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SEGREGATED EXTENSION OF GRAPHS

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Abstract. A graph in which any two adjacent vertices have distinct degrees is totally segregated. In this article segregating sequence, which is a new tool for finding segregated extension of given graph is introduced. If G is an undirected graph which contains a vertex v, then the graph $G \circ v$ is obtained from G by adding a new vertex v' which is connected to all the neighbors of v. More generally, if $v_1, v_2, \cdots, v_n$ are the vertices of G and $t = (t_1, t_2, \cdots, t_n)$ is a vector of positive integers then $H = G \circ t$ is constructed by substituting for each v_i an independent set of t_i vertices $v_i^1, v_i^2, \cdots, v_i^{t_i}$ and joining v_i^s with v_j^t if and only if v_i and v_j are adjacent in G. If G is not totally segregated and $G \circ t$ is totally segregated, then the sequence t is a *segregating sequence* of G. Here it is proved that any graph can be embedded as an induced subgraph in a totally segregated graph. Further, segregating sequence for many classes of graphs are determined.

Keywords: segregated graph; multiplication of vertices; segregated extension of graph; segregating sequence.
2010 AMS Subject Classification: 05C07, 05E30.

1. Introduction

It is known that (Konig [11]) any graph G of maximum degree $\Delta(G)$ is an induced subgraph of some $\Delta(G)$-regular graph H. Erdos and Kelly [6] determined the minimum number of vertices, the induced regulation number, which is to be added to a graph G to obtain such a $\Delta(G)$-regular supergraph H. The latter was also extended to digraphs by Beineke and Pippert [3]. The

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regulation number of a graph G is the minimum number of vertices which must be added to G to construct a $\Delta(G)$-regular supergraph H. In this case, G need not be an induced subgraph of H. Regulation number of graphs was introduced by Akiyama, Era and Harary [1] and was further studied by Akiyama and Harary [2] and Harary and Schmidt [9]. Analogous concepts for digraphs and multigraphs were introduced by Harary and Karabed [8] and Chartrand, Harary and Ollermann [5] respectively. In [4] Buckley and Harary studied the problem of embedding a highly irregular graph G as an induced subgraph in a self-centered graph H of smallest possible order so that H is regular with the same maximum degree as G.

A connected graph G is totally segregated if $deg_G u \neq deg_G v$, for every edge $uv \in E(G)$. The class of totally segregated graphs was studied by Jackson and Entringer [10]. In this paper our attempt is to find segregated extension of some graph.

2. Segregating Sequence

The concept of multiplication of vertices was given by Golumbic [7] as follows. If G is an undirected graph which contains a vertex v, then the graph $G \circ v$ is obtained from G by adding a new vertex v' which is connected to all the neighbors of v. More generally, if $v_1, v_2, \cdots, v_n$ are the vertices of G and $t = (t_1, t_2, \cdots, t_n)$ is a vector of non negative integers then $H = G \circ t$ is constructed by substituting for each v_i an independent set of t_i vertices $v_i^1, v_i^2, \cdots, v_i^{t_i}$ and joining v_i^s with v_j^t if and only if v_i and v_j are adjacent in G. We say that H is obtained from G by *multiplication of vertices*. This definition allows $t_i = 0$, in which case H includes no copy of v_i. Thus every induced subgraph of G can be obtained by multiplication of the appropriate $(0,1)$-valued vector.

Definition 2.1. *Let $G = (V,E)$ be a graph where $V = \{v_1, v_2, \cdots, v_n\}$ and $t = (t_1, t_2, \cdots, t_n)$ be a sequence of positive integers. If $G \circ t$ is totally segregated, then the sequence t is a segregating sequence of G and $G \circ t$ is the segregated extension of G which is denoted by G^S . The sequence $t = (t_1, t_2, \cdots, t_n)$ is said to be a minimal segregating sequence of the graph G if no sequence $t' = (t'_1, t'_2, \cdots, t'_n)$ with $\sum_{i=1}^{n} t'_i < \sum_{i=1}^{n} t_i$ is a segregating sequence of G. If t is minimal segregating sequence of G, $G \circ t$ is called minimal segregated-extension of G which is denoted by G^{S^-}. The sequence $t' = (t'_1, t'_2, \cdots, t'_n)$ is said to be a perfect segregating sequence of the graph G if the* *graph $G \circ t'$ is totally segregated with $\Delta(G \circ t') = \Delta(G)$ and $G \circ t'$ is the perfect segregated-extension of G which is denoted by G^{S^*}. The graph G which can be segregated using a perfect segregating sequence is called perfect segregated-extendable graph.*

Remark 2.1.

- To define segregating sequence of the graph G ordering the vertex set $V(G)$ is important.
- For a totally segregated graph G, the segregating sequence is $t = (1, 1, \cdots, 1)$. Here $G^S = G$.
- If $G^S = (V^S, E^S)$, $|V^S| = t_1 + t_2 + \cdots + t_n$

Proposition 2.1. *Any graph G has segregated extension.*

Proof. Suppose G is not totally segregated. If G is P_2, $(2,1)$ is its minimal segregating sequence. Suppose $G \ncong P_2$. An edge uv of $E(G)$ is said to be balanced if $deg_G u = deg_G v$. Since G is not totally segregated, it has at least one balanced edge. Let uv be one of the balanced edges of G . Multiply the vertex u, $\Delta(G)$ times and let the resultant graph be G_1. Then $deg_{G_1} v > \Delta(G)$. Let b be the number of balanced edges in G and b_1 be the number of balanced edges in G_1. It is clear that $b_1 < b$, since in each step balanced edges become unbalanced but no unbalanced edges become balanced. If G_1 is not totally segregated, let u_1v_1 be one of the balanced edges of G_1. Multiply the vertex u_1, $\Delta(G_1)$ times and let the resultant graph be G_2. Continue this process until no such balanced edges remain. Since G is finite, the process will end in finite number of steps. Then the resulting graph is totally segregated graph. □

Remark 2.2.

1. Let $G = (V,E)$ be a graph. If there exists a balanced edge uv such that $deg_G u = deg_G v = \Delta(G)$, then G is not perfect segregated-extendable.

2. If a graph G which is not totally segregated has a universal vertex, then it is not perfect segregated-extendable.

3. Perfect segregating sequence of a graph may not be minimal segregating sequence and minimal segregating sequence may not be perfect segregating sequence.

Example 2.1. *Take* 3 *copies of* P_4. *Let* $G = (V,E)$ *be the graph obtained by fusing* 3 *copies of end vertex of* P_4 *as in Figure 1.*

The vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$. *Then the sequence* $t = (t_i)$, *where*

$$t_i = \begin{cases} 2 & \text{for } i = 8,9,10 \\ 1 & \text{otherwise} \end{cases}$$

is the segregating sequence of G. Here note that t is a perfect segregating sequence which is not minimal.

The sequence $t = (t_i)$ *where*

$$t_i = \begin{cases} 3 & \text{if } i = 1 \\ 1 & \text{otherwise} \end{cases}$$

is a minimal sgregating sequence which is not perfect.

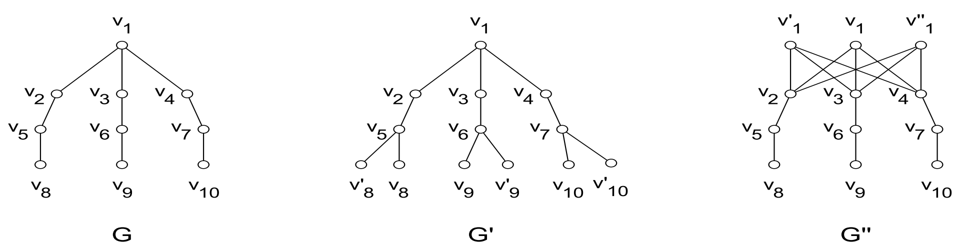

FIGURE 1. G' : Perfect segregated-extension of the graph G, G'' : Minimal segregated-extension of the graph G

Remark 2.3. Regular graphs and path P_n, $n \neq 3$ are not perfect segregated-extendable.

3. SEGREGATED EXTENSIONS OF SOME CLASSES OF GRAPHS

- **Segregating Sequence of Paths**

Let $G = P_n$ be the path on n vertices with vertex set $V = \{v_1, v_2, \cdots, v_n\}$, where v_1 and v_n are end vertices. By Remark 2.2 (1), perfect segregating sequence does not exist for path P_n, $n \neq 3$.

Remark 3.1. To make paths P_n segregated, at least one vertex among 4 consecutive vertices on path P_n, should be multiplied by a number i, where $i \geq 2$.

By Remark 3.1, The segregating sequence $t = (t_i)$ of G given below is minimal.

Case 1. $n = 4k$, $k \geq 1$.

$$t_i = \begin{cases} 2 & \text{if } i = 1, 5, \cdots, 4k-3 \\ 1 & \text{otherwise} \end{cases}$$

Case 2. $n = 4k+1$, $k \geq 1$.

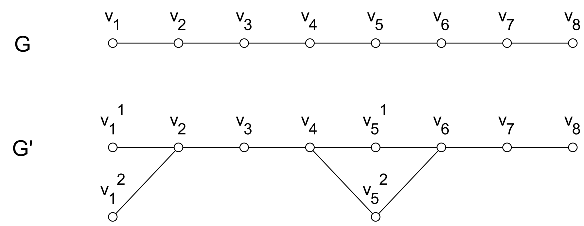

FIGURE 2. $G \cong P_8$, $G' \cong P_8 \circ t$ which is minimal segregated extension of G

$$t_i = \begin{cases} 2 & \text{if } i = 3, 7, \cdots, 4k-1 \\ 1 & \text{otherwise} \end{cases}$$

Case 3. $n = 4k+2$, $k \geq 1$.

$$t_i = \begin{cases} 2 & \text{if } i = 3, 7, \cdots, 4k-1 \\ 1 & \text{otherwise} \end{cases}$$

Case 4. $n = 4k+3$, $k \geq 1$.

$$t_i = \begin{cases} 2 & \text{if } i = 4, 8, \cdots, 4k \\ 1 & \text{otherwise} \end{cases}$$

- **Segregating Sequence of Fused Paths**

  (1) **Paths fused at one end vertex**

Let P_n be a path on n vertices and w is an end vertex. Take $m(\geq 3)$ copies of P_n. Let $G = (V, E)$ be the graph obtained by fusing the m copies of P_n at w which is denoted by $F_w(P_n)^m$. The vertex set $V = \cup_{i=1}^{m} V_i$ where $V_i = \{w, v_{i2}, v_{i3}, \cdots, v_{in}\}$. It is nothing but subdivided star.

Remark 3.2. To make $F_w(P_n)^m$ segregated, at least one vertex among 4 consecutive vertices on any branch of it at w, should be multiplied by a number i, where $i \geq 2$.

Also if $deg\,w = 3$ and if w is the only vertex with multiplicity at least 2 on the path $(w, v_{i2}, v_{i3}, v_{i4})$, $m(w) > 2$. Otherwise degree of any copy of w is 3 and $deg\,v_{i2} = 3$ which is a contradiction.

Segregating sequence of $F_w(P_n)^m$

$G = (V,E)$ where $V = \cup_{i=1}^{m} V_i$, $V_i = \{w, v_{i2}, v_{i3}, \cdots, v_{in}\}$ and V is ordered as $V = \{w, v_{12}, \cdots, v_{1n}, v_{22}, \cdots, v_{2n}, \cdots\cdots, v_{m2}, \cdots, v_{mn}\}$.

Let $t = (t_w, t_{12}, \cdots, t_{1n}, t_{22}, \cdots, t_{2n}, \cdots\cdots, t_{m2}, \cdots, t_{mn})$ be defined as follows.

Case 1. $n = 4k$, $k \geq 1$.

Subcase 1.1 $m = 3$

If $t_w = 3$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 5, 9, \cdots, 4k-3 \text{ and for all } i \\ 1 & \text{otherwise} \end{cases}$

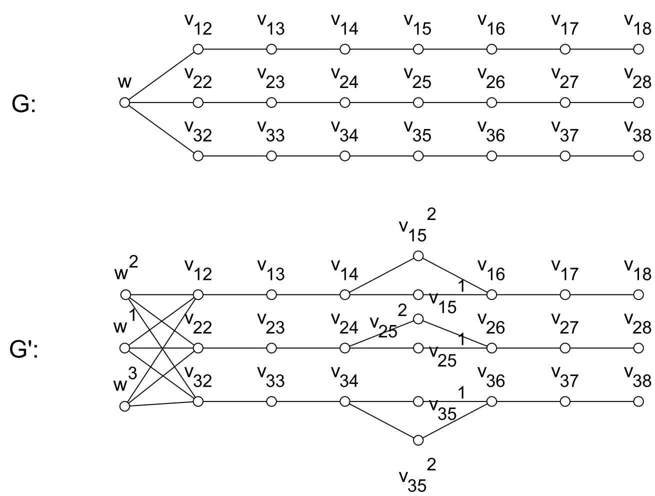

FIGURE 3. $G : F_w(P_8)^3$, $G' : F_w(P_8)^3 \circ t$ which is minimal segregated extension of G

Then t is segregating sequence and it is minimal by Remark 3.2 but not perfect.

If $t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 4, 8, \cdots, 4k \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases}$

Then t is a segregating sequence and it is perfect but not minimal.

Subcase 1.2 $m \geq 4$.

If $t_w = 2$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 5, 9, \cdots, 4k-3 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.2 and perfect.

If $t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 4, 8, \cdots, 4k \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is perfect but not minimal.

Case 2. $n = 4k+1$, $k \geq 1$.

Subcase 2.1 $m = 3$.

If $t_w = 3$ and $t_{ij} = \begin{cases} 2 & \text{if } j = 5, 9, \cdots, 4k+1 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.2 but not perfect.

In this case perfect segregating sequence does not exist.

Subcase 2.2 $m \geq 4$.

If $t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{if } j = 3, 7, \cdots, 4k-1 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.2 and perfect.

Case 3. $n = 4k+2$, $4k+3$, $k \geq 1$,$m \geq 3$.

If $t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 4, 8, \cdots, 4k \text{ and for all i} \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark ref 3.2 and perfect

(2) **Paths fused at two end vertices**

Let P_n be a path on n vertices and u, w are end vertices. Take $m(\geq 3)$ copies of P_n. Let $G = (V, E)$ be the graph obtained by fusing $m(\geq 3)$ copies of end vertices u, w of P_n separately, which is denoted by $F_{u,w}(P_n)^m$. The vertex set $V = \cup_{i=1}^{m} V_i \cup \{u, w\}$ where $V_i = \{u, v_{i2}, v_{i3}, \cdots, v_{i(n-1)}, w\}$.

Remark 3.3. To make the fused paths $F_{u,w}(P_n)^m$ segregated, at least one vertex among 4 consecutive vertices on any $u - w$ path $F_{u,w}(P_n)^m$, should be multiplied by a number i, where $i \geq 2$. Also if $deg\,u = 3$ and if u is the only vertex with multiplicity at least

2 on the path $(u, v_{i2}, v_{i3}, v_{i4})$, $m(u) > 2$. Otherwise degree of any copy of u is 3 and $deg\, v_{i2} = 3$ which is a contradiction. The case is similar for w.

Segregatig sequence of $F_{u,w}(P_n)^m$

$G = (V,E)$ where $V = \cup_{i=1}^{m} V_i$, $V_i = \{u, v_{i2}, v_{i3}, \cdots, v_{i(n-1)}, w\}$ and V is ordered as $V = \{u, v_{12}, \cdots, v_{1(n-1)}, v_{22}, \cdots, v_{2(n-1)}, \cdots\cdots, v_{m2}, \cdots, v_{m(n-1)}, w\}$.

Let $t = (t_u, t_{12}, \cdots, t_{1(n-1)}, t_{22}, \cdots, t_{2(n-1)}, \cdots\cdots, t_{m2}, \cdots, t_{m(n-1)}, t_w)$ be defined as follows.

Case 1. $n = 4k$, $k \geq 1$.

Subcase 1.1 $m = 3$

If $t_u = 3, t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 5, 9, \cdots, 4k-3 \text{ and for all } i \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.3 but not perfect.

In this case perfect segregating sequence does not exist.

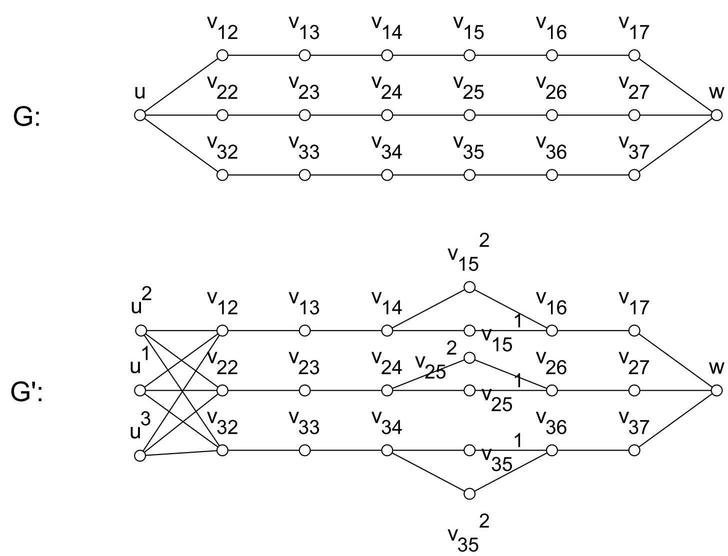

FIGURE 4. $G : F_{u,w}(P_8)^3$, $G' : F_{u,w}(P_8)^3 \circ t$ which is minimal segregated extension of G

Subcase 1.2 $m \geq 4$.

If $t_u = 2, t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 5, 9, \cdots, 4k-3 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.3 and perfect.

Case 2. $n = 4k+1$, $k \geq 1$.

Subcase 2.1 $m = 3$.

If $t_u = 1, t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{if } j = 2,6,\cdots,4k-2 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.2 but not perfect.

In this case prfect segregating sequence does not exist.

Subcase 2.2 $m \geq 4$.

If $t_u = 1, t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{if } j = 3,7,\cdots,4k-1 \text{ and for all } i. \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.2 and perfect.

Case 3. $n = 4k+2$, $k \geq 1$.

Subcase 3.1 $m = 3$.

If $t_u = 3, t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 5,9,\cdots,4k+1 \text{ and for all i} \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.2 but not perfect.

In this case perfect segregating sequence does not exist.

Subcase 3.2 $m \geq 4$.

If $t_u = 1, t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 3,7,\cdots,4k-1 \text{ and for all i} \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.2 and perfect.

Case 4. $n = 4k+3$, $k \geq 1$, $m \geq 3$.

If $t_u = 1, t_w = 1$ and $t_{ij} = \begin{cases} 2 & \text{for } j = 4,8,\cdots,4k \text{ and for all i} \\ 1 & \text{otherwise} \end{cases}$

The segregating sequence t is minimal by Remark 3.2 and perfect.

Remark 3.4. *A segregating sequence of $F_{u,w}(P_n)^3$ is perfect only when $n = 4k+3$, $k \geq 1$.*

- **Segregating Sequence of Cycles**

Let $G = C_n$ be the cycle on n vertices with vertex set $V = \{v_1, v_2, \cdots, v_n\}$. By Remark 2.2 (1), perfect segregating sequence of cycle does not exist.
Let $t = (t_1, t_2, \cdots, t_n)$ is the segregating sequence G.

Remark 3.5. To make paths C_n segregated, at least one vertex among 4 consecutive vertices on cycle C_n, should be multiplied by a number i, where $i \geq 2$.

By Remark 3.5, The segregating sequence $t = (t_i)$ of G given below is minimal.

Case 1. $n = 4k$, $k \geq 1$.

$$t_i = \begin{cases} 2 & \text{if } i = 1, 5, \cdots, 4k-3 \\ 1 & \text{otherwise} \end{cases}$$

Case 2. $n = 4k+1$, $k \geq 1$.

$$t_i = \begin{cases} 2 & \text{if } i = 1, 5, \cdots, 4k-3 \\ 3 & \text{if } i = 4k-1 \\ 1 & \text{otherwise} \end{cases}$$

Case 3. $n = 4k+2$, $k \geq 1$.

$$t_i = \begin{cases} 2 & \text{if } i = 1, 5, \cdots, 4k+1 \\ 1 & \text{otherwise} \end{cases}$$

Case 4. $n = 4k+3$, $k \geq 1$.

$$t_i = \begin{cases} 2 & \text{if } i = 1, 5, \cdots, 4k-3 \\ 3 & \text{if } i = 4k+1 \\ 1 & \text{otherwise} \end{cases}$$

- **Segregating Sequence of Complete Graphs**

Let $G = K_n$ be the complete graph on n vertices with vertex set $V = \{v_1, v_2, \cdots, v_n\}$. Then $t = (t_i)$ is the segregating sequence G where $t_i = i$. Here the segregating sequence t is the minimal but not perfect.

- **Segregating Sequence of Complete K-partite Graphs with same partite size**

Let $G = K_{r,r,\cdots,r}$ be the complete k- partite graph with partite size r. The Vertex set $V(G) = \cup_{i=1}^{k} V_i$, where $V_i = \{v_{i1}, v_{i2}, \cdots, v_{ir}\}$ denote the i^{th} partite set. Then $t = (t_{ij})$ is the segregating sequence of G, where

$$t_{ij} = \begin{cases} i & \text{if } j = 1 \\ 1 & \text{otherwise} \end{cases}$$

Here the segregating sequence t is minimal but not perfect.

- **Segregating Sequence of Petersen Graph** Let $G = (V, E)$ be Petersen graph

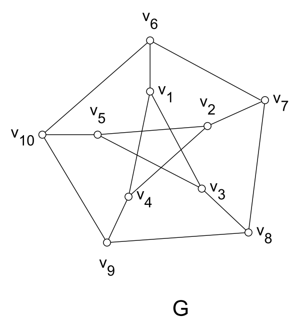

FIGURE 5. Petersen graph G.

and $V = \{v_i\}$, $i = 1, 2, \cdots, 10$. Then $t = (t_i)$ is the segregating sequence where

$$t_i = \begin{cases} 2 & \text{if } i = 1 \\ 3 & \text{if } i = 7, 9 \\ 1 & \text{otherwise} \end{cases}$$

- **Segregating Sequence of Bistar**

Let $G = (V, E)$ be bistar graph where $V = \{u, v, u_1, u_2, \cdots, u_d, v_1, v_2, \cdots, v_d\}$ and $E = \{uv.uu_i, vv_i : u_i, v_i \in V\}$.

Perfect segregating sequence does not exist for G by Remark 2.2 (1). Here $t = (t_u, t_v, \cdots, t_{u_i} \cdots, \cdots, t_{v_i}, \cdots)$ is the minimal segregating sequence of G where $t_w = \begin{cases} 2 & \text{if } w = u_1 \\ 1 & \text{otherwise} \end{cases}$

- **Segregating Sequence of Sun flower Graph**

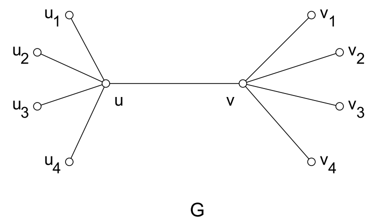

FIGURE 6. Bistar graph G.

Sun flower graph is as described in Figure 7. Let $G = W_{1,n} = (W,E), n \geq 3$ be the wheel graph where $W = \{w\} \cup V$. Let $V = \{v_1, v_2, \cdots, v_n\}$. *Sun flower graph* is the graph $\lambda(W_{1,n})$ with the vertex set $\{w\} \cup V \cup V'$ where $V' = \{v'_i : v_i \in V\}$ which is disjoint from V and with edge set $E \cup \{v_i v'_i, v_{i+1} v'_i : v_i \in V\}$ where $v_{n+1} v'_n$ is replaced by $v_1 v'_n$. Here the sequence $t = (t'_i)$ is the perfect segregating sequence of the sun flower graph

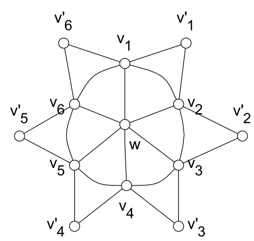

FIGURE 7. Sun flower graph $\lambda(W_{1,n})$.

$\lambda(W_{1,n})$, for $n \geq 8$.

Case 1. $n = 4k$, $k \geq 2$.

$$t'_i = \begin{cases} 2 & \text{for } i = 1,2,5,6,\cdots,4k-7,4k-6,4k-3,4k-2. \\ 1 & \text{otherwise} \end{cases}$$

Case 2. $n = 4k+1$, $k \geq 2$.

$$t'_i = \begin{cases} 2 & \text{for } i = 1,2,5,6,\cdots,4k-7,4k-6,4k-3,4k-2. \\ 3 & \text{for } i = 4k-1 \\ 1 & \text{otherwise} \end{cases}$$

Case 3. $n = 4k+2$, $k \geq 2$.

$$t'_i = \begin{cases} 2 & \text{for } i = 1,2,5,6,\cdots,4k-7,4k-6,4k-2,4k-1. \\ 3 & \text{for } i = 4k-5, 4k \\ 1 & \text{otherwise} \end{cases}$$

Case 4. $n = 4k+3$, $k \geq 2$.

$$t'_i = \begin{cases} 2 & \text{for } i = 1,2,5,6,\cdots,4k-3,4k-2,4k+1. \\ 3 & \text{for } i = 4k+2 \\ 1 & \text{otherwise} \end{cases}$$

Note that in this case the segregating sequence t is perfect as well as minimal. But for $3 \leq n \leq 7$, perfect segregating sequence does not exist.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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