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TWO DIMENSIONAL ROUGH ONLINE TESSELATION AUTOMATON AND ITS PROPERTIES

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Abstract: The concept of rough set theory was introduced in Formal Languages and Automata theory to find Rough Automata, Rough Languages and Rough Grammar. Rough Automata theory can be recognized as an efficient mathematical model in Machine learning and Data mining. In this research article, the rough sets notion is introduced in two dimensional automata theory. Rough set idea is incorporated in Two Dimensional Four Way Finite State Automata and Two Dimensional Online Tesselation Automata to give Two Dimensional Four Way Rough Finite state Automata and Two Dimensional Rough Online Tesselation Automata respectively. The Rough transition map extension of Two Dimensional Rough Online Tesselation Automata is done. The characterizations of Two Dimensional Rough Online Tesselation Automata are studied. Closure properties of Rough Languages recognized by Two Dimensional Rough Online Tesselation Automata under Boolean operations are obtained. Finally the relationship between Two Dimensional Four Way Rough Finite state Automata and Two Dimensional Rough Online Tesselation Automata is established.

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1. INTRODUCTION

Zadeh's fuzzy set theory ideas are of limited usage in Data mining, Machine Learning and Data analysis. To overcome those limitations, Pawlek introduced the concept of rough set theory as a generalized form of fuzzy sets to deal with problems having imprecise information or partial knowledge [8]. The concept of fuzziness is well studied in automata theory. Gheorge Paun did the rough set approximation of formal languages [4]. Sumitha Basu introduced Rough Grammar and Rough Languages [3]. She also introduced the concept of Rough Finite State Automata and studied some of their properties [2]. The Topologies associated with the Rough Finite State Automata and Rough Finite State Automata's characterizations were studied by S.P.Tiwari et.al's [10]. Jansirani et al's studied some of the closure properties of the Rough Languages recognized by Rough Finite State Automata. They also studied in detail, the Watson Crick Rough Automata and Watson Crick Fuzzy Automata [11].

Automata working on a Two Dimensional tape in four ways was first introduced by Blum and Hewitt. They also discussed the deterministic and non-deterministic types of Two Dimensional Finite State Automata and their corresponding Languages [5,6]. Katushi Inoue and Akira Nakamura invented the Online Tesselation Automata for two dimensional pattern matching [7]. They also studied in detail some of the properties of Online Tesselation Automata.

Motivated by the research work done on Rough Automata, here we propose to introduce the concepts of Two Dimensional Four Way Rough Finite State Automata and Two Dimensional Rough Online Tesselation Automata. The Second section deals with preliminaries. The Third section gives the concept Of Two Dimensional Rough Online Tesselation Automata. The Fourth section discusses the characterizations of Two Dimensional Rough Online Tesselation Automata and its relationship with Two Dimensional Four Way Rough Finite State Automata. The final section deals with the conclusion and future scope of this work.

2. PRELIMINARIES

In this section, the Preliminaries and the Basic Definitions needed to introduce Two Dimensional Rough Online Tesselation Automata and Two Dimensional four way Rough Finite State Automata with output are recalled.

Let X be a nonempty set and R be an equivalence relation defined on X, then the pair (X, R) is called an approximation space. If R is an equivalence relation defined on a nonempty set X and x

∈ X, then let [x] denote the set {y∈X / xRy}called an equivalence class or a block under R and X/R = {[x] / x ∈ X }.For all x ∈ X, the pair $([x],[x])$ is a rough set in (X, R) such that $\underline{\{x\}}$=[x]=$\overline{\{x\}}$ [1].

A Rough finite state Automaton is a 6-tuple $M = (Q, R, \Sigma, \delta, I, F)$,Where Q - Finite nonempty set of states, R - An equivalence relation defined on Q, Σ - Finite set of Inputs called the Alphabet, $\delta: Q \times \Sigma \longrightarrow$ A , where A $= \{(\underline{A}, \overline{A}), A \subseteq Q\}$ is called the Rough transition map such that for $(q, a) \in Q \times \Sigma$, $\delta(q, a) = (\underline{A}, \overline{A})$ being rough set in (Q,R) for some A⊆Q. I is a definable set in (Q, R) called an initial configuration, F⊆Q is the set of final states of M [3,9].

Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Rough Finite Automata. Then the set of strings definitely accepted by M (possibly accepted by M) is denoted by $\underline{\beta_M}(\overline{\beta_M})$ and defined by $\underline{\beta_M} = \{x \in \Sigma^*; \underline{\delta(I, x)} \cap F \neq \emptyset\}$, $(\overline{\beta_M}) = \{x \in \Sigma^*; \overline{\delta(I, x)} \cap F \neq \emptyset\}$ .The Rough regular language of M, is defined to be the rough set L(M) = $(\underline{\beta_M}, \overline{\beta_M})$ [3,9].

3. TWO DIMENSIONAL ROUGH ONLINE TESSELATION AUTOMATA

In this section, the Two Dimensional Rough online Tesselation Automata are introduced with an example and their Rough Transition map extensions for Blocks are also done. Some of the closure properties of the Rough Languages accepted by the Two Dimensional Rough online Tesselation Automata are studied under the operations Union, Intersection, Catenation and Projection. Finally, the Simulation of Two Dimensional Four way Rough Finite State Automata by Two Dimensional Rough Online Tesselation Automata is done.

Definition 3.1

A Two Dimensional Rough Online Tesselation Automaton is a 6-tuple $M = (Q, R, \Sigma, \delta, I, F)$,Where Q is a Finite nonempty set of states , R is an equivalence relation on defined on Q, Σ is a Finite set called the alphabet, $\delta: Q \times Q \times \Sigma \longrightarrow$ S , where S $= \{(\underline{S}, \overline{S}), S \subseteq Q\}$ is called the rough transition map such that for each $(p, q, a) \in Q \times Q \times \Sigma$, $\delta(p, q, a) = (\underline{S}, \overline{S})$ is a rough set in (Q,R) for some S⊆Q. I is said to be a definable set in (Q, R) known as the initial configuration, F⊆Q is the set of final states of M

Definition 3.2

Let $M = (Q, R, \Sigma, \delta, I, F)$be a Two Dimensional Rough Online Tesselation Automaton, then we say that a run of M on a picture p ∈ Σ^{**} consists of affiliating a rough set to each position (i, j) of the Picture p. This rough set is designed by the rough transition function δ. Such rough set depends on the rough sets already associated with the positions (i-1, j) and (i, j-1) and on the input p(i, j).
At time t=0, all the positions of the first row and the first column in the picture p is assigned with an initial state q_0 . It starts working at time t=1, after reading the input which is available at p(1,1), it assigns a rough set $\left(\underline{\delta(q_0\,, q_0\,, p(1,1))}, \overline{\delta(q_0\,, q_0\,, p(1,1))}\right)$ to the position p(1,1). At time t=2, after reading the inputs at (1,2) and (2,1), rough sets are simultaneously assigned to the positions p(1,2) and p(2,1) and so will move on to the next diagonals.
A Two Dimensional Rough Online Tesselation Automaton $M = (Q, R, \Sigma, \delta, I, F)$ accepts a picture p if there exists a run of M on p∈ Σ^{**}, such that the rough set associated with (r(p),c(p)) is a subset of Q, where r(p) denotes number of rows of p and c(p) denotes the number of columns of p.

Run of a picture P on a Two Dimensional Online Tessellation Automaton

Picture ‘P’

#	#	#	#	#
#	a	a	a	#
#	a	a	a	#
#	a	a	a	#
#	#	#	#	#

‘P’ at time t=0

q_0	q_0	q_0	q_0	q_0
q_0	a	a	a	q_0
q_0	a	a	a	q_0
q_0	a	a	a	q_0
q_0	q_0	q_0	q_0	q_0

‘P’ at time t=1

q_0	q_0	q_0	q_0	q_0
q_0	$(\underline{A}, \overline{A})$	a	a	q_0
q_0	a	a	a	q_0
q_0	a	a	a	q_0
q_0	q_0	q_0	q_0	q_0

‘P’ at time t=2

q_0	q_0	q_0	q_0	q_0
q_0	$(\underline{A}, \overline{A})$	$(\underline{B}, \overline{B})$	a	q_0
q_0	$(\underline{B}, \overline{B})$	a	a	q_0
q_0	a	a	a	q_0
q_0	q_0	q_0	q_0	q_0

Example 3.1

Let $\Sigma = \{1\}$. Let $L \subseteq \Sigma^{**}$ denote the language of all the pictures defined over Σ with the number of columns in the picture being even. That is L = {p : c(p) is even}
A Two Dimensional Rough Online Tesselation Automaton can accept the pictures of L over Σ by affiliating rough sets ($[q_1]$,$[q_1]$) and ($[q_2]$,$[q_2]$) to the positions of each odd column in the picture and each even column in the picture respectively. A picture is recognized if the positions of the rightmost column in the picture has the rough set ($[q_2]$,$[q_2]$). Define a Two Dimensional Rough Online Tesselation Automaton $M = (Q, R, \Sigma, \delta, I, F)$ as follows
Let Q = {q_0, q_1, q_2}, $\Sigma = \{1\}$, Q/R = {{q_0}, {q_1}, {q_2}}, I = {q_0}, F = {q_2}

Now the rough transitions are given by

$$(\underline{\delta(q_0,q_0,1)}, \overline{\delta(q_0,q_0,1)}) = (\underline{\delta(q_0,q_2,1)}, \overline{\delta(q_0,q_2,1)}) =$$

$$(\underline{\delta(q_1,q_0,1)}, \overline{\delta(q_1,q_0,1)}) = (\underline{\delta(q_1,q_2,1)}, \overline{\delta(q_1,q_2,1)}) = ([q_2],[q_2])$$

$$\left(\underline{\delta(q_0,q_1,1)}, \overline{\delta(q_0,q_1,1)}\right) = (\underline{\delta(q_2,q_1,1)}, \overline{\delta(q_2,q_1,1)}) = ([q_1],[q_1])$$

Definition 3.3

Let $M = (Q,R,\Sigma,\delta,I,F)$ be a Two Dimensional Rough Online Tesselation Automaton, then the block transition map $\delta^D: D \times D \times \Sigma \longrightarrow A$ of M is defined as, for all $D_1, D_2 \in D$, a $\in \Sigma$,

$\delta^D(D_1,D_2,a) = \left(\underline{\delta^D(D_1,D_2,a)}, \overline{\delta^D(D_1,D_2,a)}\right)$,where

$$\underline{\delta^D(D_1,D_2,a)} = \cup\left\{\underline{\delta(p,q,a)}: p \in B_1 \subseteq D_1 \text{ and } q \in B_2 \subseteq D_2,\ B_1,B_2 \in Q/R\right\},$$

$$\overline{\delta^D(D_1,D_2,a)} = \cup\left\{\overline{\delta(p,q,a)}: p \in B_1 \subseteq D_1 \text{ and } q \in B_2 \subseteq D_2, B_1,B_2 \in Q/R\right\}$$

Let $M = (Q,R,\Sigma,\delta,I,F)$ be a Two Dimensional Rough Online Tesselation Automaton, define $\delta^*: Q \times Q \times \Sigma^* \longrightarrow$ A , for p, q $\in$ Q , x $\in$ Σ^* *and* a $\in \Sigma$, $\delta^*(p,q,xa) = \left(\underline{\delta^*(p,q,xa)}, \overline{\delta^*(p,q,xa)}\right)$ where $\underline{\delta^*(p,q,xa)} = \underline{\delta^D(\underline{\delta^*(p,q,x)},a)}$, $\overline{\delta^*(p,q,xa)} = \overline{\delta^D(\overline{\delta^*(p,q,x)},a)}$

Let $M = (Q,R,\Sigma,\delta,I,F)$ be a Two Dimensional Rough Online Tesselation Automaton, then the block transition map of M can be extended as $\delta^{*D}: D \times D \times \Sigma^* \longrightarrow A$, for all $D_1, D_2 \in D$, x $\in \Sigma^*$,

$\delta^{*D}(D_1,D_2,x) = \left(\underline{\delta^{*D}(D_1,D_2,x)}, \overline{\delta^{*D}(D_1,D_2,x)}\right)$,where

$$\underline{\delta^{*D}(D_1,D_2,x)} = \cup\left\{\underline{\delta^*(p,q,x)}: p \in B_1 \subseteq D_1 \text{ and } q \in B_2 \subseteq D_2,\ B_1,B_2 \in Q/R\right\},$$

$$\overline{\delta^{*D}(D_1,D_2,x)} = \cup\left\{\overline{\delta^*(p,q,x)}: p \in B_1 \subseteq D_1 \text{ and } q \in B_2 \subseteq D_2, B_1,B_2 \in Q/R\right\}$$

Let $M = (Q,R,\Sigma,\delta,I,F)$ be a Two Dimensional Rough Online Tesselation Automaton, then for p, q $\in$ Q, $D_1, D_2 \in D$, x $\in \Sigma^*$ *and* a $\in \Sigma$, $\delta^{*D}(p,q,xa) = \left(\underline{\delta^{*D}(p,q,xa)}, \overline{\delta^{*D}(p,q,xa)}\right)$ where $\underline{\delta^{*D}(p,q,xa)} = \underline{\delta^D(\underline{\delta^{*D}(p,q,x)},a)}$, $\overline{\delta^{*D}(p,q,xa)} = \overline{\delta^D(\overline{\delta^{*D}(p,q,x)},a)}$

Theorem 3.1

If L_1 and L_2 are Rough Picture Languages accepted by Two Dimensional Rough Online Tesselation Automata, then $L_1 \cap L_2$ is a Rough Picture Language accepted by a Two Dimensional Rough Online Tesselation Automaton

Proof:

Let L_1 and L_2 be two Rough Picture Languages recognized by Two Rough Online Tesselation Automata $M_1 = (Q_1, R, \Sigma, \delta_1, I_1, F_1)$ and $M_2 = (Q_2, R, \Sigma, \delta_2, I_2, F_2)$.

Construct a Rough Online Tesselation Automata $M = (Q, R, \Sigma, \delta, I, F)$,where

$$Q = Q_1 \times Q_2$$

$$I = I_1 \times I_2$$

$$F = F_1 \times F_2$$

And δ is defined as, $\delta\big((p_1, p_2), (q_1, q_2), x\big) = \Big(\underline{\delta\big((p_1, p_2), (q_1, q_2), x\big)}, \overline{\delta\big((p_1, p_2), (q_1, q_2), x\big)}\Big)$

$$= (\Big(\underline{\delta_1(p_1, q_1, x)}, \underline{\delta_2(p_2, q_2, x)}\Big), (\overline{\delta_1(p_1, q_1, x)}, \overline{\delta_2(p_2, q_2, x)}))$$

Therefore M accepts the language L = $L_1 \cap L_2$

Theorem 3.2

If L_1 and L_2 are Rough Picture Languages accepted by Two Dimensional Rough Online Tesselation Automata then $L_1 \cup L_2$ is a Rough Picture Language accepted by a Two Dimensional Rough Online Tesselation Automaton

Proof:

Let L_1 and L_2 be two Rough Languages recognized by two Rough online Tesselation Automata $M_1 = (Q_1, R, \Sigma, \delta_1, I_1, F_1)$ and $M_2 = (Q_2, R, \Sigma, \delta_2, I_2, F_2)$.

Construct a Two Dimensional Rough online Tesselation Automaton $M = (Q, R, \Sigma, \delta, I, F)$,where

$$Q = Q_1 \times Q_2$$

$$I = I_1 \times I_2$$

$$F = (F_1 \times Q_2) \cup (F_2 \times Q_1)$$

And δ is defined as, $\delta\big((p_1, p_2), (q_1, q_2), x\big) = \Big(\underline{\delta\big((p_1, p_2), (q_1, q_2), x\big)}, \overline{\delta\big((p_1, p_2), (q_1, q_2), x\big)}\Big)$

$$= (\Big(\underline{\delta_1(p_1, q_1, x)}, \underline{\delta_2(p_2, q_2, x)}\Big), (\overline{\delta_1(p_1, q_1, x)}, \overline{\delta_2(p_2, q_2, x)}))$$

Therefore M accepts the language L = $L_1 \cup L_2$.

Theorem 3.3

The set of all Rough Picture Languages accepted by Two Dimensional Rough Online Tesselation Automata is closed under the operations row concatenation and column concatenation.

Proof:

Case I (row concatenation)

Let L_1 and L_2 be two Rough Languages recognized by Two Rough Online Tesselation Automata $M_1 = (Q_1, R, \Sigma, \delta_1, I_1, F_1)$ and $M_2 = (Q_2, R, \Sigma, \delta_2, I_2, F_2)$.

Construct a Two Dimensional Rough online Tesselation Automaton $M = (Q, R, \Sigma, \delta, I, F)$,where

$$Q = Q_1 \cup Q_2 \ and \ Q_1 \cap Q_2 = \emptyset$$

$$I = I_1$$

$$F = F_2$$

And the rough transition map δ is defined as for any $p, q \in Q \ and \ x \in \Sigma$,

$$\delta(p,q,x) = \left(\underline{\delta(p,q,x)}, \overline{\delta(p,q,x)}\right)$$

$$= \left\{ \begin{array}{ll} \left(\underline{\delta_1(p,q,x)}, \overline{\delta_1(p,q,x)}\right) & if\ p,q \in Q_1 and\ q \notin F_1, if\ p,q \in Q_1 and\ x \neq \lambda \\ (\underline{\delta_1(p,q,x)}, \overline{\delta_1(p,q,x)}) \cup \{I_2\} & if\ p,q \in F_1 and\ x = \lambda \\ (\underline{\delta_2(p,q,x)}, \overline{\delta_2(p,q,x)}) & if\ p,q \in F_2 \end{array} \right\}$$

Clearly L(M) is the row concatenation of L_1 and L_2.

Case II (Column Catenation)

The Rough transition map can be similarly defined for L(M) which is column concatenation of the two rough languages L_1 and L_2

Theorem 3.4

The set of all Rough Picture Languages accepted by Two Dimensional Rough Online Tesselation Automata is closed under Projection.

Proof:

Let Σ_1 and Σ_2 be two finite Alphabets corresponding to the Automata M = (Q, R, Σ_1,δ, I , F) and M_1 = (Q, R, Σ_2,δ', I , F).

Let $\theta : \Sigma_1 \rightarrow \Sigma_2$ denotes a Projection.

Let $L \subseteq \Sigma_1^{**}$ be a Rough Picture Language accepted by M = (Q, R, Σ_1,δ, I , F).

Then we say that the Rough Picture Language $L_1 = \theta(L)$ is accepted by a Two Dimensional Rough

Online Tesselation Automaton $M_1 = (Q, R, \Sigma_2, \delta', I, F)$, if the rough transition map δ' is given by $\delta'(p,q,a) = (\underline{\delta'(p,q,a)}, \overline{\delta'(p,q,a)}) = \cup_{\theta(a)=b, b\in\Sigma_1}(\underline{\delta(p,q,b)}, \overline{\delta(p,q,b)})$.

Definition 3.4

A Two Dimensional Rough Four Way Finite State Automaton is a 7-tuple $M = (Q, R, \Sigma, \delta, \Delta, I, F)$, Where Q is Finite and nonempty set of states, R denotes an equivalence relation which is defined on Q, Σ is called the Finite set of Inputs or the Alphabet, the rough transition map $\delta: Q \times \Sigma \longrightarrow S \times \Delta$, where $S = \{(\underline{S}, \overline{S}), S \subseteq Q\}$is defined as such that, for each $(q,a) \in Q \times \Sigma$, $\delta(q,a) = ((\underline{S}, \overline{S}), \{L, R, U, D\}$ is a rough set in (Q,R) for some S⊆Q. Δ = {left(L) , right(R) , up(U) , down(D)} emphasizes the tape's directions. I is a definable set in (Q, R) known as an initial configuration, F⊆Q is the set of final states of M

Definition 3.5

A Definite Configuration (Possible configuration) of a Two-Dimensional Rough Four Way Finite State Automaton $M = (Q, R, \Sigma, \delta, \Delta, I, F)$ on a given input word $W \in \Sigma^{u\times v}$ is a rough set $(\underline{(q,i,j)}, \overline{(q,i,j)})$, where the current state of A is q and the current positions of the input head in W are denoted by i and j, where1≤i≤u and 1≤j≤v.

Let us define the starting configuration of a Two-Dimensional Rough Four Way Finite State Automaton as the rough set $(\underline{(q_0,1,1)}, \overline{(q_0,1,1)})$. Let us now relate these configurations with the Rough transition function δ in the following manner.

Given a Two-Dimensional Rough Four Way Finite State Automaton M with an input word W, if M is currently in state q_c ∈Q; a∈Σ is the symbol at position (i,j) in W; and $\delta(q_c,a) = (\underline{\delta(q_c,a)}, \overline{\delta(q_c,a)})$=(S,U), where $S = \{(S, \overline{S}), S \subseteq Q\}$;then we say that configuration (p,i,j) has yielded the configuration (S,i−1,j). If i= 1, then the input head will move onto the boundary symbol # and reversion of the upward move is done. Similarly we can define the configurations for the directions D, L and R.

A Two-Dimensional Rough Finite State Automaton M recognizes a word W, if it has a configuration$(\underline{(q_M,i,j)}, \overline{(q_M,i,j)})$, where 0≤i≤u+ 1 and 0≤j≤v+ 1.

The following theorem gives the technique of simulating Two Dimensional Rough Finite State Automata Simultaneously over rows and columns.

Theorem 3.5

Let $M_1 = (Q_1, R, \Sigma, \delta_1, \Delta, I_1, F_1)$ and $M_2 = (Q_2, R, \Sigma, \delta_2, \Delta, I_2, F_2)$ be Two Dimensional Four Way Rough Finite State Automata, then a Two Dimensional Rough Online Tesselation Automaton $M = (Q, R, \Sigma, \delta, I, F)$ can be constructed such that for any input picture P , $P^M(i,j)$ is in F if and only if $\left(P_{rows}^{M_1}\right)_{columns}^{M_2}(i,j) \in F_2$

Proof:

Define a Two Dimensional Rough Online Tesselation Automaton $M = (Q, R, \Sigma, \delta, I, F)$, where

$$Q = Q_1 \times Q_2$$
$$I = I_1 \times I_2$$
$$F = F_2 \times Q_1$$

And the rough transition map δ simulates the computation of both M_1 and M_2 as follows

Let $\alpha_1 : Q \cup \{\#\} \to A_1$, where $A_1 = (\underline{A_1}, \overline{A_1})$ is a rough set for $A_1 \in Q_1$ and $\alpha_2 : Q \cup \{\#\} \to A_2$, where $A_2 = (\underline{A_2}, \overline{A_2})$ is a rough set for $A_2 \in Q_2$ be rough transition mappings such that $\alpha_1((p_1, p_2)) = ([q_1], [q_1]), \alpha_2((p_1, p_2)) = ([q_2], [q_2])$ $\alpha_1(\#) = I_1$ and $\alpha_2(\#) = I_2$ for all $p_1, p_2 \in Q$,

Now for all $(p_1, p_2), (q_1, q_2) \in Q \cup \{\#\} \times Q \cup \{\#\}$ and a $\in \Sigma$, define $\delta((p_1, p_2), (q_1, q_2), a) =$ $((\underline{\delta_1\left(\underline{\alpha_1(p_1,p_2)}, a\right)},\ \underline{\delta_2\left(\underline{\alpha_2(q_1,q_2)}\right)}), \left(\overline{\delta_1\left(\overline{(\alpha_1(p_1,p_2)}, a\right)}, \overline{\delta_2\left(\overline{\alpha_2(q_1,q_2)}\right)}\right))$

This technique of simulating Two Dimensional Rough Finite State Automata Simultaneously over rows and columns of Two Dimensional Rough Online Tesselation Automaton is very useful.

4. Characteristic Properties of Two Dimensional Rough Online Tesselation Automata

In this section, the characteristic properties known as the Source property and the Successor property of Two Dimensional Rough Online Tesselation Automata are introduced and studied.

Definition 4.1

Let $M = (Q, R, \Sigma, \delta, I, F)$ be a two Dimensional Rough Online Tesselation Automaton. The states $q_{i,j} \in Q$ and $q_{i-1,j+1} \in Q$ are said to be definite sources (respectively possible sources) to $q_{i,j+1} \in Q$ if there exists an element a $\in \Sigma^*$, such that $q_{i,j+1} \in$

$\underline{\delta(q_{i,j}, q_{i-1,j+1}, a)}$ ($q_{i,j} \in \overline{\delta(q_{i,j}, q_{i-1,j+1}, a)}$)

Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Two Dimensional Rough Online Tesselation Automaton and let $B \subseteq Q$. Then the following sets called definite source and possible source of B are

DSO(B) = { $q_{i,j+1} \in$ Q: $q_{i,j+1} \in \underline{\delta(q_{i,j}, q_{i-1,j+1}, a)}$ for some $(q_{i,j}, q_{i-1,j+1}, a) \in B \times B \times \Sigma$}

PSO(B) = { $q_{i,j+1} \in$ Q: $q_{i,j+1} \in \overline{\delta(q_{i,j}, q_{i-1,j+1}, a)}$ for some $(q_{i,j}, q_{i-1,j+1}, a) \in B \times B \times \Sigma$}

Definition 4.2

Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Two Dimensional Rough Online Tesselation Automaton. A state $q_{i,j} \in$ Q is said to be definite successor (respectively possible successor) to $q_{i,j-1} \in$Q and $q_{i-1,j} \in$ Q if there exists an element $a \in \Sigma^*$, such that $q_{i,j} \in \underline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$

($q_{i,j} \in \overline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$)

Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Two Dimensional Rough Online Tesselation Automaton and let $B \subseteq Q$. Then the following sets called definite successor and possible successor of B are,

DSU(B) = { $q_{i,j} \in$ Q: $q_{i,j} \in \underline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$ for some $(q_{i,j-1}, q_{i-1,j}, a) \in B \times B \times \Sigma$}

PSU(B) = { $q_{i,j} \in$ Q: $q_{i,j} \in \overline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$ for some $(q_{i,j-1}, q_{i-1,j}, a) \in B \times B \times \Sigma$}

Theorem 4.1

Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Two Dimensional Rough Online Tesselation Automaton and let $q_{i-1,j}$, $q_{i,j}$,$q_{i+1,j} \in Q$, then

(i) If $q_{i,j}$ is a definite source (respectively possible source) of $q_{i+1,j}$ and if $q_{i-1,j}$ is a definite source (respectively possible source) of $q_{i,j}$ then $q_{i-1,j}$ is a definite source (respectively possible source) of $q_{i+1,j}$

(ii) If $q_{i,j}$ is a definite Source of $q_{i+1,j}$, then $q_{i,j}$ is a possible source of $q_{i+1,j}$.

Proof:

(i)Case i: We now prove this theorem for definite Source

Given $q_{i,j}$ is a definite source of $q_{i+1,j}$ and $q_{i-1,j}$ is a definite source of $q_{i,j}$

So by definition, we have $q_{i+1,j} \in \underline{\delta(q_{i+1,j-1}, q_{i,j}, a)}$ and $q_{i,j} \in \underline{\delta(q_{i-1,j}, q_{i,j-1}, a)}$

Now $q_{i+1,j} \in \underline{\delta(q_{i+1,j-1}, q_{i,j}, a)} \subseteq \underline{\delta(q_{i-1,j}, q_{i,j-1}, a)}$

Therefore $q_{i+1,j} \in \underline{\delta(q_{i-1,j}, q_{i,j-1}, a)}$, Hence then $q_{i-1,j}$ is a definite source of $q_{i+1,j}$

Case ii: For possible source, we have

Given $q_{i,j}$ is a possible source of $q_{i+1,j}$ and $q_{i-1,j}$ is a possible source of $q_{i,j}$

So by definition, we have $q_{i+1,j} \in \overline{\delta(q_{i+1,j-1}, q_{i,j}, a)}$ and $q_{i,j} \in \overline{\delta(q_{i-1,j}, q_{i,j-1}, a)}$

Now $q_{i+1,j} \in \overline{\delta(q_{i+1,j-1}, q_{i,j}, a)} \subseteq \overline{\delta(q_{i-1,j}, q_{i,j-1}, a)}$

Therefore $q_{i+1,j} \in \overline{\delta(q_{i-1,j}, q_{i,j-1}, a)}$, Hence then $q_{i-1,j}$ is a possible source of $q_{i+1,j}$

(ii) Given $q_{i,j}$ is a definite source of $q_{i+1,j}$, but by the definition 4.1, we have $DSO(q_{i+1,j}) \subseteq PSO(q_{i+1,j})$. Hence $q_{i,j} \in DSO(q_{i-1,j}) \subseteq PSU(q_{i+1,j})$, so $q_{i,j} \in PSU(q_{i+1,j})$

Therefore if $q_{i,j}$ is a definite source of $q_{i+1,j}$, then $q_{i,j}$ is a possible source of $q_{i+1,j}$.

Theorem 4.2

Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Two Dimensional Rough Online Tesselation Automaton and let $q_{i-1,j}$, $q_{i,j}$,$q_{i+1,j}$ $\in Q$, then

(i) If $q_{i,j}$ is a definite successor (respectively possible successor) of $q_{i-1,j}$ and if $q_{i,j+1}$ is a definite successor (respectively possible successor) of $q_{i,j}$ then $q_{i,j+1}$ is a definite successor (respectively possible successor) of $q_{i-1,j}$

(ii) If $q_{i,j}$ is a definite successor of $q_{i-1,j}$, then $q_{i,j}$ is a possible successor of $q_{i-1,j}$.

Proof:

(i)Case i: We now prove this theorem for definite successor

Given $q_{i,j}$ is a definite successor of $q_{i-1,j}$ and $q_{i,j+1}$ is a definite successor of $q_{i,j}$

so by definition, we have $q_{i,j} \in \underline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$ and $q_{i,j+1} \in \underline{\delta(q_{i-1,j}, q_{i,j}, a)}$

now $q_{i,j+1} \in \underline{\delta(q_{i-1,j}, q_{i,j}, a)} \subseteq \underline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$

Therefore $q_{i,j+1} \in \underline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$, Hence then $q_{i,j+1}$ is a definite successor of $q_{i-1,j}$.

Case ii: For possible successor, we have

Given $q_{i,j}$ is a possible successor of $q_{i-1,j}$ and $q_{i,j+1}$ is a possible successor of $q_{i,j}$

So by definition, we have $q_{i,j} \in \overline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$ and $q_{i,j+1} \in \overline{\delta(q_{i-1,j}, q_{i,j}, a)}$

Now $q_{i,j+1} \in \overline{\delta(q_{i-1,j}, q_{i,j}, a)} \subseteq \overline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$

Therefore $q_{i,j+1} \in \overline{\delta(q_{i,j-1}, q_{i-1,j}, a)}$, Hence then $q_{i,j+1}$ is a possible successor of $q_{i-1,j}$.

(ii) Given $q_{i,j}$ is a definite successor of $q_{i-1,j}$, but by the definition 4.2, we have $DSU(q_{i-1,j}) \subseteq PSU(q_{i-1,j})$. Hence $q_{i,j} \in DSU(q_{i-1,j}) \subseteq PSU(q_{i-1,j})$, so $q_{i,j} \in PSU(q_{i-1,j})$

Therefore if $q_{i,j}$ is a definite successor of $q_{i-1,j}$, then $q_{i,j}$ is a possible successor of $q_{i-1,j}$.

Theorem 4.3

(a) Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Two Dimensional Rough Online Tesselation Automaton and let A,B ⊆ Q

(1) DSO(A) ⊆ PSO(A)

(2) A ⊆ DSO(A) ⊆ PSO(A)

(3) (i) DSO(A) ⊆ DSO(B) (ii) PSO(A) ⊆ PSO(B), if A ⊆ B

(b) Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Two Dimensional Rough Online Tesselation Automaton and let A,B ⊆ Q

(1) DSU(A) ⊆ PSU(A)

(2) A ⊆ DSU(A) ⊆ PSU(A)

(3) (i) DSU(A) ⊆ DSU(B) (ii) PSU(A) ⊆ PSU(B), if A ⊆ B

Proof:

(a)The proof of (1), (2), (3) directly follows from the definition 4.1and Theorem 4.1

(b)The proof of (1), (2), (3) directly follows from the definition 4.2 and Theorem 4.2

Theorem 4.4

Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Two Dimensional Rough Online Tesselation Automaton and ‘ ≈ ‘ be a relation on Σ , defined by a ≈ b <=> $\delta(p,q,a) = \delta(p,q,b)$ or $(\left(\underline{\delta(p,q,a)}, \overline{\delta(p,q,a)}\right) = \left(\underline{\delta(p,q,b)}, \overline{\delta(p,q,b)}\right))$, then ≈ is a congruence relation on Σ, for all p,q∈ Q and a,b∈ Σ^*

Proof:

For all p,q∈ Q and a,b∈ Σ^* such that a≈ b and c ε Σ^*

$$\delta(p,q,ac) = \left(\underline{\delta(p,q,ac)}, \overline{\delta(p,q,ac)}\right) = \left(\underline{\delta\left(\underline{\delta(p,q,a)}, c\right)}, \overline{\delta\left(\overline{\delta(p,q,a)}, c\right)}\right)$$

$$= \left(\underline{\delta\left(\underline{\delta(p,q,b)}, c\right)}, \overline{\delta\left(\overline{\delta(p,q,b)}, c\right)}\right) = \left(\underline{\delta(p,q,bc)}, \overline{\delta(p,q,bc)}\right) = \delta(p,q,bc)$$

Therefore ac≈ bc. Similarly we can prove ca≈ cb, Hence ≈ is a congruence relation on Σ.

Theorem 4.5

Let $M = (Q, R, \Sigma, \delta, I, F)$ be a Two Dimensional Rough Online Tesselation Automaton and ' ≡ ' be a relation on Q , defined by, for all p, q, r, s ∈ Q and a∈ Σ^* , p≡q and r≡s <=> $\delta(p,r,a) = \delta(q,s,a)$ or $(\left(\underline{\delta(p,r,a)}, \overline{\delta(p,r,a)}\right) = \left(\underline{\delta(q,s,a)}, \overline{\delta(q,s,a)}\right))$, then '≡' is an Equivalence relation on Q

Proof:

The reflective property concept for '≡' can be easily established.

To prove the symmetric property concept, given p≡q and r≡s <=> $\delta(p,r,a) = \delta(q,s,a)$

<=>$\left(\underline{\delta(p,r,a)}, \overline{\delta(p,r,a)}\right) = \left(\underline{\delta(q,s,a)}, \overline{\delta(q,s,a)}\right)$

<=>$\left(\underline{\delta(q,s,a)}, \overline{\delta(q,s,a)}\right) = \left(\underline{\delta(p,r,a)}, \overline{\delta(p,r,a)}\right)$

<=> $\delta(q,s,a) = \delta(p,r,a)$

<=> q≡p and s≡r.

To prove the transitive property concept, Let p≡q and r≡s, r≡s and t≡u, then we have

p≡q and r≡s <=> $\delta(p,r,a) = \delta(q,s,a)$ <=>$\left(\underline{\delta(p,r,a)}, \overline{\delta(p,r,a)}\right) = \left(\underline{\delta(q,s,a)}, \overline{\delta(q,s,a)}\right)$

q≡t and s≡u <=> $\delta(q,s,a) = \delta(t,u,a)$ <=>$\left(\underline{\delta(q,s,a)}, \overline{\delta(q,s,a)}\right) = \left(\underline{\delta(t,u,a)}, \overline{\delta(t,u,a)}\right)$

Consider $\left(\underline{\delta(p,r,a)}, \overline{\delta(p,r,a)}\right) = \left(\underline{\delta(q,s,a)}, \overline{\delta(q,s,a)}\right) = \left(\underline{\delta(t,u,a)}, \overline{\delta(t,u,a)}\right)$

Therefore $\left(\underline{\delta(p,r,a)}, \overline{\delta(p,r,a)}\right) = \left(\underline{\delta(t,u,a)}, \overline{\delta(t,u,a)}\right)$, so $\delta(p,r,a) = \delta(t,u,a)$<=> p≡t and r≡u. Hence '≡' is an Equivalence relation on Q.

5. CONCLUSION AND FUTURE SCOPE

The Two Dimensional Rough Online Tesselation Automaton is defined and its characterizations are studied. Some of the closure properties of Two Dimensional Rough Online Tesselation Automata under certain operations are discussed. The relationship between Two Dimensional Four Way Rough Finite State Automaton and Two Dimensional Rough Online Tesselation Automaton is established. Future Work intends to introduce Rough cellular automaton using rough online Tesselation automaton.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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