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## QUEUES WITH INTERRUPTION IN MARKOVIAN ENVIRONMENT

S. JAYA<sup>1,\*</sup>, B. LAKSHMI<sup>2</sup>

<sup>1</sup>Department of Mathematics, Maharaja's College, Ernakulam, Kerala-682011, India

<sup>2</sup>Department of Mathematics, CUSAT, Ernakulam, Kerala-682022, India

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**Abstract.** This paper carries out an analysis of an  $M/M/1$  queue with service interruption, where the interruptions are caused by different environmental factors which are the states of a Markov chain. Interruption due to only one factor is allowed at a time and during an interruption period, no more interruption is allowed. The server is unaware of the interruption until a random amount of time elapses. The duration of unidentified interruption is assessed with the help of the random clock that started ticking at the onset of the interruption. The interruption is identified only when the random clock is realized. A new service is commenced to the interrupted customer after fixing the interruption. The kind of service to be started after fixing the interruption depends on the environmental factor that caused the interruption. Even though any number of interruptions can occur during the service of a customer, the maximum number of interruptions is restricted to a finite number  $K$  and if the number of interruptions exceeds this maximum, the customer goes out of the system without completing the service. The superclock measures the total duration of interruption in the service of a customer. This clock starts ticking at the onset of the first interruption to a customer in service. When interruption is fixed, this clock stops. If another interruption strikes the same customer's service, the superclock starts from the earlier position where it stopped ticking and so on. Finally, when the customer leaves the system the superclock is reset to the zero position. On realization of superclock the customer goes out of the system without completing service. We analysed service process to obtain stability condition. Then we performed steady state analysis and important performance measures are obtained.

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\*Corresponding author

E-mail address: [jayasreelakam@gmail.com](mailto:jayasreelakam@gmail.com)

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## 1. INTRODUCTION

We come across different types of interruptions in all walks of life like internet browsing, banking, medical emergency, etc. Queue with interruption plays an important role in the fast growing world of network communication. The interruption can be due to different factors such as server breakdown, preemptive priority, vacation, etc. The pioneers of the concept of the queue with interruption are White and Christie[1]. The arrival rate during the breakdown of the server and service rate after the breakdown of the server could be different from what it was when the server was working. Bhaskar Sen Gupta [2] analysed a model with the above characteristics.

A survey paper by Krishnamoorthy et.al [3] gives a detailed description of the research on queueing models with service interruptions induced by break down as well as those with customer induced service break. Verghese et.al [4] introduced another form of interruption called customer induced interruption. Here the self interrupted customer enters into a finite buffer. On finding the buffer full the interrupted customer goes out of the system. On completion of their interruption, the customers in the buffer enter another buffer. They are given preemptive priority over new customers.

In almost all papers on queues with service interruptions, the service is either resumed or repeated on the removal of interruption. In the resumption of service, the interrupted service is resumed from the point of interruption, and in the case of a restart a new service begins after fixing the interruption. Keilson [5] considers a queueing model with repeat or resumption of the interrupted service. In [6] Krishnamoorthy and co authors studied a queueing model with interruption where the special stress was on deciding whether to repeat or resume an interrupted service. A paper by Krishnamoorthy, Deepak and Pramod [7] considers a queueing model with service interruption and repair where the decision on, whether to repeat or resume the interrupted service is following the realization of a phase type distributed random clock.

In [8] Krishnamoorthy et.al studied a queueing model with interruption where the interruption

is caused by  $n$  random environmental factors and the maximum number of interruptions that can happen to a customer is restricted to a finite number. If the interruption exceeds that predetermined number the customer goes out of the system without completing service. The server is unaware of the interruption for a random duration and that duration is determined with the help of a random clock started at the onset of the interruption. The total duration of interruption to a particular customer is determined with the help of a super clock started at the onset of the first interruption to a customer in service and which is reset to zero position when the customer in service leaves the system either by completing or by not completing the service. In this paper we consider a queueing system in which interruption is caused by different environmental factors which are the states of a Markov chain, ie. the environmental factor causing the previous interruption influences the environmental factor causing the next interruption. All other assumptions are the same as in [8].

The remaining discussion on the model is arranged as follows. Section 2 gives a detailed description of the model. In section 3 the service process with interruption is analysed. Section 4 is developed based on the number of interruptions. Detailed analysis of the queueing model is included in section 5. Section 6 is devoted to important performance measures. A numerical illustration is included in section 7.

## 2. MODEL DESCRIPTION

In this queueing system arrivals occur according to a Poisson process with parameter  $\lambda$ . The service time is exponentially distributed with parameter  $\mu$ . During service, interruption occurs according to a Poisson process with parameter  $\beta$ . There are  $n$  environmental factors causing interruption which form a Markov chain with initial probability vector  $p_i; i = 1, 2, \dots, n$  and transition probability matrix  $\mathbf{P} = (p_{ij}); i, j = 1, 2, \dots, n$ .

For a certain duration of time, the server is unaware of the interruption. At the start of the first interruption a random clock which is exponentially distributed with parameter  $\eta_i$ , starts ticking when the interruption is caused by the  $i^{th}$  environmental factor. The random clock measures the time elapsed from the epoch of occurrence of interruption until the identification of interruption. The realization of the random clock indicates the identification of interruption. The fixing time is exponentially distributed with parameter  $\alpha_i$ , only if the  $i^{th}$  factor is the cause

of the interruption. On fixing the interruption, a new service starts which is exponentially distributed with parameter  $\mu_i$ , provided the  $i^{th}$  factor caused the interruption.

The super clock, which is Erlang distributed with parameter  $\gamma$  and  $a$ , started at the beginning of the first interruption that strikes the customer in service, will freeze when the new service starts after interruption and it will again start functioning from the position where it stopped when another interruption occurs. Sometimes the super clock may realize before the random clock. The number of interruptions during the service of a customer is limited to  $K$ . On realization of the super clock or when the number of interruptions exceeds  $K$ , whichever occurs first to the customer in service, he goes out of the system without completing service.

### 3. ANALYSIS OF SERVICE PROCESS WITH INTERRUPTIONS

**Response time.** The service process  $\{X(t), t \geq 0\}$  where  $X(t) = (S(t), I_1(t), I_2(t), I_3(t), I_4(t))$  is a Markov Chain with  $3naK + 1$  transient states and one absorbing state. The absorbing state represents the customer moving out of the system, either after service completion or without completing service. Here  $S(t)$  denotes the status of the server at time  $t$ :

$$S(t) = \begin{cases} 0, & \text{if the service is going on at time } t, \text{ has not under gone any interruption so far,} \\ 1, & \text{if the service is interrupted at time } t, \\ 2, & \text{if service is in interruption fixing state at time } t, \\ 3, & \text{if service is continuing after interruption at time } t; \end{cases}$$

$I_1(t)$  denotes the number of interruptions occurred until time  $t$  to the current customer in service.  $I_1(t)$  varies from 0 to  $K$ .  $I_2(t)$  corresponds to the environmental factor that caused the current interruption to the service. In this model we consider  $n$  environmental factors.

Thus  $I_2(t)$  has values varying from 1 to  $n$ .  $I_3(t)$  represents the status of the random clock:

$$I_3(t) = \begin{cases} 0, & \text{if the random clock is realized} \\ 1, & \text{otherwise;} \end{cases}$$

$I_4(t)$  corresponds to the phase of the super clock:  $I_4(t)$  varies from 1 to  $a$ . The states of the process is  $\{0\} \cup \{(1, m, i, j, l) / m = 1, 2, \dots, K; i = 1, \dots, n; j = 1; l = 1, \dots, a\} \cup \{(2, m, i, j, l) / m = 1, 2, \dots, K; i = 1, \dots, n; j = 0; l = 1, \dots, a\} \cup \{(3, m, i, j, l) / m = 1, 2, \dots, K; i = 1, \dots, n; j = 0; l = 1, \dots, a\} \cup *$ , where  $*$  is the absorbing state. The infinitesimal generator of the process is given by

$$\bar{Q} = \begin{bmatrix} T & T^0 \\ 0 & 0 \end{bmatrix} \text{ where } T = \begin{bmatrix} C_0 & C_1 & 0 & 0 \\ 0 & C_2 & C_3 & 0 \\ 0 & 0 & C_4 & C_5 \\ 0 & C_6 & 0 & C_7 \end{bmatrix}_{(3naK+1) \times (3naK+1)}$$

$$\text{Let } \bar{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix}, \mathbf{p} = (p_1, p_2, \dots, p_n)$$

$$\text{then } C_0 = [-(\mu + \beta)], C_1 = \begin{bmatrix} \beta \mathbf{p} \otimes e'_a(1) & 0 \end{bmatrix}_{(1 \times naK)}, C_2 = I_K \otimes G_0.$$

$$G_0 = \begin{cases} -\gamma - \eta_r, & \text{if } i = j; i, j = (r-1)a + l; r = 1, \dots, n; l = 1, \dots, a; \\ \gamma, & \text{if } j = i + 1; i, j = (r-1)a + l; r = 1, \dots, n; l = 1, \dots, a - 1; \\ 0, & \text{otherwise;} \end{cases}$$

$$C_3 = I_K \otimes G_1, \text{ where}$$

$$G_1 = \begin{cases} \eta_r, & \text{if } i = j; i, j = (r-1)a + l; r = 1, \dots, n; l = 1, \dots, a; \\ 0, & \text{otherwise;} \end{cases}$$

$$C_4 = I_K \otimes G_2 \text{ where}$$

$$G_2 = \begin{cases} -\gamma - \alpha_r, & \text{if } i = j; i, j = (r-1)a + l; r = 1, \dots, n; l = 1, \dots, a; \\ \gamma, & \text{if } j = i + 1; i, j = (r-1)a + l; r = 1, \dots, n; l = 1, \dots, a - 1; \\ 0, & \text{otherwise;} \end{cases}$$

$$C_5 = I_K \otimes G_3 \text{ where}$$

$$G_3 = \begin{cases} \alpha_r, & \text{if } i = J; i, j = (r-1)a + l; r = 1, \dots, n; l = 1, \dots, a; \\ 0, & \text{otherwise;} \end{cases}$$

$$C_6 = \begin{bmatrix} 0 & I_{K-1} \otimes G_4 \\ 0 & 0 \end{bmatrix} \text{ where } G_4 = \beta \mathbf{P} \otimes I_a, \mathbf{P} = (p_{ij}); \text{for } i, j = 1, \dots, n.$$

$$C_7 = I_K \otimes G_5 \text{ and}$$

$$G_5 = \begin{cases} -\mu_r - \beta, & \text{if } i = J; i, j = (r-1)a + l; r = 1, \dots, n; l = 1, \dots, a \\ 0, & \text{otherwise;} \end{cases}$$

$$T^0 = \begin{bmatrix} C_{03} \\ C_{13} \\ C_{23} \\ C_{33} \end{bmatrix}_{(3nK+1) \times 1} \text{ with}$$

$$C_{03} = \begin{bmatrix} \mu \end{bmatrix}, C_{13} = \begin{bmatrix} e_{nK} \otimes \gamma e_a(a) \end{bmatrix}, C_{23} = C_{13}, C_{33} = \begin{bmatrix} e_{K-1} \otimes (\bar{\mu} \otimes e_a) \\ \bar{\mu} \otimes e_a + e_{an}\beta \end{bmatrix}_{(naK \times 1)}.$$

where  $e$  is a column vector with all its entries equal to 1 and of appropriate order.

In matrix  $T$ ,  $T_{ii} < 0, 1 \leq i \leq 3nak + 1$  and  $T_{ij} \geq 0$  for  $i = j$ . Also  $Te + T^0 = 0$ . The initial probability vector is  $\zeta = (1, 0, \dots)$ . Here the service process with interruption follows PH Distribution. So using the property that residual service time in a phase type distributed service process is also phase type we see that

- The Probability distribution  $F(\cdot)$  of  $X$  with initial probability vector  $\zeta$  is given by

$$F(x) = 1 - \zeta \cdot \exp(Tx) \cdot e, x \geq 0.$$

- The density function  $F'(x)$  in  $(0, \infty)$  is given by  $F'(x) = \zeta \exp(Tx) T^0$ .

- The Laplace Stieltjis transform  $f(s)$  of  $F(\cdot)$  is  $f(s) = \zeta (sI - T)^{-1} T^0$ .

- Probability for service completion without any interruption is,  $P(s) = \zeta (-C_0)^{-1} C_{03}$ .

- Probability for customer leaving the system due to realization of super clock,

$$P_{RS} = \zeta [(-C_0)^{-1} C_{11}] [(-C_2)^{-1} C_{13}] + \zeta [(-C_0)^{-1} C_{11}] [(-C_2)^{-1} C_{31}] [(-C_4)^{-1} C_{23}].$$

- Probability for customer leaving the system due to occurrence of maximum number of

interruption,  $P_{MI} = \zeta [(-C_0)^{-1} C_{11}] [(-C_2)^{-1} C_{31}] [(-C_4)^{-1} C_{51}] [(-C_6)^{-1} C'_{33}]$  where

$$C'_{33} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ e_{an}\beta \end{bmatrix}_{(naK \times 1)}.$$

**Lemma:** The expected time for service completion/customer leaving the system without completing service due to realization of super clock or attaining maximum  $K$  is  $E(ST) = \zeta (-T)^{-1} e$ , and hence the expected service rate is  $\mu_s = 1/E(ST)$ .

#### 4. EXPECTED NUMBER OF INTERRUPTIONS

The expected number of interruption during the service of a customer can be calculated by considering the Markov chain  $\{Y(t), t \geq 0\}$  where  $Y(t) = (I_1(t), S(t), I_2(t), I_3(t), I_4(t))$  with state space  $\{0\} \cup \{(m, 1, i, j, l) | m = 1, 2, \dots, K; i = 1, \dots, n; j = 1; l = 1, \dots, a\} \cup \{(m, 2, i, j, l) | m = 1, 2, \dots, K; i = 1, \dots, n; j = 0; l = 1, \dots, a\} \cup \{(m, 3, i, j, l) | m = 1, 2, \dots, K; i = 1, \dots, n; j = 0; l =$

$1, \dots, a, \} \cup \Omega$ , where  $\Omega$  is the absorbing state. The infinitesimal generator of the process is given by

$$Q_I = \begin{bmatrix} U & U^0 \\ 0 & 0 \end{bmatrix} \text{ where } U = \begin{bmatrix} D_0 & D_1 & & & & \\ & D_2 & D_3 & & & \\ & & D_2 & D_3 & & \\ & & & \ddots & \ddots & \\ & & & & \ddots & \ddots \\ & & & & & D_2 \end{bmatrix}_{(3naK+1) \times (3naK+1)} \quad . D_0 = C_0,$$

$$D_1 = \begin{bmatrix} \beta \mathbf{p} \otimes e'_a(1) & 0 \end{bmatrix}_{(1 \times na)} \quad D_2 = \begin{bmatrix} G_0 & G_1 & 0 \\ 0 & G_2 & G_3 \\ 0 & 0 & G_5 \end{bmatrix}_{(3na) \times (3na)},$$

$$D_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ G_4 & 0 & 0 \end{bmatrix}_{(3na) \times (3na)}$$

$$U^0 = \begin{bmatrix} D'_0 \\ D'_1 \\ D'_1 \\ \cdot \\ \cdot \\ D'_2 \end{bmatrix} \text{ where } D'_0 = C_{03}, D'_1 = \begin{bmatrix} e_n \otimes \gamma e_a(a) \\ e_n \otimes \gamma e_a(a) \\ \bar{\mu} \otimes e_a \end{bmatrix}, D'_2 = \begin{bmatrix} e_n \otimes \gamma e_a(a) \\ e_n \otimes \gamma e_a(a) \\ \bar{\mu} \otimes e_a + e_{an} \beta \end{bmatrix}.$$

The initial probability vector  $\zeta = (1 \ 0 \ 0 \ \dots)$ .

From the above matrices we get the following system characteristics:

- Probability for absorption after  $r (r \leq K)$  interruption is given by
 
$$(N_r) = \begin{cases} (-D_0)^{-1} D'_0, & \text{for } r = 0, \\ (-D_0^{-1} D_1) (-D_2^{-1} D_3)^{(r-1)} (-D_2^{-1} D'_2), & \text{for } r = K, \\ (-D_0^{-1} D_1) (-D_2^{-1} D_3)^{(r-1)} (-D_2^{-1} D'_1), & \text{otherwise.} \end{cases}$$
- Expected number of interruptions for a customer,  $E(I) = \sum_{r=1}^K r N_r$ .
- Probability for service completion after  $r (1 \leq r \leq K)$  interruptions,





where  $R$  is the minimal non- negative solution to the matrix quadratic equation

$$R^2A_2 + RA_1 + A_0=0.$$

From  $\chi Q=0$  we get

$$\mathbf{x}_0B_0 + \mathbf{x}_1B_2=0.$$

$$\mathbf{x}_0B_1 + \mathbf{x}_1(A_1 + RA_2)=0.$$

Solving the above two equations we get  $\mathbf{x}_0$  and  $\mathbf{x}_1$  subject to the normalizing condition

$$\mathbf{x}_0e + \mathbf{x}_1(I - R)^{-1}e=1.$$

### 6. PERFORMANCE MEASURES

**Expected waiting time.** The expected waiting time of a particular customer who joins the queue as the  $m^{th}$  customer, can be computed by considering the Markov Chain

$Z= \{(M(t),S(t),I_1(t),I_2(t),I_3(t),I_4(t)),t \geq 0\}$  where  $M(t)$  is the rank of the tagged customer.The tagged customers rank will decrease to 1 as the customers ahead of him leave the system. The rank of the customer is not affected by the arrival of customers following the tagged customer. Here  $S(t)$  and  $I_j(t), j = 1,2,3,4$  are as defined section 2.  $Z$  is a Markov chain with state space  $\{(m,0)|m = r, \dots, 1\} \cup \{(m,1,s,i,j,l)|m = r, \dots, 1; s = 1,2, \dots, K; i = 1, \dots, n; j = 1; l = 1, \dots, a\} \cup \{(m,2,s,i,j,l)|m = r, \dots, 1; s = 1,2, \dots, K; i = 1, \dots, n; j = 0; l = 1, \dots, a\} \cup \{(m,1,s,i,j,l)|m = r, \dots, 1; s = 1,2, \dots, K; i = 1, \dots, n; j = 0; l = 1, \dots, a\} \cup \Phi$ , where  $\Phi$  is the absorbing state. The infinitesimal generator matrix  $Q_2$  is given by  $Q_2 = \begin{bmatrix} W & W^0 \\ 0 & 0 \end{bmatrix}$

where

$$W = \begin{bmatrix} T & T^0\zeta & & & & \\ & T & T^0\zeta & & & \\ & & \ddots & \ddots & & \\ & & & T & T^0\zeta & \\ & & & & T & \end{bmatrix} \text{ and } W^0 = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ T^0 \end{bmatrix}.$$

The expected waiting time of the tagged customer, according to the position of the customer being served at the time of arrival of the tagged customer, is a column vector which is obtained from the formula  $E_W^r = -T^{-1}(I - (T^0\zeta T^{-1})^{(r-1)})(I - T^0\zeta T^{-1})^{-1}e$ .

Hence the expected waiting time of a customer waiting in the queue is  $E(W) = \sum_{r=1}^{\infty} \mathbf{x}_r E_W^r$

### Other important performance characteristics.

- Expected number of customers completing service without interruption  $E(NI) = \sum_{i=1}^{\infty} i \cdot x_{i,0}$ .
- Probability that there are  $i$  ( $i \geq 0$ ) customers in the system,  $P_i = x_i \cdot e$ .
- Expected number of customers in the system,  $E(s) = \sum_{i=1}^{\infty} i P_i$ .
- Fraction of time the server in the interrupted state,  $FT(I) = \sum_{i=1}^{\infty} (x_{i,1}e + x_{i,2}e)$ .
- Fraction of time the server is busy,  $FT(B) = \sum_{i=1}^{\infty} (x_{i,0}e + x_{i,1}e + x_{i,3}e)$ .
- Fraction of time the server in the unidentified interrupted state:  $FT(NI) = \sum_{i=1}^{\infty} x_{i,1}e$ .
- Fraction of time the server in fixing state:  $FT(FS) = \sum_{i=1}^{\infty} x_{i,2}e$ .
- Fraction of time the super clock is freezed :  $FT(SF) = \sum_{i=1}^{\infty} x_{i,3}e$ .
- Fraction of time the super clock is on:  $= \sum_{i=1}^{\infty} (x_{i,1}e + x_{i,2}e)$ .

## 7. NUMERICAL ILLUSTRATIONS

The performance of the system depending on the change in various parameters are numerically illustrated below. Assume  $\lambda = 3, \mu = 6, n = 3, a = 3; \beta = 0.5; \gamma = 0.5;$

$$\alpha_1 = 4, \alpha_2 = 3, \alpha_3 = 3,; \eta_1 = 4, \eta_2 = 4, \eta_3 = 3,; \mu_1 = 3, \mu_2 = 2, \mu_3 = 2;$$

$$\mathbf{p} = (0.3, 0.4, 0.3), \text{ and the transition probability matrix } \mathbf{P} = \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0.4 & 0.1 & 0.5 \\ 0.5 & 0.4 & 0.1 \end{bmatrix}$$

**7.1. Effect of number of interruption  $K$  on various performance measures.** From table 1, as the value of  $K$  increases expected number of customers in the system  $E(s)$  and expected waiting time  $E(W)$  increases.

**7.2. Effect of  $\beta$  on various performance measures.** By considering  $\mu = 8$  and  $K = 5$  we have the following values. From table 2, as the value of  $\beta$  increases probability for service completion with interruption  $P(s)$ , the fraction of time the server is busy  $FT(B)$  decreases, but effective service rate  $\mu_s$ , the fraction of time the server getting interrupted  $FT(I)$ , expected

TABLE 1. **Effect of change in  $K$  on  $E(s)$  &  $E(W)$**

$K$	$E(s)$	$E(W)$
1	0.6792	0.0585
2	0.7175	0.0645
3	0.7238	0.0655
4	0.7248	0.0657
5	0.7250	0.0657
6	0.7250	0.0657
7	0.7250	0.0657

TABLE 2. **Effect of change in  $\beta$  on various performance measures**

$\beta$	$P(s)$	$\mu_s$	$FT(B)$	$FT(I)$	$E(s)$	$E(I)$	$E(W)$
0.2	0.9756	0.0727	0.3713	0.0054	0.4445	0.0191	0.0321
0.4	0.9524	0.0739	0.3677	0.0106	0.5175	0.0304	0.042
0.6	0.9302	0.0751	0.3643	0.0155	0.5934	0.0376	0.0538
0.8	0.9091	0.0763	0.3611	0.0202	0.6712	0.0425	0.0653
1	0.8889	0.0774	0.3580	0.0247	0.7453	0.0465	0.0772

number of customers in the system  $E(s)$ , expected number of interruptions  $E(I)$  and expected waiting time  $E(W)$  begins to increase which are on expected lines.

**7.3. Effect of  $\mu$  on various performance measures.** By considering  $\beta = .5$  and  $K = 5$  we have the following values. From table 3, as the value of  $\mu$  increases probability for service completion with interruption  $P(s)$  increases but the fraction of time the server is busy  $FT(B)$ , effective service rate  $\mu_s$ , the fraction of time the server getting interrupted  $FT(I)$ , expected number of customers in the system  $E(s)$ , expected number of interruptions  $E(I)$  and expected waiting time  $E(W)$  increases which are on expected line.

**7.4. Effect of  $\gamma$  on various performance measures.** By considering  $\beta = .5$ ,  $\mu = 8$  and  $K = 5$  we have the following values. From table 4, as the super clock realization rate increases expected number of customers in the system and expected waiting time decreases. This is due

TABLE 3. Effect of change in  $\mu$  on various performance measures

$\mu$	$P(s)$	$\mu_s$	$FT(B)$	$FT(I)$	$E(s)$	$E(I)$	$E(W)$
6	0.9231	0.0864	0.4786	0.0171	0.7250	0.0450	0.0657
7	0.9333	0.0800	0.4148	0.0148	0.6290	0.0390	0.0557
8	0.9412	0.0745	0.3660	0.0131	0.5551	0.0344	0.0482
9	0.9474	0.0697	0.3275	0.0117	0.4967	0.0308	0.0423
10	0.9524	0.0655	0.2963	0.0106	0.4494	0.0278	0.0376
11	0.9565	0.0617	0.2705	0.0097	0.4103	0.0254	0.0338
12	0.9600	0.0584	0.2489	0.0089	0.3775	0.0234	0.0306

TABLE 4. Effect of change in  $\gamma$  on various performance measures

$\gamma$	$FT(B)$	$FT(I)$	$E(s)$	$E(I)$	$E(W)$
0.2	0.0132	0.5572	0.3661	0.0343	0.0485
0.4	0.0131	0.5561	0.3661	0.0344	0.0483
0.6	0.0130	0.5538	0.3660	0.0344	0.0480
0.8	0.0129	0.5507	0.3658	0.0345	0.0475
1	0.0127	0.5468	0.3656	0.0347	0.0469
1.2	0.0125	0.5423	0.3655	0.0350	0.0462
1.4	0.0123	0.5376	0.3653	0.0352	0.045

to customers leaving the system when super clock realizes. Expected number of interruptions increases as  $\gamma$  increases.

**7.5. Effect of  $\lambda$  on various performance measures.** By considering  $\mu = 14$  and  $K = 5$  we have the following values. From table 5, as the value of  $\lambda$  increases the fraction of time the server is busy  $FT(B)$ , the fraction of time the server gets interrupted  $FT(I)$ , expected number of customers in the system  $E(s)$ , expected number of interruptions  $E(I)$  and expected waiting time  $E(W)$  increases.

TABLE 5. **Effect of change in  $\lambda$  on various performance measures**

$\lambda$	$FT(B)$	$FT(I)$	$E(s)$	$E(W)$
1	0.0830	0.0030	0.1258	0.0205
2	0.1659	0.0059	0.2517	0.0270
3	0.2489	0.0089	0.3775	0.0306
4	0.3319	0.0119	0.5032	0.0329
5	0.4148	0.0148	0.6272	0.0345
6	0.4974	0.0183	0.7336	0.0357
7	0.5689	0.0480	0.7287	0.0359

**CONFLICT OF INTERESTS**

The author(s) declare that there is no conflict of interests.

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