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ON EXTENSION OF SOFT MAPS IN SOFT TOPOLOGICAL SPACES

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Abstract. Molodtsov introduced soft sets to deal with uncertainty. Shabir introduced soft topological spaces. In this paper the soft locally finite family of soft sets in a soft topological space is introduced and its properties are investigated. Further the soft function and soft continuous functions defined on soft topological spaces are studied using soft locally finite family of soft sets.

Keywords: soft sets; soft topology; soft function; soft continuous; soft locally finite.

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1. INTRODUCTION

The real life problems in Social sciences, Engineering, Medical sciences, Economics and Environment deal with the unpredictability and it is imprecise in nature. There are several theories in the literature such as Fuzzy set theory [1], Rough set theory[2], Interval Mathematics[3] etc dealing with uncertainties, but they have their own limitations. A new Mathematical tool dealing with uncertainty is introduced by Molodtsov [4] in 1999. Molodtsov initiated the theory of soft sets for modeling vagueness and uncertainty. Maji et.al [5, 6] studied the basic operations on soft sets and applied soft sets in decision making problems. Kharal et.al.[7] studied soft functions. Shabir et.al.[8] initiated the study of soft topological spaces. Moreover, theoretical studies of soft topological spaces have also been studied in [9],[10],[11],[12],[13].

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Soft continuous functions are studied in [11]. Soft connectedness and soft Hausdorff spaces were introduced in [14]. In this paper soft locally finite family of soft sets is introduced and its properties are investigated. Further soft functions and soft continuous functions are studied using soft locally finite family of soft sets.

2. PRELIMINARIES

Throughout this paper X,Y are universal sets and E,K are parameter spaces.

Definition 2.1. [4] *A pair (F,E) is called a soft set over X where $F: E \to 2^X$ is a mapping.*

$S(X,E)$ denotes the collection of all soft sets over X with parameter space E. We denote (F,E) by $\widetilde{F}$ in which case we write $\widetilde{F} = \{(e,F(e)) : e \in E\}$. In some occasions, we use $\widetilde{F}(e)$ for $F(e)$.

Definition 2.2. [6] *For any two soft sets $\widetilde{F}$ and $\widetilde{G}$ in $S(X,E)$, $\widetilde{F}$ is a soft subset of $\widetilde{G}$ if $F(e) \subseteq G(e)$ for all $e \in E$. If $\widetilde{F}$ is a soft subset of $\widetilde{G}$ then we write $\widetilde{F} \subseteq \widetilde{G}$.*
$\widetilde{F}$ and $\widetilde{G}$ are equal if and only if $F(e) = G(e)$ for all $e \in E$. That is $\widetilde{F} = \widetilde{G}$ if $\widetilde{F} \subseteq \widetilde{G}$ and $\widetilde{G} \subseteq \widetilde{F}$.

Definition 2.3. [6] *(i)* $\widetilde{\Phi} = \{(e,\phi) : e \in E\} = \{(e,\Phi(e)) : e \in E\} = (\Phi,E)$.
(ii) $\widetilde{X} = \{(e,X) : e \in E\} = \{(e,X(e)) : e \in E\} = (X,E)$.

Definition 2.4. [6] *The union of two soft sets $\widetilde{F}$ and $\widetilde{G}$ over X is defined as $\widetilde{F} \cup \widetilde{G} = (F \cup G, E)$ where $(F \cup G)(e) = F(e) \cup G(e)$ for all $e \in E$.*

Definition 2.5. [15] *The intersection of two soft sets $\widetilde{F}$ and $\widetilde{G}$ over X is defined as $\widetilde{F} \cap \widetilde{G} = (F \cap G, E)$ where $(F \cap G)(e) = F(e) \cap G(e)$ for all $e \in E$.*

If $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ is a collection of soft sets in $S(X,E)$ then the arbitrary union and the arbitrary intersection of soft sets are defined below.
$\bigcup\{\widetilde{F}_\alpha : \alpha \in \Delta\} = (\bigcup\{F_\alpha : \alpha \in \Delta\}, E)$ and $\bigcap\{\widetilde{F}_\alpha : \alpha \in \Delta\} = (\bigcap\{F_\alpha : \alpha \in \Delta\}, E)$ where $(\bigcup\{F_\alpha : \alpha \in \Delta\})(e) = \bigcup\{F_\alpha(e) : \alpha \in \Delta\}$ and $(\bigcap\{F_\alpha : \alpha \in \Delta\})(e) = \bigcap\{F_\alpha(e) : \alpha \in \Delta\}$, for all $e \in E$.

Definition 2.6. [8] *The complement of a soft set $\widetilde{F}$ is denoted by $(\widetilde{F})' = (F',E)$ (relative complement in the sense of Ifran Ali.et.al([15]) where $F': E \to 2^X$ is a mapping given by $F'(e) = X - F(e)$ for all $e \in E$.*

It is noteworthy to see that with respect to above complement De Morgan's laws hold for soft sets as stated below.

Lemma 2.7. [11] *Let I be an arbitrary index set and $\{\widetilde{F}_i : i \in I\} \subseteq S(X,E)$. Then $(\cup\{\widetilde{F}_i : i \in I\})' = \cap\{(\widetilde{F}_i)' : i \in I\}$ and $(\cap\{\widetilde{F}_i : i \in I\})' = \cup\{(\widetilde{F}_i)' : i \in I\}$.*

Definition 2.8. [8] *Let $\widetilde{F}$ be a soft set over X and $x \in X$. We say that $x \in \widetilde{F}$ whenever $x \in F(e)$ for all $e \in E$.*

Definition 2.9. [8] *Let Y be a non-empty subset of X. Then $\widetilde{Y}$ denotes the soft set (Y,E) over X if $Y(e) = Y$ for all $e \in E$.*

From defn 2.3(ii) it is easy to see that every soft set in S(X,E) is a soft subset of $\widetilde{X}$

Definition 2.10. [8] *Let $\widetilde{\tau}$ be a collection of soft subset of $\widetilde{X}$. Then $\widetilde{\tau}$ is said to be a soft topology on X with parameter space E if*
(i) $\widetilde{\Phi}, \widetilde{X} \in \widetilde{\tau}$, (ii) $\widetilde{\tau}$ is closed under arbitrary union, and (iii) $\widetilde{\tau}$ is closed under finite intersection.

If $\widetilde{\tau}$ is a soft topology on X with a parameter space E then the triplet $(X,E,\widetilde{\tau})$ is called a soft topological space over X with parameter space E. Identifying (X,E) with $\widetilde{X}$, $(\widetilde{X},\widetilde{\tau})$ is a soft topological space.
The members of $\widetilde{\tau}$ are called soft open sets in $(X,E,\widetilde{\tau})$. A soft set $\widetilde{F}$ in $S(X,E)$ is soft closed in $(X,E,\widetilde{\tau})$, if its complement $(\widetilde{F})'$ belongs to $\widetilde{\tau}$. $(\widetilde{\tau})'$ denotes the collection of all soft closed sets in $(X,E,\widetilde{\tau})$. The soft closure of $\widetilde{F}$ is the soft set, $\widetilde{scl}(\widetilde{F}) = \cap\{\widetilde{G} : \widetilde{G}$ is soft closed and $\widetilde{F} \subseteq \widetilde{G}\}$. The soft interior [9] of $\widetilde{F}$ is the soft set, $\widetilde{sint}(\widetilde{F}) = \cup\{\widetilde{O} : \widetilde{O}$ is soft open and $\widetilde{O} \subseteq \widetilde{F}\}$. It is easy to see that $\widetilde{F}$ is soft open $\Leftrightarrow \widetilde{F} = \widetilde{sint}\widetilde{F}$ and $\widetilde{F}$ is soft closed $\Leftrightarrow \widetilde{F} = \widetilde{scl}\widetilde{F}$

Definition 2.11. [8] *Let Z be a non-empty subset of X. Then $\widetilde{Z} \subseteq \widetilde{X}$. Let $(X,E,\widetilde{\tau})$ be a soft topological space. Define $\widetilde{\tau}\big|_Z = \{(Z\big|_F, E) : Z\big|_F(e) = F(e) \cap Z$ for every $e \in E$ and $\widetilde{F} \in \widetilde{\tau}\}$.*

Then $\widetilde{\tau}\big|_Z$ is a soft topology on Z with parameter space E and $(Z,E,\widetilde{\tau}\big|_Z) = (\widetilde{Z},\widetilde{\tau}\big|_Z)$ is a soft subspace of $(X,E,\widetilde{\tau})$. Clearly $\widetilde{Z}\big|_F = \widetilde{Z} \cap \widetilde{F}$ for all $\widetilde{F} \in \widetilde{\tau}$.

Lemma 2.12. [8] *Let $(Z,E,\widetilde{\tau}\big|_Z)$ be a soft subspace of a soft topological space $(X,E,\widetilde{\tau})$ and $\widetilde{F}$ be a soft open in $\widetilde{Z}$. If $\widetilde{Z} \in \widetilde{\tau}$ then $\widetilde{F} \in \widetilde{\tau}$*

Lemma 2.13. [8] *Let $(Z,E,\widetilde{\tau}\big|_Z)$ be a soft subspace of a soft topological space $(X,E,\widetilde{\tau})$ and $\widetilde{F}$ be a soft set over X, then*
$(i)\widetilde{F}$ is soft open in the subspace $(Z,E,\widetilde{\tau}\big|_Z)$ if and only if $\widetilde{F}=\widetilde{Z}\cap\widetilde{G}$ for some soft open set $\widetilde{G}$ in $(X,E,\widetilde{\tau})$.
$(ii)\widetilde{F}$ is soft closed in the subspace $(Z,E,\widetilde{\tau}\big|_Z)$ if and only if $\widetilde{F}=\widetilde{Z}\cap\widetilde{G}$ for some soft closed set $\widetilde{G}$ in $(X,E,\widetilde{\tau})$.

Lemma 2.14. [16] *Let $(Z,E,\widetilde{\tau}\big|_Z)$ be a soft subspace of a soft topological space $(X,E,\widetilde{\tau})$ and $\widetilde{F}$ be soft closed in $\widetilde{Z}$. If $\widetilde{Z}\in(\widetilde{\tau})'$ then $\widetilde{F}\in(\widetilde{\tau})'$*

Lemma 2.15. [14] *Let I be an arbitrary index set and $\{\widetilde{F_i}:i\in I\}\subseteq S(X,E)$. Then $\widetilde{F}\cap(\cup\{\widetilde{F_i}:i\in I\})=\cup\{\widetilde{F}\cap\widetilde{F_i}:i\in I\}$*

Definition 2.16. [7] *Let X and Y be any two universal sets. The functions $p:E\to K$ and $g:X\to Y$ induce the function $(g,p):S(X,E)\to S(Y,K)$ defined as below:*
For each $\widetilde{F}$ in $S(X,E)$ the image $(g,p)(\widetilde{F})$ is defined as,

$$(g,p)(\widetilde{F})(k)=\begin{cases}\cup\{g(F(e)):e\in p^{-1}(k)\}, & if \quad p^{-1}(k)\neq\phi\\ \phi, & otherwise.\end{cases}$$

Let $\widetilde{G}\in S(Y,K)$. Then the inverse image of $\widetilde{G}$ under the soft function (g,p) is the soft set over X denoted by $(g,p)^{-1}(\widetilde{G})$, where $(g,p)^{-1}(\widetilde{G})(e)=g^{-1}(G(p(e)))$ for all $e\in E$.

Definition 2.17. [16] *(g,p) is soft continuous from $(X,E,\widetilde{\tau})$ to $(Y,K,\widetilde{\sigma})$ if $(g,p)^{-1}(\widetilde{G})\in\widetilde{\tau}$ for every $\widetilde{G}\in\widetilde{\sigma}$.*

Definition 2.18. [8] *Let $(X,E,\widetilde{\tau})$ be a soft topological space over X, $\widetilde{G}$ be a soft set over X and $x\in X$. Then $\widetilde{G}$ is said to be a soft neighbourhood of x if there exist a soft open set $\widetilde{F}$ such that $x\in\widetilde{F}\subseteq\widetilde{G}$.*

Definition 2.19. [16] *Let $Z\subseteq X$ and $(g,p):S(X,E)\to S(Y,K)$. Define $(g,p)\big|_{\widetilde{Z}}(\widetilde{F})=(g,p)(\widetilde{F})$ for all $\widetilde{F}\in S(Z,E)$. Clearly $(g,p)\big|_{\widetilde{Z}}$ is the restriction of (g,p) to $S(Z,E)$ and $((g,p)\big|_{\widetilde{Z}})^{-1}(\widetilde{G})=(g,p)^{-1}(\widetilde{G})\cap\widetilde{Z}$ for all $\widetilde{G}$ in $S(Y,K)$*

Lemma 2.20. [11] *Let $(X,E,\widetilde{\tau})$ and $(Y,K,\widetilde{\sigma})$ be soft topological spaces. Let $(g,p):S(X,E)\to S(Y,K)$ be a soft function then the following statements are equivalent:*
$(i)(g,p)$ is soft continuous.
(ii) For each closed set $\widetilde{G}$ over $(Y,K,\widetilde{\sigma})$, $(g,p)^{-1}(\widetilde{G})$ is soft closed in X.

Lemma 2.21. [17] *For any two soft sets $\widetilde{F}$ and $\widetilde{G}$ over X, $\widetilde{F} \setminus \widetilde{G} = \widetilde{F} \cap (\widetilde{G})'$.*

Lemma 2.22. [11] *Let $\widetilde{F}$ and $\widetilde{G} \in S(X,E)$. If $\widetilde{F} \cap \widetilde{G} = \widetilde{\Phi}$ then $\widetilde{F} \subseteq (\widetilde{G})'$.*

3. Soft Locally Finite

Definition 3.1. *A family $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ of soft sets over X is* ***soft locally finite*** *in the soft topological space $(X,E,\widetilde{\tau})$ if for every $x \in X$ there exists a soft open set $\widetilde{G}$ such that $x \in \widetilde{G}$ and $\widetilde{G} \cap \widetilde{F}_\alpha \neq \widetilde{\Phi}$ holds for at most finitely many indices α.*

The following proposition follows from the definition.

Proposition 3.2. *Every finite family of soft sets over X is soft locally finite in $(X,E,\widetilde{\tau})$.*

Example 3.3. *Let $X = \{1,2,3,...\}$ be set of positive integers and $E = \{e_1,e_2\}$ be a parameter space. Let $\widetilde{\tau} = \{\widetilde{\Phi},\widetilde{X},\widetilde{F}_n\}$ where $\widetilde{F}_n = \{(e_1,X),(e_2,\{1,2,...n\})\}$ for all $n = 1,2,...$ be a soft topology on X. For $n = 1,2,,...$ let $\widetilde{G}_n = \{(e_1,X_{n+1}),(e_2,X_n)\}$ where $X_n = \{n,n+1,n+2,...\}$. Then the family $\{\widetilde{G}_n : n = 1,2,3,...\}$ is soft locally finite in $(X,\widetilde{\tau},E)$.*
Fix $x \in X$, then $x \in \widetilde{F}_x$. Now $F_x(e_1) \cap G_n(e_1) = X \cap X_{n+1} \neq \phi$ for all n. $F_x(e_2) \cap G_n(e_2) = \{1,2,...x\} \cap n \neq \phi$, when $1 \leq n \leq x$. For $n > x, F_x(e_2) \cap G_n(e_2) = \phi$. Therefore the family $\{\widetilde{G}_n, n = 1,2,....\}$ forms a soft locally finite family in $(X,E,\widetilde{\tau})$.

Example 3.4. *Let $X = \{1,2,3...\}$ be set of positive integers and $E = \{e_1,e_2\}$ be a parameter space. Let $X_n = \{n,n+1,n+2,...\}$ and let $\widetilde{\tau} = \{\widetilde{\Phi},\widetilde{X},\widetilde{F}_n\}$ where $\widetilde{F}_n = \{(e_1,X),(e_2,X_n)\}$ be a soft topology on X. Now for $n = 1,2,...$ let $\widetilde{G}_n = \{(e_1,X_{n+1}),(e_2,X_n)\}$. Then the family $\{\widetilde{G}_n : n = 1,2,...\}$ is not soft locally finite.*
Fix $x \in X$. Then $x \in \widetilde{F}_y$ for all $y \leq x$. Then $F_y(e_1) \cap G_n(e_1) = X \cap \{X_{n+1},X_{n+2}....\} \neq \phi$. $F_y(e_2) \cap G_n(e_2) = \{y,y+1,...,x+1,...\} \cap X_n \neq \phi$ when $n \in \{y,y+1,...,x+1,....\}$ for all $y \leq x$, and for all $n \geq y$, $F_y(e_2) \cap G_n(e_2) \neq \phi$. Therefore for all $y \leq x$ and $n \geq y$, $\widetilde{F}_y \cap \widetilde{G}_n \neq \phi$. Therefore every soft neighborhood of x intersects all $\widetilde{G}_n$ but for finitely many. Therefore $\{\widetilde{G}_n : n = 1,2,....\}$ is not soft locally finite.

Example 3.5. *Let $X = R$ be the set of all real numbers and $E = \{e_1,e_2\}$ be a parameter space Define soft sets $\widetilde{F}_1 = \{(e_1,Q),(e_2,R)\}$, $\widetilde{F}_2 = \{(e_1,R-Q),(e_2,R)\}$ and $\widetilde{F}_3 = \{(e_1,\phi),(e_2,R)\}$. Then $\widetilde{\tau} = \{\widetilde{\Phi},\widetilde{X},\widetilde{F}_1,\widetilde{F}_2,\widetilde{F}_3\}$ is a soft topology over X. For all $x \in R$, define $\widetilde{G}_x = \{(e_1,R),(e_2,\{x\})\}$. Then the family $\{\widetilde{G}_x : x \in R\}$ is not soft locally finite.*

Fix $y \in Q$. Then $\widetilde{F_1}$ is a soft neighborhood of y. $F_1(e_1) \cap G_x(e_1) = Q \cap R = Q \neq \phi$ for all x. $F_1(e_2) \cap G_x(e_2) = R \cap \{x\} = \{x\} \neq \phi$ for all $x \in Q$. $F_2(e_1) \cap G_x(e_1) = R - Q \cap R = R - Q \neq \phi$ for all x. $F_2(e_2) \cap G_x(e_2) = R \cap \{x\} = \{x\} \neq \phi$ for all $x \in Q$. $F_3(e_1) \cap G_x(e_1) = \phi \cap R = \phi$ for all x. $F_3(e_2) \cap G_x(e_2) = R \cap \{x\} = \{x\} \neq \phi$ for all $x \in Q$. Therefore every neighborhood of x intersects all $\widetilde{G_x}$. Therefore $\{\widetilde{G}_x : x \in R\}$ is not soft locally finite.

Example 3.6. *Let $X = R$ be the set of all real numbers and $E = \{e_1, e_2\}$ be a parameter space. Fix two real numbers x and y with $x \neq y$. Define soft sets as $\widetilde{F_1} = \{(e_1, \{x\}), (e_2, R)\}$, $\widetilde{F_2} = \{(e_1, \{y\}), (e_2, R)\}$, $\widetilde{F_3} = \{(e_1, \{x, y\}), (e_2, R)\}$ and $\widetilde{F_4} = \{(e_1, \phi), (e_2, R)\}$. Then $\widetilde{\tau} = \{\widetilde{\Phi}, \widetilde{X}, \widetilde{F_1}, \widetilde{F_2}, \widetilde{F_3}, \widetilde{F_4}\}$ is a soft topology over X. For all $z \in R$, define $\widetilde{G_z} = \{(e_1, R), (e_2, \{z\})\}$. Then the family $\{\widetilde{G_z} : z \in R\}$ is not soft locally finite.*

Fix $w \in Q$. If $w = x$, then $\widetilde{F_1}$ and $\widetilde{X}$ are the soft open neighborhoods of w. If $w = y$, then $\widetilde{F_2}$ and $\widetilde{X}$ are the soft open neighborhoods of w. If $w = \{x, y\}$ then $\widetilde{F_3}$ and $\widetilde{X}$ are the soft open neighborhoods of w. If $w = x$, $\widetilde{F_1}(e_1) \cap \widetilde{G_z}(e_1) = \{x\} \neq \phi$. $\widetilde{F_1}(e_2) \cap \widetilde{G_z}(e_2) = \{z\} \neq \phi$ If $w = y$, $\widetilde{F_2}(e_1) \cap \widetilde{G_z}(e_1) = \{y\} \cap R = \{y\} \neq \phi$. $\widetilde{F_2}(e_2) \cap \widetilde{G_z}(e_2) = R \cap \{z\} = \{z\} \neq \phi$. If $w = \{x, y\}$, $\widetilde{F_3}(e_1) \cap \widetilde{G_z}(e_1) = \{x, y\} \cap R = \{x, y\} \neq \phi$. $\widetilde{F_3}(e_2) \cap \widetilde{G_z}(e_2) = R \cap \{z\} = \{z\} \neq \phi$. $\widetilde{F_4}(e_1) \cap \widetilde{G_z}(e_1) = \phi \cap R = \phi$. $\widetilde{F_4}(e_2) \cap \widetilde{G_z}(e_2) = R \cap \{z\} = \{z\} \neq \phi$. Therefore $\{\widetilde{G_z} : z \in R\}$ is not soft locally finite family.

Theorem 3.7. *Let $\{\widetilde{F_\alpha}, \alpha \in \Delta\}$ be a soft locally finite in the soft topological space $(X, E, \widetilde{\tau})$. Then $\{\widetilde{scl}(\widetilde{F_\alpha}), \alpha \in \Delta\}$ is also soft locally finite.*

Proof. Given $\{\widetilde{F_\alpha}, \alpha \in \Delta\}$ be soft locally finite. Then for each $x \in X$ there is a soft open set $\widetilde{G}$ such that $\widetilde{G} \cap \widetilde{F} \neq \widetilde{\Phi}$ holds for at most finitely many indices α. $\widetilde{F} \cap \widetilde{G} = \widetilde{\Phi}$ for all but at most finitely many indices α. If $\widetilde{F_\alpha} \cap \widetilde{G} = \widetilde{\Phi}$, by using Lemma 2.21, we have $\widetilde{F_\alpha} \subseteq (\widetilde{G})'$. That is $\widetilde{scl}(\widetilde{F_\alpha}) \subseteq \widetilde{scl}(\widetilde{G})'$. That implies $\widetilde{scl}(\widetilde{F_\alpha}) \subseteq (\widetilde{G})'$. That is $\widetilde{scl}(\widetilde{F_\alpha}) \cap \widetilde{G} = \widetilde{\Phi}$ for all but at most finitely many indices α. Then $\widetilde{scl}(\widetilde{F_\alpha}) \cap \widetilde{G} \neq \widetilde{\Phi}$ for at most finitely many indices α. Therefore $\{\widetilde{scl}(\widetilde{F_\alpha}), \alpha \in \Delta\}$ is also soft locally finite.

$\square$

Theorem 3.8. *Let $\{\widetilde{F_i}, i \in \Delta\}$ be soft locally finite in the soft topological space $(X, E, \widetilde{\tau})$. Then $\cup \widetilde{scl}(\widetilde{F_i})$ is soft closed set.*

Proof. Take $\widetilde{F} = \cup \widetilde{scl}(\widetilde{F_i})$. By using Lemma 2.7 $(\widetilde{F})' = \cap (\widetilde{scl}(\widetilde{F_i}))'$. Let $x \in (\widetilde{F})'$. By using Theorem 3.7 $\{\widetilde{scl}(\widetilde{F_i}) : i \in \Delta\}$ is soft locally finite. Let $\widetilde{G}$ be soft open with $x \in \widetilde{G}$. Then there exist $\widetilde{scl}(\widetilde{F_{i1}}), \widetilde{scl}(\widetilde{F_{i2}}), \widetilde{scl}(\widetilde{F_{i3}}), ..., \widetilde{scl}(\widetilde{F_{in}})$ such that $\widetilde{G} \cap \widetilde{scl}(\widetilde{F_i}) \begin{cases} \neq \widetilde{\Phi} & for 1 \leq i \leq n \\ = \widetilde{\Phi} & otherwise. \end{cases}$

Now $x \in \widetilde{G} \cap (\widetilde{scl}(\widetilde{F_{i1}}))' \cap (\widetilde{scl}(\widetilde{F_{i2}}))' \cap (\widetilde{scl}(\widetilde{F_{i3}}))' \cap ...... \cap (\widetilde{scl}(\widetilde{F_{in}}))'$ is soft open and $[\widetilde{G} \cap \bigcap_{i=1}^{n} (\widetilde{scl}(\widetilde{F_i}))'] \cap \widetilde{F} = \widetilde{\Phi}$. That is $\widetilde{G} \cap \bigcap_{i=1}^{n} (\widetilde{scl}(\widetilde{F_i}))' \subseteq (\widetilde{F})'$. Therefore $(\widetilde{F})'$ is soft open, that implies $\widetilde{F}$ is soft closed. Therefore $\cup \widetilde{scl}(\widetilde{F_i})$ is soft closed.

□

Theorem 3.9. *Let $\{\widetilde{F_\alpha}, \alpha \in \Delta\}$ be a soft locally finite in the soft topological space $(X, E, \widetilde{\tau})$. Then $\cup \{\widetilde{scl}(\widetilde{F_\alpha}) : \alpha \in \Delta\} = \widetilde{scl}(\cup \widetilde{F_\alpha} : \alpha \in \Delta)$.*

Proof. $\widetilde{scl}(\widetilde{F_\alpha}) \subseteq \widetilde{scl}(\cup(\widetilde{F_\alpha}))$, $\cup\{\widetilde{scl}(\widetilde{F_\alpha})\} \subseteq \widetilde{scl}(\cup(\widetilde{F_\alpha}))$. We know that by using Theorem 3.8, $\cup(\widetilde{scl}(\widetilde{F_\alpha}))$ is soft closed. $\widetilde{scl}(\widetilde{F_\alpha})$ is the smallest closed set which contains $\widetilde{F_\alpha}$. Therefore $\widetilde{F_\alpha} \subseteq \widetilde{scl}(\widetilde{F_\alpha})$, $\cup \widetilde{F_\alpha} \subseteq \cup \widetilde{scl}(\widetilde{F_\alpha})$, $\widetilde{scl}(\cup\{\widetilde{F_\alpha}\}) \subseteq \widetilde{scl}(\cup(\widetilde{scl}(\widetilde{F_\alpha}))) = \cup(\widetilde{scl}(\widetilde{F_\alpha}))$. Therefore $\widetilde{scl}(\cup\{\widetilde{F_\alpha} : \alpha \in \Delta\}) = \cup\{\widetilde{scl}(\widetilde{F_\alpha}) : \alpha \in \Delta\}$.

□

Lemma 3.10. *If $X = \cup X_\alpha$ then $\widetilde{X} = \cup \widetilde{X_\alpha}$.*

Proof. $X_\alpha \subseteq X, \widetilde{X_\alpha}(e) = X_\alpha$ for every $e \in E$.
$(\cup \widetilde{X_\alpha})(e) = \cup \widetilde{X_\alpha}(e) = \cup X_\alpha = X = \widetilde{X}(e)$. Therefore $\widetilde{X} = \cup \widetilde{X_\alpha}$. □

Lemma 3.11. *Let $(X, E, \widetilde{\tau})$ be a soft topological spaces. Let $Z \subseteq X$. Then $(Z, E, \widetilde{\tau}|_Z)$ is a soft subspace of $(X, E, \widetilde{\tau})$. Then $(\widetilde{\tau}|_Z)_e = \widetilde{\tau}_e|_Z$.*

Proof. $B \in (\widetilde{\tau}|_Z)_e \Leftrightarrow B = \widetilde{F}(e)$ for some $\widetilde{F} \in \widetilde{\tau}|_Z \Leftrightarrow B = \widetilde{H}(e) \cap \widetilde{Z}(e)$ where $\widetilde{F} = \widetilde{H} \cap \widetilde{Z}$, $\widetilde{H} \in \widetilde{\tau}$ $\Leftrightarrow B = H(e) \cap Z$ where $H(e) \in \widetilde{\tau}_e \Leftrightarrow B \in \widetilde{\tau}_e|_Z$. □

Theorem 3.12. *Let $\{X_\alpha, \alpha \in \Delta\}$ be a family of sets that cover the set X. That is $X = \cup X_\alpha$. Let $(X, E, \widetilde{\tau})$ be a soft topological space. Assume that all the $\widetilde{X_\alpha}$ are soft open sets and let $B \subseteq X$. Then $\widetilde{B}$ is soft open if and only if each $\widetilde{B} \cap \widetilde{X_\alpha}$ is soft open in the soft subspace $(X_\alpha, E, \widetilde{\tau}|_{X_\alpha})$.*

Proof. Suppose assume that all the $\widetilde{X_\alpha}$ are soft open in X. Let $B \subseteq X$. If $\widetilde{B}$ is soft open in X then by using Lemma 2.13, $\widetilde{B} \cap \widetilde{X_\alpha}$ is soft open in X_α.
Conversely now assume that $\widetilde{B} \cap \widetilde{X_\alpha}$ is soft open in X_α for every α. By using Lemma 2.12, $\widetilde{B} \cap \widetilde{X_\alpha}$ is soft open in X . Now $\widetilde{B} = \widetilde{B} \cap \widetilde{X}$. By using Lemma 2.15, $\widetilde{B} \cap (\cup \widetilde{X_\alpha}) = \cup_\alpha (\widetilde{B} \cap \widetilde{X_\alpha})$.

Therefore $\widetilde{B}$ is soft open in X.

$\square$

Theorem 3.13. *Let $\{X_\alpha, \alpha \in \Delta\}$ be a family of sets that cover the set X. That is $X = \cup X_\alpha$. Let $(X,E,\widetilde{\tau})$ be a soft topological space. Assume that all the $\widetilde{X}_\alpha$ are soft closed sets and form a soft locally finite family and let $B \subseteq X$. Then $\widetilde{B}$ is soft closed if and only if each $\widetilde{B} \cap \widetilde{X}_\alpha$ is soft closed in the soft subspace $(X_\alpha, E, \widetilde{\tau}\big|_{X_\alpha})$.*

Proof. Suppose all the $\widetilde{X}_\alpha$ are soft closed in X and form a soft locally finite family. Let $B \subseteq X$. If $\widetilde{B}$ is soft closed in X then by using Lemma 2.13, $\widetilde{B} \cap \widetilde{X}_\alpha$ is soft closed in X_α for every α. Conversely now assume that $\widetilde{B} \cap \widetilde{X}_\alpha$ is soft closed in X_α. Then using Lemma 2.13, $\widetilde{B} \cap \widetilde{X}_\alpha$ is soft closed in X for every α . Since $\{\widetilde{B} \cap \widetilde{X}_\alpha : \alpha \in \Delta\}$ is soft locally finite and since each $\widetilde{X}_\alpha$ is soft closed in X, by using Theorem 3.8, $\cup(\widetilde{B} \cap \widetilde{X}_\alpha)$ is soft closed in X. That is by using Lemma 2.15, $\widetilde{B} \cap (\cup \widetilde{X}_\alpha)$ is soft closed in X. By using Lemma 3.10, $\widetilde{B} \cap \widetilde{X}$ is soft closed in X. That is $\widetilde{B}$ is soft closed in $\widetilde{X}$. Therefore $\widetilde{B}$ is soft closed in X .

$\square$

Proposition 3.14. $(g,p)\big|_Z = (g\big|_Z, p)$ *where* $g\big|_Z : Z \to Y$.

Proof. Let $\widetilde{F} \in S(Z,E)$. Then $\widetilde{F} \in S(X,E)$.

$$(g,p)(\widetilde{F})(k) = \begin{cases} \cup\{g(F(p(e))) : e \in p^{-1}(k)\}, & if \quad p^{-1}(k) \neq \phi \\ \phi, & otherwise. \end{cases}$$

Since $F(p(e)) \subseteq Z$,

$$(g,p)(\widetilde{F})(k) = \begin{cases} \cup\{g\big|_Z(F(p(e))) : e \in p^{-1}(k)\}, & if \quad p^{-1}(k) \neq \phi \\ \phi, & otherwise. \end{cases}$$

$= (g\big|_Z, p)(\widetilde{F})(k)$.

$(g,p)\big|_Z : S(Z,E) \to S(Y,K)$ is the restriction of (g,p) to $S(Z,E)$.

Therefore $(g,p)(\widetilde{F})(k) = (g,p)\big|_Z(\widetilde{F})(k) = (g\big|_Z, p)(\widetilde{F})(k) = (g,p)(\widetilde{F})(k) = (g,p)\big|_Z = (g\big|_Z, p)$.

$\square$

Theorem 3.15. *Let X be a non-empty set and E be a parameter space.Let $\{X_\alpha : \alpha \in \Delta\}$ be a family of subsets of X that covers X. For each α, let $(g_\alpha, p) : S(X_\alpha, E) \to S(Y,K)$ be given and assume that $g_\alpha\big|_{(X_\alpha \cap X_\beta)} = g_\beta\big|_{(X_\alpha \cap X_\beta)}$ for each $(\alpha,\beta) \in \Delta \times \Delta$. Then there exists one and* *only one* $(g,p):S(X,E)\rightarrow S(Y,K)$ *which is an extension of each* (g_α,p)*. That is for all* $\alpha\in\Delta$ $(g_\alpha,p)=(g,p)\big|_{S(X_\alpha,E)}$.

Proof. Given that $X=\cup X_\alpha$ and $(g_\alpha,p):S(X_\alpha,E)\rightarrow S(Y,K)$ and assume that $g_\alpha\big|_{(X_\alpha\cap X_\beta)}=g_\beta\big|_{(X_\alpha\cap X_\beta)}$. Define $g:X\rightarrow Y$, by $g(x)=g_\alpha(x)$ if $x\in X_\alpha$. If $x\in X_\alpha\cap X_\beta$, $g(x)=g_\alpha(x)=g_\beta(x)$. Therefore g is defined and the function $(g,p):S(X,E)\rightarrow S(Y,K)$ is defined. Now to prove that $(g_\alpha,p)=(g,p)\big|_{S(X_\alpha,E)}$. Suppose $\widetilde{F_\alpha}\in S(X_\alpha,E)$. Since $S(X_\alpha,E)\subseteq S(X,E)$, $\widetilde{F_\alpha}\in S(X,E)$.
Therefore $(g,p)(\widetilde{F_\alpha})(k)=\begin{cases}\cup\{g(F_\alpha(e)):e\in p^{-1}(k)\}, & if \quad p^{-1}(k)\neq\phi\\ \phi, & otherwise.\end{cases}$
$=\begin{cases}\cup\{g_\alpha(F_\alpha(e)):e\in p^{-1}(k)\}, & if \quad p^{-1}(k)\neq\phi\\ \phi, & otherwise.\end{cases}$
$=(g_\alpha,p)(\widetilde{F_\alpha})(k)$.
Since $(g,p)\big|_{S(X_\alpha,E)}:S(X_\alpha,E)\rightarrow S(Y,K)$ is the restriction of (g,p) to $S(X_\alpha,E)$. We write $(g,p)\big|_{S(X_\alpha,E)}=(g_\alpha,p)$. Since g is uniquely determined by g_α, (g,p) is uniquely determined by (g_α,p). □

Theorem 3.16. *Let* $(X,E,\widetilde{\tau})$ *and* $(Y,K,\widetilde{\sigma})$ *be soft topological spaces and let* $\{X_\alpha:\alpha\in\Delta\}$ *be a family of subsets of* X *that covers* X*. Assume that all the* $\widetilde{X_\alpha}$ *are soft open. For each* $\alpha\in\Delta$*, let* $(g_\alpha,p):S(X_\alpha,E)\rightarrow S(Y,K)$ *be soft continuous from* $(X_\alpha,E,\widetilde{\tau}|_{X_\alpha})$ *to* $(Y,K,\widetilde{\sigma})$ *and assume that* $g_\alpha\big|_{(X_\alpha\cap X_\beta)}=g_\beta\big|_{(X_\alpha\cap X_\beta)}$ *for each* $(\alpha,\beta)\in\Delta\times\Delta$ *and* p *is surjective. Then there exist a unique soft continuous map* $(g,p):S(X,E)\rightarrow S(Y,K)$ *from* $(X,E,\widetilde{\tau})$ *to* $(Y,K,\widetilde{\sigma})$ *which is an extension of each* (g_α,p)*.*

Proof. The existence of the function $(g,p):S(X,E)\rightarrow S(Y,K)$ follows from Theorem 3.15. Now to prove (g,p) is soft continuous. Let $\widetilde{G}$ be soft open in Y. Then $\widetilde{G}(p(e))$ is open in Y for every $e\in E$. Since for each $\alpha\in\Delta$, (g_α,p) is soft continuous. That is $(g_\alpha,p)^{-1}(\widetilde{G})$ is soft open in X_α. Again the sets $\widetilde{X_\alpha}$ are soft open in X. Therefore by using Lemma 2.12, $(g_\alpha,p)^{-1}(\widetilde{G})$ is soft open in X that is $g_\alpha^{-1}(G(p(k)))$ is open in X_α for every $k\in K$ and hence open in X for every k with $p(e)=k$. Now $(g,p)^{-1}(\widetilde{G})=(g,p)^{-1}(\widetilde{G}\cap\widetilde{X})$. $=(g,p)^{-1}(\widetilde{G}\cap(\cup\widetilde{X_\alpha}))=\cup((g,p)^{-1}(\widetilde{G}\cap\widetilde{X_\alpha}))=\cup(g_\alpha,p)^{-1}(\widetilde{G})$. Since $\cup(g_\alpha,p)^{-1}(\widetilde{G}(p(e)))$ is open in X, that is $(g_\alpha,p)^{-1}(\widetilde{G})$ is soft open in X and union of any number of soft open sets are soft open. Therefore $(g,p)^{-1}(\widetilde{G})$ is soft open in X. Therefore $(g,p):S(X,E)\rightarrow S(Y,K)$ is a unique soft continuous function from

$(X,E,\widetilde{\tau})$ to $(Y,K,\widetilde{\sigma})$ and it is an extension of (g_α,p) for each α.

□

Theorem 3.17. *Let $(X,E,\widetilde{\tau})$ and $(Y,K,\widetilde{\sigma})$ are soft topological space and let $\{X_\alpha : \alpha \in \Delta\}$ be a family of subsets of X that covers X. Assume that all the $\widetilde{X}_\alpha$ are soft closed and form a soft locally finite family. For each $\alpha \in \Delta$, let $(g_\alpha,p) : S(X_\alpha,E) \to S(Y,K)$ be soft continuous from $(X_\alpha,E,\widetilde{\tau}|_{X_\alpha})$ to $(Y,K,\widetilde{\sigma})$ and assume that $g_\alpha|_{(X_\alpha \cap X_\beta)} = g_\beta|_{(X_\alpha \cap X_\beta)}$ for each $\alpha,\beta \in \Delta\times\Delta$ and p is surjective. Then there exist a unique soft continuous map $(g,p) : S(X,E) \to S(Y,K)$ from $(X,E,\widetilde{\tau})$ to $(Y,K,\widetilde{\sigma})$ which is an extension of each (g_α,p).*

Proof. Assume that the sets $\widetilde{X}_\alpha$ are all soft closed in X and form soft locally finite family. Let $\widetilde{H}$ be soft closed in Y. Then $\widetilde{H}(p(e))$ is closed in Y for all $e \in E$. Since for every $\alpha \in \Delta$, (g_α,p) is soft continuous. That is$(g_\alpha,p)^{-1}(\widetilde{H})$ is soft closed in $(X,\widetilde{\tau}|_{X_\alpha},E)$. Again the sets $\widetilde{X}_\alpha$ are soft closed in X. Therefore by using Lemma 2.14, $(g,p)^{-1}(\widetilde{H})$ is soft closed in X. Now $(g,p)^{-1}(\widetilde{H}) = (g,p)^{-1}(\widetilde{H}\cap\widetilde{X}) = (g,p)^{-1}(\widetilde{H}\cap(\cup\widetilde{X}_\alpha)) = \cup((g,p)^{-1}(\widetilde{H}\cap\widetilde{X}_\alpha)) = \cup(g_\alpha,p)^{-1}(\widetilde{H})$. Since $\cup(g_\alpha,p)^{-1}(\widetilde{H}(p(e)))$ is closed in X. That is $(g_\alpha,p)^{-1}(\widetilde{H})$ is soft closed in X. Since it is soft locally finite and by using the Theorem 3.8, $\cup(g_\alpha,p)^{-1}(\widetilde{H})$ is soft closed in X. Therefore $(g,p)^{-1}(\widetilde{H})$ is soft closed in X. $\widetilde{H}$ is soft closed in Y, $(g,p)^{-1}(\widetilde{H})$ is soft closed in X. Therefore $(g,p) : S(X,E) \to S(Y,K)$ from $(X,E,\widetilde{\tau})$ to $(Y,K,\widetilde{\sigma})$ is a soft continuous function Therefore $(g,p) : S(X,E) \to S(Y,K)$ is a unique soft continuous function from $(X,E,\widetilde{\tau})$ to $(Y,K,\widetilde{\sigma})$ and it is an extension of (g_α,p) for each α.

□

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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