NON-FRAGILE EVENT-TRIGGERED SYNCHRONIZATION FOR SEMI-MARKOVIAN JUMPING COMPLEX DYNAMICAL NETWORKS

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Abstract. This article explores the non-fragile synchronization problem for complex dynamical networks (CDN) with semi-Markovian jumping (SMJ) parameters through event-triggered control technique. Some adequate criteria which assures the synchronization of considered semi-Markovian jumping CDNs (SMJCDNs) has been derived by making use of the Lyapunov stability theory and integral inequalities. Later, in the numerical example section Chua's circuit was taken to verify the theoretical findings.

Keywords: synchronization; semi-Markovian jump; event-triggerd control; non-fragile control.

1. INTRODUCTION

During the past decade, CDNs has gained substantial attention among the researchers owing to their appliance in many fields such as Biology, Mathematics, Sociology, Engineering and technology. Most devices in the actual globe can be modeled as CDNs such as World-wide web, Internet, electrical grids and so on. A CDN consists of huge nodes , in which all nodes represents a primary unit with specific dynamics. One of the most cardinal dynamical behavior in CDNs is synchronization. The synchronization control problem for CDNs have been examined by many Scientists and Engineers [1, 2, 3, 4, 5]. For example, Synchronization problem for CDNs

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with non-diffusive coupling has been examined in [6]. Adaptive synchronization problem for complex networks with general distributed update laws for coupling weights has been explored in [7].

Over the previous few decades, linear jumping systems have drawn significant attention due to the reality that these systems are cabable of modeling distinct types of dynamical systems that are subjected to unexpected structural variations such as random failiures and component repairs [8, 9]. Recently, Owing to its relaxed condition on probability distribution semi-Markovian jumping systems are attracted by many researchers and few articles have been published [10, 11, 12]. For example, reliable mixed passive and H_∞ filtering problem for SMJ systems has been discussed in [13]. Stability and synchronization problem for continuous time SMJ system with time-varying delay has been conferred in [14].

Nowadays, the digital controllers of digital computers are implemented to improve the usage of bandwidth and decrease the amount of signal transmission. Thus far, the considered sensors are time-triggered in the offered literature. Though, in Time-triggered controllers there must be some control waste. It should be specified that, in event-triggered control scheme the control input is released only when the triggering condition is satisfied. With the intension to overcome the disadvantage of time-triggered controllers, event-triggered control Law has considerable attention [15, 16]. In [17], event-triggered control problem for SMJ systems with transmission delays and randomly occurring uncertainties has been explained. In practical systems, due to unknown noises uncertainties or inaccuracies are unavoidable while implementing controllers. To overcome this fact non-fragile controller was taken into account [18, 19]. With the intension to attain the benefits of both controllers the hybrid controller which includes both event-triggered control and non-fragile control was designed to achieve the synchronization of SMJCDNs.

By the impact of the preceding facts, this manuscript examines the event-triggered synchronization control problem for SMJCDNs with and without non-fragile control strategy. Synchronization analysis has been performed by making use of reciprocally convex technique, Lyapunov stability theory and novel integral inequalities . Finally, synchronization of Chua's circuit was given to validate the proposed results.

2. PRELIMINARIES

Let $\{\beta(t), t \geq 0\}$ be a discrete-state continuous-time semi-Markov process and assume values in finite set $= \{1, 2, \cdots, N\}$ is given by

(1)
$$Pr\{\beta(t+l) = j | \beta(t) = i\} = \begin{cases} \alpha_{ij}(l)l + o(l) & i \neq j \\ 1 + \alpha_{ii}(l)l + o(l) & i = j \end{cases}$$

where $\Delta = \alpha_{ij}(l)$ denotes the transition probability matrix, $\lim_{l \to 0}(o(l)/l) = 0$, and $\alpha_{ij}(l) \geq 0$, for $i \neq j$, is the transition rate from mode i at time t to mode j at time $t + l$ and $\alpha_{ii}(l) = \sum_{j \in S, j \neq i} \alpha_{ij}(l)$.

Consider the SMJCDNs with coupling delays as

(2)
$$\dot{v}_i(t) = f(v_i(t)) + c_1 \sum_{j=1}^{N} \Xi_{ij} \tilde{A}(\beta(t)) v_j(t) + c_2 \sum_{j=1}^{N} \Xi_{ij} \tilde{B}(\beta(t)) v_j(t - \rho(t)) + u_i(t),$$

where $v_i(t) = (v_{i1}(t), v_{i2}(t), \cdots, v_{in}(t))^T \in \mathbb{R}^n$ denotes the state variable and $u_i(t) \in \mathbb{R}^n$ stands for the control input of the node i, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector-valued function, $\rho(t)$ is the time varying delay; c_1, c_2 are the constants indicates the coupling strength; $\tilde{A}$ and $\tilde{B} \in \mathbb{R}^{n \times n}$ represents the inner coupling matrix, the delay inner coupling matrix, respectively; $\Xi = (\Xi_{ij}) \in \mathbb{R}^{n \times n}$ symbolizes the outer coupling matrix. If there is a link among node i and node j $(i \neq j)$, then $\Xi_{ij} = 1$, otherwise $\Xi_{ij} = 0$ $(i \neq j)$. The diagonal elements of matrix Ξ are defined by

$$\Xi_{ij} = - \sum_{j=1, j \neq i}^{N} \Xi_{ij}, \text{ for all } i = 1, 2, \cdots N.$$

Choose the synchronization target node as

(3)
$$\dot{w}(t) = f(w(t))$$

and select $\psi(t) = v(t) - w(t)$ be the error. The error system of (1) can be characterized as

(4)
$$\dot{\psi}_i(t) = g(\psi_i(t)) + c_1 \sum_{j=1}^{N} \Xi_{ij} \tilde{A}(\beta(t)) \psi_j(t) + c_2 \sum_{j=1}^{N} \Xi_{ij} \tilde{B}(\beta(t)) \psi_j(t - \rho(t)) + u_i(t),$$

where $g(\psi_i(t)) = f(v_i(t)) - f(w(t))$.

The non-fragile event-triggered control rule is defined as

(5)
$$u(t) = \bar{\mathfrak{K}} \psi(t_k h)$$

where $\bar{\mathfrak{K}} = \mathfrak{K} + \Delta\mathfrak{K}(t)$, $\mathfrak{K}$ stands for the control gain matrix and $\Delta\mathfrak{K}(t)$ denotes additive gain perturbations. $\Delta\mathfrak{K}(t)$ takes the form $\Delta\mathfrak{K}(t) = \mathfrak{U}F(t)\mathfrak{V}$ where $\mathfrak{U}$ and $\mathfrak{V}$ denotes the constant matrices and $F(t)$ gratifies $F^T(t)F(t) \leq I$

In event-triggered control, the condition where the control input is to be transmitted is defined as follows:

$$(6) \qquad (\psi^T(i_k h)O\psi(i_k h)) \geq \sigma v^T(t_k h)Ov(t_k h)$$

where $0 \leq \sigma \leq 1$ is a known parameter, $O > 0$ is the matrix to be determined and $\psi(i_k h) = v(t_k h) - v(i_k h)$.

Substituting (5) into (4) gives

$$\dot{\psi}_i(t) = g(\psi_i(t)) + c_1 \sum_{j=1}^{N} \Xi_{ij}\tilde{A}(\beta(t))\psi_j(t) + c_2 \sum_{j=1}^{N} \Xi_{ij}\tilde{B}(\beta(t))\psi_j(t - \rho(t))$$

$$(7) \qquad\qquad + \mathfrak{K}_i\psi_i(t - \eta(t)) + \mathfrak{K}\psi(i_k h),$$

Thus, (7) can be written as

$$(8) \qquad \dot{\psi}(t) = \bar{g}(\psi(t)) + c_1 A_i\psi(t) + c_2 B_i\psi(t - \rho(t)) + \mathfrak{K}\psi(t - \eta(t)) + \mathfrak{K}\psi(i_k h),$$

where $\psi(t) = [\psi_1(t), \psi_2(t), \cdots, \psi_N(t)]$, $\bar{g}(\psi(t)) = [\bar{g}(\psi_1(t)), \bar{g}(\psi_2(t)), \cdots, \bar{g}(\psi_N(t))]$, $A = \Xi_{ij} \otimes \tilde{A}(\beta(t))$, $B = \Xi_{ij} \otimes \tilde{B}(\beta(t))$ and $\mathfrak{K} = diag\{\mathfrak{K}_1, \mathfrak{K}_2, \cdots, \mathfrak{K}_N\}$.

Assumption 2.1. *Let $\rho : \mathbb{R}^n \to \mathbb{R}^n$ be a continuous vector valued function and gratifies the following condition:*

$$[\rho(p) - \rho(q) - U(p - q)]^T[\rho(p) - \rho(q) - V(p - q)]^T \leq 0.$$

for all $p, q \in \mathbb{R}^n$, where U and V are constant matrices of appropriate dimensions.

Lemma 2.2. *[20]For any two scalars $\upsilon_2 \geq \upsilon_1 > 0$, constant matrix $H \in \mathbb{R}^{n \times n}, H = H^T > 0$, such that the integrations concerned are well defined:*

$$-(\upsilon_2 - \upsilon_1)\int_{t-\upsilon_2}^{t-\upsilon_1} \xi^T(s)H\xi(s)\mathrm{d}s \leq -\left(\int_{t-\upsilon_2}^{t-\upsilon_1} \xi(s)\mathrm{d}s\right)^T H\left(\int_{t-\upsilon_2}^{t-\upsilon_1} \xi(s)\mathrm{d}s\right)$$

Lemma 2.3. [21] *For any vectors p_1 and p_2, real scalars $a \geq 0, b \geq 0$, any matrix $\mathscr{W}$, and symmetric matrix $\mathscr{P} > 0$, satisfying $\begin{bmatrix} \mathscr{P} & \mathscr{W} \\ * & \mathscr{P} \end{bmatrix} \geq 0$ and $a + b = 1$, the succeeding inequality holds:*

$$-\frac{1}{a} p_1^T \mathscr{P} p_1 - \frac{1}{b} p_2^T \mathscr{P} p_2 \leq - \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}^T \begin{bmatrix} \mathscr{P} & \mathscr{W} \\ * & \mathscr{P} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

Lemma 2.4. [22] *Let $\Sigma = \Sigma^T$, U and V be the real constant matrices of appropriate dimensions $\Sigma + UF(t)V + V^T F(t)U^T < 0$ for F satisfying $F^T(t)F(t) \leq I$, iff there exists a scalar $\delta > 0$ such that $\Sigma + \delta^{-1}UU^T + \delta V^T V < 0$.*

Lemma 2.5. [23] *For any matrix $E \in \mathbb{R}^{n \times m}$, $E = E^T > 0$, differentiable function θ from $[a, b] \to \mathbb{R}^n$, the succeeding inequality holds:*

$$\int_a^b \dot{z}^T(s)E\dot{z}(s)\mathrm{d}s \geq \frac{\vartheta^T \left[Y_1^T E Y_1 + \pi^2 Y_2^T E Y_2 \right] \vartheta}{b - a}$$

where $\vartheta = [z^T(b) \ z^T(a) \ \int_a^b \frac{z^T(s)}{b-a}\mathrm{d}s]^T$, $Y_1 = [I \ -I \ 0]$ and $Y_2 = [I/2 \ I/2 \ -I]$.

3. Main Results

In this section, the event-triggered non fragile control for SMJCDNs has been developed through the following theorems.

Theorem 3.1. *The SMJCDNs (8) is asymptotically synchronized if there exist matrices $\mathfrak{P}_i > 0, \mathfrak{Q}_1 > 0, \mathfrak{Q}_2 > 0, \mathfrak{R}_1 > 0, \mathfrak{R}_2 > 0, \mathfrak{Z}_1 > 0, \mathfrak{Z}_2 > 0$ and matrix $\mathfrak{S}, \mathfrak{L}$ and $\mathfrak{G}$ such that the following LMIs hold*

$$(9) \qquad \Upsilon(\delta) < 0, i = 1, 2, \cdots, s$$

where

$$\Upsilon_{1,1} = \mathfrak{Q}_1 + \mathfrak{Q}_2 + \rho \mathfrak{R}_2 + \mathfrak{Z}_1 - \mathfrak{R}_1 - (\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2) - v\bar{\mathfrak{X}} + \sum_{j=1}^N \alpha_{i,j}\mathfrak{P}_j, \ \Upsilon_{1,2} = \mathfrak{P}_i + c_1\mathfrak{G}(\Xi \otimes A_i),$$

$$\Upsilon_{1,3} = \mathfrak{R}_1 - \mathfrak{S}, \ \Upsilon_{1,4} = \mathfrak{S}, \ \Upsilon_{1,7} = \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \ \Upsilon_{1,10} = \frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{1,11} = -v\bar{\mathfrak{Y}}, \ \Upsilon_{2,2} = -2\mathfrak{G} - \rho^2\mathfrak{R}_1,$$

$$\Upsilon_{2,3} = c_2\mathfrak{G}(\Xi_2 \otimes B_i), \ \Upsilon_{2,7} = \mathfrak{L}, \ \Upsilon_{2,11} = \mathfrak{G}, \ \Upsilon_{2,12} = \mathfrak{L}, \ \Upsilon_{3,3} = (1-\mu)\mathfrak{Q}_1 - 2\mathfrak{R}_1 + \mathfrak{S} + \mathfrak{S}^T,$$

$$\Upsilon_{3,4} = -\mathfrak{S} + \mathfrak{R}_2, \ \Upsilon_{4,4} = -\mathfrak{R}_2, \ \Upsilon_{5,5} = -\frac{1}{\rho}\mathfrak{R}_2, \ \Upsilon_{6,6} = -\frac{1}{\rho}\mathfrak{R}_2, \ \Upsilon_{7,7} = -(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2) + \sigma O,$$

$$\Upsilon_{7,8} = \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \ \Upsilon{7,9} = -\frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{7,10} = -\frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{8,8} = -(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2),$$

$$\Upsilon_{8,9} = \frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{10,10} = \pi^2\mathfrak{Z}_2, \ \Upsilon_{11,11} = -\nu I, \ \Upsilon_{12,12} = -O$$

Along with, the gain matrices are attained as $\mathfrak{K} = \mathfrak{G}\mathfrak{L}^{-1}$.

Proof: Consider the Lyapunov-Krasovskii functional as

$$\mathscr{V}(t) = \sum_{i=1}^{5} \mathscr{V}_i(t) \tag{10}$$

where

$$\mathscr{V}_1(t) = \psi^T(t)\mathfrak{P}_i\psi(t) + \int_{t-\rho(t)}^{t} \psi^T(s)\mathfrak{Q}_1\psi(\lambda)d\lambda + \int_{t-\rho}^{t} \psi^T(\lambda)\mathfrak{Q}_2\psi(\lambda)d\lambda, \tag{11}$$

$$\mathscr{V}_2(t) = \rho\int_{-\rho}^{0}\int_{t+\theta}^{t} \dot{\psi}^T(\lambda)\mathfrak{R}_1\dot{\psi}(\lambda)d\lambda d\theta + \int_{-\rho}^{0}\int_{t+\theta}^{t} \psi^T(\lambda)\mathfrak{R}_2\psi(\lambda)d\lambda d\theta \tag{12}$$

$$\mathscr{V}_3(t) = \int_{t-\eta}^{} \psi^T(\lambda)\mathfrak{Z}_1\psi(\lambda)d\lambda + \eta\int_{-\eta}^{0}\int_{t+\theta}^{t} \dot{\psi}^T(\lambda)\mathfrak{Z}_2\dot{\psi}(\lambda)d\lambda d\theta \tag{13}$$

Finding the time-derivative of (10) along the trajectory of system (9), one can get

$$L\mathscr{V}_1(t) = 2\psi^T(t)\mathfrak{P}_i\dot{\psi}(t) + \psi^T(t)\sum_{j=1}^{N} \alpha_{ij}(\delta)\mathfrak{P}_j\psi(t) + \psi^T(t)\mathfrak{Q}_1\psi(t)$$

$$- (1-\mu)\psi^T(t-\rho(t))\mathfrak{Q}_1\psi(t-\rho(t)) + \psi^T(t)\mathfrak{Q}_3\psi(t) - \psi^T(t-\rho)\mathfrak{Q}_3\psi(t-\rho) \tag{14}$$

$$L\mathscr{V}_2(t) = \rho^2\dot{\psi}^T(t)\mathfrak{R}_1\dot{\psi}(t) - \rho\int_{t-\rho}^{t} \dot{\psi}^T(s)\mathfrak{R}_1\dot{\psi}(\lambda)d\lambda + \rho\psi^T(t)\mathfrak{R}_2\psi(t) - \int_{t-\rho}^{t} \psi^T(\lambda)\mathfrak{R}_2\psi(\lambda)d\lambda \tag{15}$$

$$L\mathscr{V}_3(t) = \psi^T(\lambda)\mathfrak{Z}_1\psi(\lambda) - \psi^T(t-\eta)\mathfrak{Z}_1\psi(t-\eta) + \eta^2\dot{\psi}^T(t)\mathfrak{Z}_2\dot{\psi}(t) - \eta\int_{t-\eta}^{t} \dot{\psi}^T(\lambda)\mathfrak{Z}_2\dot{\psi}(\lambda)d\lambda \tag{16}$$

From (15) and lemma 2.2 we have

$$-\int_{t-\rho}^{t} \psi^T(\lambda)\mathfrak{R}_2\psi(\lambda)d\lambda = -\int_{t-\rho(t)}^{t} \psi^T(\lambda)\mathfrak{R}_2\psi(\lambda)d\lambda - \int_{t-\rho}^{t-\tau(t)} \psi^T(\lambda)\mathfrak{R}_2\psi(\lambda)d\lambda$$

$$\leq -\frac{1}{\rho}\begin{bmatrix} \int_{t-\rho}^{t-\rho(t)} \psi(\lambda)d\lambda \\ \int_{t-\rho(t)}^{t} \psi(\lambda)d\lambda \end{bmatrix}^T \begin{bmatrix} \mathfrak{R}_2 & 0 \\ 0 & \mathfrak{R}_2 \end{bmatrix} \begin{bmatrix} \int_{t-\rho}^{t-\rho(t)} \psi(\lambda)d\lambda \\ \int_{t-\rho(t)}^{t} \psi(\lambda)d\lambda \end{bmatrix} \tag{17}$$

Let us consider

$$\varphi_1(t) = \int_{t-\rho(t)}^{t} \dot{\psi}(\lambda)d\lambda, \varphi_2(t) = \int_{t-\rho}^{t-\rho(t)} \dot{\psi}(\lambda)d\lambda, \tag{18}$$

with $0 < \rho(t) < \rho$ and by lemma 2.2 and 2.3, we have

$$-\rho \int_{t-\rho}^{t} \dot{\psi}^T(\lambda)$$

$$\mathfrak{R}_1 \dot{\psi}(\lambda)d\lambda = -\int_{t-\rho(t)}^{t} \dot{\psi}^T(\lambda)\mathfrak{R}_1 \dot{\psi}(\lambda)d\lambda - \int_{t-\rho}^{t-\rho(t)} \dot{\psi}^T(\lambda)\mathfrak{R}_1 \dot{\psi}(\lambda)d\lambda$$

$$\leq -\frac{1}{\rho(t)} \left(\int_{t-\rho(t)}^{t} \dot{\psi}(\lambda)d\lambda \right)^T \mathfrak{R}_1 \left(\int_{t-\rho(t)}^{t} \dot{\psi}(\lambda)d\lambda \right) - \frac{1}{\rho - \rho(t)} \left(\int_{t-\rho}^{t-\rho(t)} \dot{\psi}(\lambda)d\lambda \right)^T \mathfrak{R}_1$$

$$\left(\int_{t-\rho}^{t-\rho(t)} \dot{\psi}(\lambda)d\lambda \right)$$

(19)

$$= \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \end{bmatrix}^T \begin{bmatrix} \mathfrak{R}_1 & \mathfrak{S} \\ \star & \mathfrak{R}_1 \end{bmatrix} \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \end{bmatrix}$$

In specific when $\rho(t) = 0$ or $\rho(t) = \rho(t)$, we have $\varphi_1(t) = 0$ or $\varphi_2(t) = 0$. Thus,

$$(20) \qquad -\rho \int_{t-\rho}^{t} \dot{\psi}^T(\lambda)\mathfrak{R}_1 \dot{\psi}(\lambda)d\lambda \leq \kappa^T(t)\Omega\kappa(t)$$

where $\kappa(t) = \begin{bmatrix} \psi^T(t) & \psi^T(t-\rho(t)) & \psi^T(t-\rho) \end{bmatrix}^T, \Omega = \begin{bmatrix} -\mathfrak{R}_1 & \mathfrak{R}_1 - \mathfrak{S} & \mathfrak{S} \\ \star & -2\mathfrak{R}_1 + \mathfrak{S} + \mathfrak{S}^T & -\mathfrak{S} + \mathfrak{R}_1 \\ \star & \star & -\mathfrak{R}_1 \end{bmatrix}$

By making use of lemma 2.5, (16) can be rewritten as

$$-\eta \int_{t-\eta}^{t-\eta(t)} \dot{\psi}^T(\lambda)\mathfrak{Z}_2 \dot{\psi}(\lambda)d\lambda$$

$$\leq - \begin{bmatrix} \psi(t-\eta(t)) \\ \psi(t-\eta) \\ \frac{1}{\eta}\int_{t-\eta}^{t-\eta(t)}\psi(\lambda)d\lambda \end{bmatrix}^T \begin{bmatrix} \mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & -\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & \frac{-\pi^2}{2}\mathfrak{Z}_2 \\ -\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & Z_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & \frac{-\pi^2}{2}\mathfrak{Z}_2 \\ -\frac{\pi^2}{2}\mathfrak{Z}_2 & -\frac{\pi^2}{2}\mathfrak{Z}_2 & \pi^2\mathfrak{Z}_2 \end{bmatrix}$$

$$(21) \qquad \times \begin{bmatrix} \psi(t-\eta(t)) \\ \psi(t-\eta) \\ \frac{1}{\eta}\int_{t-\eta}^{t-\eta(t)}\psi(\lambda)d\lambda \end{bmatrix}$$

and

$$- \eta \int_{t-\eta(t)}^{t} \dot{\psi}^T(\lambda) \mathfrak{Z}_2 \psi(\lambda) d\lambda$$

$$\leq - \begin{bmatrix} \psi(t) \\ \psi(t-\eta(t)) \\ \frac{1}{\eta}\int_{t-\eta(t)}^{t}\psi(\lambda)d\lambda \end{bmatrix}^T \begin{bmatrix} \mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & -\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & -\frac{-\pi^2}{2}\mathfrak{Z}_2 \\ -\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & \mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & -\frac{-\pi^2}{2}\mathfrak{Z}_2 \\ -\frac{-\pi^2}{2}\mathfrak{Z}_2 & -\frac{-\pi^2}{2}\mathfrak{Z}_2 & \pi^2\mathfrak{Z}_2 \end{bmatrix}$$

$$(22) \qquad \begin{bmatrix} \psi(t) \\ \psi(t-\eta(t)) \\ \frac{1}{\eta}\int_{t-\eta(t)}^{t}\psi(\lambda)d\lambda \end{bmatrix}$$

For any matrix G, we have

$$2\{\dot{\psi}^T(t)G[-\dot{\psi}(t)+g(\psi(t))+c_1(\Xi_1\otimes A_i)\psi(t)+c_2(\Xi_2\otimes B_i)\psi(t-\rho(t))+\mathfrak{K}\psi(t-\eta(t))$$

$$(23) \qquad +\mathfrak{K}\psi(i_{kh})]\} = 0$$

By assumption 2.1, for any $\nu > 0$, we have

$$(24) \qquad \nu \begin{bmatrix} \psi(t) \\ g(\psi(t)) \end{bmatrix}^T \begin{bmatrix} \bar{\mathfrak{X}} & \bar{\mathfrak{Y}} \\ * & I \end{bmatrix} \begin{bmatrix} \psi(t) \\ g(\psi(t)) \end{bmatrix} \leq 0$$

From (11)-(24), we have

$$(25) \qquad E\{L\mathscr{V}(t)\} \leq \Psi^T(t)\Upsilon(\delta)\Psi(t) < 0$$

where $\Psi^T(t) = \begin{bmatrix} \psi^T(t) & \dot{\psi}(t) & \psi(t-\rho(t)) & \psi(t-\rho) & \frac{1}{\rho}\int_{t-\rho}^{t-\rho(t)}\psi^T(\lambda)d\lambda & \int_{t-\rho(t)}^{t}\psi^T(\lambda)d\lambda \end{bmatrix}$ $\psi^T(t-\eta(t)) \ \psi(t-\eta) \ \frac{1}{\eta}\int_{t-\eta}^{t-\eta(t)}\psi^T(\lambda)d\lambda \ \int_{t-\eta(t)}^{t}\psi^T(\lambda)d\lambda \ \bar{g}(\psi(t)) \ \psi(i_kh) \end{bmatrix}$ and $\Upsilon(\delta)$ is given in (9).

Theorem 3.2. *The SMJCDNs, (8) is asymptotically synchronized if there exist matrices $\mathfrak{P}_i > 0, \mathfrak{Q}_1 > 0, \mathfrak{Q}_2 > 0, \mathfrak{R}_1 > 0, \mathfrak{R}_2 > 0, \mathfrak{Z}_1 > 0, \mathfrak{Z}_2 > 0$, matrices $\mathfrak{S}, \mathfrak{L}, \mathfrak{G}$ such that the subsequent LMIs hold*

$$(26) \qquad \Upsilon_{(i,j),\bar{w}} < 0, i = 1, 2, \cdots, s$$

where

$$\Upsilon_{(1,1),\bar{w}} = \mathfrak{Q}_1 + \mathfrak{Q}_2 + \rho\mathfrak{R}_2 + \mathfrak{Z}_1 - \mathfrak{R}_1 - (\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2) - v\bar{\mathfrak{X}} + \sum_{j=1}^{N} \alpha_{i,j}\mathfrak{P}_j, \ \Upsilon_{1,2} = \mathfrak{P}_i,$$

$$\Upsilon_{1,3} = \mathfrak{R}_1 - \mathfrak{S}, \ \Upsilon_{1,4} = \mathfrak{S}, \ \Upsilon_{1,7} = \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \ \Upsilon_{1,10} = \frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{1,11} = -v\bar{\mathfrak{Y}},$$

$$\Upsilon_{2,2} = -2\mathfrak{S} - \rho^2\mathfrak{R}_1, \ \Upsilon_{2,3} = c_2\mathfrak{S}(\Xi_2 \otimes B_i), \ \Upsilon_{2,7} = \mathfrak{L}, \ \Upsilon_{2,11} = \mathfrak{S}, \ \Upsilon_{2,12} = \mathfrak{L},$$

$$\Upsilon_{3,3} = (1-\mu)\mathfrak{Q}_1 - 2\mathfrak{R}_1 + \mathfrak{S} + \mathfrak{S}^T, \ \Upsilon_{3,4} = -\mathfrak{S} + \mathfrak{R}_2, \ \Upsilon_{4,4} = -\mathfrak{R}_2, \ \Upsilon_{5,5} = -\frac{1}{\rho}\mathfrak{R}_2,$$

$$\Upsilon_{6,6} = -\frac{1}{\rho}\mathfrak{R}_2, \ \Upsilon_{7,7} = -(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2) + \sigma O, \ \Upsilon_{7,8} = \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \ \Upsilon_{7,9} = -\frac{\pi^2}{2}\mathfrak{Z}_2,$$

$$\Upsilon_{7,10} = -\frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{8,8} = -(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2), \ \Upsilon_{8,9} = \frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{10,10} = \pi^2\mathfrak{Z}_2,$$

$$\Upsilon_{11,11} = -vI, \ \Upsilon_{12,12} = -O$$

Along with, the gain matrices are attained as $K = \mathfrak{S}\mathfrak{L}^{-1}$.

In the upcoming theorem, the results in the preceding theorem was enlarged with non-fragile controller for the system (8).

Theorem 3.3. *The SMJCDNs (8) is asymptotically synchronized if there exist matrices* $\mathfrak{P}_i >$ $0, \mathfrak{Q}_1 > 0, \mathfrak{Q}_2 > 0, \mathfrak{R}_1 > 0, \mathfrak{R}_2 > 0, \mathfrak{Z}_1 > 0, \mathfrak{Z}_2 > 0$ *and matrix* $\mathfrak{S}, \mathfrak{L}, \mathfrak{S}$ *and scalars* ς_1, ς_2 *such that the subsequent LMIs hold*

(27)
$$\begin{bmatrix} \Upsilon_{(i,j),\bar{w}} & \mathfrak{U}_1 & \varsigma_1\mathfrak{V}_1 & \mathfrak{U}_2 & \varsigma_2\mathfrak{V}_2 \\ * & -\varsigma_1I & 0 & 0 & 0 \\ * & * & -\varsigma_1I & 0 & 0 \\ * & * & * & -\varsigma_2I & 0 \\ * & * & * & * & -\varsigma_2I \end{bmatrix} < 0, i = 1, 2, \cdots, s$$

where

$$\Upsilon_{(1,1),\bar{w}} = \mathfrak{Q}_1 + \mathfrak{Q}_2 + \rho \mathfrak{R}_2 + \mathfrak{Z}_1 - \mathfrak{R}_1 - (\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2) - v\bar{\mathfrak{X}} + \sum_{j=1}^{N} \alpha_{ij,\bar{w}}\mathfrak{P}_j, \ \Upsilon_{1,2} = \mathfrak{P}_i,$$

$$\Upsilon_{1,3} = \mathfrak{R}_1 - \mathfrak{S}, \ \Upsilon_{1,4} = \mathfrak{S}, \ \Upsilon_{1,7} = \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \ \Upsilon_{1,10} = \frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{1,11} = -v\bar{\mathfrak{Y}},$$

$$\Upsilon_{2,2} = -2\mathfrak{S} - \rho^2\mathfrak{R}_1, \ \Upsilon_{2,3} = c_2\mathfrak{S}(\Xi_2 \otimes B_i), \ \Upsilon_{2,7} = \mathfrak{L}, \ \Upsilon_{2,11} = \mathfrak{S},$$

$$\Upsilon_{2,12} = \mathfrak{L}, \ \Upsilon_{3,3} = (1-\mu)\mathfrak{Q}_1 - 2\mathfrak{R}_1 + \mathfrak{S} + \mathfrak{S}^T, \ \Upsilon_{3,4} = -\mathfrak{S} + \mathfrak{R}_2,$$

$$\Upsilon_{4,4} = -\mathfrak{R}_2, \ \Upsilon_{5,5} = -\frac{1}{\rho}\mathfrak{R}_2, \ \Upsilon_{6,6} = -\frac{1}{\rho}\mathfrak{R}_2, \ \Upsilon_{7,7} = -(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2) + \sigma O,$$

$$\Upsilon_{7,8} = \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \ \Upsilon_{7,9} = -\frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{7,10} = -\frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{8,8} = -(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2),$$

$$\Upsilon_{8,9} = \frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{10,10} = \pi^2\mathfrak{Z}_2, \ \Upsilon_{11,11} = -vI, \ \Upsilon_{12,12} = -O$$

Along with, the gain matrices are attained as $\mathfrak{L} = \mathfrak{S}\mathfrak{K}^{-1}$.

Proof: By making use of $\Delta K(t) = \mathfrak{U}F(t)\mathfrak{V}$, LMI in (26) can be written as

$$\Upsilon_{(i,j),\bar{w}} + \mathfrak{U}_1 F(t)\mathfrak{V}_1 + \mathfrak{V}_1^T F(t)\mathfrak{U}_1^T + \mathfrak{U}_2 F(t)\mathfrak{V}_2 + \mathfrak{V}_2^T F(t)\mathfrak{U}_2,$$

where

$$\mathfrak{U}_1 = \begin{bmatrix} \underbrace{0\cdots 0}_{6} & G\mathfrak{U} & \underbrace{0\cdots 0}_{5} \end{bmatrix}, \ \mathfrak{U}_2 = \begin{bmatrix} \underbrace{0\cdots 0}_{11} & G\mathfrak{U} \end{bmatrix}, \ \mathfrak{V}_1 = \begin{bmatrix} 0 & \mathfrak{V} & \underbrace{0\cdots 0}_{10} \end{bmatrix}, \mathfrak{V}_2 = \begin{bmatrix} \underbrace{0\cdots 0}_{11} & \mathfrak{V} \end{bmatrix}$$

Then from lemma 2.4, we have $\Upsilon_{(i,j),\bar{w}} + \varsigma^{-1}\mathfrak{U}_1\mathfrak{U}_1^T + \varsigma_1\mathfrak{V}_1^T\mathfrak{V}_1 + \varsigma^{-1}\mathfrak{U}_2\mathfrak{U}_2^T + \varsigma_1\mathfrak{V}_2^T\mathfrak{V}_2$.

Thus, one can get

$$(28) \quad \begin{bmatrix} \Upsilon_{(i,j),\bar{w}} & \mathfrak{U}_1 & \varsigma_1\mathfrak{V}_1 & \mathfrak{U}_2 & \varsigma_2\mathfrak{V}_2 \\ * & -\varsigma_1 I & 0 & 0 & 0 \\ * & * & -\varsigma_1 I & 0 & 0 \\ * & * & * & -\varsigma_2 I & 0 \\ * & * & * & * & -\varsigma_2 I \end{bmatrix}, i = 1, 2, \cdots, s$$

If the semi-Markovian jumping parameters are not considered then the the above theorem can be modified as

Theorem 3.4. *The SMJCDNs (8) is asymptotically synchronized if there exist matrices $\mathfrak{P}_i > 0, \mathfrak{Q}_1 > 0, \mathfrak{Q}_2 > 0, \mathfrak{R}_1 > 0, \mathfrak{R}_2 > 0, \mathfrak{Z}_1 > 0, \mathfrak{Z}_2 > 0$ and matrix $\mathfrak{S}, \mathfrak{L}, \mathfrak{G}$ and scalars ς_1, ς_2 such that the subsequent LMIs hold*

(29)
$$\begin{bmatrix} \Upsilon_{i,j} & \mathfrak{U}_1 & \varsigma_1\mathfrak{V}_1 & \mathfrak{U}_2 & \varsigma_2\mathfrak{V}_2 \\ * & -\varsigma_1 I & 0 & 0 & 0 \\ * & * & -\varsigma_1 I & 0 & 0 \\ * & * & * & -\varsigma_2 I & 0 \\ * & * & * & * & -\varsigma_2 I \end{bmatrix} < 0, i = 1, 2, \cdots, s$$

where

$$\Upsilon_{(1,1)} = \mathfrak{Q}_1 + \mathfrak{Q}_2 + \rho\mathfrak{R}_2 + \mathfrak{Z}_1 - \mathfrak{R}_1 - (\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2) - v\bar{\mathfrak{X}}, \ \Upsilon_{1,2} = \mathfrak{P},$$

$$\Upsilon_{1,3} = \mathfrak{R}_1 - \mathfrak{S}, \ \Upsilon_{1,4} = \mathfrak{S}, \ \Upsilon_{1,7} = \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \ \Upsilon_{1,10} = \frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{1,11} = -v\bar{\mathfrak{Y}},$$

$$\Upsilon_{2,2} = -2\mathfrak{G} - \rho^2\mathfrak{R}_1, \ \Upsilon_{2,3} = c_2\mathfrak{G}(\Xi_2 \otimes B), \ \Upsilon_{2,7} = \mathfrak{L}, \ \Upsilon_{2,11} = \mathfrak{G},$$

$$\Upsilon_{2,12} = \mathfrak{L}, \ \Upsilon_{3,3} = (1-\mu)\mathfrak{Q}_1 - 2\mathfrak{R}_1 + \mathfrak{S} + \mathfrak{S}^T, \ \Upsilon_{3,4} = -\mathfrak{S} + \mathfrak{R}_2,$$

$$\Upsilon_{4,4} = -\mathfrak{R}_2, \ \Upsilon_{5,5} = -\frac{1}{\rho}\mathfrak{R}_2, \ \Upsilon_{6,6} = -\frac{1}{\rho}\mathfrak{R}_2, \ \Upsilon_{7,7} = -(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2) + \sigma O,$$

$$\Upsilon_{7,8} = \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \ \Upsilon_{7,9} = -\frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{7,10} = -\frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{8,8} = -(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2),$$

$$\Upsilon_{8,9} = \frac{\pi^2}{2}\mathfrak{Z}_2, \ \Upsilon_{10,10} = \pi^2\mathfrak{Z}_2, \ \Upsilon_{11,11} = -vI, \ \Upsilon_{12,12} = -O$$

Along with, the gain matrices are attained as $\mathfrak{L} = \mathfrak{G}\mathfrak{K}^{-1}$.

4. NUMERICAL EXAMPLE

This section affords numerical examples to validate the results.

Example 4.1. *Let us consider the isolated node of the dynamical network as the Chua's circuit:*

(30)
$$\dot{w} = f(w) = \begin{bmatrix} a(w_2 - w_1 - h(w_1)) \\ w_1 - w_2 + w_3 \\ \gamma w_2 \end{bmatrix}$$

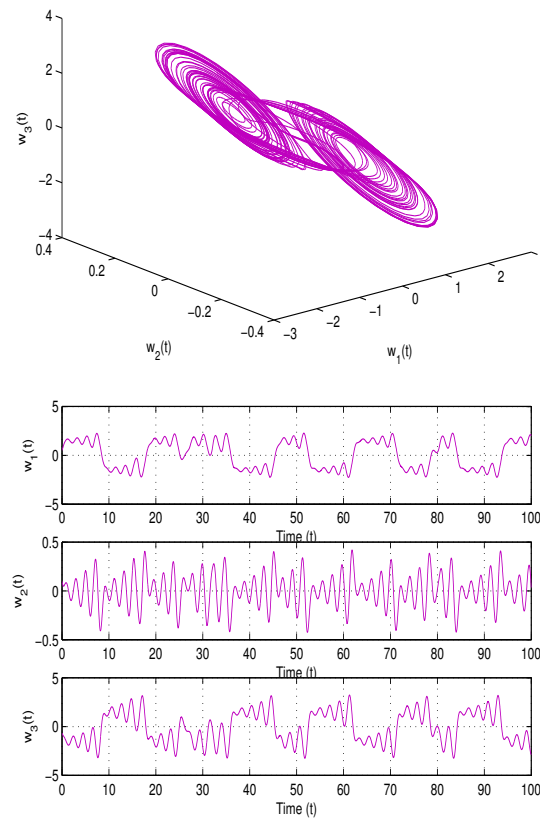FIGURE 1. Choatic attractor and State trajectories of Chua's system

where $h(w_1) = nw_1 + 0.5(m-n)(|s_1+1| - |s_1-1|)$ and the parameters $a = 9, \gamma = 100/7, m = -8/7, n = -5/7$. Consider the SMJCDN (8) with

$$A_1 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, A_2 = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix} B_1 = \begin{bmatrix} -1.2 & 0 \\ 0 & -1.2 \end{bmatrix}, B_2 = \begin{bmatrix} 2.1 & 0 \\ 0 & 2.1 \end{bmatrix}$$

$$\Xi_1 = \begin{bmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 \\ 0.1 & 0.1 & -0.2 \end{bmatrix}, \Xi_2 = \begin{bmatrix} -3 & 1.5 & 1.5 \\ 0.5 & -1 & 0.5 \\ 0.8 & 0.8 & -1.6 \end{bmatrix}$$

The transition rates are defined as $\alpha_{11}(\delta) \in [-2.2, -1.5]$ and $\alpha_{22}(\delta) \in [-2.2, -1.5]$. Without loss of generality, we have $\alpha_{11,1} = -2.2, \alpha_{11,2} = -1.5, \ \alpha_{22,1} = -2.2, \ \alpha_{22,2} = -1.5$ While
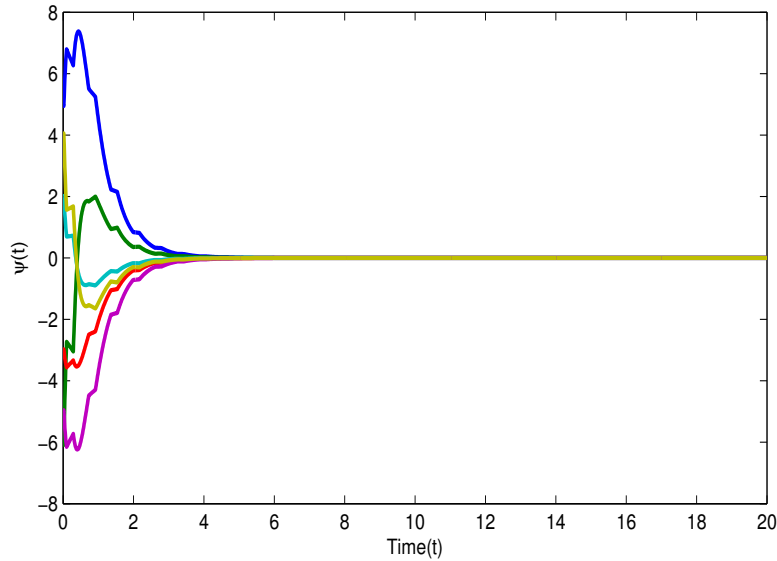
FIGURE 2. State trajectories of the error system

resolving the LMIs in Theorem 4, the gain matrices and triggering matrix are

$$\mathfrak{K}_1 = \begin{bmatrix} -0.0161 & -0.2161 \\ 0.0284 & 0.1575 \end{bmatrix}, \mathfrak{K}_2 = \begin{bmatrix} -0.0263 & 0.0165 \\ 0.0292 & 0.1504 \end{bmatrix}, \mathfrak{K}_3 = \begin{bmatrix} -0.0233 & 0.0164 \\ 0.0290 & 0.1525 \end{bmatrix},$$

$$O = \begin{bmatrix} 0.2731 & 0.0000 & 0 & 0 & 0 & 0 \\ 0.0000 & 0.2731 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2731 & 0.0000 & 0 & 0 \\ 0 & 0 & 0.0000 & 0.2731 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2731 & 0.0000 \\ 0 & 0 & 0 & 0 & 0.0000 & 0.2731 \end{bmatrix}$$

The Chaotic attractor of the Chua's circuit and state trajectories are given in Figure 1 and Figure 2. By providing the designed controller to the error system, the state trajectories are depicted in Figure 3. The releasing instants for the event-triggered controller was given in Figure 4.

5. CONCLUSION

In this paper, the synchronization problem for SMJCDNs with non-fragile controller and event-triggered controller was discussed in order to achieve the benefits of both controllers. The MATLAB LMI tool box is used to solve the derived LMIs. Eventually, the applicability of the designed controller was examined for synchronization of Chua's circuit through the simulation results.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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