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A STUDY ON SOFT ROUGH MINIMAL CONTINUOUS AND SOFT ROUGH MAXIMAL CONTINUOUS

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Abstract: The notions of soft rough minimal open set, soft rough maximal open set, soft rough minimal closed set, soft rough maximal closed set, soft rough minimal continuous and soft rough maximal continuous are introduced and studied. We also investigate some related properties of these concepts.

Keywords: soft rough minimal open set; soft rough maximal open set; soft rough minimal closed set; soft rough maximal closed set; soft rough minimal continuous; soft rough maximal continuous.

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1. INTRODUCTION

Rough set was introduced by Pawlak [1] for dealing with vagueness and granularity in information systems. This theory deals with the approximation of an arbitrary subset of a universe by two definable or observable subsets called lower and upper approximation. The reference space in rough set theory is the approximation space whose topology is generated by the equivalence classes of R. Rough set theory was proposed as a new approach to processing of

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incomplete data. One of the aims of the rough set theory is a description of imprecise concepts. Suppose we are given a finite non empty set U of objects called universe. Each object of U is characterized by a description, for example a set of attributes values. In rough set theory an equivalence relation (reflexive, symmetric, and transitive relation) on the universe of objects is defined based on their attribute values. In particular, this equivalence relation constructed based on the equality relation on attribute values.

In the year 1999, Russian specialist Molodtsov [2], started the idea on soft sets as another scientific instrument to manage vulnerabilities while demonstrating issues in building material sciences, software engineering, financial aspects, sociologies and restorative sciences as a general mathematical tool for dealing with uncertain objects. To solve complicated problems in economics, engineering, environmental science and social science, methods in classical mathematics are not always successful because of various types of uncertainties presented in these problems. While probability theory, fuzzy set theory [4], rough set theory [1,5], and other mathematical tools are well known and often useful approaches to describing uncertainty, each of these theories has its inherent difficulties as pointed out by Molodtsov in [2,6]. The concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. This so-called soft set theory is free from the difficulties affecting existing methods.

Feng Feng et., al [3] used soft set theory to generalize Pawlak rough set model. Based on the novel granulation structures they introduced soft approximation spaces, soft rough approximation and soft rough sets and studied some of the properties of soft approximation spaces and the same properties of Pawlak approximation space.

Presently, works on soft set theory are progressing rapidly. Maji et al. [7] defined several operations on soft sets and made a theoretical study on the theory of soft sets. Aktas and Cagman [8] compared soft sets to the related concepts of rough sets. M.Lellies Thivagar et.,al [9] introduced rough topology and Muhammad Shabir, et.,al [10] introduced soft topological spaces.

2. PRELIMINARIES

Definition 2.1 [2]: Let A be a subset of E. A $pair$ $(F\ ,A)$ is called the Soft Set over X. Where $F:A\rightarrow P(X)$ $defined\ by\ F(e)\rightarrow P(X)\ for\ all\ e\ \in A.$ In other words, $F(e)$ may be considered as the set of $e-approximate\ element$ of the soft set (F,A).

Definition 2.2 [12]: Suppose U be a finite nonempty set called the Universe and R be an equivalence relation on U. The pair (U,R) is called the approximation space. Let X be a subset of U and $R(x)$ denoted the equivalence class determined by x.

i) The lower approximation space of a subset $X\ of\ U$ is defined as:

$$\underline{R}(X)=\cup_{x\in U}\ R(x):R(x)\subseteq U$$

ii) The upper approximation space of a subset $X\ of\ U$ is defined as:

$$\overline{R}(X)=\cup_{\mathrm{x}\in \mathrm{U}}\ \mathrm{R(x)}:\mathrm{R(x)}\cap \mathrm{X}\neq\emptyset$$

iii) The boundary region of X with respect to R is defined as:

$$B_R(X)=\ \overline{R}(X)-\underline{R}(X)$$

The set X is said to be rough with respect to R, if $\overline{R}(X)\neq\underline{R}(X)$, i. e., $B_R(X)\neq\emptyset$

Definition 2.3: [10] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if

1) $\emptyset\ and\ \tilde{X}$ belong to τ
2) The union of any number of soft sets in τ belong to τ
3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X.

Definition 2.4: [9] Let U be the universe of objects, R be an equivalence relation on U and $\tau_R=\{U,\emptyset,R_*(X),R^*(X),B_R(X)\}$ where $X\subseteq U$. τ_R satisfies the following axioms:

i) $U\ and\ \emptyset$ belong to τ

ii) The union of elements of any sub collection of τ_R is in τ_R

iii) The intersection of the elements of any finite sub collection of τ_R is in τ_R

τ_R forms a topology on U called as a rough topology on U with respect to X. We call (U,τ_R,X) as a rough topological space.

Definition 2.5: [11] Let $\mathfrak{S} = (f, A)$ be a Soft set over U. The pair $S = (U, \mathfrak{S})$ is called a Soft approximation space. Based on S, we define the following two operators:

$$\underline{apr_S}(X) = \{u \in U; \exists a \in A\ (u \in f(a) \subseteq X)\},$$

$$\overline{apr_S}(X) = \{u \in U; \exists a \in A(u \in f(a), f(a) \cap X \neq \emptyset)\},$$

assigning to every subset X ⊆ U two sets $\underline{apr_S}(X)$ and $\overline{apr_S}(X)$ called the lower and upper soft rough approximations of X in S , respectively. If $\underline{apr_S}(X) = \overline{apr_S}(X)$, X is said to be soft definable; Otherwise X is called a soft rough set.

Clearly, $\overline{apr_S}(X)$ and $\underline{apr_S}(X)$ can be expressed equivalently as:

$$\underline{apr_S}(X) = \cup_{a \in A} \{f(a); f(a) \subseteq X\},$$

$$\overline{apr_S}(X) = \cup_{a \in A} \{f(a); f(a) \cap X \neq \emptyset\}.$$

Definition 2.6: [12] Let U be the Universe, $P = (U, S)$ be a soft approximation space and $\tau_{SR}(X) = \left\{U, \emptyset, \underline{R_P}(X), \overline{R_P}(X), Bnd(X)\right\}$, where $X \subseteq U$. $\tau_{SR}(X)$ satisfies the following axioms:

(i) U and empty set ∈ $\tau_{SR}(X)$,

(ii) The union of the elements of any sub collection of $\tau_{SR}(X)$ is in $\tau_{SR}(X)$,

(iii) The intersection of the elements of any finite sub collection of $\tau_{SR}(X)$ is in $\tau_{SR}(X)$.

$\tau_{SR}(X)$ forms a topology on U called as the soft rough topology on U with respect to X. $(U, \tau_{SR}(X), E)$ is called a Soft Rough Topological Space.

Definition 2.7: [13] A proper nonempty open subset U of a topological space X is said to be a minimal open set if any open set which is contained in U is ∅ or U.

Definition 2.8: [14] A proper nonempty open subset U of a topological space X is said to be a maximal open set if any open set which is contains U is X or U.

Definition 2.9: [15] A proper nonempty closed subset F of a topological space X is said to be a minimal closed set if any closed set which is contained in F is ∅ or F.

Definition 2.10: [15] A proper nonempty closed subset F of a topological space X is said to be a maximal closed set if any closed set which contains in F is X or F.

Theorem 2.11: [15] Let X be a topological space and $F \subset X$. F is minimal closed set if and only if $X - F$ is maximal open set.

Theorem 2.12 [15]: Let X be a topological space and $U \subset X$. U is a minimal open set if and only if $X - U$ is maximal closed set.

Definition 2.13 [16]: Let X and Y be the topological spaces. A map $f: X \to Y$ is called minimal continuous if $f^{-1}(M)$ is an open set in X for every minimal open set M in Y.

Definition 2.14 [16]: Let X and Y be the topological spaces. A map $f: X \to Y$ is called maximal continuous if $f^{-1}(M)$ is an open set in X for every maximal open set M in Y.

Definition 2.15 [17]: A proper nonempty soft open subset F_K of a soft topological space $(F_A, \tilde{\tau})$ is said to be minimal soft open set if any soft open set which is contained in F_K is $F_\emptyset$ or F_K

Definition 2.16 [17]: A proper nonempty soft open subset F_K of a soft topological space $(F_A, \tilde{\tau})$ is said to be maximal soft open set if any soft open set which contains F_K is F_A or F_K.

Definition 2.17 [17]: A proper nonempty soft closed subset F_C of a soft topological space $(F_A, \tilde{\tau})$ is said to be minimal soft closed set if any soft closed set which is contained in F_C is $F_\emptyset$ or F_C.

Definition 2.18 [17]: A proper nonempty soft closed subset F_C of a soft topological space $(F_A, \tilde{\tau})$ is said to be maximal soft closed set if any soft closed set which contains F_C is F_A or F_C.

Definition 2.19 [18]: Let $(U, \tau R(X))$ be a rough topological space. A non empty rough closed (resp. rough open) set A of U is said to be a rough minimal closed (resp. rough minimal open) set if only if any rough closed (resp. rough open) set which is contained in A is $\emptyset$ or A.

Definition 2.20 [18]: Let $(U, \tau R(X))$ be a rough topological space. A non empty rough closed (resp. rough open) set A of U is said to be a rough maximal closed (resp. rough maximal open) set if only if any R closed (resp. R open) set which is contain in A is $\emptyset$ or U.

Definition 2.21 [19]: Consider A and B are soft topological spaces. The mapping $g: A \to B$ is known as

i) Soft minimal continuous (soft min-continuous) if $g^{-1}((M, P))$ is a soft open set in A for each soft minimal open set (M, P) in B.

ii) Soft maximal continuous (soft max-continuous) if $g^{-1}((M,P))$ is a soft open set in A for each soft maximal open set (M,P) in B.

Definition 2.22 [12]: Let $(U,\tau_{SR}(X),E) and\ (V,\tau_{SR'}(Y),E)$ be two soft rough topological spaces. The mapping $f:(U,\tau_{SR}(X),E) \rightarrow (V,\tau_{SR'}(X),Y)$ is called a soft rough continuous on U, if the inverse image of each soft rough-open set in V is soft rough open in U.

3. SOFT ROUGH MINIMAL OPEN SET AND SOFT ROUGH MAXIMAL OPEN SET AND SOFT ROUGH MINIMAL CLOSED SET AND SOFT ROUGH MAXIMAL CLOSED SET

Let U be an initial universe, X⊂U, P(U) be the set of all subsets of U, E be the set of parameters, A⊂E, P=(U,S) be the approximation space in U and $(U,\tau_{SR}(X),E)$ be the soft rough topological space with parameter E.

Definition 3.1: A proper nonempty soft rough open subset $\tau_{SR}(x)$, of a soft rough topological space $(U,\tau_{SR}(X),E)$ is said to be soft rough minimal open set if any soft rough open set which is contained in $\tau_{SR}(x)$ is $\emptyset$ or $\tau_{SR}(x)$. The family of all soft rough minimal open sets in a soft rough topological space $(U,\tau_{SR}(X),E)$ is denoted by $SR\ Mi\ O(\tau_{SR}(X))$

Example 3.2: Let $U = \{x_1,x_2,x_3,x_{4,}x_5,x_6,x_7,x_8\}$, $E = \{e_1,e_2,e_3,e_4,e_5,e_6\}$ and $A = \{e_1,e_2,e_3,e_4\} \subseteq E$. Consider the soft approximation $P = (U,S)$, where $S = (F,A)$is a soft set over U given by: $F(e_1) = \{x_2,x_3\}, F(e_2) = \{x_1,x_4,x_5\}, F(e_3) = \{x_4\}, F(e_4) = \{x_6,x_7,x_8\}$. For $X = \{x_2,x_4\}$, we have $\underline{R_P}(X) = \{x_4\}, \overline{R_P}(X) = \{x_1,x_2,x_3,x_4,x_5\}, and\ Bnd_P(X) = \{x_1,x_2,x_3,x_5\}$. Then $\tau_{SR}(X) = \{U,\emptyset,\{x_4\},\{x_1,x_2,x_3,x_4,x_5\},\{x_1,x_2,x_3,x_5\}\}$ is a soft rough topology. Then $SR\ Mi\ O(\tau_{SR}(X))$={x_4}.

Theorem 3.3: Let $(U,\tau_{SR}(X),E)$ be a soft rough topological space.

i) Let $\tau_{SR}(x)$ be a soft rough minimal open set and $\tau_{SR}(y)$ is a soft rough open set on U with respect to X. Then, $\tau_{SR}(x) \cap \tau_{SR}(y) = \emptyset\ or\ \tau_{SR}(x) \subset \tau_{SR}(y)$.

ii) Let $\tau_{SR}(x)\ and\ \tau_{SR}(z)$ be soft rough minimal open sets. Then, $\tau_{SR}(x) \cap \tau_{SR}(z) = \emptyset$ or $\tau_{SR}(x) = \tau_{SR}(z)$.

Proof: i) Let $\tau_{SR}(x)$ be any soft rough minimal open set and $\tau_{SR}(y)$ be a soft rough open set on U with respect to X. If $\tau_{SR}(x) \cap \tau_{SR}(y) = \emptyset$ then, it is obvious. Consider $\tau_{SR}(x) \cap \tau_{SR}(y) \neq \emptyset$, then $\tau_{SR}(x) \cap \tau_{SR}(y) \subset \tau_{SR}(x)$, $\tau_{SR}(x) \cap \tau_{SR}(y)$ is a soft rough open set. Therefore, $\tau_{SR}(x) \cap \tau_{SR}(y)$=$\emptyset$ or $\tau_{SR}(x) \cap \tau_{SR}(y) = \tau_{SR}(x)$. By our assumption, $\tau_{SR}(x) \cap \tau_{SR}(y) \neq \emptyset$, we have $\tau_{SR}(x) \cap \tau_{SR}(y) = \tau_{SR}(x)$ that implies $\tau_{SR}(x) \subset \tau_{SR}(y)$.

ii)If $\tau_{SR}(x) \cap \tau_{SR}(z) = \emptyset$ then by i), $\tau_{SR}(x) \subset \tau_{SR}(z)$ and $\tau_{SR}(z) \subset \tau_{SR}(x)$. Therefore $\tau_{SR}(x) = \tau_{SR}(z)$

Definition 3.4: A proper nonempty soft rough open subset $\tau_{SR}(x)$ of a soft rough topological space $(U, \tau_{SR}(X), E)$ is said to be soft rough maximal open set if any soft rough open set which contains $\tau_{SR}(x)$ is U or $\tau_{SR}(x)$. The family of all soft rough maximal open sets in a soft rough topological space $(U, \tau_{SR}(X), E)$ is denoted by $SR\ Ma\ O(\tau_{SR}(X))$

Example 3.5: Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. Consider the soft approximation $P = (U, S)$, where $S = (F, A)$is a soft set over U given by: $F(e_1) = \{x_2, x_5, x_3\}, F(e_2) = \{x_1, x_4, x_2\}, F(e_3) = \{x_4\}, F(e_4) = \{x_6, x_7, x_8\}$. For $X = \{x_2, x_4\}$, we have $\underline{R_P}(X) = \{x_4\}, \overline{R_P}(X) = \{x_1, x_2, x_3, x_4, x_5\}, and\ Bnd_P(X) = \{x_1, x_2, x_3, x_5\}$ .Then $\tau_{SR}(X) = \{U, \emptyset, \{x_4\}, \{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_5\}\}$ is a soft rough topology. Then $SR\ Ma\ O\big(\tau_{SR}(X)\big) = \{x_1, x_2, x_3, x_4, x_5\}$.

Lemma 3.6: Let $(U, \tau_{SR}(X), E)$ be a soft rough topological space

i) Let $\tau_{SR}(x)$ be a soft rough maximal open set and $\tau_{SR}(y)$ is a soft rough open set on U with respect to X. Then, $\tau_{SR}(x) \cup \tau_{SR}(y) = U\ or\ \tau_{SR}(y) \subset \tau_{SR}(x)$.

ii) Let $\tau_{SR}(x)\ and\ \tau_{SR}(z)$ be soft rough maximal open sets. Then, $\tau_{SR}(x) \cup \tau_{SR}(z) = U$ or $\tau_{SR}(x) = \tau_{SR}(z)$.

Proof: i) Let $\tau_{SR}(x)$ be any soft rough maximal open set and $\tau_{SR}(y)$ be any soft rough open set on U with respect to X. If $\tau_{SR}(x) \cup \tau_{SR}(y) = U$ then it obvious. Consider, $\tau_{SR}(x) \cup \tau_{SR}(y) \neq U$, then $\tau_{SR}(x) \subset \tau_{SR}(x) \cup \tau_{SR}(y)$, $\tau_{SR}(x) \cup \tau_{SR}(y)$ is soft rough open set. Therefore $\tau_{SR}(x) \cup \tau_{SR}(y) = U$ or $\tau_{SR}(x) \cup \tau_{SR}(y) = \tau_{SR}(x)$ By our assumption, $\tau_{SR}(x) \cup \tau_{SR}(y) \neq U$, we have $\tau_{SR}(x) \cup \tau_{SR}(y) = \tau_{SR}(x)$ that implies $\tau_{SR}(y) \subset \tau_{SR}(x)$.

ii) If $\tau_{SR}(x) \cup \tau_{SR}(z) \neq U$ then by (i) $\tau_{SR}(x) \subset \tau_{SR}(z)$ and $\tau_{SR}(z) \subset \tau_{SR}(x)$. Therefore $\tau_{SR}(x) = \tau_{SR}(z)$.

Theorem 3.7: Let $\tau_{SR}(x)\ \tau_{SR}(y)\ and\ \tau_{SR}(z)$ be soft rough maximal open sets such that $\tau_{SR}(x) \neq \tau_{SR}(y)$. If $\tau_{SR}(x) \cap \tau_{SR}(y) \subset \tau_{SR}(z)$, then either $\tau_{SR}(x) = \tau_{SR}(z) or\ \tau_{SR}(y) = \tau_{SR}(z)$

Proof: Let $\tau_{SR}(x)\ \tau_{SR}(y)\ and\ \tau_{SR}(z)$ be a soft rough maximal open sets such that $\tau_{SR}(x) \neq \tau_{SR}(y)$and $\tau_{SR}(x) \cap \tau_{SR}(y) \subset \tau_{SR}(z)$. If $\tau_{SR}(y) = \tau_{SR}(z)$, then the result is obvious.

Consider $\tau_{SR}(y) \neq \tau_{SR}(z)$.

We have, $\tau_{SR}(x) \cap \tau_{SR}(z) = \tau_{SR}(x) \cap (\tau_{SR}(z) \cap U)$ [since, $\tau_{SR}(z) \cap U = \tau_{SR}(z)$]

$=\tau_{SR}(x) \cap (\tau_{SR}(z) \cap (\tau_{SR}(x) \cup \tau_{SR}(y)))$ [from lemma 3.6(ii)]

$=(\tau_{SR}(x) \cap ((\tau_{SR}(z) \cap \tau_{SR}(x)) \cup (\tau_{SR}(z) \cap (\tau_{SR}(y)))$

$=(\tau_{SR}(x) \cap (\tau_{SR}(z) \cap \tau_{SR}(x))) \cup ((\tau_{SR}(x) \cap (\tau_{SR}(z) \cap \tau_{SR}(y)))$

$=(\tau_{SR}(x) \cap \tau_{SR}(z)) \cup (\tau_{SR}(z) \cap \tau_{SR}(x) \cap \tau_{SR}(y))$

$=(\tau_{SR}(x) \cap \tau_{SR}(z)) \cup (\tau_{SR}(x) \cap \tau_{SR}(y))$ [since, $\tau_{SR}(x) \cap \tau_{SR}(y) \subset \tau_{SR}(z)$]

$$\tau_{SR}(x) \cap \tau_{SR}(z) = \tau_{SR}(x) \cap (\tau_{SR}(z) \cup \tau_{SR}(y))$$

If $\tau_{SR}(z) \neq \tau_{SR}(y)$ then, $\tau_{SR}(z) \cup \tau_{SR}(y) = U$(by lemma 3.6(ii)) and hence $\tau_{SR}(x) \cap \tau_{SR}(z) = \tau_{SR}(x)$ that implies $\tau_{SR}(x) \subset \tau_{SR}(z)$ .Since $\tau_{SR}(x)$ and $\tau_{SR}(z)$ are soft rough maximal open sets, $\tau_{SR}(x) = \tau_{SR}(z)$.

Theorem 3.8: Let $\tau_{SR}(x), \tau_{SR}(y)\ and\ \tau_{SR}(z)$be soft rough maximal open sets, which are different from each other. Then, $\tau_{SR}(x) \cap \tau_{SR}(y) \not\subset \tau_{SR}(x) \cap \tau_{SR}(z)$

Proof: Consider, $\tau_{SR}(x) \cap \tau_{SR}(y) \subset \tau_{SR}(x) \cap \tau_{SR}(z)$.

$$\left(\tau_{SR}(x) \cap \tau_{SR}(y)\right) \cup (\tau_{SR}(y) \cap \tau_{SR}(z)) \subset (\tau_{SR}(x) \cap \tau_{SR}(z)) \cup (\tau_{SR}(y) \cap \tau_{SR}(z))$$

$\tau_{SR}(y) \cap (\tau_{SR}(x) \cup \tau_{SR}(z)) \subset (\tau_{SR}(x) \cup \tau_{SR}(y)) \cap \tau_{SR}(z)$.

By lemma 3.6(ii), we have $\tau_{SR}(x) \cup \tau_{SR}(z) = U$ and $\tau_{SR}(x) \cup \tau_{SR}(y) = U$ such that $\tau_{SR}(y) \subset \tau_{SR}(z)$. Thus $\tau_{SR}(y) = \tau_{SR}(z)$, which is contradiction to our assumption that $\tau_{SR}(x) \neq \tau_{SR}(y) \neq \tau_{SR}(z)$. Hence, we have, $\tau_{SR}(x) \cap \tau_{SR}(y) \not\subset \tau_{SR}(x) \cap \tau_{SR}(z)$.

Definition 3.9: A proper nonempty soft rough closed subset $\tau_{SR}(x)$, of a soft rough topological space $(U, \tau_{SR}(X), E)$ is said to be soft rough minimal closed set if any soft rough closed set which is contained in $\tau_{SR}(x)$ is $\emptyset$ or $\tau_{SR}(x)$. The family of all soft rough minimal closed sets in a soft rough topological space $(U, \tau_{SR}(X), E)$ is denoted by $SR\ Mi\ C\big(\tau_{SR}(X)\big)$.

Example 3.10: Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. Consider the soft approximation $P = (U, S)$, where $S = (F, A)$is a soft set over U given by: $F(e_1) = \{x_2, x_3\}, F(e_2) = \{x_1, x_4, x_5\}, F(e_3) = \{x_4\}, F(e_4) = \{x_7, x_8\}$. For $X = \{x_2, x_4\}$, we have $\underline{R_P}(X) = \{x_4\}, \overline{R_P}(X) = \{x_1, x_2, x_3, x_4, x_5\}, and\ Bnd_P(X) = \{x_1, x_2, x_3, x_5\}$ .Then $\tau_{SR}(X) = \{U, \emptyset, \{x_4\}, \{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_5\}\}$ is a soft rough topology and $\tau_{SR}(\overline{X}) = \{U, \emptyset, , \{x_1, x_2, x_3, x_5, x_6, x_7, x_8\}, \{x_6, x_7, x_8\}, \{x_4, x_6, x_7, x_8\}\}$. Then $SR\ Mi\ C\big(\tau_{SR}(X)\big) = \{x_6, x_7, x_8\}$.

Theorem 3.11: Let $(U, \tau_{SR}(X), E)$ be a soft rough topological space.

i) Let $\tau_{SR}(\overline{x})$ be a soft rough minimal closed set and $\tau_{SR}(\overline{y})$ is a soft rough closed set on U with respect to X. Then, $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{y}) = \emptyset\ or\ \tau_{SR}(\overline{x}) \subset \tau_{SR}(\overline{y})$.

ii) Let $\tau_{SR}(\overline{x})\ \ and\ \tau_{SR}(\overline{z})$ be soft rough minimal closed sets. Then, $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{z}) = \emptyset$ or $\tau_{SR}(\overline{x}) = \tau_{SR}(\overline{z})$.

Proof: i) Let $\tau_{SR}(\overline{x})$ be any soft rough minimal closed set and $\tau_{SR}(\overline{y})$ be a soft rough closed set on U with respect to X. If $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{y}) = \emptyset$ then, it is obvious. Consider$\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{y}) \neq \emptyset$, then $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{y}) \subset \tau_{SR}(\overline{x})$, $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{y})$ is a soft rough closed set. Therefore, $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{y}) = \emptyset$ or $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{y}) = \tau_{SR}(\overline{x})$. By our assumption $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{y}) \neq \emptyset$, we have $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{y}) = \tau_{SR}(\overline{x})$ that implies $\tau_{SR}(\overline{x}) \subset \tau_{SR}(\overline{y})$.

ii)If $\tau_{SR}(\overline{x}) \cap \tau_{SR}(\overline{z}) = \emptyset$ then by i), $\tau_{SR}(\overline{x}) \subset \tau_{SR}(\overline{z})$ and $\tau_{SR}(\overline{z}) \subset \tau_{SR}(\overline{x})$. Therefore $\tau_{SR}(\overline{x}) = \tau_{SR}(\overline{z})$.

Definition 3.12: A proper nonempty soft rough closed subset $\tau_{SR}(x)$, of a soft rough topological space $(U, \tau_{SR}(X), E)$ is said to be maximal soft rough closed set if any soft rough closed set which contains $\tau_{SR}(x)$ is U or $\tau_{SR}(x)$. The family of all soft rough maximal closed sets in a soft rough topological space $(U, \tau_{SR}(X), E)$ is denoted by $SR\ Ma\ C(\tau_{SR}(X))$.

Example 3.13: Let $U = \{x_1, x_2, x_3, x_{4,} x_5, x_6, x_7, x_8\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. Consider the soft approximation $P = (U, S)$, where $S = (F, A)$is a soft set over U given by: $F(e_1) = \{x_2, x_3\}, F(e_2) = \{x_1, x_4, x_5\}, F(e_3) = \{x_4\}, F(e_4) = \{x_7, x_8\}$. For $X = \{x_2, x_4\}$, we have $\underline{R_P}(X) = \{x_4\}, \overline{R_P}(X) = \{x_1, x_2, x_3, x_4, x_5\}, and\ Bnd_P(X) = \{x_1, x_2, x_3, x_5\}$ .Then $\tau_{SR}(X) = \{U, \emptyset, \{x_4\}, \{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_5\}\}$ is a soft rough topology. $\tau_{SR}(\overline{X}) = \{U, \emptyset, , \{x_1, x_2, x_3, x_5, x_6, x_7, x_8\}, \{x_6, x_7, x_8\}, \{x_4, x_6, x_7, x_8\}\}$. Then $SR\ Ma\ C(\tau_{SR}(X)) = \{x_1, x_2, x_3, x_5, x_6, x_7, x_8\}$.

Lemma 3.14: Let $(U, \tau_{SR}(X), E)$ be a soft rough topological space.

i) Let $\tau_{SR}(\overline{x})$ be a soft rough maximal closed set and $\tau_{SR}(\overline{y})$ is a soft rough closed set on U with respect to X. Then, $\tau_{SR}(\overline{x}) \cup \tau_{SR}(\overline{y}) = U\ or\ \tau_{SR}(\overline{y}) \subset \tau_{SR}(\overline{x})$.

ii) Let $\tau_{SR}(\overline{x})\ \ and\ \tau_{SR}(\overline{z})$ be soft rough maximal closed sets. Then, $\tau_{SR}(\overline{x}) \cup \tau_{SR}(\overline{z}) = U$ or$\tau_{SR}(\overline{x}) = \tau_{SR}(\overline{z})$.

Proof: i) Let $\tau_{SR}(\overline{x})$ be any soft rough maximal closed set. If $\tau_{SR}(\overline{x}) \cup \tau_{SR}(\overline{y}) = U$ then the proof is obvious. Consider $\tau_{SR}(\overline{y})$ be any soft rough closed set such that $\tau_{SR}(\overline{x}) \cup \tau_{SR}(\overline{y}) \neq U$. Since $\tau_{SR}(\overline{x}) \subset \tau_{SR}(\overline{x}) \cup \tau_{SR}(\overline{y})$, we have $\tau_{SR}(\overline{x}) \cup \tau_{SR}(\overline{y}) = \tau_{SR}(\overline{x})\ or\ \tau_{SR}(\overline{x}) \cup \tau_{SR}(\overline{y}) = U$ which implies $\tau_{SR}(\overline{x}) \cup \tau_{SR}(\overline{y}) = \tau_{SR}(\overline{x})$. Hence, $\tau_{SR}(\overline{y}) \subset \tau_{SR}(\overline{x})$.

ii) From the above result, $\tau_{SR}(\overline{x}) \subset \tau_{SR}(\overline{z})$ and $\tau_{SR}(\overline{z}) \subset \tau_{SR}(\overline{x})$. Therefore $\tau_{SR}(\overline{x}) = \tau_{SR}(\overline{z})$.

The relationship between soft rough minimal open set, soft rough maximal open set, soft rough minimal closed set and soft rough maximal closed set:

Theorem 3.15: Let $\tau_{SR}(x)$ is a nonzero soft rough subset of a soft rough topological space $(U, \tau_{SR}(X), E)$.then the following are holds.

i) $\tau_{SR}(x)$ is a soft rough minimal closed set if and only if $U - \tau_{SR}(x)$ is soft rough maximal open set.

ii) $\tau_{SR}(x)$ is a soft rough maximal closed set if only if $U - \tau_{SR}(x)$ is a soft rough minimal open set.

Proof: It follows from definitions.

Example 3.16: Let $U = \{x_1, x_2, x_3, x_{4,}x_5, x_6, x_7, x_8\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. Consider the soft approximation $P = (U, S)$, where $S = (F, A)$is a soft set over U given by: $F(e_1) = \{x_2, x_3\}, F(e_2) = \{x_1, x_4, x_5\}, F(e_3) = \{x_4\}, F(e_4) = \{x_7, x_8\}$. For $X = \{x_2, x_4\}$, we have $\underline{R_P}(X) = \{x_4\}, \overline{R_P}(X) = \{x_1, x_2, x_3, x_4, x_5\}, and\ Bnd_P(X) = \{x_1, x_2, x_3, x_5\}$ .Then $\tau_{SR}(X) = \{U, \emptyset, \{x_4\}, \{x_1, x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_3, x_5\}\}$ is a soft rough topology.

$$SR\ Mi\ O(\tau_{SR}(X): \{x_4\}$$

$$SR\ Ma\ O(\tau_{SR}(X): \{x_1, x_2, x_3, x_4, x_5\}$$

$$SR\ Mi\ C(\tau_{SR}(X): \{x_6, x_7, x_8\}$$

$$SR\ Ma\ C(\tau_{SR}(X): \{x_1, x_2, x_3, x_5, x_6, x_7, x_8\}$$

4. SOFT ROUGH MINIMAL CONTINUOUS AND SOFT ROUGH MAXIMAL CONTINUOUS

Definition 4.1: Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR}(Y), E)$ be the soft rough topological spaces .The mapping $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is called soft rough minimal continuous if $f^{-1}(F(E))$ is a soft rough open set in $(U, \tau_{SR}(X), E)$ for each soft rough minimal open set $F(E)$ in $(V, \tau_{SR}(Y), E)$.

Example 4.2: $U = \{x_1, x_2, x, x_4\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1, e_2, e_3\}$ ⊂E and $S = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2\}), (e_1, \{x_3, x_4\})\}$. If X⊂U such that $X = \{x_1, x_2\}$, then we have $\underline{R_P}(X) = \{x_2\}$, $\overline{R_P}(X) = \{x_1, x_2, x_3\} and\ Bnd_P(X) = \{x_1, x_3\}$. Thus $\tau_{SR}(X) = \{U, \Phi, \{x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_3\}\}$ is a soft rough topology. Let $V = \{x_1, x_2, x, x_4\}$, $S' = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2\}), (e_3, \{x_1, x_4\})\}$ and $Y = \{x_2, x_4\} \subset V$, then $\underline{R_P}(Y) = \{x_2\}$, $\overline{R_P}(Y) = \{x_1, x_2, x_4\}\ and\ Bnd_P(Y) = \{x_1, x_4\}$. Thus $\tau_{SR'}(Y) = \{V, \emptyset, \{x_2\}, \{x_1, x_2, x_4\}, \{x_1, x_4\}\}$ is a soft rough topology.

Define $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is an identity map, then $f^{-1}\{x_2\} = \{x_2\}$ is open in $(U, \tau_{SR}(X), E)$, $\{x_2\}$ is open in $(V, \tau_{SR}(Y), E)$. Then f is a soft rough minimal continuous.

Definition 4.3: Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR}(Y), E)$ be the soft rough topological spaces. The map $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is called the soft rough maximal continuous if $f^{-1}(F(E))$ is a soft rough open set in $(U, \tau_{SR}(X), E)$ for each soft rough maximal open set $F(E)$ in$(V, \tau_{SR}(Y), E)$.

Example 4.4: $U = V = \{x_1, x_2, x, x_4\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1, e_2, e_3\}$⊂E.

$S = \{(e_1, \{x_1, x_2\}), (e_2, \{x_4\}), (e_1, \{x_2, x_3\})\}$. If X⊂U such that $X = \{x_1, x_2\}$,then we have $\underline{R_P}(X) = \{x_1, x_2\}$, $\overline{R_P}(X) = \{x_1, x_2, x_3\} and\ Bnd_P(X) = \{x_3\}$. Thus $\tau_{SR}(X) = \{U, \emptyset, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_3\}\}$ is a soft rough topology. Let $V = \{x_1, x_2, x, x_4\}$, $S' = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2, x_3\}), (e_3, \{x_4\})\}$ and $Y = \{x_1, x_3\} \subset V$, then we have $\underline{R_P}(Y) = \{x_1, x_3\}$, $\overline{R_P}(Y) = \{x_1, x_2, x_3\}\ and\ Bnd_P(Y) = \{x_2\}$. Thus $\tau_{SR'}(Y) = \{V, \emptyset, \{x_1, x_3\}, \{x_1, x_2, x_3\}, \{x_2\}\}$ is a soft rough topology.

Define $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ be an identity map, then $f^{-1}\{x_1, x_2, x_3\} = \{x_1, x_2, x_3\}$ is open in $(U, \tau_{SR}(X), E)$, $\{x_1, x_2, x_3\}$ is open in $(V, \tau_{SR}(Y), E)$. Then f is soft rough maximal continuous.

Theorem 4.5: Every soft rough continuous map is soft rough minimal continuous but not conversely.

Proof: Let $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ be a soft rough continuous map. Let $SR\ Mi\ O(\tau_{SR}(Y))$ be the soft rough minimal open set in $(V, \tau_{SR}(Y), E)$. Soft rough minimal open set is also a soft rough open set in soft rough topological space and by the definition of soft rough continuous, $f^{-1}((V, \tau_{SR}(Y), E))$ is a soft rough open set in $(U, \tau_{SR}(X), E)$. Converse is not true since every soft rough open set is not a soft rough minimal open set.

Example 4.6: In example 4.2, f is not soft rough continuous, since $f^{-1}\{x_1, x_2, x_4\} = \{x_1, x_2, x_4\}$ is not open in $(U, \tau_{SR}(X), E)$.

Theorem 4.7: Every soft rough continuous map is soft rough maximal continuous but not conversely.

Proof: Similar to that of Theorem 4.5

Example 4.8: In example 4.4, f is not soft rough continuous because $f^{-1}\{x_2\} = \{x_2\}$ is open in $(U, \tau_{SR}(X), E)$.

Remark 4.9: Soft rough minimal continuous maps are independent of soft rough maximal continuous map.

In Example 4.2 f is a soft rough minimal continuous but it is not a soft rough maximal continuous. In Example 4.4 f is soft rough maximal continuous but it is not a soft rough minimal continuous.

Theorem 4.10: Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR}(Y), E)$ be the soft rough topological spaces. A map $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is soft rough minimal continuous if and only if the inverse image of each soft maximal closed set in $(V, \tau_{SR}(Y), E)$ is a soft rough closed set in $(U, \tau_{SR}(X), E)$.

Proof: The proof follows from the definition with the fact that the complement of soft rough minimal open set is soft rough maximal closed set.

Theorem 4.11: The composition of soft rough continuous map and a soft rough minimal continuous map is soft rough minimal continuous map.

Proof: Consider $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is a soft rough continuous map and $g: (V, \tau_{SR}(Y), E) \to (W, \tau_{SR}(Z), E)$ is a soft rough minimal continuous map. Consider a soft rough minimal open set $SR\ Mi\ O(\tau_{SR}(Z))$ in $(W, \tau_{SR}(Z), E)$. By definition of soft rough minimal continuous map, $g^{-1}(SR\ Mi\ O(\tau_{SR}(Z)))$ is a soft rough open set in$(V, \tau_{SR}(Y), E)$. $f^{-1}\left(g^{-1}(SR\ Mi\ O(\tau_{SR}(Z)\ ))\right)$ is also a soft rough open set in $(U, \tau_{SR}(X), E)$ since f is a soft rough continuous map. Therefore, $f^{-1}\left(g^{-1}\left(SR\ Mi\ O(\tau_{SR}(Z))\right)\right) = (g \circ f)^{-1}(SR\ Mi\ O(\tau_{SR}(Z))$. Thus $g \circ f$ is a soft rough minimal continuous.

Theorem 4.12: Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR}(Y), E)$ be the soft rough topological spaces. A map $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is soft rough maximal continuous if and only if the inverse image of each soft minimal closed set in $(V, \tau_{SR}(Y), E)$ is a soft rough closed set in $(U, \tau_{SR}(X), E)$.

Proof: The proof follows from the definition and with the fact that the complement of soft rough maximal open set is soft rough minimal closed set.

Theorem 4.13: The composition of is soft rough continuous map and a soft rough maximal continuous maps is soft rough maximal continuous map.

Proof: Similar to theorem 4.11.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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