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AN M/M/1 QUEUE WITH SINGLE VACATION IN RANDOM ENVIRONMENT AND MULTIPLE VACATIONS IN DETERMINISTIC ENVIRONMENT

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Abstract. The model under consideration is a single server multiple vacation queueing system where the vacation is classified into two categories namely type I vacation and type II vacation. The server opts type I vacation following non-empty busy period of providing service to at the minimum of one customer. On returning from type I vacation if the server finds the system empty, it goes for type II vacation. In type I vacation, depending on the environment, there are n distinct kinds of vacations. All types of vacation are vulnerable to Interruption. Each type of vacation can be interrupted when the number of customers in the system reaches predefined thresholds, where each vacation has a different threshold. The long run system probabilities, mean and variance of the number of customers in the system, etc. are computed. Using Little's formula the expression for waiting time is obtained and numerically illustrated. An optimization problem is discussed with numerical illustration.

Keywords:multiple vacation; vacation interruption; random environment; busy period.

2010 AMS Subject Classification: 60K25, 60k30.

1. INTRODUCTION

In queueing theory, vacation is the absence of server from the service center or the unavailability of server for a random duration of time at a stretch. Occasional operation of a service

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may be economically invoking when entire time service would result in substantial server idle time or would prevent the utilization of the server in different productive capacities. Queueing models with vacation have been studied extensively in the past and the same has been successfully applied in diverse areas such as manufacturing, computer/communication networks etc. Excellent survey on the earlier works of vacation models have been reported by Doshi [1], Takagi [2] and Tian and Zhang [3]. These paper and books provide a vast description of the queueing systems with server vacation.

The most commonly used vacation policies are single vacation policy, multiple vacation policy and working vacation policy. In multiple vacation policy, when the server returns from a vacation and finds the queue empty, it immediately takes another vacation. If there is atleast one customer the server will start functioning according to the service policy. Zhang and Tian [4] analysed a Geo/G/1 queue with Multiple Adaptive Vacation (MAV) where the server can take at most a certain number (n) of vacations continuously. In a working vacation policy the server works at a lower rate during vacation period rather than completely stopping the service during vacation period. As the risk of loosing customers and the dissatisfaction of customers are less during working vacation, the research interest on working vacation models grew fast. Servi and Finn [5] were the pioneers of the concept of working vacation. Appreciable work has been done on queue with interruption since the concept was first introduced by Levy and Yechiali [6]. Another significant aspect is vacation interruption. It was introduced and developed by Li and Tian [7], [8]. Zhang and Hou [9] studied an $M/G/1$ queue with multiple working vacation and vacation interruption. Zhang [10], presented an analysis on the multi server vacation model with three threshold policy.

Ibe and Isijola [11] considers a model with two types of vacations, where type I vacation is taken after a non zero busy period of serving at least one customer and type II vacation is taken after a zero busy period, on completing type I vacation. The distribution of both vacations are different. The authors extend the idea in paper [12] by introducing two new concepts, partial vacation interruption and total vacation interruption. In partial vacation interruption only type II vacation is interrupted. The interruption to vacation occurs when the number of customers in the system reaches a threshold value K. In total vacation interruption, type I and type II vacations are interrupted when the number of customers in the system reaches the threshold values K_1 and K_2 respectively, where $K_1 \geq K_2$.

In almost all vacation models in literature, the focus is mainly on various vacation policies like multiple adaptive vacation, multi server vacation, working vacation and vacation interruption. In the model under consideration the server utilizes the vacation time to extend service to multiple entities which can be provided with available infrastructure and for server maintenance, depending on the need of the situation. Here the service is referred as primary service and service during vacation is referred as secondary service. During vacation time the server finds n-number of other organizations in need of secondary service.The server chooses on among them based on circumstances which we call as environment. On returning from the vacation if the server finds the primary queue empty it utilizes the time for system correction and up gradation which is referred as type II vacation. If the priority is for maintenance, the server will go for maintenance instead of secondary service. Interruption occurs in the secondary service when the queue length in the primary queue exceeds a threshold value depending on the environment.

The remaining discussions on this model is arranged as follows. Section 2 provides a detailed description of the model. The analysis of the model is consolidated in section 3. An optimization problem is discussed in section 4. Numerical illustrations are provided in section 5.

2. Model Description

The model under consideration is a single server queueing system in which arrival occurs according to a Poisson process with parameter λ. The service time is exponentially distributed with parameter μ. There are two type of vacations in this model. The type I vacation is taken at the end of non-zero busy period. There are n different category of type I vacation based on n environmental factors. These n category of vacation are numbered 1 to n based on the descending order of duration of vacation. On returning from type I vacation, if the server finds the queue empty, it goes for type II vacation. The type II vacation is numbered as the $(n+1)^{th}$ category of vacation. The i^{th} vacation duration is exponentially distributed with parameter γ_i, $1 \leq i \leq n+1$. At the end of a non-zero busy period, depending on the environment, the server opts for a vacation of i^{th} kind with probability $p_i, 1 \leq i \leq n$. When the number of customers in the system exceeds K_{n+1} during the $n+1^{th}$ kind of vacation, the server returns from vacation and starts serving customers. The i^{th} type of vacation is interrupted when the number of customers in the queue reaches K_i, where $K_1 > K_2 > \ldots > K_{n+1}$.

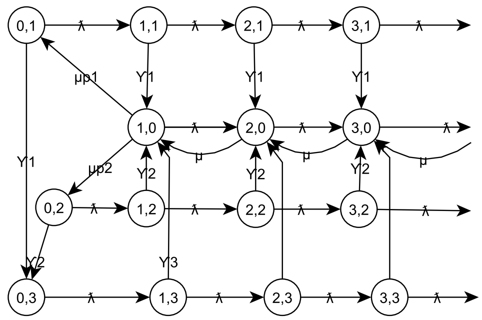

FIGURE 1. Model description

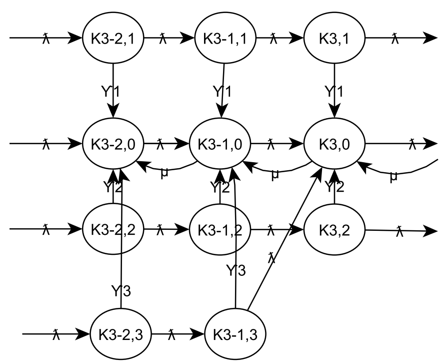

FIGURE 2. Model description

3. Analysis of the Model

The state of the system is defined as (n,k), where n is the number of customers in the system and k is the status of server:

$$k = \begin{cases} 0, & \text{when service is going on ;} \\ i, & \text{server is on the } i^{th} \text{ category of type I vacation; } 1 = 1,2,\ldots,n; \\ n+1, & \text{server is on the } n+1^{th} \text{ category of vacation(type II).} \end{cases}$$

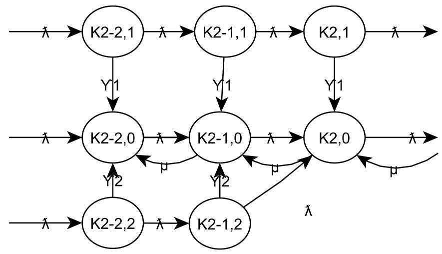

FIGURE 3. Model description

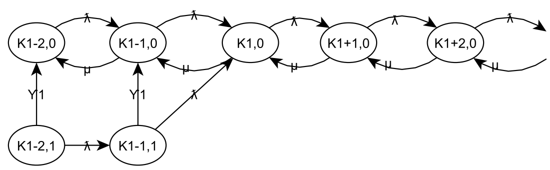

FIGURE 4. Model description

Let $P_{i,k}$ denote the steady state probability that the system contains i customers when the server is in the k^{th} state. First consider the case of $n = 2$. From the global balance in figure:1

(1) $$(\lambda+\gamma_1)P_{0,1} = \mu p_1 P_{1,0},$$

(2) $$(\lambda+\gamma_2)P_{0,2} = \mu p_2 P_{1,0},$$

(3) $$\lambda P_{03} = \gamma_1 P_{0,1} + \gamma_2 P_{0,2},$$

(4) $$(\lambda+\gamma_3)P_{k,3} = \lambda P_{k-1,3},\ 1 \le k < K_3,$$

(5) $$(\lambda+\gamma_2)P_{k,2} = \lambda P_{k-1,2},\ 1 \le k < K_2,$$

(6) $$(\lambda+\gamma_1)P_{k,1} = \lambda P_{k-1,1},\ 1 \le k < K_1.$$

From the local balance

(7) $$\lambda(P_{k,0}+P_{k,1}+P_{k,2}+P_{k,3}) = \mu P_{k+1,0},\ 1 \le k < K_3,$$

$$\lambda(P_{k,0}+P_{k,1}+P_{k,2}) = \mu P_{k+1,0},\ K_3 \le k < K_2, \tag{8}$$

$$\lambda(P_{k,0}+P_{k,1}) = \mu P_{k+1,0},\ K_2 \le k < K_1, \tag{9}$$

$$\lambda(P_{k,0}) = \mu P_{k+1,0},\ k \ge K_1. \tag{10}$$

From (1) $P_{0,1} = \beta_1 P_{10}$, From (2) $P_{0,2} = \beta_2 P_{10}$, From (3) $P_{0,3} = \beta_3 P_{10}$. In the above $\beta_1 = \frac{\mu p_1}{\lambda+\gamma_1}$, $\beta_2 = \frac{\mu p_2}{\lambda+\gamma_2}$ and $\beta_3 = (\frac{\gamma_1}{\lambda}\frac{\mu p_1}{\lambda+\gamma_1} + \frac{\gamma_2}{\lambda}\frac{\mu p_2}{\lambda+\gamma_2})$. (4) $\Rightarrow P_{k,3} = \alpha_3^k \beta_3 P_{10}$, for $1 \le k \le K_3 - 1$. (5) $\Rightarrow P_{k,2} = \alpha_2^k \beta_2 P_{10}$, for $1 \le k \le K_2 - 1$.

(6) $\Rightarrow P_{k,1} = \alpha_1^k \beta_1 P_{10}$, for $1 \le k \le K_1 - 1$.

(7) $\Rightarrow P_{k,0} = \left[\rho^{k-1} + \sum_{i=1}^{3} \rho\alpha_i\beta_i \left[\frac{\rho^{k-1}-\alpha_i^{k-1}}{\rho-\alpha_i}\right]\right] P_{10}$, for $2 \le k \le K_3$.

From(8)

$$P_{k,0} = \left[\rho^{k-1} + \sum_{i=1}^{2} \rho\alpha_i\beta_i \left[\frac{\rho^{k-1}-\alpha_i^{k-1}}{\rho-\alpha_i}\right] + \rho^{k-K_3+1}\alpha_3\beta_3 \left[\frac{\rho^{K_3-1}-\alpha_3^{K_3-1}}{\rho-\alpha_3}\right]\right] P_{10},$$

for $K_3 + 1 \le k \le K_2$.

From (9)

$$P_{k,0} = \left[\rho^{k-1} + \rho\alpha_1\beta_1 \left[\tfrac{\rho^{k-1}-\alpha_1^{k-1}}{\rho-\alpha_1}\right] + \sum_{i=2}^{3} \rho^{k-K_i+1}\alpha_i\beta_i \frac{\rho^{K_i-1}-\alpha_i^{K_i-1}}{\rho-\alpha_i}\right] P_{10},$$

for $K_2 + 1 \le k \le K_1$.

From(10), $P_{k,0} = [\rho^{k-1} + \sum_{i=1}^{3} \rho^{k-K_i}\alpha_i\beta_i \frac{\rho^{K_i-1}-\alpha_i^{K_i-1}}{\rho-\alpha_i}]P_{10}$, for $k \ge K_1 + 1$.

Since the total probability equals one, $\sum_{k=1}^{\infty} P_{k,0} + \sum_{k=0}^{K_1-1} P_{k,1} + \sum_{k=0}^{K_2-1} P_{k,2} + \sum_{k=0}^{K_3-1} P_{k,3} = 1$

$\Rightarrow [A_1 + A_2 + A_3 + A_4]P_{10} = 1$,

where $A_1 = \frac{1}{1-\rho}$, $A_2 = \sum_{i=1}^{3}\left[\frac{\rho^2\alpha_i\beta_i}{1-\rho}\right]\left[\frac{\rho^{K_i-1}-\alpha_i^{K_i-1}}{\rho-\alpha_i}\right]$,

$A_3 = \sum_{i=1}^{3}\left[\frac{\alpha_i\beta_i\rho}{\rho-\alpha_i}\right]\left[\frac{\alpha_i-\alpha_i^{K_i}}{1-\alpha_i} - \frac{\rho-\rho^{K_i}}{1-\rho}\right]$, $A_4 = \sum_{i=1}^{3}\sum_{k=1}^{K_i}\alpha_i^{k-1}\beta_i$, $P_{10} = \frac{1}{[A_1+A_2+A_3+A_4]}$.

Also the expected number of customers in the system,

$E(N) = \sum_{k=1}^{\infty} kP_{k,0} + \sum_{k=0}^{K_1-1} kP_{k,1} + \sum_{k=0}^{K_2-1} kP_{k,2} + \sum_{k=0}^{K_3-1} kP_{k,0} = [B_1 + B_2 + B_3 + B_4]P_{10}$ where

$B_1 = \frac{1}{(1-\rho)^2}$,

$$B_2=\sum_{i=1}^{3}\frac{\alpha_i\beta_i\rho}{(\alpha_i-\rho)}\left[\frac{K_i\alpha_i^{K_i+1}-(K_i+1)\alpha_i^{K_i}-\alpha_i^2+2\alpha_i}{(1-\alpha_i)^2}-\frac{K_i\rho^{K_i+1}-(K_i+1)\rho^{K_i}-\rho^2+2\rho}{(1-\rho)^2}\right],$$

$$B_3=\sum_{i=1}^{3}\frac{\alpha_i\beta_i}{(\alpha_i-\rho)}\left[\alpha_i^{K_i-1}-\rho^{K_i-1}\right]\left[\frac{(K_i\rho^2)}{(1-\rho)}+\frac{\rho^2}{(1-\rho)^2}\right],B_4=\sum_{i=1}^{3}\beta_i\left[\frac{(K_i-1)\alpha_i^{K_i+1}-K_i\alpha_i^{K_i}+\alpha_i}{(1-\alpha_i)^2}\right].$$

Now consider the case of $n=3$. Then $[C_1+C_2+C_3+C_4]P_{10}=1$,
where $C_1=\frac{1}{1-\rho}$, $C_2=\sum_{i=1}^{4}\left[\frac{\rho^2\alpha_i\beta_i}{1-\rho}\right]\left[\frac{\rho^{K_i-1}-\alpha_i^{K_i-1}}{\rho-\alpha_i}\right]$,
$C_3=\sum_{i=1}^{4}\left[\frac{\alpha_i\beta_i\rho}{\rho-\alpha_i}\right]\left[\frac{\alpha_i-\alpha_i^{K_i}}{1-\alpha_i}-\frac{\rho-\rho^{K_i}}{1-\rho}\right]$,
$C_4=\sum_{i=1}^{4}\sum_{k=1}^{K_i}\alpha_i^{k-1}\beta_i$, $P_{10}=\frac{1}{[C_1+C_2+C_3+C_4]}$.

Also the expected number of customers in the system,

$E(N)=[D_1+D_2+D_3+D_4]P_{10}$ where $D_1=\frac{1}{(1-\rho)^2}$,

$$D_2=\sum_{i=1}^{4}\frac{\alpha_i\beta_i\rho}{(\alpha_i-\rho)}\left[\frac{K_i\alpha_i^{K_i+1}-(K_i+1)\alpha_i^{K_i}-\alpha_i^2+2\alpha_i}{(1-\alpha_i)^2}-\frac{K_i\rho^{K_i+1}-(K_i+1)\rho^{K_i}-\rho^2+2\rho}{(1-\rho)^2}\right],$$

$$D_3=\sum_{i=1}^{4}\frac{\alpha_i\beta_i}{(\alpha_i-\rho)}\left[\alpha_i^{K_i-1}-\rho^{K_i-1}\right]\left[\frac{(K_i\rho^2)}{(1-\rho)}+\frac{\rho^2}{(1-\rho)^2}\right],D_4=\sum_{i=1}^{4}\beta_i\left[\frac{(K_i-1)\alpha_i^{K_i+1}-K_i\alpha_i^{K_i}+\alpha_i}{(1-\alpha_i)^2}\right].$$

So depending on the environmental factor, for n category of type I vacation,

$$\sum_{k=1}^{K_{n+1}}P_{k,0}+\sum_{i=2}^{n+1}\sum_{k=K_i+1}^{K_{i-1}}P_{k,0}+\sum_{k=K_1+1}^{\infty}P_{k,0}+\sum_{i=1}^{n+1}\sum_{k=0}^{K_i-1}P_{k,i}=1\Rightarrow[S_1+S_2+S_3+S_4]P_{10}=1.$$

$P_{10}=\frac{1}{[S_1+S_2+S_3+S_4]}$, where $S_1=\frac{1}{1-\rho}$, $S_2=\sum_{i=1}^{n+1}\left[\frac{\rho^2\alpha_i\beta_i}{1-\rho}\right]\left[\frac{\rho^{K_i-1}-\alpha_i^{K_i-1}}{\rho-\alpha_i}\right]$,

$S_3=\sum_{i=1}^{n+1}\left[\frac{\alpha_i\beta_i\rho}{\rho-\alpha_i}\right]\left[\frac{\alpha_i-\alpha_i^{K_i}}{1-\alpha_i}-\frac{\rho-\rho^{K_i}}{1-\rho}\right]$, $S_4=\sum_{i=1}^{n+1}\sum_{k=1}^{K_i}\alpha_i^{k-1}\beta_i$, $P_{10}=\frac{1}{[S_1+S_2+S_3+S_4]}$

Expected number of customers in the system,

$$E(N)=\sum_{k=1}^{K_{n+1}}kP_{k,0}+\sum_{i=2}^{n+1}\sum_{k=K_i+1}^{K_{i-1}}kP_{k,0}+\sum_{k=K_1+1}^{\infty}kP_{k,0}+\sum_{i=1}^{n+1}\sum_{k=0}^{K_i-1}kP_{k,i}$$

$=[I_1+I_2+I_3+I_4]P_{10}$, where $I_1=\frac{1}{(1-\rho)^2}$,

$$I_2=\sum_{i=1}^{n+1}\frac{\alpha_i\beta_i\rho}{(\alpha_i-\rho)}\left[\frac{K_i\alpha_i^{K_i+1}-(K_i+1)\alpha_i^{K_i}-\alpha_i^2+2\alpha_i}{(1-\alpha_i)^2}-\frac{K_i\rho^{K_i+1}-(K_i+1)\rho^{K_i}-\rho^2+2\rho}{(1-\rho)^2}\right],$$

$$I_3=\sum_{i=1}^{n+1}\frac{\alpha_i\beta_i}{(\alpha_i-\rho)}\left[\alpha_i^{K_i-1}-\rho^{K_i-1}\right]\left[\frac{(K_i\rho^2)}{(1-\rho)}+\frac{\rho^2}{(1-\rho)^2}\right],$$

$$I_4=\sum_{i=1}^{n+1}\beta_i\left[\frac{(K_i-1)\alpha_i^{K_i+1}-K_i\alpha_i^{K_i}+\alpha_i}{(1-\alpha_i)^2}\right].$$

Using Little's Law $E(N)=\lambda W$, expected waiting time in the system, $E(W)=\frac{E(N)}{\lambda}$.

Variance of the number of customers in the system, $V(N)=E(N^2)-(E(N))^2$.

$$E(N^2)=\sum_{k=1}^{K_{n+1}}k^2P_{k,0}+\sum_{i=2}^{n+1}\sum_{k=K_i+1}^{K_{i-1}}k^2P_{k,0}+\sum_{k=K_1+1}^{\infty}k^2P_{k,0}+\sum_{i=1}^{n+1}\sum_{k=0}^{K_i-1}k^2P_{k,i}$$

$$=[R_1+R_2+R_3+R_4]P_{10},\text{ where } R_1=\frac{(1+\rho)}{(1-\rho)^3},$$

$$R_2=\sum_{i=1}^{n+1}\frac{\alpha_i\beta_i\rho}{(\alpha_i-\rho)}\left[\frac{K_i^2\alpha_i^{K_i+2}-(2K_i^2+2K_i-1)\alpha_i^{K_i+1}+(K_i+1)^2\alpha_i^{K_i}-\alpha_i^3+3\alpha_i^2-4\alpha}{(\alpha_i-1)^3}\right.$$
$$\left.-\frac{K_i^2\rho^{K_i+2}-(2K_i^2+2K_i-1)\rho^{K_i+1}+(K_i+1)^2\rho^{K_i}-\rho^3+3\rho^2-4\rho}{(\rho-1)^3}\right],$$

$$R_3=\sum_{i=1}^{n+1}\left[\frac{\alpha_i\beta_i\rho^2}{(1-\rho)}\right]\left[\frac{\alpha_i^{K_i-1}-\rho^{K_i-1}}{\alpha_i-\rho}\right]\left[K_i^2+\frac{2K_i}{(1-\rho)}+\frac{1+\rho}{(1-\rho)^2}\right],$$

$$R_4=\sum_{i=1}^{n+1}\beta_i\left[\frac{(K_i^2-2K_i+1)\alpha_i^{K_i+2}-(2K_i^2-2K_i-1)\alpha_i^{K_i+1}+(K_i)^2\alpha_i^{K_i}-\alpha_i^2-\alpha_i}{(\alpha_i-1)^3}\right].$$

4. Optimization Problem

For the effective utilization of the model discussed, optimization of the threshold values $(K_i's)$ is inevitable. So an optimization problem is discussed in this section and Numerical illustrations are provided. Let C_0 be the unit time revenue obtained from providing service, $C_i, 1\le i\le n+1$ be the unit time revenue obtained from i^{th} category of vacation, C be the holding cost per unit time per customer and C_i' be the fixed cost for switching the service from i^{th} category of vacation to normal service. So the expected total profit, $TP=T_1+T_2-\overline{C}-\widehat{C}$ where, T_1 is the total revenue from service, T_2 is the revenue from vacation, $\overline{C}$ is the holding cost of waiting customers and $\widehat{C}$ is the total switching cost. Here $T_1=\frac{1}{\mu-\lambda}C_0$,

$$T_2=\sum_{i=1}^{n}p_iC_i\left[\left(\frac{\lambda}{\lambda+\gamma_i}\right)^{K_i}\frac{K_i}{\lambda}+\sum_{r=0}^{K_i-1}\frac{\lambda^r}{(\lambda+\gamma_i)^{r+1}}\right]+$$
$$\sum_{i=1}^{n}\frac{p_i\gamma_iC_{n+1}}{\lambda+\gamma_i}\left[\left(\frac{\lambda}{\lambda+\gamma_{n+1}}\right)^{K_{n+1}}\frac{K_{n+1}}{\lambda}+\sum_{s=1}^{\infty}\frac{s}{\gamma_{n+1}}\frac{(\gamma_{n+1})^s}{(\lambda+\gamma_{n+1})^{s+1}}\right].$$

$$\overline{C}=c.E(N).$$

$$\widehat{C}=\sum_{i=1}^{n}p_iC_i'\left[\left(\frac{\lambda}{\lambda+\gamma_i}\right)^{K_i}\frac{1}{K_i}+\sum_{r=1}^{K_i-1}\frac{\lambda^r\gamma_i}{(\lambda+\gamma_i)^{r+1}}\right](\mu-\lambda)+$$
$$\sum_{i=1}^{n}p_iC_i'\frac{\gamma_i}{(\lambda+\gamma_i)}\left[\left(\frac{\lambda}{\lambda+\gamma_{n+1}}\right)^{K_{n+1}}\frac{1}{K_{n+1}}+\left(1-\left(\frac{\lambda}{\lambda+\gamma_{n+1}}\right)^{K_{n+1}}\right)\right](\mu-\lambda).$$

5. Numerical Illustrations

As an example consider a model with the duration of first category of type I vacation, $\gamma_1=0.1$, the duration of second category of type I vacation, $\gamma_2=0.2$, the duration of type II vacation, $\gamma_3=0.3$. Let $p_1=0.6$ be the probability for the server proceeding to first category of type I vacation and $p_2=0.4$ be the probability for the server proceeding to second category of type I vacation. Then the effect of various values of traffic intensity (ρ), K_1, K_2, and K_3 on the expected number of customers in the system and expected waiting time are plotted below (fig.5-fig.10).

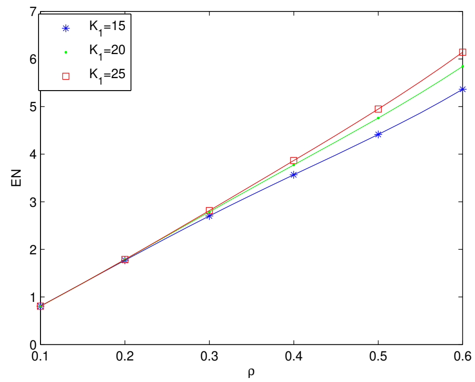

FIGURE 5. Effect of various values of K_1 and ρ on $E(N)$ when $K_2 = 10, K_3 = 5$

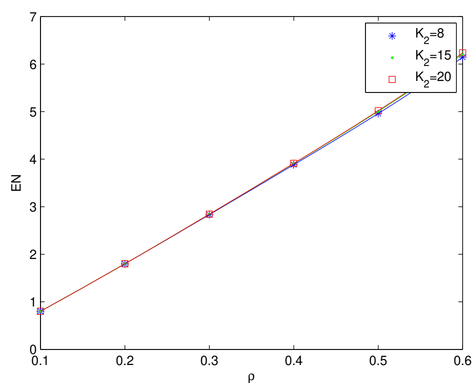

FIGURE 6. Effect of various values of K_2 and ρ on $E(N)$ when $K_1 = 25, K_3 = 8$

From fig.5, 6 and 7 we note that as ρ and K_1 increase the expected number of customers in the system $E(N)$ also increases. Increase in ρ means either arrival rate increases or service rate decreases. When arrival rate increases the number of customers in the system also increases. When service rate decreases then also the number of customers in the system increases due to slow service.

When K_1 increases it is trivially seen that the number of customers in the system will increase as the customers should wait for the return of the server from vacation until the threshold value

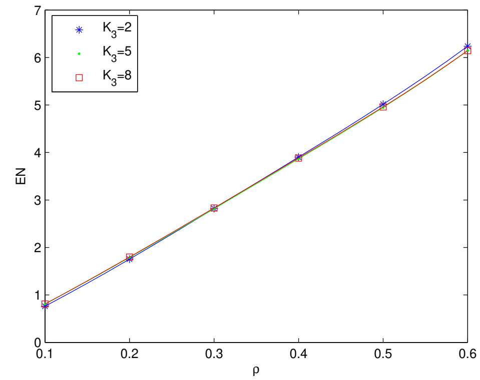

FIGURE 7. Effect of various values of K_3 and ρ on $E(N)$ when $K_1 = 25, K_2 = 10$

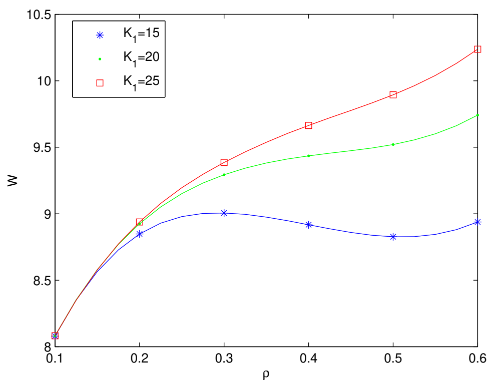

FIGURE 8. Effect of various values of K_1 and ρ on W when $K_2 = 10, K_3 = 5$

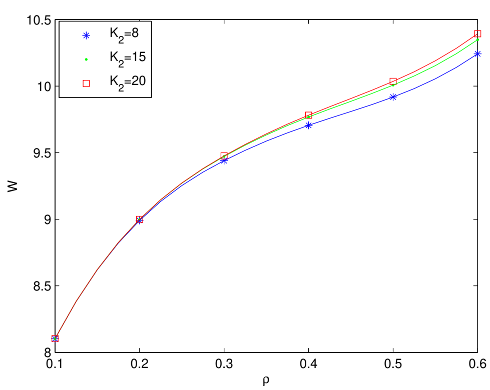

FIGURE 9. Effect of various values of K_2 and ρ on W when $K_1 = 25, K_3 = 8$

K_1 is reached.

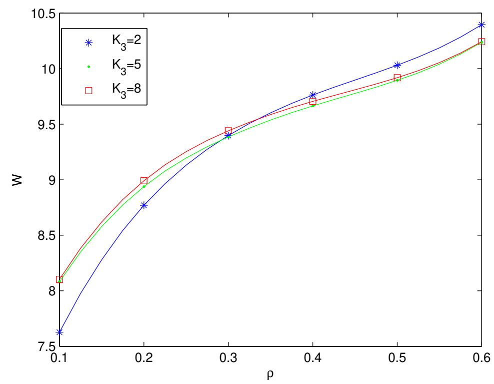

FIGURE 10. Effect of various values of K_3 and ρ on W when $K_2 = 10, K_1 = 25$

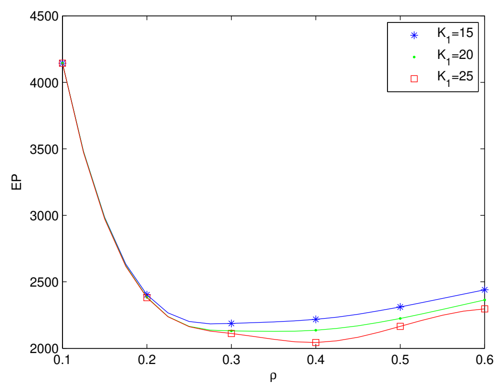

FIGURE 11. Effect of various values of K_1 and ρ on W when $K_2 = 10, K_3 = 5$

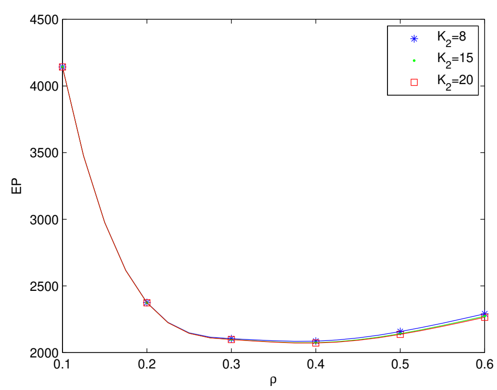

FIGURE 12. Effect of various values of K_2 and ρ on W when $K_1 = 25, K_3 = 8$

From fig.8 and fig.9 it is clear that as ρ increases expected waiting time also increases. From fig.10 it is clear that for small values of ρ the value of K_1 does not make much difference in the expected waiting time. As K_1 increases the expected waiting time also increases. This is due to the delay of the server return from vacation due to the increased threshold value K_1. From fig.12, for small values of ρ, as K_3 increases waiting time also increases but as the value of ρ increases, the waiting time is greater for smaller values of K_3. As the duration of vacations decrease, expected waiting time increases with increase in the value of K_3.

By assuming $C_0 = \$250, C_1 = \$250, C_2 = \$100, C_3 = \$50, C = \$25$ and $C_1' = C_2' = C_3' = \$100$ the effect of various values of traffic intensity(ρ), K_1, K_2, and K_3 on expected profit EP are plotted below (fig.11-fig.13).

From fig.11, 12 and fig.13 it is clear that as ρ increases the expected profit decreases, reaches a minimum value and then begins to increase. As ρ increases either arrival rate increases or service rate decreases. Increase in arrival rate causes frequent interruption of vacation and switching of service which is very expensive and it reduces the profit. Also the increase in arrival rate or the decrease in service rate reduces the chance of occurrence of vacation. This reduces the loss due to switching of service.

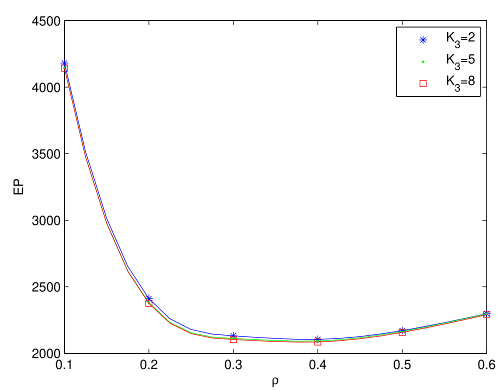

FIGURE 13. Effect of various values of K_3 and ρ on W when $K_2 = 10, K_1 = 25$

For small values of λ expected profit shows convexity (fig.14). As λ increases the expected profit decreases(fig.15).

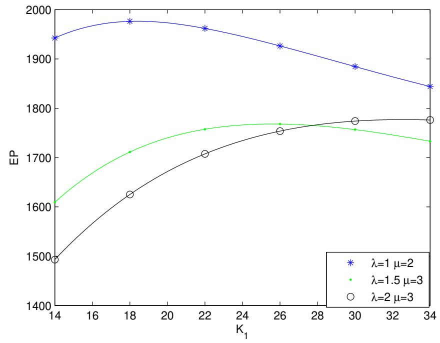

FIGURE 14. Effect of various values of $\lambda \& \mu$ on EP when $K_2 = 10, K_3 = 5$

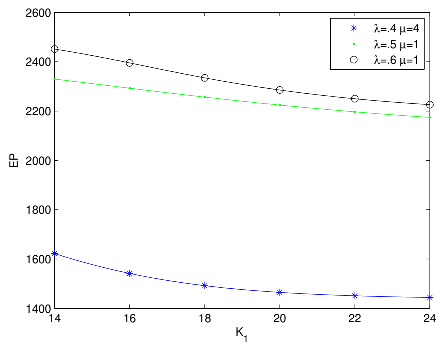

FIGURE 15. Effect of threshold values and traffic intensity on waiting time

6. CONCLUSIONS

A multiple vacation M/M/1 queueing system with two types of vacation is considered here. The server goes for type II vacation after a zero busy period provided there is no customer in the system. The type II vacation is numbered as the $(n+1)^{th}$ category of vacation. All the vacations can be interrupted depending on the threshold value $K_i, 1 \leq i \leq n+1$. If the number of customers in the system reaches $K_i, 1 \leq i \leq n$, while the server is in the i^{th} category of vacation that vacation is interrupted and the server starts service. We obtained the expression for the mean and variance of the number of customers in the system. An optimization of expected profit is discussed. Numerical examples are graphically illustrated. The graphical representations clearly indicates that by suitably picking out the threshold values, maximization of profit and minimization of waiting time are viable.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] B.T. Doshi, Queueing systems with vacations–A survey, Queueing Syst. 1 (1986), 29–66.

[2] H. Takagi, Queueing Analysis–A Foundation of Performance Evaluation, Vol. 1. Vacation and Priority Systems, North-Holland, Amsterdam, 1991.

[3] N. Tian, Z.G. Zhang, A two threshold vacation policy in multiserver queueing systems, Eur. J. Oper. Res. 168 (2006), 153–163.

[4] Z.G. Zhang, N. Tian, Discrete time Geo/G/1 queue with multiple adaptive vacations, Queueing Syst. 38 (2001), 419–429.

[5] L.D. Servi, S.G. Finn, M/M/1 queues with working vacations (M/M/1/WV), Perform. Eval. 50 (2002), 41–52.

[6] [1]Y. Levy, U. Yechiali, Utilization of Idle Time in an M / G /1 Queueing System, Manage. Sci. 22 (1975), 202–211.

[7] J.-H. Li, N.-S. Tian, Z.-Y. Ma, Performance analysis of GI/M/1 queue with working vacations and vacation interruption, Appl. Math. Model. 32 (2008), 2715–2730.

[8] J. Li, N. Tian, The M/M/1 queue with working vacations and vacation interruptions, J. Syst. Sci. Syst. Eng. 16 (2007), 121–127.

[9] M. Zhang, Z. Hou, Performance analysis of M/G/1 queue with working vacations and vacation interruption, J. Comput. Appl. Math. 234 (2010), 2977–2985.

[10] Z.G. Zhang, On the three threshold policy in the multi-server queueing system with vacations, Queueing Syst. 51 (2005), 173–186.

[11] O.C. Ibe, O.A. Isijola, M/M/1 Multiple Vacation Queueing Systems with Differentiated Vacations, Model. Simul. Eng. 2014 (2014), 158247.

[12] O.A. Isijola-Adakeja, O.C. Ibe, M/M/1 Multiple Vacation Queueing Systems With Differentiated Vacations and Vacation Interruptions, IEEE Access. 2 (2014), 1384–1395.