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CUBIC NEAR-RING

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Abstract. Y. B. Jun et al introduced a remarkable structure namely cubic sets that combines fuzzy set and interval-valued fuzzy set. Motivated by the above theory our aim in this paper is to introduce the notion of cubic near-ring. The notions of R-intersection, R-union, P-intersection and P-union are investigated. We prove that R-intersection of two cubic near-ring is again a cubic near-ring. It is shown by means of counter examples that the R-union, P-intersection and P-union of two cubic near-ring is not a cubic near-ring.

Keywords: fuzzy near-ring; interval-valued fuzzy near-ring; cubic set; cubic ring.

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1. Introduction

The fundamental concept of fuzzy set was introduced by Zadeh [10]. After the introduction of the concept of fuzzy sets by Zadeh several researchers were conducted on the generalization of the notion of fuzzy set. In 1975 Zadeh [11] introduced the concept of interval-valued fuzzy subsets. Where the values of membership functions are intervals of numbers instead of the numbers. In 1982, W. J. Liu [6] introduced the concept of fuzzy ring. In 2010, K. Hur and H.

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W. Kang [2] introduced interval-valued fuzzy subgroups and rings. Jun et al [5] introduced the new concept called cubic sets. This structure encompass interval-valued fuzzy set and fuzzy set. Also Jun et al [4] introduced the notion of cubic subgroups. The purpose of this paper to introduce the notion of cubic near-ring. The notions of *R*-intersection, *R*-union, *P*-intersection and *P*-union are introduced and we provide some results on it.

2. PRELIMINARIES

In this section, we listed some basic definitions related to cubic near-ring.

Definition 2.1. [7] *A near-ring is an algebraic system (N,+, .) consisting of a non-empty set N together with two binary operations called "+" and "." such that* $(N,+)$ *is a group not necessarily abelian and* $(N, .)$ *is a semigroup connected by the following distributive law:* $(x+y).z = x.z+y.z$ *valid for all* $x,y,z \in N$*. Precisely speaking, it is a right near-ring because it satisfies the right distributive law. We will use the word "near-ring" to mean "right near-ring".*

Definition 2.2. [11] *Let X be a non-empty set. A mapping* $\bar{\mu} : X \to D[0,1]$ *is called interval-valued fuzzy set, where* $D[0,1]$ *denote the family of all closed subintervals of* $[0,1]$.

Definition 2.3. *Let N be a near-ring and* μ *be a fuzzy set of N. Then* μ *is called a fuzzy near-ring of N. If it satisfies the following conditions,*

i) $\mu(x-y) \geq min\{\mu(x), \mu(y)\}$,

ii) $\mu(xy) \geq min\{\mu(x), \mu(y)\}, \forall x,y \in N$.

Definition 2.4. *Let N be a near-ring and* μ *be an interval-valued fuzzy set of N. Then* μ *is called an interval-valued fuzzy near-ring of N. If it satisfies the following conditions,*

i) $\bar{\mu}(x-y) \geq min\{\bar{\mu}(x), \bar{\mu}(y)\}$,

ii) $\bar{\mu}(xy) \geq min\{\bar{\mu}(x), \bar{\mu}(y)\}, \forall x,y \in N$.

Definition 2.5. [3] *Let X be a non-empty set. A cubic set* $\mathscr{A}$ *is a structure of the form* $\mathscr{A} = \{\langle x, \bar{\mu}(x), \lambda(x)\rangle : x \in X\}$ *and denoted by* $\mathscr{A} = \langle \bar{\mu}_A, \lambda \rangle$, $\bar{\mu}_A = \left[\mu_A^-, \mu_A^+\right]$ *is an interval-valued fuzzy set (briefly, IVF) in X and* $\lambda : X \to [0,1]$ *is a fuzzy set in X.*

Definition 2.6. [5] *The complement of* $\mathscr{A} = \langle \bar{\mu}_A, \lambda \rangle$ *is defined to be the cubic set* $\mathscr{A}^c = \{\langle x, (\bar{\mu}_A)^c(x), 1-\lambda(x)\rangle \,|x \in X\}$.

Definition 2.7. [4] *A cubic set* $\mathscr{A} = \langle \bar{\mu}_A, \lambda \rangle$ *is called a cubic subgroup of* X. *If it satisfies the following conditions,*

i) $\bar{\mu}_A(xy) \geq min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$,

ii) $\bar{\mu}_A(x^{-1}) \geq \bar{\mu}_A(x)$,

iii) $\lambda(xy) \leq max\{\lambda(x), \lambda(y)\}$,

iv) $\lambda(x^{-1}) \leq \lambda(x)\ \forall x, y \in N$.

Definition 2.8. [5] *For any* $\mathscr{A}_i = \{\langle x, \bar{\mu}_i(x), \lambda_i(x)\rangle \,|x \in X\}$ *where* $i \in \wedge$ *(index set), we have the following,*

i) $\bigcap_{R, i\in\wedge} \mathscr{A}_i = \{\langle x, (\bigcap_{i\in\wedge} \bar{\mu}_i)(x), (\bigcup_{i\in\wedge} \lambda_i)(x)\rangle \,|x \in X\}$ *(R- intersection)*

ii) $\bigcup_{R, i\in\wedge} \mathscr{A}_i = \{\langle x, (\bigcup_{i\in\wedge} \bar{\mu}_i)(x), (\bigcap_{i\in\wedge} \lambda_i)(x)\rangle \,|x \in X\}$ *(R- union)*

iii) $\bigcap_{P, i\in\wedge} \mathscr{A}_i = \{\langle x, (\bigcap_{i\in\wedge} \bar{\mu}_i)(x), (\bigcap_{i\in\wedge} \lambda_i)(x)\rangle \,|x \in X\}$ *(P- intersection)*

iv) $\bigcup_{P, i\in\wedge} \mathscr{A}_i = \{\langle x, (\bigcup_{i\in\wedge} \bar{\mu}_i)(x), (\bigcup_{i\in\wedge} \lambda_i)(x)\rangle \,|x \in X\}$ *(P- union)*

3. MAIN RESULTS

We now introduce the notion of cubic near-ring as follows,

Definition 3.1. *Let* N *be a near-ring,* $(N, \bar{\mu})$ *be an interval-valued fuzzy near-ring and* (N, γ) *be a fuzzy near-ring. A cubic set* $\mathscr{A} = \langle \bar{\mu}, \gamma \rangle$ *is called a cubic near-ring of* N *if it satisfies the following conditions,*

i) $\bar{\mu}(x-y) \geq min\ \{\bar{\mu}(x), \bar{\mu}(y)\}$,

ii) $\bar{\mu}(xy) \geq min\ \{\bar{\mu}(x), \bar{\mu}(y)\}$,

iii) $\gamma(x-y) \leq max\ \{\gamma(x), \gamma(y)\}$,

iv) $\gamma(xy) \leq max\ \{\gamma(x), \gamma(y)\}, \forall x, y \in N$.

Example 3.2. *Let* $N = \{0, a, b, c\}$ *be the near ring with* $(N, +)$ *as the Klein's four group and* $(N, .)$ *as defined below (scheme 10:(0,0,0,1) See* [7], *p.408).*

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	c	a
b	0	0	0	b
c	0	0	0	c

Then $(N,+,.)$ is a near-ring.

Define an interval-valued fuzzy set $\bar{\mu}$ in N by

$\bar{\mu}(0) = [0.6, 0.7]$, $\bar{\mu}(a) = [0.4, 0.5]$, $\bar{\mu}(b) = [0.5, 0.6]$, $\bar{\mu}(c) = [0.4, 0.5]$

Then $\bar{\mu}$ is an interval-valued fuzzy near-ring.

Define a fuzzy set γ in N by $\gamma(0) = 0.2, \gamma(a) = 0.45, \gamma(b) = 0.4, \gamma(c) = 0.45$. Then γ is a fuzzy near-ring.

Hence $\mathscr{A} = \langle \bar{\mu}, \gamma \rangle$ is a cubic near-ring.

Remark 3.3. *Every cubic ring is a cubic near-ring. But the converse need not be true.*

Proof. Let $N =$ The Dihedral group $D_8 = \{0, a, 2a, 3a, b, a+b, 2a+b, 3a+b\}$ $\forall a, 2a, 3a, b, a+b, 2a+b, 3a+b \in N$(scheme 41: (10,10,10,10,10,10,10,10) See [7], p.416).

+	0	a	2a	3a	b	a+b	2a+b	3a+b
0	0	a	2a	3a	b	a+b	2a+b	3a+b
a	a	2a	3a	0	a+b	2a+b	3a+b	b
2a	2a	3a	0	a	2a+b	3a+b	b	a+b
3a	3a	0	a	2a	3a+b	b	a+b	2a+b
b	b	3a+b	2a+b	a+b	0	3a	2a	a
a+b	a+b	b	3a+b	2a+b	a	0	3a	2a
2a+b	2a+b	a+b	b	3a+b	2a	a	0	3a
3a+b	3a+b	2a+b	a+b	b	3a	2a	a	0

.	0	a	2a	3a	b	a+b	2a+b	3a+b
0	0	0	0	0	b	b	b	b
a	0	0	0	0	b	b	b	b
2a	0	0	0	0	b	b	b	b
3a	0	0	0	0	b	b	b	b
b	0	0	0	0	b	b	b	b
a+b	0	0	0	0	b	b	b	b
2a+b	0	0	0	0	b	b	b	b
3a+b	0	0	0	0	b	b	b	b

Then $(N,+,.)$ is a near-ring.

Define an interval-valued fuzzy set $\bar{\mu}$ in N by

$$\bar{\mu}(x) = \begin{cases} [0.8, 0.9] & \text{if } x = 0, b, 3a+b \\ [0.7, 0.8] & \text{if } x = a, 2a, 3a, a+b, 2a+b \end{cases}$$

Then $\bar{\mu}$ is an interval-valued fuzzy near-ring.

Define a fuzzy set γ in N by

$$\gamma(x) = \begin{cases} 0.43 & \text{if } x = 0, b, 3a+b \\ 0.57 & \text{if } x = a, 2a, 3a, a+b, 2a+b \end{cases}$$

Then γ is a fuzzy near-ring. Hence $\mathscr{A} = \langle \bar{\mu}, \gamma \rangle$ is a cubic near-ring.

Since $(N,+)$ is not abelian, N is not a ring. Therefore a cubic near-ring $\mathscr{A} = (\bar{\mu}, \gamma)$ is not a cubic ring. □

Theorem 3.4. *Let $\mathscr{A}_1 = \langle \bar{\mu}_1, \gamma_1 \rangle$ and $\mathscr{A}_2 = \langle \bar{\mu}_2, \gamma_2 \rangle$ be two cubic near-rings. Then their R-intersection $(\mathscr{A}_1 \cap \mathscr{A}_2)_R = \langle \bar{\mu}_1 \cap \bar{\mu}_2, \gamma_1 \cup \gamma_2 \rangle$ is a cubic near-ring.*

Proof. Define $(\bar{\mu}_1 \cap \bar{\mu}_2)$ as,

$$(\bar{\mu}_1 \cap \bar{\mu}_2)(x-y) = \min\{\bar{\mu}_1(x-y), \bar{\mu}_2(x-y)\}$$

and

$$(\bar{\mu}_1 \cap \bar{\mu}_2)(xy) = \min\{\bar{\mu}_1(xy), \bar{\mu}_2(xy)\}.$$

Now,

$$i)\ (\bar{\mu}_1 \cap \bar{\mu}_2)(x-y) = min\{\bar{\mu}_1(x-y), \bar{\mu}_2(x-y)\},$$

$$\geq min\{min[\bar{\mu}_1(x), \bar{\mu}_1(y)], min[\bar{\mu}_2(x), \bar{\mu}_2(y)]\},$$

$$= min\{min[\bar{\mu}_1(x), \bar{\mu}_2(x)], min[\bar{\mu}_1(y), \bar{\mu}_2(y)]\},$$

$$= min\{(\bar{\mu}_1 \cap \bar{\mu}_2)(x), (\bar{\mu}_1 \cap \bar{\mu}_2)(y)\}.$$

$$ii)\ (\bar{\mu}_1 \cap \bar{\mu}_2)(xy) = min\{\bar{\mu}_1(xy), \bar{\mu}_2(xy)\},$$

$$\geq min\{min[\bar{\mu}_1(x), \bar{\mu}_1(y)], min[\bar{\mu}_2(x), \bar{\mu}_2(y)]\},$$

$$= min\{min[\bar{\mu}_1(x), \bar{\mu}_2(x)], min[\bar{\mu}_1(y), \bar{\mu}_2(y)]\},$$

$$= min\{(\bar{\mu}_1 \cap \bar{\mu}_2)(x), (\bar{\mu}_1 \cap \bar{\mu}_2)(y)\}.$$

Define $(\gamma_1 \cup \gamma_2)$ as

$$(\gamma_1 \cup \gamma_2)(x-y) = max\{\gamma_1(x-y), \gamma_2(x-y)\},$$

$$(\gamma_1 \cup \gamma_2)(xy) = max\{\gamma_1(xy), \gamma_2(xy)\}.$$

$$iii)\ (\gamma_1 \cup \gamma_2)(x-y) = max\{\gamma_1(x-y), \gamma_2(x-y)\},$$

$$\geq max\{max[\gamma_1(x), \gamma_1(y)], max[\gamma_2(x), \gamma 2(y)]\},$$

$$= max\{max[\gamma_1(x), \gamma_2(x)], max[\gamma_1(y), \gamma_2(y)]\},$$

$$= max\{(\gamma_1 \cup \gamma_2)(x), (\gamma_1 \cup \gamma_2)(y)\}.$$

$$iv)\ (\gamma_1 \cup \gamma_2)(xy) = max\{\gamma_1(xy), \gamma_2(xy)\},$$

$$\geq max\{max[\gamma_1(x), \gamma_1(y)], max[\gamma_2(x), \gamma 2(y)]\},$$

$$= max\{max[\gamma_1(x), \gamma_2(x)], max[\gamma_1(y), \gamma_2(y)]\},$$

$$= max\{(\gamma_1 \cup \gamma_2)(x), (\gamma_1 \cup \gamma_2)(y)\}.$$

Thus $(\mathscr{A}_1 \cap \mathscr{A}_2)_R = \langle \bar{\mu}_1 \cap \bar{\mu}_2, \gamma_1 \cup \gamma_2 \rangle$ is a cubic near-ring. □

Remark 3.5. *i) Let $\mathscr{A}_1 = \langle \bar{\mu}_1, \gamma_1 \rangle$ and $\mathscr{A}_2 = \langle \bar{\mu}_2, \gamma_2 \rangle$ be two cubic near-rings. Then their R-union, $(\mathscr{A}_1 \cup \mathscr{A}_2)_R = \langle \bar{\mu}_1 \cup \bar{\mu}_2, \gamma_1 \cap \gamma_2 \rangle$ is not a cubic near-ring.*

ii) Let $\mathscr{A}_1 = \langle \bar{\mu}_1, \gamma_1 \rangle$ and $\mathscr{A}_2 = \langle \bar{\mu}_2, \gamma_2 \rangle$ be two cubic near-rings. Then their P-intersection, $(\mathscr{A}_1 \cap \mathscr{A}_2)_P = \langle \bar{\mu}_1 \cap \bar{\mu}_2, \gamma_1 \cap \gamma_2 \rangle$ is not a cubic near-ring.

iii) Let $\mathscr{A}_1 = \langle \bar{\mu}_1, \gamma_1 \rangle$ and $\mathscr{A}_2 = \langle \bar{\mu}_2, \gamma_2 \rangle$ be two cubic near-rings. Then their P-union,$(\mathscr{A}_1 \cup \mathscr{A}_2)_P = \langle \bar{\mu}_1 \cup \bar{\mu}_2, \gamma_1 \cup \gamma_2 \rangle$ is not a cubic near-ring.

Proof. The following example shows that the R-union, P-intersection and P-union of two cubic near-ring is not a cubic near-ring.

Let $N = \{0, a, b, a+b\}$ be the near ring with $(N,+)$ as the Klein's four group and $(N,.)$ as defined below (scheme 16:(0,0,0,14) See [7], p.408).

+	0	a	b	a+b
0	0	a	b	a+b
a	a	0	a+b	b
b	b	a+b	0	a
a+b	a+b	b	a	0

.	0	a	b	a+b
0	0	0	0	0
a	0	0	0	0
b	0	0	0	a
a+b	0	0	0	a

Then $(N,+,.)$ is a near-ring.

i) Define $\bar{\mu}_1 : N \to D[0,1]$ by

$$\bar{\mu}_1(x) = \begin{cases} [0.3, 0.4] & \text{if } x = 0 \\ [0.2, 0.3] & \text{if } x = a \\ [0.1, 0.2] & \text{if } x = b, a+b \end{cases}$$

Define $\bar{\mu}_2 : N \to D[0,1]$ by

$$\bar{\mu}_2(x) = \begin{cases} [0.6, 0.7] & \text{if } x = 0 \\ [0.1, 0.2] & \text{if } x = a, a+b \\ [0.3, 0.4] & \text{if } x = b \end{cases}$$

Define $\gamma_1 : N \to D[0,1]$ by

$$\gamma_1(x) = \begin{cases} 0.25 & \text{if } x = 0 \\ 0.35 & \text{if } x = a \\ 0.4 & \text{if } x = b, a+b \end{cases}$$

Define $\gamma_2 : N \to D[0,1]$ by

$$\gamma_2(x) = \begin{cases} 0.27 & \text{if } x = 0 \\ 0.45 & \text{if } x = a, a+b \\ 0.32 & \text{if } x = b \end{cases}$$

Define $(\bar{\mu}_1 \cup \bar{\mu}_2)(x)$ = max $\{\bar{\mu}_1(x), \bar{\mu}_2(x)\}, \forall x, y \in N$ then

$$(\bar{\mu}_1 \cup \bar{\mu}_2)(0) = [0.6, 0.7]$$

$$(\bar{\mu}_1 \cup \bar{\mu}_2)(a) = [0.2, 0.3]$$

$$(\bar{\mu}_1 \cup \bar{\mu}_2)(b) = [0.3, 0.4]$$

$$(\bar{\mu}_1 \cup \bar{\mu}_2)(a+b) = [0.1, 0.2]$$

Since

$$\begin{aligned} (\bar{\mu}_1 \cup \bar{\mu}_2)(a+b) &\geq min\{(\bar{\mu}_1 \cup \bar{\mu}_2)(a), (\bar{\mu}_1 \cup \bar{\mu}_2)(b)\}, \\ &= \{[0.2, 0.3], [0.3, 0.4]\}, \\ &= [0.2, 0.3] \end{aligned}$$

But $(\bar{\mu}_1 \cup \bar{\mu}_2)(a+b) = [0.1, 0.2]$,

$[0.1, 0.2] \ngeq [0.2, 0.3]$ Which is absurd.

This shows that the union of two interval-valued fuzzy near-ring is not a interval-valued fuzzy near-ring.

Therefore $(\mathscr{A}_1 \cup \mathscr{A}_2)_R = \langle \bar{\mu}_1 \cup \bar{\mu}_2, \gamma_1 \cap \gamma_2 \rangle$ is not a cubic near-ring.

ii) Define $\gamma_1 : N \to D[0,1]$ by

$$\gamma_1(x) = \begin{cases} 0.3 & \text{if } x = 0 \\ 0.4 & \text{if } x = a \\ 0.5 & \text{if } x = b, a+b \end{cases}$$

Define $\gamma_2 : N \to D[0,1]$ by

$$\gamma_2(x) = \begin{cases} 0.1 & \text{if } x = 0 \\ 0.6 & \text{if } x = a, a+b \\ 0.2 & \text{if } x = b \end{cases}$$

Define $(\gamma_1 \cap \gamma_2)(x) = \min \{\gamma_1(x), \gamma_2(x)\}, \forall\, x, y \in N$

Then $(\gamma_1 \cap \gamma_2)(x) = \begin{cases} 0.1 & \text{if } x = 0 \\ 0.4 & \text{if } x = a \\ 0.2 & \text{if } x = b \\ 0.5 & \text{if } x = a+b \end{cases}$

$$\begin{aligned} (\gamma_1 \cap \gamma_2)(a+b) &\leq max\{(\gamma_1 \cap \gamma_2)(a), (\gamma_1 \cap \gamma_2)(b)\} \\ &= max\{0.4, 0.2\}, \\ &= 0.4 \end{aligned}$$

But $(\gamma_1 \cap \gamma_2)(a+b) = 0.5$

$0.5 \nleq 0.4$ Which is a contradiction.

This shows that intersection of two fuzzy near-rings need not be a fuzzy near-ring.

Hence the P-intersection $(\mathscr{A}_1 \cap \mathscr{A}_2)_P = \langle \bar{\mu}_1 \cap \bar{\mu}_2, \gamma_1 \cap \gamma_2 \rangle$ is not a cubic near-ring.

iii) From i) the union of two interval-valued fuzzy near-ring is not a interval-valued fuzzy near-ring.

Therefore $(\mathscr{A}_1 \cup \mathscr{A}_2)_P = \langle \bar{\mu}_1 \cup \bar{\mu}_2, \gamma_1 \cup \gamma_2 \rangle$ is not a cubic near-ring. □

Theorem 3.6. *If $\mathscr{A} = \langle \bar{\mu}, \gamma \rangle$ is a cubic near-ring of N then $\mathscr{A}^c$ is also a cubic near-ring of N.*

Proof. Since $\mathscr{A} = \{(x, \bar{\mu}(x), \gamma(x)) \,|x \in N\}$ is a cubic near-ring of N.

Define $\mathscr{A}^c = \{\langle x, (\bar{\mu}_A)^c(x), \gamma^c(x)\rangle \,|x \in N\}$

$$
\begin{aligned}
i)\ (\bar{\mu})^c(x-y) &= 1 - \bar{\mu}(x-y) \\
&\leq 1 - min\{\bar{\mu}(x), \bar{\mu}(y)\}, \\
&= max\{1-\bar{\mu}(x), 1-\bar{\mu}(y)\}, \\
(\bar{\mu})^c(x-y) &\leq max\{(\bar{\mu})^c(x), (\bar{\mu})^c(y)\}.
\end{aligned}
$$

$$
\begin{aligned}
ii)\ (\bar{\mu})^c(xy) &= 1 - \bar{\mu}(xy) \\
&\leq 1 - min\{\bar{\mu}(x), \bar{\mu}(y)\}, \\
&= max\{1-\bar{\mu}(x), 1-\bar{\mu}(y)\}, \\
(\bar{\mu})^c(xy) &\leq max\{(\bar{\mu})^c(x), (\bar{\mu})^c(y)\}.
\end{aligned}
$$

$$
\begin{aligned}
iii)\ (\gamma)^c(x-y) &= 1 - \gamma(x-y) \\
&\geq 1 - max\{\gamma(x), \gamma(y)\}, \\
&= min\{1-\gamma(x), 1-\gamma(y)\}, \\
(\gamma)^c(x-y) &\geq min\{(\gamma)^c(x), (\gamma)^c(y)\}.
\end{aligned}
$$

$$
\begin{aligned}
iv)\ (\gamma)^c(xy) &= 1 - \gamma(xy) \\
&\geq 1 - max\{\gamma(x), \gamma(y)\}, \\
&= min\{1-\gamma(x), 1-\gamma(y)\}, \\
(\gamma)^c(xy) &\geq min\{(\gamma)^c(x), (\gamma)^c(y)\}.
\end{aligned}
$$

Therefore $\mathscr{A}^c = \langle(\bar{\mu})^c(x), (\gamma)^c(x)\rangle$ is also a cubic near-ring. □

4. CUBIC BI-IDEALS OF CUBIC NEAR-RINGS

Definition 4.1. [7] *Let N be a near-ring.Given two subsets A and B of N,we define the following products $AB = \{ab|a \in A, b \in B\}$ and $A \star B = \left\{\left(a' + b\right)a - a'a|a, a' \in A, b \in B\right\}$.*

Definition 4.2. [7] *A subgroup B of $(N,+)$ is said to be bi-ideal of N if $BNB \cap B \star NB \subseteq B$.*

Definition 4.3. *A cubic subgroup $\mathscr{A} = \langle \bar{\mu}, \omega \rangle$ of N is called cubic bi-ideal of N, if for all $x,y,z \in N$. If it satisfies the following conditions:*

i) $\bar{\mu}(x-y) \geq min\ \{\bar{\mu}(x), \bar{\mu}(y)\}$

ii) $\omega(x-y) \leq max\ \{\omega(x), \omega(y)\}$

iii) $\bar{\mu}(xyz) \geq min\ \{\bar{\mu}(x), \bar{\mu}(z)\}$

iv) $\omega(xyz) \leq max\ \{\omega(x), \omega(z)\}$

Example 4.4. *Let $N = \{0,a,b,c\}$ be Klein's four group. Define multiplication in N as follows: (scheme $13: (0,7,13,9)$ See* [7]*, p.408).*

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

Then $(N,+,.)$ is a Cubic near-ring.

Let $\bar{\mu} : N \to D[0,1]$ be an interval-valued fuzzy subset defined by $\bar{\mu}(0) = [0.4, 0.5]$, $\bar{\mu}(a) = [0.2, 0.3] = \bar{\mu}(c)$, $\bar{\mu}(b) = [0.3, 0.4]$ Then $\bar{\mu}$ is an interval-valued fuzzy bi-ideal of N.

Let $\omega : N \to [0,1]$ be a fuzzy subset defined by $\omega(0) = 0.1, \omega(a) = 0.4 = \omega(c), \omega(b) = 0.35$. Then ω is a fuzzy bi-ideal of N.

Hence $\mathscr{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic bi-ideal of N.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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