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MHD MIXED CONVECTIVE SLIP FLOW OF CASSON FLUID OVER A POROUS INCLINED PLATE WITH JOULE HEATING, VISCOUS DISSIPATION AND THERMAL RADIATION

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Abstract: The present study examines numerically heat and mass transfer on mixed convective Casson fluid flow over an inclined plate in presence of non-uniform magnetic field. The impact of Joule heating, viscous dissipation and thermal radiation are taken into consideration. The resulting ordinary differential equations obtained from the governing partial differential equations of the flow are solved by using MATLAB bvp4c method. The effects of relevant parameters on velocity, temperature and concentration are discussed graphically, while skin-friction coefficient, rate of heat transfer and mass transfer are presented in tabular structure.

Keywords: Casson fluid; slip conditions; MHD; Joule heating; viscous dissipation.

2010 AMS Subject Classification: 76W05.

1. INTRODUCTION

The problem of mixed convective magneto-hydrodynamic flows over a vertical plate encountered in various engineering and technological applications such as electronic devices cooled by fans, heat exchangers in a low velocity environment, drying technologies, fluid flows in ocean and in atmosphere [1, 2]. Bejan [3] and Pop and Ingham [4] described the subject in their monographs. Some relevant literatures are found in [5-14].

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The impact of non-Newtonian fluid is significant in various industrial and engineering applications. Casson fluid is a shear thinning non-Newtonian fluid introduced by Casson [15] in 1959 during his investigation on flow equations for pigment oil suspensions of printing ink type. This constitutive equation can be used to explore rheological character of materials such as honey, jelly, tomato sauce concentrated fruit juice, soup, mercury amalgams etc. Hayat et al. [16] examined mixed convective flow of Casson fluid in a stretching sheet with convective boundary conditions employing homotopy analysis. Raju et al. [17] examined Casson fluid for the influence of magnetic field over a stretching sheet and obtained that the induced magnetic parameter raises the heat transfer rate. Imran et al. [18] examined slip effects on convective flow of Casson fluid past an oscillating vertical surface. They found that the velocity field decrease due to the influence of the slip parameter. Shaw et al. [19] studied the Casson fluid flow under convective boundary conditions over a horizontal plate using similarity transformations. They found that the influence of Casson parameter reduced the momemtum boundary layer thickness. Reddy et al. [20] presented a description on collective effects of frictional and irregular heat over Maxwell and Casson fluids. They found that the velocity profiles for Casson fluid are maximum than that of Maxwell fluid.

Das et al. [10] analyzed the effects of Joule heating and viscous dissipation on MHD mixed convective flow for Newtonian fluid over a porous plate with non-uniform magnetic field and slip boundary conditions. They found that fluid velocity increases as the magnetic parameter increases and the impact of slip parameter causes deceleration in fluid velocity.

In this paper, the work of Das et al. [10] is extended to Casson fluid to study heat and mass transfer with heat radiation and nth-order chemical reaction on the MHD mixed convective flow. Solutions are obtained numerically by using MATLAB bvp4c method and it is observed that the results in absence of Casson parameter, chemical reaction and heat radiation are agreed with that of Das et al. [10].

2. Mathematical Formulation

Consider a steady, mixed convective flow of incompressible, electrically conducting Casson fluid over a porous inclined plate with slip conditions in presence of nth-order chemical reaction. The plate makes an acute angle γ with the vertical measured in the clockwise direction. The x-axis is considered along the plate, y-axis is normal to the plate and z-axis perpendicular to x, y

-axes. The effects of viscous dissipation and Joule heating are considered.

A non-uniform magnetic field $B=\frac{B_0}{\sqrt{x}}$ is applied along the y-axis. The induced magnetic field is neglected Cowling [21] by assuming Reynolds number is very small. Let (u,v,w) and (J_x,J_y,J_z) be the components of velocity and current density vector respectively.

In view of the above assumptions, the conservation of mass, momentum, energy and mass transfer of the problem can be written following (Chen [6], Das et al. [10]) as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}+g\beta_T^*\left(T-T_\infty\right)\cos\gamma+g\beta_c^*\left(C-C_\infty\right)\cos\gamma-\frac{B}{\rho}J_z, \tag{2}$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{3}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2+\frac{1}{\sigma}J_z^2-\frac{\partial q_r}{\partial y}, \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-K_c\left(C-C_\infty\right)^n, \tag{5}$$

where $\beta, p, \rho, g, \mu, \upsilon, \beta_T^*, \beta_c^*, T, C, C_p, \gamma, D_m, K_c, n$ is the Casson parameter, pressure, density, acceleration due to gravity, dynamic viscosity, kinematic viscosity, coefficient of heat transfer, coefficient of mass transfer, fluid temperature, species concentration, specific heat at constant pressure, inclination of the plate with vertical, mass diffusivity, coefficient of chemical reaction and order of chemical reaction respectively. The last three terms in the equation (4) indicate the effects of viscous dissipation, Joule heating and thermal radiation respectively.

The boundary conditions ([Das et al. [10]) are

$$y=0: u=L_1\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right), v=-v_w, T=T_w+L_2\left(\frac{\partial T}{\partial y}\right), C=C_w+L_3\left(\frac{\partial C}{\partial y}\right);$$

$$y\to\infty: u\to U, T\to T_\infty, C\to C_\infty. \tag{6}$$

where $T_w = T_\infty + \frac{T_0}{x}, C_w = C_\infty + \frac{C_0}{x}$ are the variable temperature and concentration at the plate, T_0, C_0 are constant and T_∞, C_∞ are free stream temperature and concentration respectively. $L_{1,}, L_2, L_3$ are slip factors related velocity, thermal and concentration. The no slip condition can be recovered by taking $L_1 = L_2 = L_3 = 0.$

Now, from Ohm's law (Cowling [21]), we get $J_x = \sigma(E_x - Bw), J_y = 0, J_z = \sigma(E_z + Bu).$

At infinity, there is no electrical current as magnetic field is unaltered, therefore $J_x \to 0, J_y \to 0$ when $y \to \infty.$ Considering $u \to U, w \to 0$ at $y \to \infty,$ we get

$E_x = 0, E_y = 0, E_z = -BU$, and thus $J_x = 0, J_y = 0, J_z = \sigma B(u - U)$.

Using the above assumptions, the equations (2) and (4) become

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + g\beta_T^*(T - T_\infty)\cos\gamma + g\beta_c^*(C - C_\infty)\cos\gamma - \frac{\sigma B^2}{\rho}(u - U), \tag{7}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B(u - U)^2 - \frac{\partial q_r}{\partial y}. \tag{8}$$

Using Rosseland approximation [22], we get

$$q_r = -\frac{4\sigma_1}{3\alpha_1}\frac{\partial q_r}{\partial y}, \tag{9}$$

where σ_1 Stefan-Boltzmann constant, α_1 is the mean absorption coefficient.

Assuming temperature differences within the flow are small, using Taylor's series, we can express T^4 as

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{10}$$

We take

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial y}, \tag{11}$$

then equation (1) is satisfied, where $\psi(x, y)$ is the stream function.

The following similarity variables are used

$$\eta = y\sqrt{\frac{U}{\upsilon x}}, \psi = \sqrt{U\upsilon x}\, f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{12}$$

Using (9)-(12) in equations (7)-(8), the following equations are obtained

$$\left(1 + \frac{1}{\beta}\right) f''' + \frac{1}{2} ff'' - M^2(f' - 1) + \lambda_1 \theta \cos\gamma + \lambda_2 \cos\gamma = 0, \tag{13}$$

$$\left(1 + \frac{R}{\Pr}\right)\theta'' + \frac{1}{2}\Pr f\theta' + Ec\Pr\left[\left(1 + \frac{1}{\beta}\right) f''^2 + M^2(f' - 1)^2\right] = 0, \tag{14}$$

$$\phi'' + \frac{1}{2} Sc\phi' - KSc\phi^n = 0, \tag{15}$$

where $M^2 = \frac{\sigma B^2}{\rho U}$ is the magnetic field parameter, $Ec = \frac{U^2}{C_p(T_w - T_\infty)}$ is the Eckert number, $\Pr = \frac{\mu C_p}{\kappa}$ is the Prandtl number, $R = \frac{16\sigma_1 T_\infty^3}{3\alpha_1 \kappa}$ is the radiation of heat transfer, $Sc = \frac{\upsilon}{D_m}$ is the Schmidt number, $K = \frac{K_c(C_w - C_\infty)^{n-1} x}{U}$ is the local chemical reaction parameter, $\lambda_1 = \frac{g\beta_T^* T_0}{U^2}$ and $\lambda_2 = \frac{g\beta_c^* C_0}{U^2}$ are mixed convective parameter.

The boundary conditions are

$$\eta = 0: f(0) = s, f'(0) = \delta\left(1 + \frac{1}{\beta}\right) f''(0), \theta(0) = 1 + \gamma_1 \theta'(0), \phi'(0) = 1 + \gamma_2 \phi'(0);$$

$$\eta \to \infty: f' \to 1, \theta \to 0, \phi \to 0. \tag{16}$$

where $s = -2v_w\sqrt{\frac{x}{\upsilon U}}$ is the suction parameter, $\delta = L_1\sqrt{\frac{U}{\upsilon x}}$ is the velocity slip parameter, $\gamma_1 = L_2\sqrt{\frac{U}{\upsilon x}}$ is the thermal slip parameter, $\gamma_2 = L_3\sqrt{\frac{U}{\upsilon x}}$ is the concentration slip parameter.

The governing higher order non-linear differential equations (13)-(15) are solved with boundary conditions (16) using MATLAB bvp4c method.

3. RESULTS AND DISCUSSION

To get insight into the flow problem, we assign the following numerical values to the physical parameter for computation as follows:

$$\beta = 2, \delta = 0.6, M^2 = 0.5, \Pr = 1, \gamma = \frac{\pi}{4}, n = 2, Ec = 0.2, K = 1, R = 0.5, s = 0.2, \gamma_1 = 0.5, \gamma_2 = 0.5,$$

$$\lambda_1 = 0.2, \lambda_2 = 0.2.$$

In the absence of chemical reaction and thermal radiation and when $\beta \to \infty$, the values of skin friction and Nusselt number are compared with that of the values of Das et al. [10] and it was found that they are in good agreement which is shown in Table1.

Figures 1-5 represents the variation of velocity profile against y for different values of Casson parameter β, magnetic field parameter M^2, velocity slip parameter δ, radiation parameter R, and inclination of the plate γ with the vertical respectively. The fluid velocities in all the Figures 1-5 ascend from the plate and then approaches to 1 in the free stream. From Fig. 1, it is observed that the fluid flow is accelerated due to the growth of Casson fluid parameter, which causes the yield stress upgrades. Figure 2 shows that the fluid velocity accelerated with an increase in M^2. This behavior is physically explained by the fact that an increase in M^2 increases Lorentz body force which arise in the momentum equation (2) makes the momentum boundary layer thickness thinner. The fluid which is retarded due to viscous effects, receives a push due to the magnetic field $\left(B = \frac{B_0}{\sqrt{x}} \right)$ which counteracts the viscous force and as a result the fluid velocity increases as elucidated by Das et al. [10].

Figure 3 shows the effect of slip velocity on fluid velocity $f'(\eta)$. It is seen that an increase in slip parameter δ increases the fluid which slip past at the plate and the fluid experiences less drag and as a consequence fluid flow enhances near the plate and this effect diminishes far away from the plate.

Figure 4 shows that the fluid velocity reduces due to an increasing value of R and which implies the wideness of momentum boundary layer. Figure 5 shows that $f'(\eta)$ decrease for increasing inclination γ of the plate. The inclination γ of the plate is measured from the vertical position and maximized fluid velocity is found when the position of the plate is vertical $(\gamma = 0)$ and minimized fluid velocity is found when the position of the plate is horizontal $\left(\gamma = \frac{\pi}{2}\right)$.

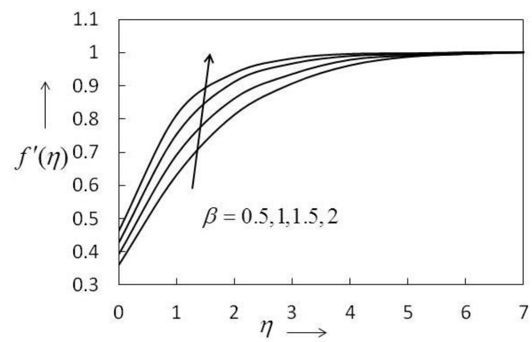

Figure1: Velocity profile for different β.

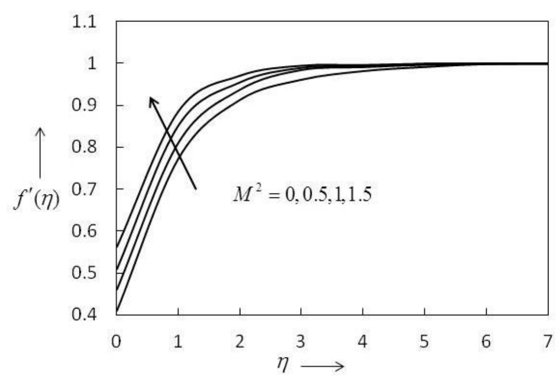

Figure 2: Velocity profile for different M^2.

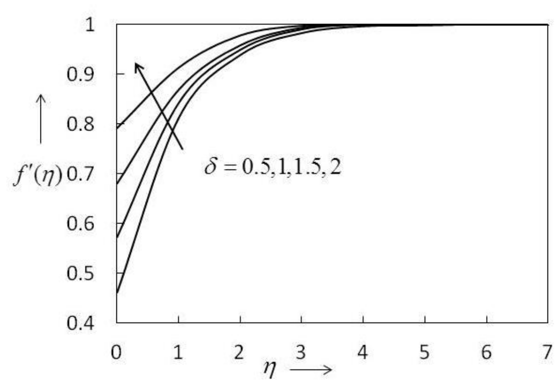

Figure 3: Velocity profile for different δ.

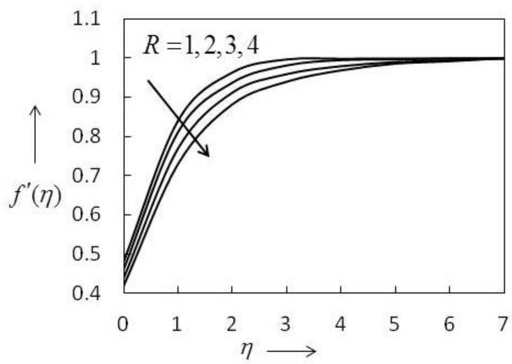

Figure 4: Velocity profile for different R.

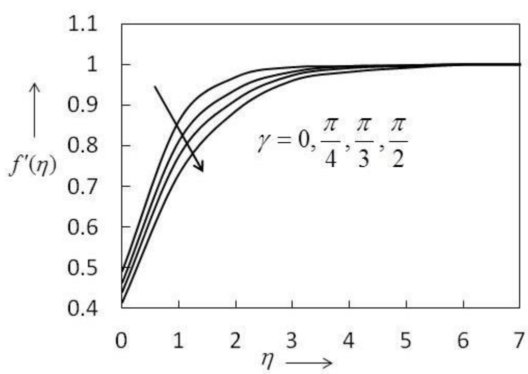

Figure 5: Velocity profile for different γ.

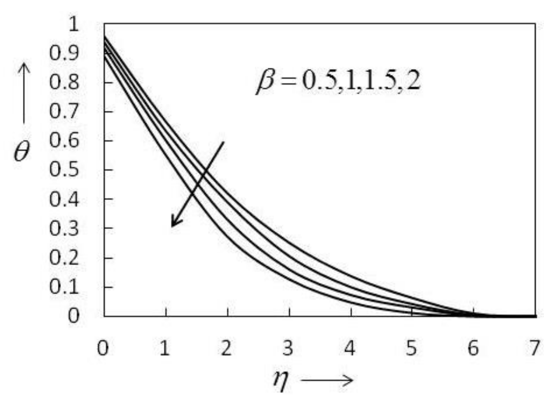

Figure 6: Temperature profile for different β.

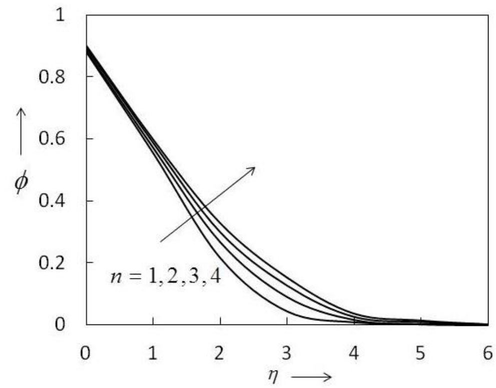

Figure 7: Concentration profile for different n.

Figure 6 shows that the fluid temperature θ decrease with an increase in β. Thus the thickness of thermal boundary layer increases with the growth of β. Figure 7 shows that the species concentration profile increases when the order n of the rate of chemical reaction increases and thus the thickness of concentration boundary layer decreases.

Table 1: Comparison of present work

M^2	δ	Ec	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
			Das et al. [10]		Present work ($\beta \to \infty, R = 0,$ and without mass transfer)	
0.5	0.5	0.1	0.82450	0.90292	0.8245	0.9030
1.5	0.5	0.1	0.95597	0.89573	0.9560	0.8958
0.5	1.0	0.1	0.59019	0.91717	0.5902	0.9182
0.5	0.5	0.2	0.82473	0.88438	0.8247	0.8845

Table 2: Numerical values of skin friction, heat transfer and mass transfer rates:

β	M^2	R	δ	K	n	$\left(1+\frac{1}{\beta}\right)f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	0.5	0.5	0.5	1.0	2	0.7670	0.9566	0.6272
1.0	0.5	0.5	0.5	1.0	2	0.7748	0.9478	0.6346
1.5	0.5	0.5	0.5	1.0	2	0.7816	0.9412	0.6498
	1.0					0.8096	0.9508	0.6308
	1.5					0.8537	0.9484	0.6348
		0				0.7602	0.9449	0.6313
		1.0				0.7709	0.9614	0.6185
			0			0.9256	0.9425	0.6306
			1.0			0.5958	0.9614	0.6214
				0.2		0.7768	0.9347	0.6379
				0.6		0.7710	0.9425	0.6312
					1	0.7695	0.9424	0.6415
					3	0.7626	0.9611	0.6015

It is observed from Table 2 that the shear stress $\left(1+\frac{1}{\beta}\right)f''(0)$ at the plate $\eta = 0$ increases for increasing values of $\beta, M^2,$ and R, whereas it decreases with the increasing values of $\delta, K,$ and n. Also, it is seen that the rate of heat transfer $-\theta'(0)$ decreases with the increasing values of β and M^2, whereas it rises with the increasing values of R, δ, K and n. Further, the rate of mass transfer $-\phi'(0)$ increases with the increasing values of β and M^2, whereas it decreases with the increasing values of R, δ, K and n.

3. CONCLUSIONS

The above study leads to the following conclusions

1. The fluid velocity enhance when the Casson fluid parameter, strength of magnetic field, velocity slip parameter increases while it decreases when the thermal radiation, and inclination of the plate with the vertical increases.
2. The fluid temperature of the boundary layer increases with the increasing values of Casson fluid parameter. The fluid concentration inside the boundary layer decrease when rate of order of chemical reaction increases.
3. Skin friction increases when the Casson fluid parameter, strength of magnetic field, thermal radiation increases, while it decreases when velocity slip parameter, chemical reaction parameter, and rate of order of reaction increases.
4. Rate of heat transfer increases when the thermal radiation parameter, velocity slip parameter, chemical reaction parameter and order of chemical reaction rate increases, while it decreases with an increasing strength of magnetic field parameter and Casson fluid parameter.
5. The rate of mass transfer increases when the strength of magnetic field, Casson fluid parameter increases, while it decreases when the thermal radiation parameter, velocity slip parameter, chemical reaction parameter, order of chemical reaction rate increase.

CONFLICT OF INTERESTS

The author declares that there is no conflict of interests.

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