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# MATHEMATICAL MODELING AND OPTIMAL CONTROL OF ABSTAINING ATTITUDE OF COULD COMPUTING ADOPTION

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**Abstract.** Communally popularized in the year 2002 with the start of Amazon web services, cloud computing has transformed the way IT services and resources are furnished to the clients. Cloud computing is sprawling the IT view. Pushed by diverse completing and converging influences, cloud computing is progressing as a viable IT service provider model at an amazing pace. It also provides diverse advantages compared to traditional on-premise computing models, including minimized costs and advanced agility and flexibility. Customers can usually demand cloud services through a web browser or web service. Adopting cloud computing, customers can economize cost of hardware implementation, software licenses and system management. On the other hand, it also has some issues which influence the cloud computing adoption by organizations. In the present paper, we propose two optimal strategies using two controls $u_1$ and $u_2$ to minimize the contact between abstainers and susceptible and maximize the users. Pontryagin's maximum principle is used to characterize the optimal controls and the optimality system

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is solved by an iterative method. Finally, numerical simulations are performed to verify the theoretical analysis using Matlab.

**Keywords:** cloud computing; abstainers; optimal control; existence of optimal control; discrete Pontryagin's maximum principle; maximizing the use of cloud services.

**2010 AMS Subject Classification:** 68Q01, 93A30.

## 1. INTRODUCTION

In the forthcoming years, the term Cloud Computing has become an arising and fastest technology in the world. Cloud Computing is one of the most talked about technologies and for the reason that the several benefits furnished by it has got lots of consideration from media and investigators [1, 2]. In the software industry and also in the ICT services market, cloud computing is presenting more and more crucial role. Cloud computing is the delivery of computing resources such as storage, application software and CPU, through the Internet. Massive virtualised datacenters are being set up across the world to deliver quick and effective services, virtually enormous computing and storage capability, rising reliability and availability of services and reducing the cost of services to the clients. The clients are the consumers of service in cloud environment. They demand for services from the cloud service vendor. In cloud computing, consumers are permitted to select their service in real time and personalize them conforming to their needs. The client series of cloud computing is large it does not only consists of computers but all devices that can connect to the internet and utilize the services available via internet for instance mobile phones, laptops, tablets, and workstations [4]. This technology contributed to the growth and advancement of other types of services on the market; financial and accounting services, human resources services, educational services etc. Actually, there are highly few companies in developing regions which are surely using cloud computing services [3, 5, 9]. It is in its early stages [11, 12]. In addition, this technology has become outspread at higher education in developed countries, but in contrary in developing countries [13]. Hence, its implementation is influenced and can impact the society and economy prosperity. In this work, we suggest an epidemiological approach [14] to study the renunciation behavior to adopt cloud computing services. In epidemiology, we in most cases utilize category model to describe the development of an infectious disease. Correspondingly, all along cloud computing use process, the population can be classified into different compartments. Moreover, the contact among people is important in cloud computing implementation; it is very similar to the contagion phenomenon since abstinents can impact the susceptible users through their network (mass media or others) to abstain from the cloud computing use. For this aim, the epidemic method is more convenient to model the abstinent behavior of cloud computing adoption.

We set down a mathematical model SAC that represents the changes of cloud computing adoption by customers and the negative impact of abstainers, who renounce from cloud computing using considering its risks, on the susceptible users.

The first objective of this article is to carry out a concise literature review on this topic with a view to shortly outline the fundamental definitions and theoretical perspectives on cloud computing and also the main advantages and issues of cloud computing services; the second objective is to realize a more elaborate and particular analysis through SAC model concerning adoption of cloud computing services by clients that are divided into three types: susceptible users, abstainers and efficient users as well as to accomplish the optimal control problem for the proposal model where we provide some results relating to the existence of the optimal controls and we identified these optimal controls applying Pontryagin's maximum principle.Finally, numerical simulations are showed to verify the theoretical analysis using Matlab.

## 2. LITERATURE REVIEW

### Definition of Cloud Computing

Cloud computing is a model for allowing on-demand network access with regard to share computing resources like network bandwidth, storage, applications, etc , that is able to be rapidly scalable with minimal service provider maintenance [4]. As per the National Institute of Standard and Technology (NIST) cloud computing is represented with five characteristics, three service models and four deployment models. Cloud computing five characteristics are composed of on demand self service, broad network access, resource pooling, rapid elasticity and measured service. On-demand self service offers automatic computing capacity control to systems, without demanding human interaction. Broad network access enables heterogeneous users, like mobile laptops, phones, to link to cloud systems via the network. Resource pooling in cloud systems is accessible as pooling resources for divers clients which is capable to automatically assign and reassign conforming to client request. Rapid elasticity provides quickly and flexibly supplying of resources. It can rapidly scale out and effectively release to immediately scale in with a view to maintain client's systems. Measure service affords handling, administrating and reporting of resource utilization. Three cloud service models correspond to software, platform and infrastructure models. Software as a Service (SaaS) is the model of delivering the resources for clients with regard to exploit the vendor's application running on a cloud infrastructure. The applications can be accessed from an user interface like a web browser or web service. For instance "Google Apps". Platform as a Service (PaaS) permits customers to implement their own infrastructures or applications utilizing programming languages and tools maintained by the cloud service vendor. Infrastructure as a Service (IaaS) provides processing, storage network bandwidth and other fundamental computing resources which allow customers to deploy and run operating systems or applications. Four cloud deployment models, public, private community and hybrid, are partitioned according to the needs. Public cloud is regulated for the general public. The cloud system owner provides cloud services to customers. Private cloud is made to an individual organization. It can be controlled by either the organization or a third party. Community cloud is the cloud infrastructure that is shared by multiple organizations and maintains a particular community. Hybrid cloud is the combination of two or more cloud infrastructures that are bound together.

**Benefits of Cloud Computing**

Major progress in cloud computing use is predicted. The global cloud computing market size is expected to grow from USD 371.4 billion in 2020 to USD 832.1 billion by 2025, at a Compound Annual Growth Rate (CAGR) of 17.5% during the forecasting period [6]. These expectations for growth are depended on the awareness of the many benefits of cloud computing. Cloud computing has captivated attention of organizations in last few years through its noticeable opportunities of cost benefit in establishing and maintaining hardware and software infrastructure especially for start up and small and medium enterprises. Cloud computing advantages are

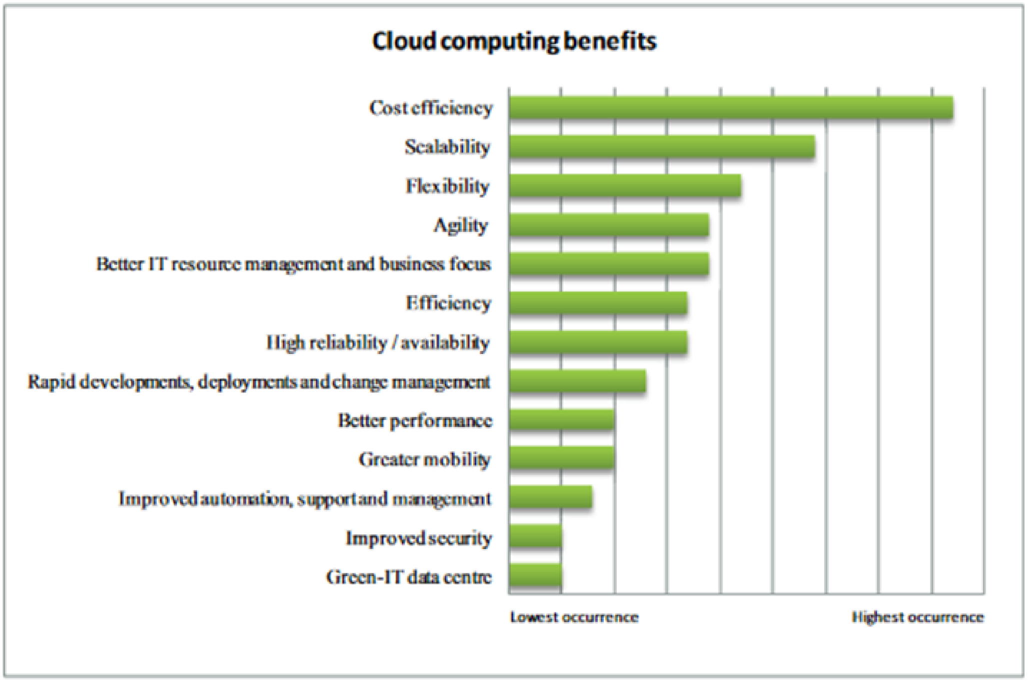


FIGURE 1. Cloud computing advantages [8]

detailed in figure 1, organized from the highest occurrence (accordingly mentioned most in literature) to the lowest. The other advantages of cloud computing are: Access to a great assortment of applications without needing to download or install anything at all, reduced prices, efficiency of employment, quality of the service (QoS) acknowledged under the contract, outsourced IT control, clear maintenance and upgrade, access to the application from any computer over the Internet, scalability through on request resources, pay-as-you-go cost model [10].

**Cloud Computing challenges** In spite of its many advantages, cloud computing services have their challenges for instance issues relating to the security and the privacy of data in the cloud and deficiency of interoperability between cloud platforms [7]. Figure 2 represents the listing of classified problems.

Interoperability between various cloud service vendor is a considerable problem. With the lack of a standard cloud computing API, it is difficult for a client to move from one cloud service vendor to another [8].

Moreover, The users of this technology are anxious about the confidentiality and security of their sensitive client data. This anxiety is due to threir ignorance of the physical location of data.

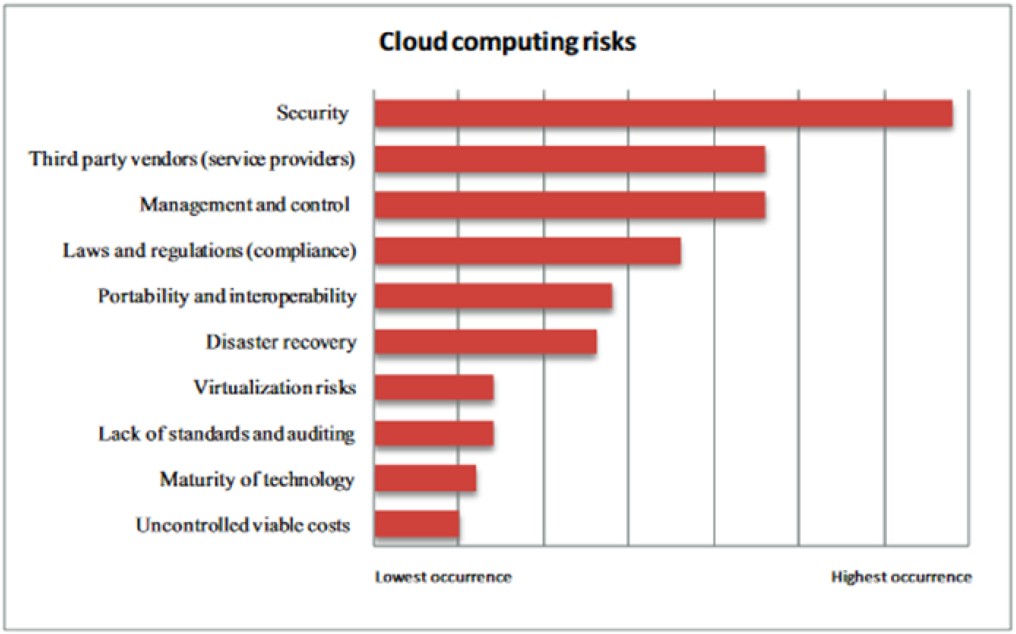


FIGURE 2. Cloud computing advantages [8]

Hence, suitable security procedure should be used by the cloud service supplier to afford the confidentiality need of users. Furthermore, Deploying application and data in distributed infrastructures rises the ability of illegal access and increases anxieties like confidentiality, identification control, authentication, conformity, availability of data, encryption and network security.

## 3. PRESENTATION OF MODEL

We set down a mathematical model SAC that represents the changes of cloud computing adoption by customers and the negative impact of abstainers, who renounce from cloud computing using considering its risks, on the susceptible users. We categorize the population tagged by N into three categories:

The susceptible users (S) are designated to utilize cloud computing services, and they have not yet started to use this technology. The category of susceptible users is raised by the entrance of individuals into the class S at a rate $\Lambda$ and it is weakened when susceptible users adopt cloud computing services at rate $\xi$. It is expected that susceptible users can pick up abstainer attitude (and become the abstainers of using this technology) through efficient influence with these abstainers at a rate $\beta$. By way of explanation, it is supposed that the receiving of an abstainer behavior is comparable to acquiring disease infection. Sooner or later, susceptible users endure usual death (at a rate $\mu$ ).

The abstainers (A) have an attitude of abstaining from using cloud computing services. The population of abstainers raises when the susceptible users renounce to implement this technology through efficient influence with abstainers at a rate $\beta$. It decreases by usual death (at the rate $\mu$ ) and when some abstainers quit their position of refusing the implementation of cloud computing technology and convert to adopt it then become customers at a rate $\gamma$ .

The customers (C) are the consumers of service in cloud environment. This category is raised at a rate $\xi$. when susceptible users adopt this technology and when the abstainers become users at a rate $\gamma$ . This class is diminished by usual death (at the rate $\mu$).

The mathematical SAC model is

$$(1) \qquad \begin{cases} S(i+1) = S(i) + \Lambda - \beta \frac{S(i)A(i)}{N} - (\mu + \xi)S(i), \\ \\ A(i+1) = A(i) + \beta \frac{S(i)A(i)}{N} - (\gamma + \mu +)A(i), \\ \\ C(i+1) = C(i) + \gamma A(i) + \xi S(t) - \mu C(i), \end{cases}$$

with initial conditions $S(0) > 0 \quad A(0) > 0, \quad C(0) > 0$

## 4. AN OPTIMAL CONTROL PROBLEM

Any country pursuing to increase its economic growth is in crucial need of rising many indicators as well as raising the rate of cloud computing adoption. For this, it must formulate some optimal approachs for a consciousness program which enables these countries to improve the rate of users with an optimum effort. Hence, the goal of this suggested strategy of control is to reduce the number of abstainers and increase the number of consumers all along the time interval $[t_0, t_f]$. In the model (1), we involve two controls $u_1$ and $u_2$ for $t \in [T_0, t_f]$: the control $u_1$ designates the awareness campaign in order to increase the knowledge of this

technology and its benefits via the use of communication channels like mass media and interpersonal communication channel. Mass media involves internet, newspapers, television or radio. Interpersonal communication channel can comprise communication among two or more people. This is beneficial to motivate the susceptible users to adopt cloud computing services, and reduce the impact of infuenced to susceptible. The control $u_2$ designates the suggestions made in view to address some of the issues and concerns that abstenents have regarding cloud computing. To mitigate security concerns, rather than utilizing a public cloud, an alternative is a private cloud. These are implemented on a proprietary principles and apply many of the same regulations as public cloud computing. Private clouds are considered more secure [15].
The data location risk in the cloud may need regulatory intervention to focus on problems concerning personal confidentiality [5]. This is conducive to influence the abstinents to use services provided by this technology. Therefore, the controlled mathematical system is delivered via the ensuing system of differential equations and the object functioanl.

To sum up, our goal is to minimize the objectif functional presented as follow:

$$J(u_1,u_2) = A(T_f) - C(T_f) + \sum_{i=0}^{t_f-1} (A(i) - C(i) + \frac{M}{2}u_1^2(i) + \frac{L}{2}u_2^2(i)) \tag{2}$$

subject to

$$\begin{cases} S(i+1) = S(i) + \Lambda - \beta(1-u_1(i))\frac{S(i)A(i)}{N} - (\mu+\xi)S(i), \\ \\ A(i+1) = A(i) + \beta(1-u_1(i))\frac{S(i)A(i)}{N} - (\gamma+\mu)A(i) - u_2(i)A(i), \\ \\ C(i+1) = C(i) + \gamma A(i) + \xi S(i) - \mu C(i) + u_2(i)A(i), \end{cases} \tag{3}$$

With initial conditions $S(0) > 0 \quad A(0) > 0, \quad C(0) > 0$ and where the parametres $M,L$ are positives. The first term of $J$ is the principal goal, decreasing the number the first control $u_1$ and the second control$u_2$. The second term is systemic cost of the awareness program, and in order to balance the size of terms in $J$ we use the positive constants $M,L$. We search an optimal pair $(u_1^*, u_2^*)$ such as

$$J(u_1^*, u_2^*) = min\{J(u_1,u_2) \quad (u_1,u_2) \in U_{ad}\} \tag{4}$$

where $U_{ad}$ is the set of admissible controls defined by

$$U_{ad} = \{(u_{1,i}, u_{2,i}) : 0 < u_{1,min} \leq u_{i,1} \leq u_{1,max} < 1,\ 0 < u_{2,min} \leq u_{i,2} \leq u_{2,max} < 1; \quad i = 0, 1 \ldots t_f - 1\} \tag{5}$$

**4.1. Existence theorem.** To demonstrate the existence of the optimal pair for $J$, we use a result [16, 17, 18]

There exists an optimal control $(u^*_{1,i}, u^*_{2,i})$ such as

$$J(u^*_{1,i}, u^*_{2,i}) = \min_{(u_{1,i}, u_{2,i}) \in U_{ad}} J(u_{1,i}, u_{2,i})$$

subjet to the control system (3) with initial conditions.

*Proof.* Since the coefficients of the state equations are bounded and there are a finite number of time steps $S, A, C$ are uniformly bounded for all $(u_{1,i}, u_{2,i})$ in the control set $U_{ad}$, thus $J(u_{1,i}, u_{2,i})$ is bounded for all $(u_{1,i}, u_{2,i}) \in U_{ad}$.
Since $J(u_{1,i}, u_{2,i})$ is bounded,

$$\inf_{(u_{1,i}, u_{2,i}) \in U_{ad}} J(u_{1,i} u_{2,i})$$

is finite, and there exists a sequence $(u^j_{1,i}, u^j_{2,i}) \in U_{ad}$ such as

$$\lim_{j \to +\infty} (u^j_{1,i}, u^j_{2,i}) = \inf_{(u_{1,i}, u_{2,i}) \in U_{ad}} J(u_{1,i}, u_{2,i})$$

and corresponding sequences of states

$$S^j \to S$$

$$A^j \to A$$

$$C^j \to C$$

Since there is a finite number of uniformly bounded sequences, there exist $(u^*_{1,i}, u^*_{2,i}) \in U_{ad}$ and $(S^*,\ A^*,\ C^*,) \in \mathbb{R}^{T_f+1}$ such as, on a subsequence,

$$(u^j_{1,i}, u^j_{2,i}) \to (u^*_{1,i}, u^*_{2,i})$$

Finally, due to the finite dimensional structure of system (3) and the objective function $J(u_{1,i}, u_{2,i})$, $(u^*_{1,i}, u^*_{2,i})$ is an optimal control with corresponding states $(S^*,\ A^*,\ C^*)$. Therefore

$$\min_{(u_{1,i}, u_{2,i}) \in U_{ad}} J(u_{1,i}, u_{2,i})$$

□

**4.2. Characterization of the optimal controls.** A necessary condition for an optimal control is obtained by applied of the Pontryagin's Maximum Principle [20, 21, 22]. This principle converts (2-3) into a problem of minimizing an Hamiltonian, $\mathbf{H_i}$ at time step i is

$$\mathbf{H_i} = A(i) - C(i) + \frac{M}{2}u_1^2(i) + \frac{L}{2}u_2^2(i) + \lambda_{1,i+1}S(i+1) + \lambda_{2,i+1}A(i+1) + \lambda_{3,i+1}C(i+1) \tag{6}$$

(7)

$$\begin{aligned}\mathbf{H_i} = A(i) - C(i) + \frac{M}{2}u_1^2(i) + \frac{L}{2}u_2^2(i) + \lambda_{1,i+1}(S(i) + \Lambda - \beta(1-u_1(i))\frac{S(i)A(i)}{N} - (\mu+\xi)S(i))\\ + \lambda_{2,i+1}(A(i) + \beta(1-u_1(i))\frac{S(i)A(i)}{N} - (\gamma+\mu)A(i) - u_2(i)A(i))\\ + \lambda_{3,i+1}(C(i) + \gamma A(i) + \xi S(i) - \mu C(i) + u_2(i)A(i))\end{aligned}$$

Let $(u^*_{1,i}, u^*_{2,i})$ the optimal pair solution of (2), and $(S^*_i, A^*_i, C^*_i)$ the solution of (3) according to $(u^*_{1,i}, u^*_{2,i})$, there exists adjoints functions $\lambda_{1,i}, \lambda_{2,i}, and \lambda_{3,i}$ such as

$$\begin{cases} \lambda_{1,i} = \lambda_{1,i+1}(1-\mu-\xi) + \lambda_{3,i+1}\xi + \beta(1-u_{1,i})\frac{A(i)}{N}(\lambda_{2,i+1} - \lambda_{1,i+1}) \\ \lambda_{2,i} = 1 + (\lambda_{2,i+1} - \lambda_{1,i+1})\beta(1-u_{1,i})\frac{S(i)}{N} + \lambda_{2,i+1}(1-\mu-\gamma-u_{2,i}) + \lambda_{3,i+1}(\gamma+u_{2,i}) \\ \lambda_{3,i} = -1 + \lambda_{3,i+1}(1-\mu) \end{cases}$$

with transversality conditions

$$\lambda_{1,t_f} = 0,\ \lambda_{2,t_f} = 1,\ \lambda_{3,t_f} = -1$$

and

$$u_1^*(i) = \max\left\{\min\left\{\frac{\beta S^*(i)A^*(i)(\lambda_{2,i+1} - \lambda_{1,i+1})}{M*N}, 1\right\}, 0\right\}$$

$$u_2^*(i) = \max\left\{\min\left\{\frac{A^*(i)(\lambda_{3,i+1} - \lambda_{2,i+1})}{L}, 1\right\}, 0\right\}$$

*Proof.* Applying Pontryagin's Principale, the following adjoints system is calculated as follow

$$\lambda_{1,i+1} = \frac{\partial \mathbf{H_i}}{\partial S_i}, \quad \lambda_{2,i+1} = \frac{\partial \mathbf{H_i}}{\partial A_i}, \quad \lambda_{3,i+1} = \frac{\partial \mathbf{H_i}}{\partial C_i}, \tag{8}$$

so

$$\lambda_{1,i+1} = \lambda_{1,i+1}(1-\mu-\xi) + \lambda_{3,i+1}\xi + \beta(1-u_{1,i})\frac{A(i)}{N}(\lambda_{2,i+1} - \lambda_{1,i+1}) \tag{9}$$

and

$$\lambda_{1,t_f} = 0 \tag{10}$$

$$\lambda_{2,i+1} = 1 + (\lambda_{2,i+1} - \lambda_{1,i+1})\beta(1-u_{1,i})\frac{S(i)}{N} + \lambda_{2,i+1}(1-\mu-\gamma-u_{2,i}) + \lambda_{3,i+1}(\gamma+u_{2,i}) \tag{11}$$

and

$$\lambda_{2,t_f} = 1 \tag{12}$$

$$\lambda_{3,i+1} = -1 + \lambda_{3,i+1}(1-\mu) \tag{13}$$

and

$$\lambda_{3,t_f} = -1 \tag{14}$$

considering the optimality conditions

$$\frac{\partial \mathbf{H_i}}{\partial u_{1,i}} = 0, \quad \frac{\partial \mathbf{H_i}}{\partial u_{2,i}} = 0 \tag{15}$$

so

$$\frac{\partial \mathbf{H_i}}{\partial u_{1,i}} = Mu_{1,i} + \lambda_{1,i+1}\beta\frac{S(i)A(i)}{N} - \lambda_{2,i+1}\beta\frac{S(i)A(i)}{N} = 0, \tag{16}$$

and taking account the bounds of $u^*_{1,i}$ we obtain

$$u^*_{1,i} = \max\left\{\min\left\{\beta\frac{S^*(i)A^*(i)}{M*N}(\lambda_{2,i+1} - \lambda_{1,i+1}), 1\right\}, 0\right\} \tag{17}$$

in the same way we calculate $u^*_{2,i}$,

$$\frac{\partial \mathbf{H_i}}{\partial u_{2,i}} = Lu_{2,i} - \lambda_{2,i+1}A(i) + \lambda_3 I(i) \tag{18}$$

so

$$u_{2,i}^{*} = \max\left\{\min\left\{\frac{(\lambda_{3,i+1}-\lambda_{2,i+1})A^{*}(i)}{L},1\right\},0\right\} \tag{19}$$

□

## OPTIMALITY SYSTEM

The optimality system is presented as follow

$$\left\{\begin{array}{l}
S(i+1) = S(i) + \Lambda - \beta(1-u_1(i))\frac{S(i)A(i)}{N} - (\mu+\xi)S(i), \\
A(i+1) = A(i) + \beta(1-u_1(i))\frac{S(i)A(i)}{N} - (\gamma+\mu)A(i) - u_2(i)A(i), \\
C(i+1) = C(i) + \gamma A(i) + \xi S(i) - \mu C(i) + u_2(i)A(i), \\
\lambda_{1,i} = \lambda_{1,i+1}(1-\mu-\xi) + \lambda_{3,i+1}\xi + \beta(1-u_{1,i})\frac{A(i)}{N}(\lambda_{2,i+1}-\lambda_{1,i+1}) \\
\lambda_{2,i} = 1 + (\lambda_{2,i+1}-\lambda_{1,i+1})\beta(1-u_{1,i})\frac{S(i)}{N} + \lambda_{2,i+1}(1-\mu-\gamma-u_{2,i}) + \lambda_{3,i+1}(\gamma+u_{2,i}) \\
\lambda_{3,i} = -1 + \lambda_{3,i+1}(1-\mu) \\
S(0) = S_0,\ A(0) = A_0,\ C(0) = C_0 \\
\lambda_{1,t_f} = 0,\ \lambda_{2,t_f} = 1,\ \lambda_{3,t_f} = -1
\end{array}\right. \tag{20}$$

and

$$\left\{\begin{array}{l}
u_1^{*}(i) = \max\left\{\min\left\{\frac{\beta S^{*}(i)A^{*}(i)(\lambda_{2,i+1}-\lambda_{1,i+1})}{M*N},1\right\},0\right\} \\
u_2^{*}(i) = \max\left\{\min\left\{\frac{A^{*}(i)(\lambda_{3,i+1}-\lambda_{2,i+1})}{L},1\right\},0\right\}
\end{array}\right. \tag{21}$$

## 5. NUMERICAL SIMULATION

In this section, we present numerical results for (20). In order to solve numerically the optimality system, we use an iterative method.

The code according to our system is written and compiled in Matlab using the following data

| $\Lambda$ | $\mu$ | $\beta$ | $\xi$ | $\gamma$ |
|---|---|---|---|---|
| 65 | 0.065 | 0.3 | 0.03 | 0.01 |

With S(1)=100 I(1)=200; C(1)=100.

**Case 1:** Using only the optimal control $u_1^*$
In this approach, we aim attention at the effort of the consciousness campaign on susceptible users in regard to sensitize them of the benefits of cloud computing use and to protect them from the negative impact of abstinents. So, this strategy aim to decrease the contact between abstainers and susceptible users.

In figure 2, we observe that with the first control that reduce the negative impact of abstainers

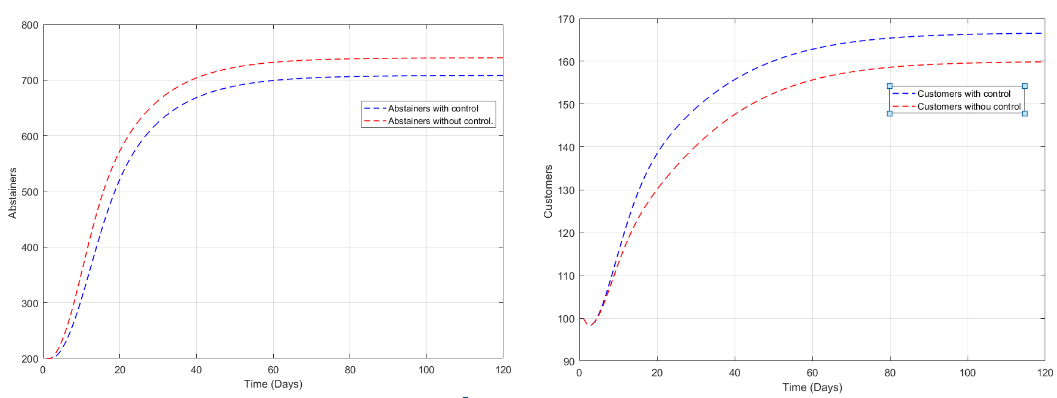


FIGURE 3. The evolution of abstainers and customers with control $u_1^*$

to susceptible(Abstainers can influence susceptible users to renounce to cloud computing adoption.), the number of abstainers with control is lower than the number of abstainers without controls in the end of the simulation. In addition, the number of customers with control is bigger than the number of customers without control. These transformations are essential but not sufficing. So for this aim, we must also designate through this control approach the abstinents.

**Case 2:** Using only the optimal control $u_2^*$

In this control we seek to increase the number of customers and decrease the number of abstainers by solving some issues of the cloud computing services. In figure 3, by the use of the second

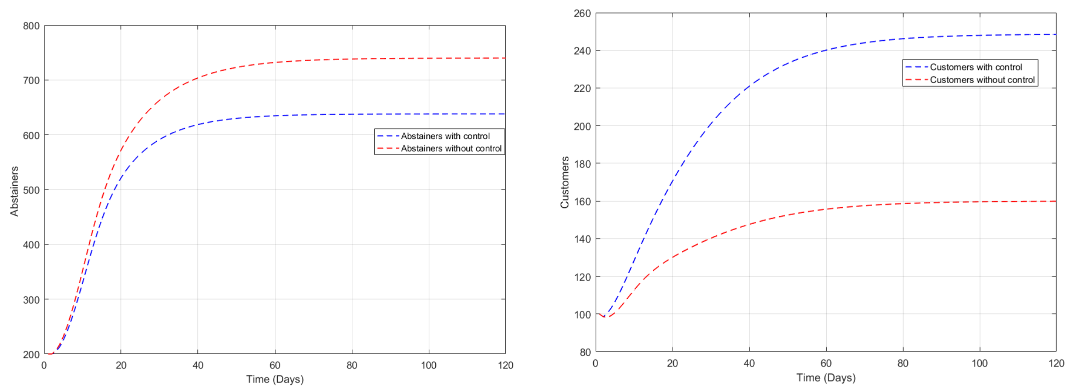


FIGURE 4. The evolution of abstainers and customers with control $u_2^*$

control $u_2$ we observe that the number of abstainers with control is lower than the number of abstainers without controls in the end of the simulation, In addition the number of customers with control is bigger than the number of customers without control too.

**Case 3:** use of $u_1^*$, $u_2^*$

we associate the optimal control $u_1^*, u_2^*$. In this approach, the two optimum controls $u_1^*$, $u_2^*$ are turned on at the same time in order to enhance the numerical results of case 1 and 2. The optimal controls $u_1^*$, $u_2^*$ describes the effort to persuade the abstainers to change their point of view and minimize their negative impact on the susceptible users. The results while combining the two strategies are better than use just only the first or the second control. The number of abstainers with controls is lower than the number of abstainers without control and even better than the number of abstainers with $u_1$ or with $u_2$. In the other hand, the number of customers with control in figure (5) is bigger than the number of customers without control, and the results in the end of the simulation are better than the number of customers with $u_1$ or with $u_2$.

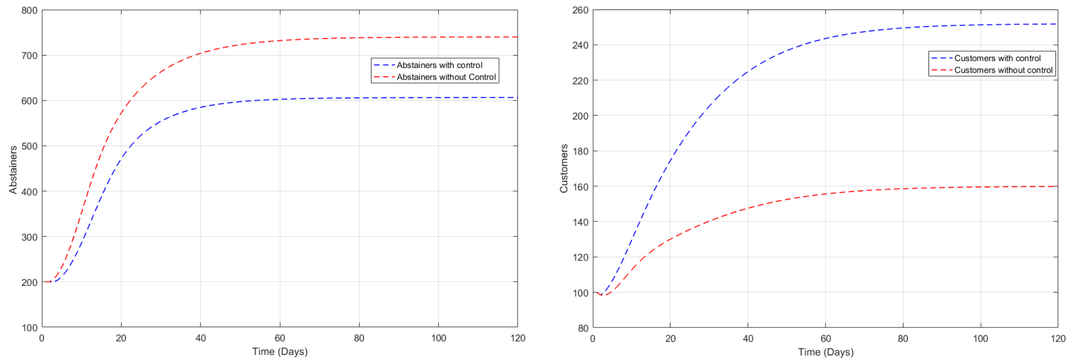


FIGURE 5. The evolution of abstainers and customers with controls $u_1^*, u_2^*$

At the end, we deduce that the proposal approach is more efficient when we associated two optimum controls $u_1^*$ and $u_2^*$.

## 6. CONCLUSION

Cloud computing previsions for success show valuable improvements for and uses of cloud computing services. To make cloud environments more powerful, proper controls, mitigating aforementioned risks should be accomplished.

In the present paper, We establish a mathematical model SAC that serves as the changes of cloud computing adoption by clients and the negative effect of abstainers, who abstain from cloud computing using considering its risks, on the susceptible users. We also proposed a number of controls that could be regarded for the mitigation of cloud computing risks in order to allow a high adoption of its services by organizations and we identified these optimal controls applying Pontryagin's maximum principle as well as and the optimum system is resolved by an iterative program. Also, the numerical simulation was realized through Matlab. We conclude that the suggested method is more efficient when we associated two optimum controls $u_1^*$ and $u_2^*$.

## CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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