \frac{c^2}{\alpha_1^2}}, -(1 - \mu_1)k\sqrt{1 - \frac{c^2}{\alpha_1^2}} \right\}$$

$$C_4 = \left\{ 0, -\frac{(Q+R\mu_1)c^2}{\beta\alpha_1^2}, -\left[\{P+Q+\mu_1(Q+R)\} \frac{c^2}{\alpha_1^2} + 2Nk^2 \right], -\sqrt{1 - \frac{c^2}{\alpha_1^2}} \{1 - \beta(1 - \mu_1)\}, \right. \\ \left. -\{1 - \beta(1 - \mu_1)\}, 2iN\sqrt{1 - \frac{c^2}{\alpha_1^2}}, -(1 - \mu_1)k\sqrt{1 - \frac{c^2}{\alpha_1^2}} \right\}$$

$$C_5 = \left\{ 0, -\frac{(Q+R\mu_2)c^2}{\beta\alpha_2^2}, -\left[\{P+Q+\mu_2(Q+R)\} \frac{c^2}{\alpha_2^2} + 2Nk^2 \right], \sqrt{1 - \frac{c^2}{\alpha_2^2}} \{1 - \beta(1 - \mu_2)\}, \right. \\ \left. -\{1 - \beta(1 - \mu_2)\}, -2iN\sqrt{1 - \frac{c^2}{\alpha_2^2}}, -(1 - \mu_2)k\sqrt{1 - \frac{c^2}{\alpha_2^2}} \right\}$$

$$C_6 = \left\{ 0, -\frac{(Q+R\mu_2)c^2}{\beta\alpha_2^2}, -\left[\{P+Q+\mu_2(Q+R)\} \frac{c^2}{\alpha_2^2} + 2Nk^2 \right], -\sqrt{1 - \frac{c^2}{\alpha_2^2}} \{1 - \beta(1 - \mu_2)\}, \right. \\ \left. -\{1 - \beta(1 - \mu_2)\}, 2iN\sqrt{1 - \frac{c^2}{\alpha_2^2}}, (1 - \mu_2)k\sqrt{1 - \frac{c^2}{\alpha_2^2}} \right\}$$

$$C_7 = \left\{ 0, 0, -2iNk^2 \sqrt{1 - \frac{c^2}{\alpha_3^2}}, -i \left\{ \beta \left(1 + \frac{\rho_{12}}{\rho_{22}} \right) - 1 \right\}, -i \sqrt{1 - \frac{c^2}{\alpha_3^2}} \left\{ 1 - \beta \left(1 + \frac{\rho_{12}}{\rho_{22}} \right) \right\}, \left(2 - \frac{c^2}{\alpha_3^2} \right), -ik \left(1 + \frac{\rho_{12}}{\rho_{22}} \right) \right\}$$

$$C_8 = \left\{ 0, 0, 2iNk^2 \sqrt{1 - \frac{c^2}{\alpha_3^2}}, -i \left\{ \beta \left(1 + \frac{\rho_{12}}{\rho_{22}} \right) - 1 \right\}, i \sqrt{1 - \frac{c^2}{\alpha_3^2}} \left\{ 1 - \beta \left(1 + \frac{\rho_{12}}{\rho_{22}} \right) \right\}, \left(2 - \frac{c^2}{\alpha_3^2} \right), -ik \left(1 + \frac{\rho_{12}}{\rho_{22}} \right) \right\}$$

And considering by Snell's law, we have

$$\sqrt{\frac{c^2}{\alpha^2} - 1} = \tan \theta, \quad \sqrt{\frac{c^2}{\alpha_1^2} - 1} = \tan \theta_1, \quad \sqrt{\frac{c^2}{\alpha_2^2} - 1} = \tan \theta_2, \quad \sqrt{\frac{c^2}{\alpha_3^2} - 1} = \tan \theta_3.$$

For Reflection and Refraction of fast dilatational wave (P_f) corresponding to ϕ_1 propagating with the phase velocity α_1 , we divide all the homogeneous equations with A_2 and we have non homogeneous equations as follows

$$\begin{bmatrix} C_1^T & C_2^T & C_3^T & C_5^T & C_6^T & C_7^T & C_8^T \end{bmatrix} \times \begin{bmatrix} \frac{A_0}{A_2} & \frac{B_0}{A_2} & \frac{A_1}{A_2} & \frac{A_3}{A_2} & \frac{A_4}{A_2} & \frac{A_5}{A_2} & \frac{A_6}{A_2} \end{bmatrix}^T = \begin{bmatrix} -C_4^T \end{bmatrix} \quad (25)$$

Solving (25) for $\frac{A_0}{A_2}$, $\frac{B_0}{A_2}$ and $\frac{A_1}{A_2}$, we have

$$\frac{A_0}{A_2} = \frac{\Delta_1}{\Delta}, \quad \frac{B_0}{A_2} = \frac{\Delta_2}{\Delta}, \quad \frac{A_1}{A_2} = \frac{\Delta_3}{\Delta} \quad (26)$$

where,

$$\Delta = \begin{vmatrix} C_1^T & C_2^T & C_3^T & C_5^T & C_6^T & C_7^T & C_8^T \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} -C_4^T & C_2^T & C_3^T & C_5^T & C_6^T & C_7^T & C_8^T \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} C_1^T & -C_4^T & C_3^T & C_5^T & C_6^T & C_7^T & C_8^T \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} C_1^T & C_2^T & -C_4^T & C_5^T & C_6^T & C_7^T & C_8^T \end{vmatrix}$$

The coefficient $\frac{A_0}{A_2}$ gives the transmission coefficient of P_f wave from medium II to medium I,

$\frac{B_0}{A_2}$ gives the reflection of P_f wave in medium I whereas $\frac{A_1}{A_2}$ gives the reflection of P_f wave in medium II.

For Reflection and Refraction of slow dilatational (P_s) wave corresponding to ϕ_2 propagating with the phase velocity α_2 , we divide all the homogeneous equations with A_4 and we have non homogeneous equations as follows

$$\begin{bmatrix} C_1^T & C_2^T & C_3^T & C_4^T & C_5^T & C_7^T & C_8^T \end{bmatrix} \times \begin{bmatrix} \frac{A_0}{A_4} & \frac{B_0}{A_4} & \frac{A_1}{A_4} & \frac{A_2}{A_4} & \frac{A_3}{A_4} & \frac{A_5}{A_4} & \frac{A_6}{A_4} \end{bmatrix}^T = \begin{bmatrix} -C_6^T \end{bmatrix} \quad (27)$$

Solving (27) for $\frac{A_0}{A_4}$, $\frac{B_0}{A_4}$ and $\frac{A_3}{A_4}$, we have

$$\frac{A_0}{A_4} = \frac{\Delta_1'}{\Delta'}, \quad \frac{B_0}{A_4} = \frac{\Delta_2'}{\Delta'}, \quad \frac{A_3}{A_4} = \frac{\Delta_3'}{\Delta'} \quad (28)$$

where,

$$\Delta' = |C_1^T \ C_2^T \ C_3^T \ C_4^T \ C_5^T \ C_7^T \ C_8^T|$$

$$\Delta_1' = |-C_6^T \ C_2^T \ C_3^T \ C_4^T \ C_5^T \ C_7^T \ C_8^T|$$

$$\Delta_2' = |C_1^T \ -C_6^T \ C_3^T \ C_4^T \ C_5^T \ C_7^T \ C_8^T|$$

$$\Delta_3' = |C_1^T \ C_2^T \ C_3^T \ C_4^T \ -C_6^T \ C_7^T \ C_8^T|$$

The coefficient $\frac{A_0}{A_4}$ gives the transmission coefficient of P_s wave from medium II to medium I,

$\frac{B_0}{A_4}$ gives the reflection of P_s wave in medium I whereas $\frac{A_3}{A_4}$ gives the reflection of P_s wave in medium II.

For Reflection and Refraction of shear wave (SV) propagating with the phase velocity α_3 , we divide all the homogeneous equations with A_6 and we have non homogeneous equations as follows

$$[C_1^T \ C_2^T \ C_3^T \ C_4^T \ C_5^T \ C_6^T \ C_7^T] \times \left[\frac{A_0}{A_6} \ \frac{B_0}{A_6} \ \frac{A_1}{A_6} \ \frac{A_2}{A_6} \ \frac{A_3}{A_6} \ \frac{A_4}{A_6} \ \frac{A_5}{A_6} \right]^T = [-C_8^T] \quad (29)$$

Solving (29) for $\frac{A_0}{A_6}$, $\frac{B_0}{A_6}$ and $\frac{A_5}{A_6}$, we have

$$\frac{A_0}{A_6} = \frac{\Delta_1''}{\Delta''}, \quad \frac{B_0}{A_6} = \frac{\Delta_2''}{\Delta''}, \quad \frac{A_5}{A_6} = \frac{\Delta_3''}{\Delta''} \quad (30)$$

where,

$$\Delta'' = |C_1^T \ C_2^T \ C_3^T \ C_4^T \ C_5^T \ C_6^T \ C_7^T|$$

$$\Delta_1'' = |-C_8^T \ C_2^T \ C_3^T \ C_4^T \ C_5^T \ C_6^T \ C_7^T|$$

$$\Delta_2'' = |C_1^T \ -C_8^T \ C_3^T \ C_4^T \ C_5^T \ C_6^T \ C_7^T|$$

$$\Delta_3'' = |C_1^T \ C_2^T \ C_3^T \ C_4^T \ C_5^T \ C_6^T \ -C_8^T|$$

The coefficient $\frac{A_0}{A_6}$ gives the transmission coefficient of shear wave from medium II to medium I,

$\frac{B_0}{A_6}$ gives the reflection of shear wave in medium I whereas $\frac{A_5}{A_6}$ gives the reflection of shear wave in medium II.

4. NUMERICAL RESULTS AND DISCUSSION

Following Ewing et al. [13], for the water layer (medium I), for seismic body wave velocity, density and elastic parameters, the following values are taken

$$\alpha = 1.463 \times 10^5 \text{ cm/sec}$$

$$\lambda_0 = 0.214 \times 10^{11} \text{ dyne/cm}^2$$

$$\rho_0 = 1 \text{ gm/cm}^3$$

For kerosene saturated sandstone (medium II), the following values for the relevant parameters are taken keeping in view by Yew and Jogi [14] and Fatt [15] as

$$P = 0.99663 \times 10^{11} \text{ dyne/cm}^2$$

$$Q = 0.07435 \times 10^{11} \text{ dyne/cm}^2$$

$$R = 0.03262 \times 10^{11} \text{ dyne/cm}^2$$

$$N = 0.2765 \times 10^{11} \text{ dyne/cm}^2$$

$$\rho_{11} = 1.926137 \text{ gm/cm}^3$$

$$\rho_{12} = -0.002137 \text{ gm/cm}^3$$

$$\rho_{22} = 0.215337 \text{ gm/cm}^3$$

For P_f , P_s and SV waves, the velocities of these waves for the above constants are

$$\alpha_1 = 2.349 \times 10^5 \text{ cm/sec}$$

$$\alpha_2 = 1.086 \times 10^5 \text{ cm/sec}$$

$$\alpha_3 = 1.196 \times 10^5 \text{ cm/sec}$$

The graphs have been represented for reflection and refraction coefficients with the angle of incidence for different values of porosity using the help of Mathematica-7. In Fig.2, it is observed that transmission coefficient for all values of porosity of material are merging at the initial and normal angle for fast dilatational wave. The transmission coefficient increases rapidly like impulse function and then decreases rapidly between the angle of incidence 0-10 degree and

then slowly decreases and tend to zero and then nature get reversed. The porosity affects the transmission coefficient remarkably but the nature and pattern of curves are preserved. In Fig.3, we can observe same type of nature as in Fig.2 for reflection of fast dilatational wave in medium I. In Fig.4, it can be seen that the reflection coefficient for reflection of fast dilatational wave in medium I that initially for some angle of incidence 0-10 degree approximately does not exist. The reflection coefficient decreases as angle of incidence increases and tends to zero near the normal angle and then marginally increases for all values of porosity but remarkable decrease in reflection coefficient can be observed. In Fig.5, all the curves for transmission coefficient of slow dilatational wave against the angle of incidence emerges from zero and starts increasing with angle of incidence and after certain value of incidence near normal decreases rapidly and meets to zero and then increases very rapidly but as we increase the porosity the magnitude of transmission coefficient decreases and nature of curves are preserved. In Fig.6, we can observe same type of nature as in Fig.5 for reflection of slow dilatational wave in medium I. In Fig.7, at the initial and normal angle for all values of porosity of material the reflection coefficient merges at both ends with magnitude equals to one for slow dilatational wave in liquid saturated porous medium. The reflection coefficient decreases as angle of incidence increases and meets to zero at certain value of angle of incidence and then increases rapidly. The porosity induces the rapidness in the curve for increasing values of porosity but nature and pattern are preserved. In Fig.8, one can see that transmission coefficient for SV-waves is negligible but to show effects of porosity range taken very small otherwise for all values of porosity coincides and same nature followed by reflection of SV-waves in medium I in Fig.9. The reflection coefficient for SV-waves does not exist in medium II.

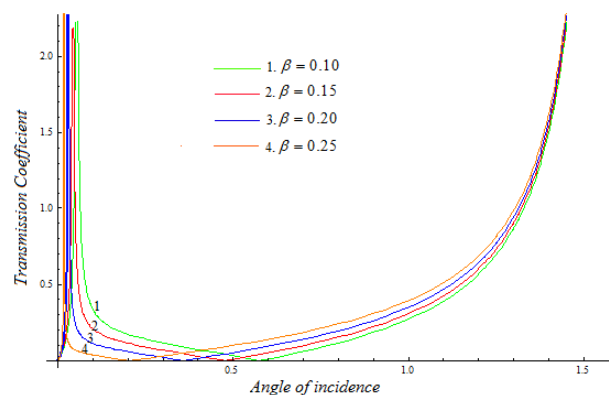


Fig.2: Transmission Coefficient versus angle of incidence for P_f waves from Medium II to Medium I.

EFFECT OF POROSITY ON THE REFLECTION AND REFRACTION COEFFICIENTS

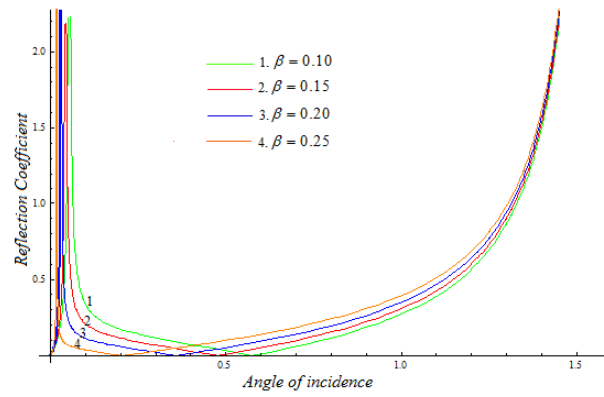


Fig.3: Reflection Coefficient versus angle of incidence for P_f waves in Medium II.

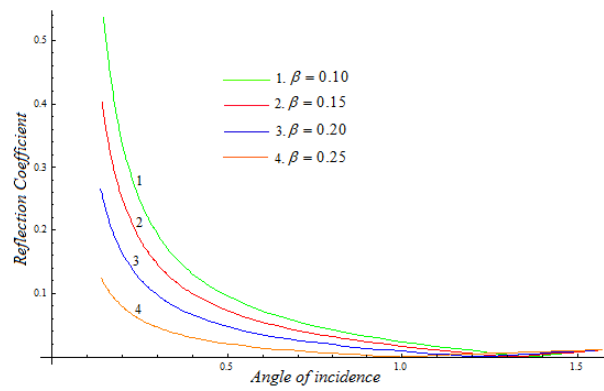


Fig.4 Reflection Coefficient versus angle of incidence for P_f waves in Medium I.

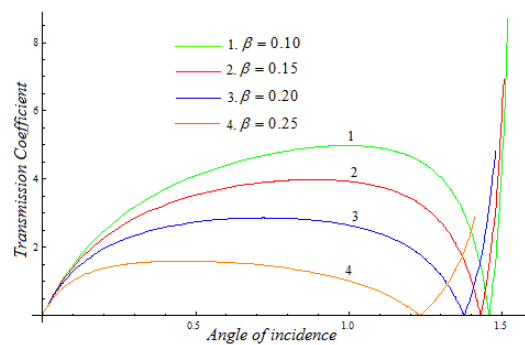


Fig.5: Transmission Coefficient versus angle of incidence for P_s waves from Medium II to Medium I.

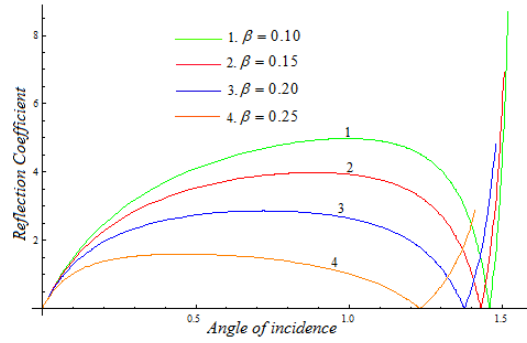


Fig.6: Reflection Coefficient versus angle of incidence for P_s waves in Medium II.

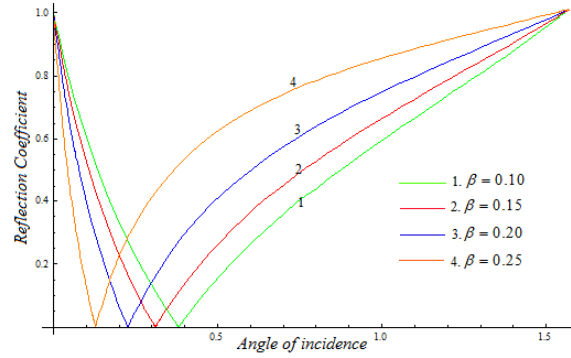


Fig.7: Reflection Coefficient versus angle of incidence for P_s waves in Medium I.

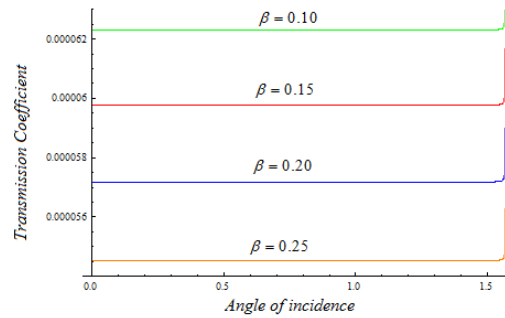


Fig.8: Transmission Coefficient versus angle of incidence for SV-waves from Medium II to Medium I.

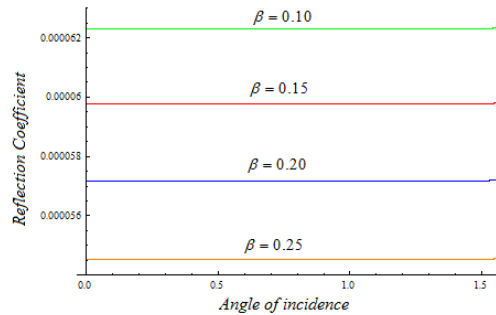


Fig.9: Reflection Coefficient versus angle of incidence for SV-waves in Medium II.

5. CONCLUSIONS

The reflection and refraction of Rayleigh wave has been contemplated in a liquid saturated porous layer beneath uniform liquid layer. Analytical expression for reflection and refraction coefficients of three types of wave's fast dilatational wave, slow dilatational wave and SV-wave are procured. It can be found that the reflection and refraction coefficient dependency on porosity and angle of incidence are well. For the numerical interpretation of the results a specific model is chosen. From numerical results it can be clearly visible that the porosity of the medium highly affects the reflection and refraction of these waves. These variations of reflection and refraction help us understand which type porous material hidden in the ocean floor.

CONFLICT OF INTERESTS:

The author(s) declares that there is no conflict of interests.

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