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RATIONAL TYPE CONTRACTION IN CONTROLLED METRIC SPACES

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Abstract: The aim of this paper is to establish a fixed point theorem for rational type contraction in a complete controlled metric space. Our results extend/generalize many pre-existing results in literature. We also provide example which show the usefulness of these results.

Keywords: fixed point theory; rational type contraction; controlled metric space.

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1. Introduction and Preliminaries

Dass and Gupta [26] established first fixed point theorem for rational contractive type conditions in metric space.

Theorem 1.1 (see [26]). Let (X,d) be a complete metric space, and let $\mathcal{T}: X \to X$ be a self-mapping. If there exist $\alpha, \beta \in [0,1)$ with $\alpha + \beta < 1$ such that

$$d(\mathcal{T}x, \mathcal{T}y) \leq \alpha d(x,y) + \beta \frac{[1 + d(x,\mathcal{T}x)]d(y,\mathcal{T}y)}{1 + d(x,y)} \quad (1.1)$$

for all $x, y \in X$, then $\mathcal{T}$ has a unique fixed point $x^* \in X$.

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Nazam *et al.* [27] proved a real generalization of Dass-Gupta fixed point theorem in the frame work of dualistic partial metric spaces.

Czerwik [1] reintroduced a new class of generalized metric spaces, called as b-metric spaces, as generalizations of metric spaces.

Definition 1 ([1]). Let X be a nonempty set and $s \geq 1$. A function $d_b: X \times X \longrightarrow [0, \infty)$ is said to be a b -metric if for all $\sigma, \varsigma, \omega \in X$,

(b1). $d_b(\sigma, \varsigma) = 0$ iff $\sigma = \varsigma$

(b2). $d_b(\sigma, \varsigma) = d_b(\varsigma, \sigma)$ for all $\sigma, \varsigma \in X$

(b3). $d_b(\sigma, \omega) \leq s[d_b(\sigma, \varsigma) + d_b(\varsigma, \omega)]$

The pair (X, d_b) is then called a b-metric space. Subsequently, many fixed point results on such spaces were given (see [2–7]).

Kamran et al. [8] initiated the concept of extended b-metric spaces.

Definition 2. Let X be a nonempty set and $p: X \times X \longrightarrow [1, \infty)$ be a function. A function $d_e: X \times X \longrightarrow [0, \infty)$ is called an extended b -metric if for all $\sigma, \varsigma, \omega \in X$,

(e1). $d_e(\sigma, \varsigma) = 0$ iff $\sigma = \varsigma$

(e2). $d_e(\sigma, \varsigma) = d_e(\varsigma, \sigma)$ for all $\sigma, \varsigma \in X$

(e3). $d_e(\sigma, \omega) \leq p(\sigma, \omega)[d_e(\sigma, \varsigma) + d_e(\varsigma, \omega)]$

The pair(X, d_e) is called an extended b-metric space.

Very recently, a new kind of a generalized b-metric space was introduced by Mlaiki et al. [9].

Definition 3 ([9]). Let X be a nonempty set and $p: X \times X \longrightarrow [1, \infty)$ be a function. A function $d_c: X \times X \longrightarrow [0, \infty)$ is called a controlled metric if for all $\sigma, \varsigma, \omega \in X$,

(c1). $d_c(\sigma, \varsigma) = 0$ iff $\sigma = \varsigma$

(c2). $d_c(\sigma, \varsigma) = d_c(\varsigma, \sigma)$ for all $\sigma, \varsigma \in X$

(c3). $d_c(\sigma, \omega) \leq p(\sigma, \omega)[d_c(\sigma, \varsigma) + d_c(\varsigma, \omega)]$

The pair (X, d_c) is called a controlled metric space (see also [10]).

The Cauchy and convergent sequences in controlled metric type spaces are defined in this way.

Definition 4 ([9]). Let (X, d_c) be a controlled metric space and $\{\sigma_n\}_{n\geq 0}$be a sequence in D. Then,

1. The sequence $\{\sigma_n\}$ converges to some σ in X; if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(\sigma_n, \sigma) < \varepsilon$ for all $n \geq N$. In this case, we write $\lim_{n\to\infty} \sigma_n = \sigma$.
2. The sequence $\{\sigma_n\}$ is Cauchy; if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(\sigma_n, \sigma_m) < \varepsilon$ for all $n, m \geq N$.
3. The controlled metric space (X, d_c) is called complete if every Cauchy sequence is convergent.

Definition 5 ([9]). Let (X, d_c) be a controlled metric space. Let $\sigma \in X$ and $\varepsilon > 0$.

1. The open ball $B(\sigma, \varepsilon)$ is
$$B(\sigma, \varepsilon) = \{\varsigma \in X : d_c(\varsigma, \sigma) < \varepsilon\}.$$
2. The mapping $E: X \longrightarrow X$ is said to be continuous at $\sigma \in X$; if for all $\varepsilon > 0$, there exists $\delta > 0$ such that $E\big(B(\sigma, \varepsilon)\big) \subseteq B(E\sigma, \varepsilon)$.

The main purpose of this paper is to present some fixed point theorems for mappings involving rational expressions in the context of complete controlled metric spaces. Our result extends and generalizes some well-known results in the literature. We also provide examples to show significance of the obtained results involving rational type contractive conditions.

2. Results on Rational Type Contractions

Theorem 2.1 Let (X, d_c) be a complete controlled metric space. Let $\mathrm{E}: X \longrightarrow X$ be so that there are $\gamma_i \in (0,1), \forall\, i \in \{1,2,3,4,5\}$ with $\lambda = \frac{\gamma_1+\gamma_2}{1-\sum_{i=3}^{5}\gamma_i} < 1$,

$$d_c(E\sigma, E\varsigma) \leq \gamma_1 d_c(\sigma, \varsigma) + \gamma_2 d_c(\sigma, E\sigma) + \gamma_3 d_c(\varsigma, E\varsigma) + \gamma_4 \frac{d_c(\sigma, E\sigma).d_c(\varsigma, E\varsigma)}{1+d_c(\sigma,\varsigma)}$$
$$+\gamma_5 \frac{d_c(\varsigma, E\varsigma)[1+d_c(\sigma, E\sigma)]}{1+d_c(\sigma,\varsigma)} \qquad (2.1)$$

for all $\sigma, \varsigma \in X$. For $\sigma_0 \in X$, take $\sigma_n = \mathrm{E}^n \sigma_0$. Assume that

$$\sup_{m\geq 1} \lim_{i\to\infty} \frac{p(\sigma_{i+1}, \sigma_{i+2}) p(\sigma_{i+1}, \sigma_m)}{p(\sigma_i, \sigma_{i+1})} < \lambda^{-1} \qquad (2.2)$$

Suppose that $\lim_{n\to\infty} p(\sigma_n, \sigma)$ and $\lim_{n\to\infty} p(\sigma, \sigma_n)$ exist, are finite, and $(\gamma_3 + \gamma_5) \lim_{n\to\infty} p(\sigma, \sigma_n) < 1$ for every $\sigma \in X$, then E possesses a unique fixed point.

Proof. Let $\sigma_0 \in X$ be initial point. The considered sequence $\{\sigma_n\}$ verifies $\sigma_{n+1} = E\sigma_n$ for all $n \in \mathbb{N}$. Obviously, if there exists $n_0 \in \mathbb{N}$ for which $\sigma_{n_0+1} = \sigma_{n_0}$, then $E\sigma_{n_0} = \sigma_{n_0}$, and the proof is finished. Thus, we suppose that $\sigma_{n+1} \neq \sigma_n$ for every $n \in \mathbb{N}$. Thus, by (2.1), we have

$$\begin{aligned} d_c(\sigma_n, \sigma_{n+1}) &= d_c(E\sigma_{n-1}, E\sigma_n) \\ &\leq \gamma_1 d_c(\sigma_{n-1}, \sigma_n) + \gamma_2 d_c(\sigma_{n-1}, E\sigma_{n-1}) + \gamma_3 d_c(\sigma_n, E\sigma_n) \\ &+\gamma_4 \frac{d_c(\sigma_{n-1}, E\sigma_{n-1}).d_c(\sigma_n, E\sigma_n)}{1+d_c(\sigma_{n-1},\sigma_n)} + \gamma_5 \frac{d_c(\sigma_n, E\sigma_n)[1+d_c(\sigma_{n-1}, E\sigma_{n-1})]}{1+d_c(\sigma_{n-1},\sigma_n)} \\ &= \gamma_1 d_c(\sigma_{n-1}, \sigma_n) + \gamma_2 d_c(\sigma_{n-1}, \sigma_n) + \gamma_3 d_c(\sigma_n, \sigma_{n+1}) \\ &+\gamma_4 \frac{d_c(\sigma_{n-1}, \sigma_n).d_c(\sigma_n, \sigma_{n+1})}{1+d_c(\sigma_{n-1},\sigma_n)} + \gamma_5 \frac{d_c(\sigma_n, \sigma_{n+1})[1+d_c(\sigma_{n-1}, \sigma_n)]}{1+d_c(\sigma_{n-1},\sigma_n)} \\ &\leq (\gamma_1 + \gamma_2) d_c(\sigma_{n-1}, \sigma_n) + (\gamma_3 + \gamma_4 + \gamma_5) d_c(\sigma_n, \sigma_{n+1}) \end{aligned}$$

The last inequality gives

$$d_c(\sigma_n, \sigma_{n+1}) \leq \frac{\gamma_1+\gamma_2}{1-(\gamma_3+\gamma_4+\gamma_5)} d_c(\sigma_{n-1}, \sigma_n) \tag{2.3}$$

Thus, we have

$$d_c(\sigma_n, \sigma_{n+1}) \leq \lambda d_c(\sigma_{n-1}, \sigma_n) \leq \lambda^2 d_c(\sigma_{n-2}, \sigma_{n-1}) \leq \cdots \leq \lambda^n d_c(\sigma_0, \sigma_1) \tag{2.4}$$

For all $n, m \in \mathbb{N}$ and $n < m$, we have

$$\begin{aligned} d_c(\sigma_n, \sigma_m) &\leq p(\sigma_n, \sigma_{n+1}) d_c(\sigma_n, \sigma_{n+1}) + p(\sigma_{n+1}, \sigma_m) d_c(\sigma_{n+1}, \sigma_m) \\ &\leq p(\sigma_n, \sigma_{n+1}) d_c(\sigma_n, \sigma_{n+1}) + p(\sigma_{n+1}, \sigma_m) p(\sigma_{n+1}, \sigma_{n+2}) d_c(\sigma_{n+1}, \sigma_{n+2}) \\ &+p(\sigma_{n+1}, \sigma_m) p(\sigma_{n+2}, \sigma_m) d_c(\sigma_{n+2}, \sigma_m) \\ &\leq p(\sigma_n, \sigma_{n+1}) d_c(\sigma_n, \sigma_{n+1}) + p(\sigma_{n+1}, \sigma_m) p(\sigma_{n+1}, \sigma_{n+2}) d_c(\sigma_{n+1}, \sigma_{n+2}) \\ &+p(\sigma_{n+1}, \sigma_m) p(\sigma_{n+2}, \sigma_m) p(\sigma_{n+2}, \sigma_{n+3}) d_c(\sigma_{n+2}, \sigma_{n+3}) \\ &+p(\sigma_{n+1}, \sigma_m) p(\sigma_{n+2}, \sigma_m) p(\sigma_{n+3}, \sigma_m) d_c(\sigma_{n+3}, \sigma_m) \\ &\leq p(\sigma_n, \sigma_{n+1}) d_c(\sigma_n, \sigma_{n+1}) + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} p(\sigma_j, \sigma_m)\right) p(\sigma_i, \sigma_{i+1}) d_c(\sigma_i, \sigma_{i+1}) \\ &+ \prod_{i=n+1}^{m-1} p(\sigma_j, \sigma_m)\, d_c(\sigma_{m-1}, \sigma_m) \end{aligned} \tag{2.5}$$

This implies that

$$\begin{aligned} d_c(\sigma_n, \sigma_m) &\leq p(\sigma_n, \sigma_{n+1}) d_c(\sigma_n, \sigma_{n+1}) \\ &+ \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} p(\sigma_j, \sigma_m)\right) p(\sigma_i, \sigma_{i+1}) d_c(\sigma_i, \sigma_{i+1}) \end{aligned}$$

$$+\prod_{i=n+1}^{m-1} p\big(\sigma_j,\sigma_m\big)\, d_c(\sigma_{m-1},\sigma_m)$$

$$\leq p(\sigma_n,\sigma_{n+1})\lambda^n d_c(\sigma_0,\sigma_1)$$

$$+\sum_{i=n+1}^{m-2}\Big(\prod_{j=n+1}^{i} p\big(\sigma_j,\sigma_m\big)\Big)\, p(\sigma_i,\sigma_{i+1})\lambda^i d_c(\sigma_0,\sigma_1)$$

$$+\prod_{i=n+1}^{m-1} p\big(\sigma_j,\sigma_m\big)\, \lambda^{m-1} d_c(\sigma_0,\sigma_1)$$

$$\leq p(\sigma_n,\sigma_{n+1})\lambda^n d_c(\sigma_0,\sigma_1)$$

$$+\sum_{i=n+1}^{m-1}\Big(\prod_{j=n+1}^{i} p\big(\sigma_j,\sigma_m\big)\Big)\, p(\sigma_i,\sigma_{i+1})\lambda^i d_c(\sigma_0,\sigma_1) \tag{2.6}$$

Let

$$u_r=\sum_{i=0}^{r}\Big(\prod_{j=0}^{i} p\big(\sigma_j,\sigma_m\big)\Big)\, p(\sigma_i,\sigma_{i+1})\lambda^i d_c(\sigma_0,\sigma_1) \tag{2.7}$$

Consider

$$v_i=\sum_{i=0}^{r}\Big(\prod_{j=0}^{i} p\big(\sigma_j,\sigma_m\big)\Big)\, p(\sigma_i,\sigma_{i+1})\lambda^i d_c(\sigma_0,\sigma_1) \tag{2.8}$$

In view of condition (2.2) and the ratio test, we ensure that the series $\sum_i v_i$ converges. Thus, $\lim\limits_{n\to\infty} u_n$ exists. Hence, the real sequence $\{u_n\}$ is Cauchy. Now, using (2.6), we get

$$d_c(\sigma_n,\sigma_m)\leq d_c(\sigma_0,\sigma_1)[\lambda^n p(\sigma_n,\sigma_{n+1})+(u_{m-1}-u_n)] \tag{2.9}$$

Above, we used $p(\sigma,\varsigma)\geq 1$. Letting $n,m\longrightarrow\infty$ in (2.9), we obtain

$$\lim_{n,m\to\infty} d_c(\sigma_n,\sigma_m)=0 \tag{2.10}$$

Thus, the sequence $\{\sigma_n\}$ is Cauchy in the complete controlled metric space (X,d_c). So, there is some $\sigma^*\in X$.So that

$$\lim_{n\to\infty} d_c(\sigma_n,\sigma^*)=0; \tag{2.11}$$

that is, $\sigma_n\longrightarrow\sigma^*$ as $n\longrightarrow\infty$. Now, we will prove that σ^* is a fixed point of E. By (2.1) and condition (iii), we get

$$d_c(\sigma^*,E\sigma^*)\leq p(\sigma^*,\sigma_{n+1})d_c(\sigma^*,\sigma_{n+1})+p(\sigma_{n+1},E\sigma^*)d_c(\sigma_{n+1},E\sigma^*)$$

$$=p(\sigma^*,\sigma_{n+1})d_c(\sigma^*,\sigma_{n+1})+p(\sigma_{n+1},E\sigma^*)d_c(E\sigma_n,E\sigma^*)$$

$$\leq p(\sigma^*,\sigma_{n+1})d_c(\sigma^*,\sigma_{n+1})$$

$$+p(\sigma_{n+1},E\sigma^*)\Big[\gamma_1 d_c(\sigma_n,\sigma^*)+\gamma_2 d_c(\sigma_n,E\sigma_n)+\gamma_3 d_c(\sigma^*,E\sigma^*)+$$

$$\gamma_4\frac{d_c(\sigma_n,E\sigma_n).d_c(\sigma^*,E\sigma^*)}{1+d_c(\sigma_n,\sigma^*)}+\gamma_5\frac{d_c(\sigma^*,E\sigma^*)[1+d_c(\sigma_n,E\sigma_n)]}{1+d_c(\sigma_n,\sigma^*)}\Big]$$

$$= p(\sigma^*, \sigma_{n+1}) d_c(\sigma^*, \sigma_{n+1})$$

$$+p(\sigma_{n+1}, E\sigma^*)\left[\gamma_1 d_c(\sigma_n, \sigma^*) + \gamma_2 d_c(\sigma_n, \sigma_{n+1}) + \gamma_3 d_c(\sigma^*, E\sigma^*) + \gamma_4 \frac{d_c(\sigma_n,\sigma_{n+1}).d_c(\sigma^*,E\sigma^*)}{1+d_c(\sigma_n,\sigma^*)} + \gamma_5 \frac{d_c(\sigma^*,E\sigma^*)[1+d_c(\sigma_n,\sigma_{n+1})]}{1+d_c(\sigma_n,\sigma^*)}\right] \quad (2.12)$$

Taking the limit as $n \longrightarrow \infty$ and using (2.10), (2.11) and the fact that $\lim_{n\to\infty} p(\sigma_n, \sigma)$ and $\lim_{n\to\infty} p(\sigma, \sigma_n)$ exist, are finite, we obtain that

$$d_c(\sigma^*, E\sigma^*) \leq \left[(\gamma_3 + \gamma_5) \lim_{n\longrightarrow\infty} p(\sigma_{n+1}, E\sigma^*)\right] d_c(\sigma^*, \mathrm{E}\sigma^*) \quad (2.13)$$

Suppose that $\sigma^* \neq E\sigma^*$, having in mind that $\left[(\gamma_3 + \gamma_5) \lim_{n\longrightarrow\infty} p(\sigma_{n+1}, E\sigma^*)\right] < 1$, so

$$0 < d_c(\sigma^*, E\sigma^*) \leq \left[(\gamma_3 + \gamma_5) \lim_{n\longrightarrow\infty} p(\sigma_{n+1}, E\sigma^*)\right] d_c(\sigma^*, \mathrm{E}\sigma^*) < d_c(\sigma^*, \mathrm{E}\sigma^*) \quad (2.14)$$

It is a contradiction. This yields that $\sigma^* = E\sigma^*$. Now, we prove the uniqueness of σ^*. Let ς^* be another fixed point of E in X, then $E\varsigma^* = \varsigma^*$. Now, by (2.1), we have

$$d_c(\sigma^*, \varsigma^*) = d_c(E\sigma^*, E\varsigma^*)$$

$$\leq \gamma_1 d_c(\sigma^*, \varsigma^*) + \gamma_2 d_c(\sigma^*, E\sigma^*) + \gamma_3 d_c(\varsigma^*, E\varsigma^*) + \gamma_4 \frac{d_c(\sigma^*,E\sigma^*).d_c(\varsigma^*,E\varsigma^*)}{1+d_c(\sigma^*,\varsigma^*)}$$

$$+\gamma_5 \frac{d_c(\varsigma^*,E\varsigma^*)[1+d_c(\sigma^*,E\sigma^*)]}{1+d_c(\sigma^*,\varsigma^*)}$$

$$= \gamma_1 d_c(\sigma^*, \varsigma^*) + \gamma_2 d_c(\sigma^*, \sigma^*) + \gamma_3 d_c(\varsigma^*, \varsigma^*) + \gamma_4 \frac{d_c(\sigma^*,E\sigma^*).d_c(\varsigma^*,E\varsigma^*)}{1+d_c(\sigma^*,\varsigma^*)}$$

$$+\gamma_5 \frac{d_c(\varsigma^*,E\varsigma^*)[1+d_c(\sigma^*,E\sigma^*)]}{1+d_c(\sigma^*,\varsigma^*)}$$

$$= \gamma_1 d_c(\sigma^*, \varsigma^*) + \gamma_2 d_c(\sigma^*, \sigma^*) + \gamma_3 d_c(\varsigma^*, \varsigma^*) + \gamma_4 \frac{d_c(\sigma^*,\sigma^*).d_c(\varsigma^*,\varsigma^*)}{1+d_c(\sigma^*,\varsigma^*)}$$

$$+\gamma_5 \frac{d_c(\varsigma^*,\varsigma^*)[1+d_c(\sigma^*,\sigma^*)]}{1+d_c(\sigma^*,\varsigma^*)}$$

$$\leq \gamma_1 d_c(\sigma^*, \varsigma^*) \quad (2.15)$$

It is a contradiction. This yields that $\sigma^* = \varsigma^*$. It completes the proof.

3. EXAMPLE

Example 3.1 Consider $X = \{0,1,2\}$. Take the controlled metric d_c defined as

$$d_c(0,1) = \tfrac{1}{2}, d_c(0,2) = \tfrac{11}{20}, d_c(1,2) = \tfrac{3}{2},$$

where $p: X \times X \longrightarrow [0,\infty)$ is symmetric such that

$$p(0,0) = p(1,1) = p(2,2) = p(1,2) = 1, p(0,2) = 2, p(0,1) = \tfrac{3}{2}$$

Given $E : X \longrightarrow X$ as

$$E0 = 2 \text{ and } E1 = E2 = 1.$$

Consider $\gamma_1 = \frac{1}{11}$ and $\gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = \frac{2}{11}$. Then

$$\lambda = \frac{\gamma_1 + \gamma_2}{1 - \sum_{i=3}^{5} \gamma_i} = \frac{\frac{1}{11} + \frac{2}{11}}{1 - 3\left(\frac{2}{11}\right)} = \frac{3}{5} < 1,$$

Take $\sigma_0 = 0$, then $\sigma_1 = 2$, and $\sigma_n = 1$, for all $n \geq 2$, we have

$$\sup_{m\geq 1} \lim_{i\to\infty} \frac{p(\sigma_{i+1},\sigma_{i+2})p(\sigma_{i+1},\sigma_m)}{p(\sigma_i,\sigma_{i+1})} = 1 < \frac{5}{3} = \lambda^{-1}$$

Clearly, (2.2) is satisfied. On the other hand, note that (2.1) holds for all $\sigma, \varsigma \in X$. All other hypotheses of Theorem 2.1 are verified, and so E has a unique fixed point, which is $\sigma^* = 1$.

AUTHORS' CONTRIBUTIONS

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

REFERENCES

[1] S. Czerwik, Contraction mappings in b-metric spaces, Acta Math. Inform. Univ. Ostrav. 1 (1993), 5-11

[2] H. Afshari, H. Aydi, E. Karapinar, On generalized α-ψ-Geraghty contractions on b-metric spaces, Georgian Math. J. 27 (2020), 9-21.

[3] E. Ameer, H. Aydi, M. Arshad, M. De la Sen, Hybrid Ćirić type graphic Υ,Λ-contraction mappings with applications to electric circuit and fractional differential equations, Symmetry. 12 (2020), 467.

[4] A. Azam, N. Mehmood, J. Ahmad, S. Radenović, Multivalued fixed point theorems in cone b-metric spaces, J. Inequal. Appl. 2013 (2013), 582.

[5] H. Huang, S. Radenović, Some fixed point results of generalized Lipschitz mappings on cone b-metric spaces over Banach algebras, J. Comput. Anal. Appl. 20 (2016), 566–583.

[6] E. Karapinar, S. Czerwik, H. Aydi, (α,ψ)-Meir-Keeler contraction mappings in generalized-metric spaces, J. Funct. Spaces, 2018 (2018), Article ID 3264620.

[7] H. Qawaqneh, M. Md Noorani, et al. Fixed point results for multi-valued contractions in b−metric spaces and an application, Mathematics. 7 (2019), 132.

[8] T. Kamran, M. Samreen, Q. UL Ain, A generalization of b-metric space and some fixed point theorems, Mathematics. 5 (2017), 19.

[9] N. Mlaiki, H. Aydi, N. Souayah, T. Abdeljawad, Controlled metric type spaces and the related contraction principle, Mathematics. 6 (2018), 194.

[10] T. Abdeljawad, N. Mlaiki, H. Aydi, N. Souayah, Double controlled metric type spaces and some fixed point results, Mathematics. 6 (2018), 320.

[11] D. Wardowski, Fixed points of a new type of contractive mappings in complete metric spaces, Fixed Point Theory Appl. 2012 (2012), 94.

[12] J. Ahmad, A. Al-Rawashdeh, A. Azam, New fixed point theorems for generalized F-contractions in complete metric spaces, Fixed Point Theory Appl. 2015 (2015), 80.

[13] E. Ameer, H. Aydi, M. Arshad, H. Alsamir, M. Noorani, Hybrid multivalued type contraction mappings in αK-complete partial b-metric spaces and applications, Symmetry. 11 (2019), 86.

[14] L. Budhia, H. Aydi, A. H. Ansari, D. Gopal, Some new fixed point results in rectangular metric spaces with an application to fractional-order functional differential equations, Nonlinear Anal.: Model. Control, 25 (2020), 580–597.

[15] N. Hussain, J. Ahmad, A. Azam, On Suzuki-Wardowski type fixed point theorems, J. Nonlinear Sci. Appl. 8 (2015), 1095–1111.

[16] N. Hussain, J. Ahmad, L. Ćirić, A. Azam, Coincidence point theorems for generalized contractions with application to integral equations, Fixed Point Theory Appl. 2015 (2015), 78.

[17] Z. Ma, A. Asif, H. Aydi, S.U. Khan, M. Arshad, Analysis of F-contractions in function weighted metric spaces with an application, Open Math. 18 (2020), 582–594.

[18] M. Olgun, T. Alyildiz, O. Bicer, I. Altun, Fixed point results for F-contractions on space with two metrics, Filomat, 31 (2017), 5421–5426.

[19] V. Parvaneh, M.R. Haddadi, H. Aydi, On best proximity point results for some type of mappings, J. Funct. Spaces, 2020 (2020), Article ID 6298138.

[20] P. Patle, D. Patel, H. Aydi, S. Radenović, On H+type multivalued contractions and applications in symmetric and probabilistic spaces, Mathematics. 7 (2019), 144.

[21] H. Sahin, I. Altun, D. Turkoglu, Two fixed point results for multivalued F-contractions on M-metric spaces, Rev. R. Acad. Cienc. Exactas Fís. Nat., Ser. A Mat., RACSAM. 113 (2019), 1839–1849.

[22] W. Shatanawi, E. Karapinar, H. Aydi, A. Fulga, Wardowski type contractions with applications on Caputo type nonlinear fractional differential equations, U.P.B. Sci. Bull., Ser. A, 82 (2020), 157 –170.

[23] S. Reich, Fixed points of contractive functions. Boll. Unione Mat. Ital. 5 (1972), 26-42.

[24] S. Reich, Kannan's fixed point theorem. Boll. Un. Mat. Ital. 4 (1971), 1–11.

[25] B. Samet, C. Vetro, P. Vetro, Fixed point theorems for $\alpha-\psi$-contractive type mappings, Nonlinear Anal., Theory Meth. Appl. 75 (2012), 2154 –2165.

[26] B.K. Dass, S. Gupta, An extension of Banach contraction principle through rational expressions, Indian J. Pure Appl. Math. 6 (1975), 1455-1458.

[27] M. Nazam, H. Aydi, M. Arshad, A real generalization of the Dass-Gupta fixed point theorem, TWMS J. Pure Appl. Math. 11 (2020), 109-118.