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A CERTAIN CHARACTER CONNECTED WITH SEPARATION AXIOMS IN BINARY TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce and study a new class of axioms called the binary semi-T_0, binary semi-T_1, binary semi-T_2, binary semi-T_3, and binary semi-T_4 spaces. Further, we have given an appropriate examples to understand the abstract concepts clearly.

Keywords: binary semi-T_0; binary semi-T_1; binary semi-T_2; binary semi-T_3; binary semi-T_4 spaces.

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1. Introduction

The concept of binary topology from X to Y is introduced by Nithyanantha Jothi and Thangavelu [1]. He also introduced the concepts of binary closed, binary closure, binary interior and binary continuity. Further, the concepts of base and sub base of a binary topological space are introduced and investigated. Also, in 2012, the authors[2] introduced the concept of binary-T_0, binary-T_1, binary-T_2, binary-T_3, and binary-T_4 spaces. The authors[3] introduced binary semi open sets in binary topological spaces and obtained some basic results. Recently,

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sathishmohan et.al.,[5] introduce and study the concept of binary generalized semi closed sets and binary semi generalized closed sets in binary topological spaces. Also, the authors[6] introduced the concept of binary generalized semi(binary semi generalized) closure and interior of a sets in binary topological spaces. The purpose of this paper, is to introduce binary semi-T_0, binary semi-T_1, binary semi-T_2, binary semi-T_3, and binary semi-T_4 spaces in binary topological spaces and characterize their basic properties.

2. Preliminaries

Definition 2.1. *Let X and Y be any two nonempty sets. A binary topology* [1] *from X to Y is a binary structure $\mathscr{M} \subseteq \mathscr{P}(X) \times \mathscr{P}(Y)$ that satisfies the axioms.*

(1) (ϕ,ϕ) *and* $(X,Y) \in \mathscr{M}$.

(2) $(A_1 \cap A_2, B_1 \cap B_2) \in \mathscr{M}$ *whenever* $(A_1,B_1) \in \mathscr{M}$ *and* $(A_2,B_2) \in \mathscr{M}$.

(3) If $\{(A_\alpha,B_\alpha) : \alpha \in \Delta\}$ *is a family of members of* $\mathscr{M}$ *then* $(\bigcup\limits_{\alpha\in\Delta} A_\alpha, \bigcup\limits_{\alpha\in\Delta} B_\alpha) \in \mathscr{M}$.

Definition 2.2. [1] *If $\mathscr{M}$ is a binary topology from X to Y then the triplet $(X,Y,\mathscr{M})$ is called a binary topological space and the members of $\mathscr{M}$ are called the binary open subsets of the binary topological space $(X,Y,\mathscr{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X,Y,\mathscr{M})$. If Y=X then $\mathscr{M}$ is called a binary topology on X in which case we write $(X,X,\mathscr{M})$ as a binary topological space.*

Definition 2.3. [1] *Let X and Y be any two nonempty sets and let (A,B) and (C,D) $\in \mathscr{P}(X) \times \mathscr{P}(Y)$. We say that (A,B) $\subseteq$ (C,D) if A $\subseteq$ C and B $\subseteq$ D.*

Definition 2.4. [1] *Let $(X,Y,\mathscr{M})$ be a binary topological space and A $\subseteq$ X, B $\subseteq$ Y. Then (A,B) is called binary closed in $(X,Y,\mathscr{M})$ if $(X-A, Y-B) \in \mathscr{M}$.*

Proposition 2.5. [1] *Let $(X,Y,\mathscr{M})$ be a binary topological space and (A,B) $\subseteq$ (X,Y). Let $(A,B)^{1^*} = \cap\{A_\alpha : (A_\alpha,B_\alpha)$ is binary closed and (A,B) $\subseteq (A_\alpha,B_\alpha)\}$ and $(A,B)^{2^*} = \cap\{B_\alpha : (A_\alpha,B_\alpha)$ is binary closed and (A,B) $\subseteq (A_\alpha,B_\alpha)\}$. Then $((A,B)^{1^*},(A,B)^{2^*})$ is binary closed and (A,B) $\subseteq ((A,B)^{1^*},(A,B)^{2^*})$.*

Definition 2.6. [1] *The ordered pair $((A,B)^{1^*},(A,B)^{2^*})$ is called the binary closure of (A,B), denoted by b-cl(A,B) in the binary space $(X,Y,\mathscr{M})$ where (A,B) $\subseteq$ (X,Y).*

Definition 2.7. [1] *Let X and Y be any two nonempty sets and let (A,B) and (C,D)* $\in \mathscr{P}(X) \times \mathscr{P}(Y)$. *We say that (A,B)* $\not\subset$ *(C,D) if one of the following holds:*

(1) $A \subseteq C$ *and* $B \not\subset D$

(2) $A \not\subset C$ *and* $B \subseteq D$

(3) $A \not\subset C$ *and* $B \not\subset D$.

Definition 2.8. [1] *(i)* $(A,B)^{1^\circ} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ *is binary open and* $(A_\alpha, B_\alpha) \subseteq (A,B)\}$.
(ii) $(A,B)^{2^\circ} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ *is binary open and* $(A_\alpha, B_\alpha) \subseteq (A,B)\}$.

Definition 2.9. [1] *Let* $(X,Y,\mathscr{M})$ *be a binary topological space and (A,B)* $\subseteq$ *(X,Y). The ordered pair* $((A,B)^{1^\circ}, (A,B)^{2^\circ})$ *is called the binary interior of (A,B) denoted by b-int(A,B).*

Definition 2.10. [2] *A binary topological spaces* $(X,Y,\mathscr{M})$ *is called a binary-*T_0 *if for any two jointly distinct points* $(x_1,y_1),(x_2,y_2) \in X \times Y$, *there exists (A,B)*$\in \mathscr{M}$ *such that exactly one of the following holds.*
(i) $(x_1,y_1) \in (A,B), (x_2,y_2) \in (X-A, Y-B)$
(ii) $(x_1,y_1) \in (X-A, Y-B), (x_2,y_2) \in (A,B)$.

Definition 2.11. [2] *A binary topological spaces* $(X,Y,\mathscr{M})$ *is called a binary-*T_1 *if for every two jointly distinct points* $(x_1,y_1),(x_2,y_2) \in X \times Y$, *there exists (A,B) and (C,D)*$\in \mathscr{M}$, *with* $(x_1,y_1) \in (A,B)$ *and* $(x_2,y_2) \in (C,D)$ *such that* $(x_2,y_2) \in (X-A, Y-B), (x_1,y_1) \in (X-C, Y-D)$.

Definition 2.12. [2] *The binary points* $(x_1,y_1),(x_2,y_2) \in X \times Y$ *are distinct if* $x_1 \neq x_2, y_1 \neq y_2$.

Definition 2.13. [2] *A binary topological spaces* $(X,Y,\mathscr{M})$ *is called a binary-*T_2 *if for any two jointly distinct points* $(x_1,y_1),(x_2,y_2) \in X \times Y$, *there exists jointly disjoint binary open sets (A,B) and (C,D) such that* $(x_1,y_1) \in (A,B)$ *and* $(x_2,y_2) \in (C,D)$.

Definition 2.14. [4] *A binary topological spaces* $(X,Y,\mathscr{M})$ *is called a binary-*T_3 *or binary regular if* $(X,Y,\mathscr{M})$ *is binary-*T_1 *and for every (x,y)*$\in X \times Y$ *and every binary closed set (A,B)*$\subseteq X \times Y$ *such that (x,y)*$\in (X-A, Y-B)$ *there exists jointly disjoint binary open sets* $(U_1,V_1),(U_2,V_2)$ *such that (x,y)*$\in (U_1,V_1)$, *(A,B)*$\subseteq (U_2,V_2)$.

Definition 2.15. [4] *A binary topological spaces $(X,Y,\mathscr{M})$ is called a binary-T_4 or binary normal if $(X,Y,\mathscr{M})$ is binary-T_1 and for every pair of jointly disjoint binary closed sets $(A_1,B_1),(A_2,B_2)$ there exists jointly disjoint binary open sets $(U_1,V_1),(U_2,V_2)$ such that $(A_1,B_1) \subseteq (U_1,V_1)$ and $(A_2,B_2) \subseteq (U_2,V_2)$*

Definition 2.16. [2] *Two binary open sets (A,B) and (C,D) are said to be disjoint if (A∩C, B∩D)=(ϕ,ϕ). That is A∩C=ϕ and B∩D=ϕ.*

Definition 2.17. [1] *Let $(X,Y,\mathscr{M})$ be a binary topological space and let (x,y) ∈ X×Y. The binary open set (A,B) is called a binary neighbourhood of (x,y) if x ∈ A and y ∈ B.*

3. BINARY SEMI-T_0, T_1, T_2 SPACES

In this section, we establish the intellection of binary semi-T_0, binary semi-T_1 and binary semi-T_2 spaces and study some of their characterizations.

Definition 3.1. *A binary topological spaces $(X,Y,\mathscr{M})$ is called a binary semi-T_0(briefly, bs-T_0) if for any two jointly distinct points $(x_1,y_1),(x_2,y_2) \in X \times Y$, there exists binary semi open set (A,B) such that exactly one of the following holds.*
(i) $(x_1,y_1) \in (A,B),(x_2,y_2) \in (X-A,Y-B)$
(ii) $(x_1,y_1) \in (X-A,Y-B),(x_2,y_2) \in (A,B)$.

Definition 3.2. *A binary topological spaces $(X,Y,\mathscr{M})$ is called a binary semi-T_1(briefly, bs-T_1) if for every two jointly distinct points $(x_1,y_1),(x_2,y_2) \in X \times Y$ with $x_1 \neq x_2, y_1 \neq y_2$, there exists binary semi open sets (A,B) and (C,D) with $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (C,D)$ such that $(x_2,y_2) \in (X-A,Y-B),(x_1,y_1) \in (X-C,Y-D)$.*

Definition 3.3. *A binary topological spaces $(X,Y,\mathscr{M})$ is called a binary semi-T_2(briefly, bs-T_2) if for every two jointly distinct points $(x_1,y_1),(x_2,y_2) \in X \times Y$, with $x_1 \neq x_2, y_1 \neq y_2$, there exists disjoint binary semi open sets (A,B) and (C,D) such that $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (C,D)$.*

Theorem 3.4. *Let $(X,Y,\mathscr{M})$ be a binary topological spaces, then for every binary-T_0 space is binary semi-T_0 space.*

Proof: *Let (X,Y) be a binary-T_0 space, (x_1,y_1) and (x_2,y_2) be a two distinct points of (X,Y),* *as (X,Y) is binary-T_0 space there exists binary open set (A,B) such that $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (X-A,Y-B)$. Since every binary open set is binary semi open and hence (A,B) is binary semi open set such that $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (X-A,Y-B)$. Hence (X,Y) is binary semi-T_0 space.*

Example 3.5. *Let X= $\{a,b\}$, Y =$\{a,b,c\}$. Clearly $\mathscr{M}$ =$\{(\phi,\phi),(\phi,\{a\}),(\{a\},\{a\}),(\{a\},\{a,b\}),(\{b\},\phi),(\{b\},\{a\}),(\{b\},\{c\}),(\{b\},\{a,c\}),(X,\{a\}),(X,\{a,b\}),(X,\{a,c\}),(X,Y)\}$ is a binary topology from X to Y. We have binary semi open=$\{(\phi,\phi),(\phi,\{a\}),(\phi,\{a,b\}),(\{a\},\{a\}),(\{a\},\{a,b\}),(\{b\},\phi),(\{b\},\{a\}),(\{b\},\{c\}),(\{b\},\{a,b\}),(\{b\},\{a,c\}),(\{b\},Y),(X,\{a\}),(X,\{a,b\}),(X,\{a,c\}),(X,Y)\}$. Let (x_1,y_1)=$(\{b\},\{a\})$ and (x_2,y_2)=$(\{a\},\{c\})$, $(x_1,y_1),(x_2,y_2) \in (X,Y)$ and $(x_1,y_1) \neq (x_2,y_2)$ there exists binary semi open set (A,B)=$(\{b\},\{a,b\})$ then it is binary semi-T_0 space but not binary-T_0 space*

Theorem 3.6. *Let $(X,Y,\mathscr{M})$ be a binary topological spaces, then for every binary-T_1 space is binary semi-T_1 space.*

Proof: *Let (X,Y) be a binary-T_1 space and let $x_1 \neq x_2, y_1 \neq y_2$ in (X,Y). Then there exists distinct binary open sets (A,B) and (C,D) such that $(x_1,y_1) \in (A,B)$, $(x_2,y_2) \in (X-A,Y-B)$ and $(x_2,y_2) \in (C,D)$, $(x_1,y_1) \in (X-C,Y-D)$. As every binary open set is binary semi open and hence (A,B) and (C,D) are distinct binary semi open sets with $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (C,D)$ such that $(x_2,y_2) \in (X-A,Y-B),(x_1,y_1) \in (X-C,Y-D)$. Hence (X,Y) is binary semi-T_1 space.*

Example 3.7. *Let X= $\{a,b\}$, Y =$\{a,b,c\}$. Clearly $\mathscr{M}$ =$\{(\phi,\phi),(\phi,\{c\}),(\{a\},\{a\}),(\{a\},\{a,c\}),(\{b\},\{c\}),(X,\{a,c\}),(X,Y)\}$ is a binary topology from X to Y. We have binary semi open=$\{(\phi,\phi),(\phi,\{c\}),(\phi,\{b,c\}),(\{a\},\{a\}),(\{a\},\{a,b\}),(\{a\},\{a,c\}),(\{a\},Y),(\{b\},\{c\}),(\{b\},\{b,c\}),(X,\{a,c\}),(X,Y)\}$. Let (A,B)=$(\{b\},\{c\})$ and (C,D)=$(\{a\},\{a,b\})$. Let (x_1,y_1)=$(\{b\},\{c\})$ and (x_2,y_2)=$(\{a\},\{b\})$, $(x_1,y_1),(x_2,y_2) \in$(X,Y) and $(x_1,y_1) \neq (x_2,y_2)$ then it is clear that $(x_1,y_1) \in$(A,B), $(x_2,y_2) \notin$ (A,B) and $(x_2,y_2) \in$(C,D) and $(x_1,y_1) \notin$(C,D). Then we can say that it is binary semi-T_1 space but not binary-T_1 space.*

Theorem 3.8. *Let $(X,Y,\mathscr{M})$ be a binary topological spaces, then for every binary-T_2 space is binary semi-T_2 space.*

Proof: *Let (X,Y) be a binary-T_2 space and let $x_1 \neq x_2, y_1 \neq y_2$ in (X,Y). Then there exists disjoint binary open sets (A,B) and (C,D) such that $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (C,D)$. As every binary open set is binary semi open and hence (A,B) and (C,D) are disjoint binary semi open set such that $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (C,D)$. Hence (X,Y) is binary semi-T_2 space.*

Example 3.9. *From the Example 3.7, Let (x_1,y_1)=($\{b\}$,$\{c\}$) and (x_2,y_2)=($\{a\}$,$\{a\}$). Let (A,B)=*
($\{b\}$,$\{b,c\}$) and (C,D)=($\{a\}$,$\{a\}$), (x_1,y_1),(x_2,y_2) $\in$(X,Y) and $(x_1,y_1) \neq (x_2,y_2)$ then it is clear that (x_1,y_1) $\in$(A,B), and (x_2,y_2) $\in$(C,D). Then we can say that it is binary semi-T_2 space but not binary-T_2 space.

Theorem 3.10. *Let $(X,Y,\mathscr{M})$ be a binary topological spaces, then binary semi-T_1 space is binary semi-T_0 space.*

Proof: *Let (X,Y) be a binary semi-T_1 space and let (x_1,y_1) and (x_2,y_2) be two distinct points of (X,Y), as (X,Y) is binary semi-T_1 space there exists binary semi open sets (A,B) and (C,D) such that $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (X-A,Y-B)$ and $(x_1,y_1) \notin (C,D)$ and $(x_2,y_2) \in (C,D)$. Since every binary open set is binary semi open and hence (A,B) is binary semi open set such that $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (X-A,Y-B)$. Hence (X,Y) is binary semi-T_0.*

Example 3.11. *Let X= $\{a,b\}$, Y =$\{a,b,c\}$. Clearly $\mathscr{M}$ =$\{$(ϕ, ϕ),(ϕ,$\{c\}$),($\{a\}$,$\{a\}$),($\{a\}$,$\{a,c\}$), ($\{b\}$,$\{c\}$),(X,$\{a,c\}$),(X,Y)$\}$ is a binary topology from X to Y. We have binary semi open=$\{$(ϕ, ϕ), (ϕ,$\{c\}$),(ϕ,$\{b,c\}$),($\{a\}$,$\{a\}$),($\{a\}$,$\{a,b\}$),($\{a\}$,$\{a,c\}$),($\{a\}$,Y),($\{b\}$,$\{c\}$),($\{b\}$,$\{b,c\}$),(X,$\{a,c\}$), (X,Y)$\}$. Let (A,B)=($\{b\}$,$\{c\}$) and (C,D)=($\{a\}$,$\{a,b\}$). Let (x_1,y_1)=($\{b\}$,$\{c\}$) and (x_2,y_2)=($\{a\}$, $\{b\}$), (x_1,y_1),(x_2,y_2) $\in$(X,Y) and $(x_1,y_1) \neq (x_2,y_2)$ then it is clear that (x_1,y_1) $\in$(A,B), (x_2,y_2) $\notin$(A,B) and (x_2,y_2) $\in$(C,D) and (x_1,y_1) $\notin$(C,D). Then we can say that it is binary semi-T_1 space but not binary-T_1 space.*

Theorem 3.12. *Let $(X,Y,\mathscr{M})$ be a binary topological spaces, then binary semi-T_2 space is binary semi-T_0 space.*

Proof: *Let (X,Y) be a binary semi-T_2 space and let (x_1,y_1) and (x_2,y_2) be two distinct points of (X,Y), as (X,Y) is binary semi-T_2 space there exists binary semi open sets (A,B) and (C,D) such that $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (C,D)$, since (A,B) and (C,D) are disjoint. Since every binary* *open set is binary semi open and hence (A,B) is binary semi open set such that* $(x_1, y_1) \in (A, B)$ *and* $(x_2, y_2) \in (X - A, Y - B)$. *Hence (X,Y) is binary semi-*T_0.

Example 3.13. *From the Example 3.7, Let* (x_1, y_1)*=(*$\{b\}$,$\{c\}$*) and* (x_2, y_2)*=(*$\{a\}$,$\{a\}$*), Let (A,B) =(*$\{b\}$,$\{b,c\}$*) and (C,D)=(*$\{a\}$,$\{a\}$*),* (x_1, y_1),(x_2, y_2) $\in$*(X,Y) and* $(x_1, y_1) \neq (x_2, y_2)$ *then it is clear that* (x_1, y_1) $\in$*(A,B), and* (x_2, y_2) $\in$*(C,D). Then we can say that it is binary semi-*T_2 *space but not binary-*T_0 *space.*

Theorem 3.14. *If a binary topological spaces,* $(X, Y, \mathscr{M})$ *is binary semi-*T_2 *then* $(X, Y, \mathscr{M})$ *is binary semi-*T_1.

Proof: *Suppose* $(X, Y, \mathscr{M})$ *is binary semi-*T_2. *Let* $(x_1, x_2) \in X$ *and* $(y_1, y_2) \in Y$ *with* $x_1 \neq x_2, y_1 \neq y_2$. *Since* $(X, Y, \mathscr{M})$ *is binary semi-*T_2*, there exists disjoint binary semi open sets* (U_1, V_1),(U_2, V_2) *with* $(x_1, y_1) \in (U_1, V_1)$,$(x_2, y_2) \in (U_2, V_2)$. *Since* (U_1, V_1) *and* (U_2, V_2) *are disjoint, we have* $(x_1, y_1) \in (X - U_2, Y - V_2)$ *and* $(x_2, y_2) \in (X - U_1, Y - V_1)$*.This shows that* $(X, Y, \mathscr{M})$ *is binary semi-*T_1.

Theorem 3.15. *A binary topological space* $(X, Y, \mathscr{M})$ *is a binary semi-*T_0 *space if and only if binary semi closure of distinct points are distinct.*

Proof: *Let* (x_1, y_1) *and* (x_2, y_2) *be distinct points of (X,Y). Since (X,Y) is a binary semi-*T_0 *space there exists a binary semi open set (U,V), such that* $x_1, y_1 \in U_1, V_1$ *and* $x_2, y_2 \notin U_2, V_2$. *Consequently ((X,Y)-(U,V)) is a binary semi closed set containing* (x_2, y_2) *but not* (x_1, y_1). *But b-scl(*$\{x_2, y_2\}$*) is the intersection of all binary semi closed set containing* (x_2, y_2). *Hence* $(x_2, y_2) \in$ *b-scl*$(\{x_2, y_2\})$. *But* $(x_1, y_1) \notin$ *b-scl*$(\{x_2, y_2\})$ *as* $(x_1, y_1) \notin ((X, Y) - (U, V))$. *Therefore b-scl(*$\{x_1, y_1\}$*)* $\neq$ *b-scl*$(\{x_2, y_2\})$.

Conversely, let b-scl($\{x_1, y_1\}$*)* $\neq$ *b-scl*$(\{x_2, y_2\})$ *for* $(x_1, y_1) \neq (x_2, y_2)$. *Then there exists atleast one point* $(z_1, z_2) \in (X, Y)$ *such that* $(z_1, z_2) \in$ *b-scl*$(\{x_1, y_1\})$ *but* $(z_1, z_2) \notin$ *b-scl*$(\{x_2, y_2\})$. *We claim* $(x_1, y_1) \notin$ *b-scl*$(\{x_2, y_2\})$ *because if* $(x_1, y_1) \in$ *b-scl*$(\{x_2, y_2\})$, $(x_1, y_1) \subseteq$ *b-scl*$(\{x_2, y_2\})$ *implies b-scl(*$\{x_1, y_1\}$*)* $\subseteq$ *b-scl*$(\{x_2, y_2\})$, *so* $(z_1, z_2) \in$ *b-scl*$(\{x_2, y_2\})$, *which is a contradiction. Hence* $(x_1, y_1) \notin$ *b-scl*$(\{x_2, y_2\})$, *which implies* $(x_1, y_1) \in (X, Y) -$ *b-scl*$(\{x_2, y_2\})$, *which is a binary semi open set containing* (x_1, y_1) *but not* (x_2, y_2). *Hence (X,Y) is a binary semi-*T_0 *space.*

Theorem 3.16. *A binary topological space $(X,Y,\mathscr{M})$ is a binary semi-T_1 space if and only if every binary point is binary semi closed.*

Proof: *Assume that $(X,Y,\mathscr{M})$ is a binary semi-T_1. Let (x,y)$\in X\times Y$. Let $(\{x\},\{y\})\in\mathscr{P}(X)\times\mathscr{P}(Y)$. We shall show that $(\{x\},\{y\})$ is binary semi closed. It is enough to show that $(X-\{x\},Y-\{y\})$ is binary semi open. Let (a,b)$\in(X-\{x\},Y-\{y\})$. This implies that $a\in X-\{x\}$ and $b\in Y-\{y\}$. Hence $a\neq x$ and $b\neq y$. That is (a,b) and (x,y) are jointly distinct binary points of X×Y. Since $(X,Y,\mathscr{M})$ is binary semi-T_1, there exists binary semi open sets (A,B) and (C,D), (a,b)∈(A,B) and (x,y)∈(C,D) such that (a,b)$\in(X-C,Y-D)$ and (x,y)$\in(X-A,Y-B)$. Therefore, (A,B)$\subseteq(X-\{x\},Y-\{y\})$. Hence $(X-\{x\},Y-\{y\})$ is a binary neighbourhood of (a,b). This implies $(\{x\},\{y\})$ is binary semi closed.*

Conversely, assume that $(\{x\},\{y\})$ is binary semi closed for every (x,y)$\in X\times Y$. Let (x_1,y_1) and $(x_2,y_2)\in X\times Y$ with $x_1\neq x_2$, $y_1\neq y_2$. Therefore, $(x_2,y_2)\in(X-\{x_1\},Y-\{y_1\})$ and $(X-\{x_1\},Y-\{y_1\})$ is binary semi open. Also $(x_1,y_1)\in(X-\{x_2\},Y-\{y_2\})$ and $(X-\{x_2\},Y-\{y_2\})$ is binary semi open. This shows that $(X,Y,\mathscr{M})$ is a binary semi-T_1.

Theorem 3.17. *If a binary topological space $(X,Y,\mathscr{M})$ is called a binary semi-T_0, then $(X,\mathscr{M}_X)$ is semi-T_0 and $(Y,\mathscr{M}_Y)$ is semi-T_0.*

Proof: *Since $(\mathscr{M})$ is a binary topology from X to Y, we have $(\mathscr{M}_X)=\{A\subseteq X:(A,B)\in(\mathscr{M})$ for some $B\subseteq Y\}$ is a topology on X and $(\mathscr{M}_Y)=\{B\subseteq Y:(A,B)\in(\mathscr{M})$ for some $A\subseteq X\}$ is a topology on Y. Let $(x_1,x_2)\in X$ and $(y_1,y_2)\in Y$ with $x_1\neq x_2$, $y_1\neq y_2$. Since $(X,Y,\mathscr{M})$ is binary semi-T_0, there exists semi open set (A,B) such that either $(x_1,y_1)\in(A,B)$, $(x_2,y_2)\in(X-A,Y-B)$ or $(x_1,y_1)\in(X-A,Y-B)$, $(x_2,y_2)\in(A,B)$. This implies that either $x_1\in A$, $x_2\in X-A$, $y_1\in B$, $y_2\in Y-B$ or $x_1\in X-A$, $x_2\in A$, $y_1\in Y-B$, $y_2\in B$. This implies that $(X,\mathscr{M}_X)$ is semi-T_0 and $(Y,\mathscr{M}_Y)$ is semi-T_0.*

Theorem 3.18. *If a binary topological space $(X,Y,\tau_{\mathscr{M}(X)}\times\sigma_{\mathscr{M}(Y)})$ is called a binary semi-T_0, then the topological spaces (X,τ) and (Y,σ) are semi-T_0.*

Proof: *Suppose that $(X,Y,\tau_{\mathscr{M}(X)}\times\sigma_{\mathscr{M}(Y)})$ is binary semi-T_0. Let $(x_1,x_2)\in X$ and $(y_1,y_2)\in Y$ with $x_1\neq x_2$, $y_1\neq y_2$. Since $(X,Y,\tau_{\mathscr{M}(X)}\times\sigma_{\mathscr{M}(Y)})$ is binary semi-T_0, there exists (A,B) $\in\tau_{\mathscr{M}(X)}\times\sigma_{\mathscr{M}(Y)}$ such that either $(x_1,y_1)\in(A,B)$, $(x_2,y_2)\in(X-A,Y-B)$ or $(x_1,y_1)\in(X-A,Y-B)$, $(x_2,y_2)\in(A,B)$. This implies that either $x_1\in A$, $x_2\in X-A$, $y_1\in B$, $y_2\in Y-B$ or*

$x_1 \in X-A$, $x_2 \in A$, $y_1 \in Y-B$, $y_2 \in B$. This implies either $x_1 \in A$, $x_2 \in X-A$ or $x_1 \in X-A$, $x_2 \in A$ and $y_1 \in B$, $y_2 \in Y-B$ or $y_1 \in Y-B$, $y_2 \in B$. Since $(A,B) \in \tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(Y)}$, we have $A \in \tau$ and $B \in \sigma$. Hence (X,τ) and (Y,σ) are semi-T_0.

Theorem 3.19. *If a binary topological space $(X,Y,\mathscr{M})$ is called a binary semi-T_1, then $(X,\mathscr{M}_X)$ is semi-T_1 and $(Y,\mathscr{M}_Y)$ is semi-T_1.*

Proof: *Since $(\mathscr{M})$ is a binary topology from X to Y, we have $(\mathscr{M}_X) = \{A \subseteq X : (A,B) \in (\mathscr{M})$ for some $B \subseteq Y\}$ is a topology on X and $(\mathscr{M}_Y) = \{B \subseteq Y : (A,B) \in (\mathscr{M})$ for some $A \subseteq X\}$ is a topology on Y. Let $(x_1,x_2) \in X$ and $(y_1,y_2) \in Y$ with $x_1 \neq x_2$, $y_1 \neq y_2$. Since $(X,Y,\mathscr{M})$ is binary semi-T_1, there exists binary semi open sets $(U_1,V_1),(U_2,V_2)$ with $(x_1,y_1) \in (U_1,V_1)$, $(x_2,y_2) \in (U_2,V_2)$, such that $(x_1,y_1) \in (X-U_2,Y-V_2)$, $(x_2,y_2) \in (X-U_1,Y-V_1)$. This implies that $x_1 \in U_1$, $x_2 \in U_2$ and $y_1 \in V_1$, $y_2 \in V_2$ such that $x_1 \in X-U_2$, $x_2 \in X-U_1$ and $y_1 \in Y-V_2$, $y_2 \in Y-V_1$. Hence $(X,\mathscr{M}_X)$ is semi-T_1 and $(Y,\mathscr{M}_Y)$ is semi-T_1*

4. Binary Semi-T_3, T_4 Spaces

In this section, we initiate binary semi-T_3,T_4 spaces by utilizing binary semi open sets and examination some of their properties.

Definition 4.1. *A binary topological spaces $(X,Y,\mathscr{M})$ is called a binary semi-T_3 or binary semi regular if $(X,Y,\mathscr{M})$ is binary semi-T_1 and for every $(x,y) \in X \times Y$ and every binary semi closed set $(A,B) \subseteq X \times Y$ such that $(x,y) \in (X-A,Y-B)$ there exists jointly disjoint binary semi open sets $(U_1,V_1),(U_2,V_2)$ such that $(x,y) \in (U_1,V_1)$, $(A,B) \subseteq (U_2,V_2)$.*

Definition 4.2. *A binary topological spaces $(X,Y,\mathscr{M})$ is called a binary semi-T_4 or binary semi normal if $(X,Y,\mathscr{M})$ is binary semi-T_1 and for every pair of jointly disjoint binary semi closed sets $(A_1,B_1),(A_2,B_2)$ there exists jointly disjoint binary semi open sets $(U_1,V_1),(U_2,V_2)$ such that $(A_1,B_1) \subseteq (U_1,V_1)$ and $(A_2,B_2) \subseteq (U_2,V_2)$*

Theorem 4.3. *Every binary regular space is binary semi regular space.*

Proof: *Let (X,Y) is binary regular and (A,B) be a binary closed set not containing (x,y) implies (A,B) be a binary semi closed set not containing (x,y). As (X,Y) is binary semi regular there* *exists jointly disjoint binary semi open sets* $(U_1,V_1),(U_2,V_2)$ *such that (x,y)* $\in (U_1,V_1)$, *(A,B)* $\subseteq (U_2,V_2)$. *Hence (X,Y) is binary semi regular.*

Example 4.4. *Let X =* $\{a,b\}$, *Y =* $\{a,b,c\}$. *Clearly* $\mathscr{M} = \{(\phi,\phi),(\{b\},\{a\}),(\{\phi,\{b,c\}),(\{b\},Y),(X,Y)\}$ *is a binary topology from X to Y. We have binary semi open set =* $\{(\phi,\phi),(\phi,\{b,c\}),(\{a\},\{b,c\}),(\{b\},\{a\}),(\{b\},Y),(X,\{a\}),(X,Y)\}$. *Let* $(A,B)=(\{a\},\phi)$, $(x,y)=(\{b\},\{a\})$, $(U_1,V_1) = (\{b\},$
$\{a\})$ *and* $(U_2,V_2) = (\{a\},\{b,c\})$ *then it is binary semi regular space but not binary regular space.*

Theorem 4.5. *Every binary semi regular space is binary semi-*T_0 *space.*

Proof: *Let (X,Y) is binary semi regular. As (X,Y) is binary semi regular every singleton set* $\{x_1,y_1\}$ *is binary semi closed subset of (X,Y) and* $\{x_2,y_2\}$ *be any point* $(X,Y)-\{x_1,y_1\}$ *then* $x_1 \neq x_2$, $y_1 \neq y_2$. *By definition of binary semi regularity there exists two jointly disjoint binary semi open sets* (U_1,V_1) *and* (U_2,V_2) *such that* $(x_1,y_1) \subseteq (U_1,V_1)$ *and* $(x_2,y_2) \notin (U_2,V_2)$, *implies* $(x_1,y_1) \in (U_1,V_1)$ *and* $(x_2,y_2) \notin (U_2,V_2)$ *Hence (X,Y) is binary semi-*T_0 *space.*

Example 4.6. *Let X=* $\{a,b\}$, *Y =*$\{a,b,c\}$. *Clearly* $\mathscr{M} = \{(\phi,\phi),(\phi,\{a\}),(\{a\},\{a\}),(\{a\},\{a,b\}),(\{b\},\phi),(\{b\},\{a\}),(\{b\},\{c\}),(\{b\},\{a,c\}),(X,\{a\}),(X,\{a,b\}),(X,\{a,c\}),(X,Y)\}$ *is a binary topology from X to Y. We have binary semi open=*$\{(\phi,\phi),(\phi,\{a\}),(\phi,\{a,b\}),(\{a\},\{a\}),(\{a\},\{a,b\}),(\{b\},\phi),(\{b\},\{a\}),(\{b\},\{c\}),(\{b\},\{a,b\}),(\{b\},\{a,c\}),(\{b\},Y),(X,\{a\}),(X,\{a,b\}),(X,\{a,c\}),(X,Y)\}$. *Let* $(x_1,y_1)=(\{b\},\{a\})$ *and* $(x_2,y_2)=(\{a\},\{c\})$, $(x_1,y_1),(x_2,y_2) \in (X,Y)$ *and* $(x_1,y_1) \neq (x_2,y_2)$ *there exists binary semi open set (A,B)=*$(\{b\},\{a,b\})$ *then it is binary semi-*T_0 *space but not binary semi regular space.*

Theorem 4.7. *Every binary semi regular space is binary semi-*T_2 *space.*

Proof: *Let (X,Y) is binary semi regular. As (X,Y) is binary semi regular every singleton set* $\{x_1,y_1\}$ *is binary semi closed subset of (X,Y) and* $\{x_2,y_2\}$ *be any point* $(X,Y)-\{x_1,y_1\}$ *then* $x_1 \neq x_2$, $y_1 \neq y_2$. *By definition of binary semi regularity there exists two jointly disjoint binary semi open sets* (U_1,V_1) *and* (U_2,V_2) *such that* $(x_1,y_1) \subseteq (U_1,V_1)$ *and* $(x_2,y_2) \in (U_2,V_2)$.
$\Rightarrow (x_1,y_1) \in (U_1,V_1)$ *and* $(x_2,y_2) \in (U_2,V_2)$. *Hence (X,Y) is binary semi-*T_2 *space.*

Example 4.8. *From the Example 3.7, Let* (x_1,y_1)*=(*$\{b\},\{c\}$*) and* (x_2,y_2)*=(*$\{a\},\{a\}$*). Let* (U_1,V_1)
=($\{b\},\{b,c\}$*) and* (U_2,V_2)*=(*$\{a\},\{a\}$*),* $(x_1,y_1),(x_2,y_2)$ *∈(X,Y) and* $(x_1,y_1) \neq (x_2,y_2)$ *then it is clear that* (x_1,y_1) *∈(A,B), and* (x_2,y_2) *∈(C,D). Then we can say that it is binary semi-*T_2 *space but not binary semi regular space.*

Theorem 4.9. *Let the topological spaces (X,τ) and* (Y,σ) *are semi-*T_3 *spaces if and only if the binary topological space* $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(Y)})$ *is called a binary semi-*T_3*.*

Proof: *Suppose (X,τ) and* (Y,σ) *are semi-*T_3 *spaces. Let (x,y)* $\in X \times Y$ *and (A,B)* $\subseteq X \times Y$ *be a binary semi closed (x,y)∈* $(X - A \times Y - B)$*. Therefore, x∈X, y∈Y and A⊆X, B⊆Y. Since (X,τ) is semi-*T_3*, there exists disjoint semi open sets* $U_1,U_2 \in \tau$*, x∈* U_1 *and A* $\subseteq U_2$*. Also, since (Y,σ) is semi-*T_3*, there exists disjoint semi open sets* $V_1,V_2 \in \sigma$*, y∈* V_1 *and B* $\subseteq V_2$*. This implies that (x,y)∈* (U_1,V_1) *and (A,B)∈* (U_2,V_2)*. Since* U_1 *and* U_2 *are disjoint semi open sets, we have* $U_1 \cap U_2 = \phi$*. Also since* V_1 *and* V_2 *are disjoint semi open sets we have* $V_1 \cap V_2 = \phi$*. Thus* $(U_1 \cap U_2, V_1 \cap V_2) = (\phi,\phi)$*. Hence* (U_1,V_1) *and* (U_2,V_2) *are disjoint binary semi open sets. This implies that* $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(Y)})$ *is binary semi-*T_3*.*

Conversely, assume that $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(X)})$ *is binary semi-*T_3*. Let x∈X and A be a semi closed subset of (X,τ). Let y∈Y and B be a semi closed subset of (Y,σ). Therefore, (x,y)∈* $X \times Y$ *and (A,B) is binary semi closed in* $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(X)})$*. Since* $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(X)})$ *is binary semi-*T_3*, there exists disjoint semi open sets* (U_1,V_1) *and* (U_2,V_2) *such that (x,y)∈* (U_1,V_1) *and (A,B)⊆* (U_2,V_2)*. Hence x∈* U_1 *and A⊆* U_2*, y∈* V_1 *and B⊆* V_2*. This proves that (X,τ) and* (Y,σ) *are semi-*T_3 *spaces*

Theorem 4.10. *Every binary normal space is binary semi normal space.*

Proof: *Let (X,Y) be a binary normal space and* (A_1,B_1) *and* (A_2,B_2) *be pair of jointly disjoint binary closed. As every binary closed set is binary semi closed set.* (A_1,B_1) *and* (A_2,B_2) *are binary semi closed sets and (X,Y) is binary semi normal, therefore there exists disjoint binary semi open sets* (U_1,V_1) *and* (U_2,V_2) *such that* $(A_1,B_1) \subseteq (U_1,V_1)$ *and* $(A_2,B_2) \subseteq (U_2,V_2)$*. Thus for every pair of disjoint binary closed sets* (A_1,B_1) *and* (A_2,B_2) *there exists disjoint binary semi open sets* (U_1,V_1) *and* (U_2,V_2) *such that* $(A_1,B_1) \subseteq (U_1,V_1)$ *and* $(A_2,B_2) \subseteq (U_2,V_2)$*. Hence (X,Y) is binary semi normal.*

Theorem 4.11. *Every binary semi normal space is binary semi regular space.*

Proof: *Let (X,Y) be a binary semi normal, Let (F,G) be any binary semi closed set and let (x,y) be a point of (X,Y) such that (x,y)$\notin$(F,G). As $\{x,y\}$ is a binary semi closed subset of (X,Y) such that $\{x,y\} \cap (F,G) = \phi$. Then by binary semi normality, there exists binary semi open sets (U_1,V_1) and (U_2,V_2) such that $\{x,y\} \subseteq (U_1,V_1)$, (F,G) $\subseteq (U_2,V_2)$ and $(U_1,V_1) \cap (U_2,V_2) = \phi$. Also $\{x,y\} \subseteq (U_1,V_1) \Longrightarrow$ (x,y) $\in (U_1,V_1)$.*

Thus there exists binary semi open sets (U_1,V_1) and (U_2,V_2) such that (x,y) $\in (U_1,V_1)$, (F,G) $\subseteq (U_2,V_2)$ and $(U_1,V_1) \cap (U_2,V_2) = \phi$ it follows that the space is (X,Y) is binary semi regular.

Example 4.12. *Let X= $\{a,b\}$, Y =$\{a,b,c\}$. Clearly $\mathscr{M}$ = $\{(\phi,\phi),(\{b\},\{a\}),(\phi,\{b,c\}),(\{b\},Y),(X,Y)\}$ is a binary topology from X to Y. We have binary semi open set = $\{(\phi,\phi),(\phi,\{b,c\}),(\{a\},\{b,c\}),(\{b\},\{a\}),(\{b\},Y),(X,\{a\}),(X,Y)\}$. Let $(A,B)=(\{a\},\phi)$, $(x,y)=(\{b\},\{a\})$, $(U_1,V_1) = (\{b\},$*

$\{a\})$ and $(U_2,V_2) = (\{a\},\{b,c\})$ then it is binary semi regular space but not binary semi normal space.

Theorem 4.13. *A binary semi closed subspace of a binary semi normal space is binary semi normal.*

Proof: *Let (K,L) be a binary semi closed subspace of a binary semi normal space. Let (A_1,B_1) and (A_2,B_2) be disjoint binary semi closed subset of (K,L). Since (K,L) is binary semi closed in (X,Y). (A_1,B_1) and (A_2,B_2) are binary semi closed in (X,Y). Since (X,Y) is binary semi normal, there exists disjoint binary semi open sets (U_1,V_1) and (U_2,V_2) in (X,Y), such that $(A_1,B_1) \subseteq (U_1,V_1)$ and $(A_2,B_2) \subseteq (U_2,V_2)$. Since (K,L) contains both (A_1,B_1) and (A_2,B_2), we have $(A_1,B_1) \subseteq (K,L) \cap (U_1,V_1)$, $(A_2,B_2) \subseteq (K,L) \cap (U_2,V_2)$ and $((K,L) \cap (U_1,V_1)) \cap (K,L) \cap (U_2,V_2) = (\phi,\phi)$. Since (U_1,V_1) and (U_2,V_2) are binary semi open in (X,Y). $(K,L) \cap (U_1,V_1)$ and $(K,L) \cap (U_2,V_2)$ are binary semi open in (K,L). Thus in the subspace (K,L), we have disjoint binary semi open sets $((K,L) \cap (U_1,V_1))$ containing (A_1,B_1) and $((K,L) \cap (U_2,V_2))$ containing (A_2,B_2). Hence the subspace (K,L) is binary semi normal.*

Theorem 4.14. *Let the topological spaces (X,τ) and (Y,σ) are semi-T_4 spaces if and only if the binary topological space $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(Y)})$ is called a binary semi-T_4.*

Proof: *Suppose (X,τ) and (Y,σ) are semi-T_4 spaces. (A_1,B_1) and (A_2,B_2) be disjoint pair of binary semi closed sets in $(X,Y,\mathscr{M})$. Then A_1,A_2 are disjoint semi closed sets in (X,τ) and B_1,B_2 are disjoint semi closed sets in (Y,σ). Since (X,τ) is semi-T_4, there exists disjoint semi open sets in $U_1,U_2 \in \tau$, $A_1 \subseteq U_1$ and $A_2 \subseteq U_2$. Also, since (Y,σ) is semi-T_4 there exists disjoint semi open sets $V_1,V_2 \in \sigma$, $B_1 \subseteq V_1$ and $B_2 \subseteq V_2$. This implies that $(A_1,B_1) \subseteq (U_1,V_1)$ and $(A_2,B_2) \subseteq (U_2,V_2)$. Since U_1 and U_2 are disjoint semi open sets, we have $U_1 \cap U_2 = \phi$. Also since V_1 and V_2 are disjoint semi open sets, we have $V_1 \cap V_2 = \phi$. Thus $(U_1 \cap U_2, V_1 \cap V_2) = (\phi,\phi)$. Hence (U_1,V_1) and (U_2,V_2) are disjoint binary semi open sets . This implies that $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(Y)})$ is a binary semi-T_4.*

Conversely, assume that $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(Y)})$ is binary semi-T_4. Let A_1,A_2 be disjoint semi closed sets in (X,τ) and B_1,B_2 be disjoint semi closed sets in (Y,σ). Then (A_1,B_1), (A_2,B_2) are binary semi closed in $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(Y)})$. Since $(X,Y,\tau_{\mathscr{M}(X)} \times \sigma_{\mathscr{M}(Y)})$ is binary semi-T_4, there exists disjoint binary semi open sets (U_1,V_1) and (U_2,V_2) such that $(A_1,B_1) \subseteq (U_1,V_1)$ and $(A_2,B_2) \subseteq (U_2,V_2)$. That is, $A_1 \subseteq U_1$, $A_2 \subseteq U_2$ and $B_1 \subseteq V_1$, $B_2 \subseteq V_2$. Hence (X,τ) and (Y,σ) are semi-T_4 spaces.

CONCLUSION

The separation axioms namely semi-T_0, semi-T_1, semi-T_2, semi-T_3 and semi-T_4 are extended to binary topological spaces. It is editorialize deserving to perceive that binary semi-T_4 $\Rightarrow$ binary semi-T_3 $\Rightarrow$ binary semi-T_2 $\Rightarrow$ binary semi-T_1 $\Rightarrow$ binary semi-T_0.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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