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ON $\widehat{D}$-CLOSED MAPS AND $\widehat{D}$-OPEN MAPS IN TOPOLOGICAL SPACES

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Abstract. In this paper, we introduce $\widehat{D}$-closed map from a topological space X to a topological space Y as the image of every closed set is $\widehat{D}$-closed and also we prove that the composition of two $\widehat{D}$-closed maps need not be $\widehat{D}$-closed map. We also obtain some properties of $\widehat{D}$-closed maps.

Keywords: $\widehat{D}$-open set; $\widehat{D}$-closed set; quasi $\widehat{D}$-closed maps; $\widehat{D}$-open maps; $\widehat{D}$-closed maps; strongly $\widehat{D}$-closed maps.

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1. INTRODUCTION

T. Noiri., H. Maki and J. Umehara [6] introduced the concept of gp-closed and pre-gp-closed map using gp-closed sets. G. B. Navalagi [10] introduced the concepts of strongly α-closed maps and quasi α-closed maps in topological space by using α-closed set in topological spaces. In this paper, a new class of maps called $\widehat{D}$-closed maps have been introduced and studied their relations with various generalized closed maps. We prove that the composition of two $\widehat{D}$-closed maps need not be $\widehat{D}$-closed map. We also obtain some properties of $\widehat{D}$-closed maps and quasi

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$\widehat{D}$-closed, strongly $\widehat{D}$-closed and the relationships between these maps. K. Dass and G. Suresh [11] introduced new class of sets called $\widehat{D}$-closed sets in topological spaces.

2. Preliminaries

Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f : (X,\tau) \to (Y,\sigma)$ (or simply $f : X \to Y$) denotes a function f of a space (X,τ) into a space (Y,σ). Let A be a subset of a space X. The closure, the interior and complement of A are denoted by $cl(A)$, $int(A)$ and A^c respectively.

Definition 2.1. *A subset A of a topological space (X,τ) is called*

i) a pre-open set [5] *if $A \subset int(cl(A))$ and a pre-closed set if $cl(int(A)) \subset A$,*

ii) a semi-open set [2] *if $A \subset cl(int(A))$ and a semi-closed set if $int(cl(A)) \subset A$,*

iii) a semi-pre-open set [7] *(β-open* [1]*) if $A \subset cl(int(cl(A)))$ and a semi-preclosed set ($= \beta$-closed) if $int(cl(int(A))) \subset A$.*

Definition 2.2. *Let (X,τ) be a topological space and $A \subset X$*

i) an ω-closed set [8] *(= $\widehat{g}$-closed* [9]*) if $cl(A) \subset U$ whenever $A \subset U$ and U is semi-open in (X,τ),*

ii) a D-closed set [4] *if $pcl(A) \subset int(U)$ whenever $A \subset U$ and U is ω-open in (X,τ).*

Complements of the above mentioned sets are called their respectively open sets

Definition 2.3. *A subset A of (X,τ) is called an $\widehat{D}$-closed* [11] *set if $spcl(A) \subset U$ whenever $A \subset U$ and U is D-open in (X,τ). The class of all $\widehat{D}$-closed sets in (X,τ) is denoted by $\widehat{D}c(\tau)$. That is, $\widehat{D}c(\tau) = \{A \subset X : A \text{ is } \widehat{D} - \text{closed in } (X,\tau)\}$.*

Definition 2.4. *Let (X,τ) be a topological space and $A \subset X$*

(1) semi-pre interior of A denoted by $spint(A)$ is the union of all semi-pre open subsets of A

(2) semi-pre closure of A denoted by $spcl(A)$ is the intersection of all semi-pre closed subsets of A

Definition 2.5. *A space X is called a $T_{\widehat{D}}$-space if every $\widehat{D}$-closed set is closed.*

Theorem 2.6. [11] *A subset A of a topological space (X,τ) is said to be $\widehat{D}$-open if and only if $F \subset spint(A)$ whenever $A \supset F$ and F is D-closed in (X,τ).*

Proposition 2.7. [11] *In a topological space X, assume that $\widehat{D}o(\tau)$ is closed under any union. Then $\widehat{D}cl(A)$ is an $\widehat{D}$-closed set for every subset A of X.*

3. $\widehat{D}$-CLOSED MAPS

Definition 3.1. *A map $f : X \to Y$ is said to be $\widehat{D}$-closed if the image of every closed set of X is $\widehat{D}$-closed in Y.*

Theorem 3.2. *A surjective map $f : X \to Y$ is $\widehat{D}$-closed if and only if for each subset S of Y and each open set U containing $f^{-1}(S)$, there exists an $\widehat{D}$-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.*

Proof. **Necessity** Suppose that f is $\widehat{D}$-closed. Let S be any subset of Y and U an open set of X containing $f^{-1}(S)$. Put $V = (f(U^c))^c$. Then V is $\widehat{D}$-open in Y containing S and $f^{-1}(V) \subset U$.
Sufficiency. Let F be any closed set of X. Put $B = (f(F))^c$, then we have $f^{-1}(B) \subset F^c$ and F^c is open in X. Be hypothesis there exists an $\widehat{D}$-open set in V of Y such that $B \subset V$ and $f^{-1}(V) \subset F^c$ and so $F \subset (f^{-1}(V))^c = f^{-1}(V^c)$. Therefore, we obtain $f(F) = V^c$. Since V^c is $\widehat{D}$-closed, $f(F)$ is $\widehat{D}$-closed in Y. This gives f is $\widehat{D}$-closed. □

Remark 3.3. *Necessity of above theorem is proved without assuming that f is surjective. Therefore we can obtain the following corollary.*

Corollary 3.4. *If $f : X \to Y$ is $\widehat{D}$-closed, then for any closed set F of Y and for any open set U of X containing $f^{-1}(F)$ there exists a semi-preopen set V of Y such that $F \subset V$ and $f^{-1}(V) \subset U$.*

Proof. By Theorem 3.2, there exists an $\widehat{D}$-open W of Y such that $F \subset W$ and $f^{-1}(W) \subset W$. Since F is closed, F is D-closed. By theorem 2.6 $F \subset spint(w)$. Put $V = spint(W)$ then V is semi-preopen in Y such that $F \subset V$ and $f^{-1}(spint(W)) \subset f^{-1}(W) \subset U$ and hence $f^{-1}(V) \subset U$. □

Remark 3.5. *The following example shows that composition of two $\widehat{D}$-closed maps is not $\widehat{D}$-closed.*

Example 3.6. *Let $X = Y = Z = \{p,q,r\}$, $\tau = \{\phi,\{p\},\{q\},\{p,q\},X\}$, $\sigma = \{\phi,\{p,q\},Y\}$ and $\eta = \{\phi,\{p\},Z\}$. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ are identity maps. Then clearly f and g are $\widehat{D}$-closed maps but $g\circ f : X\to Z$ is not $\widehat{D}$-closed, since $\{p,r\}$ is closed in X and $(g\circ f)\{p,r\} = g(f(\{p,r\})) = g(\{p,r\}) = \{p,r\}$ is not $\widehat{D}$-closed in Z.*

Proposition 3.7. *If $f : X\to Y$ and $g : Y\to Z$ are $\widehat{D}$-closed maps with Y is a $T_{\widehat{D}}$–space, then $g\circ f : X\to Z$ is also an $\widehat{D}$-closed map.*

Proof. Clearly follows from Definitions. □

Proposition 3.8. *If $f : X\to Y$ from a space X to a $T_{\widehat{D}}$-space Y. Then the following are equivalent:*

(1) f is $\widehat{D}$-closed
(2) f is closed

Proof. Follows by Definition 2.5 □

Proposition 3.9. *Let $f : X\to Y$ and $g : Y\to Z$ be two maps such that $g\circ f : X\to Z$ is $\widehat{D}$-closed.*

i) If f is continuous surjection, then g is $\widehat{D}$-closed;
ii) If g is $\widehat{D}$-irresolute and injective, then f is $\widehat{D}$-closed;
iii) If f is $\widehat{D}$-continuous surjection and X is a $T_{\widehat{D}}$-space then g is $\widehat{D}$-closed

Proof. i) Let A be a closed set of Y. Since f is continuous, $f^{-1}(A)$ is closed in X. Also since $g\circ f$ is $\widehat{D}$-closed and f is surjective, $(g\circ f)f^{-1}(A) = g(A)$ is $\widehat{D}$-closed in Z. Hence g is $\widehat{D}$-closed.

ii) Let B be a closed set of X. Since $g\circ f$ is $\widehat{D}$-closed, $(g\circ f)(B)$ is $\widehat{D}$-closed in Z. Also since g is $\widehat{D}$-irresolute, $g^{-1}(g\circ f)(A)$ is $\widehat{D}$-closed in Y. Since g is injective, $f(B)$ is $\widehat{D}$-closed in Y. Hence, f is $\widehat{D}$-closed.

iii) Let A be a closed set of Y. Since f is $\widehat{D}$-continuous, $f^{-1}(A)$ is $\widehat{D}$-closed in X. Also since X is a $T_{\widehat{D}}$-space, we have $f^{-1}(A)$ is closed in X. Since $(g\circ f)$ is closed and f is surjective, then $(g\circ f)f^{-1}(A) = g(A)$ is $\widehat{D}$-closed in Z. Hence, g is $\widehat{D}$-closed.

□

Definition 3.10. *A space X is said to be ultra $\widehat{D}$-regular if for each closed set F of X and each point $x \notin F$ there exists disjoint $\widehat{D}$-open sets U and V such that $F \subset U$ and $x \in V$.*

Theorem 3.11. *In a topological space X, assume that $\widehat{D}o(\tau)$ is closed under any union. Then the following statements are equivalent:*

a) X is ultra $\widehat{D}$-regular,

b) for every point x of X every open set V containing x, there exists an $\widehat{D}$-open set A such that $x \in A \subset \widehat{D}cl(A) \subset V$.

Proof. $a \Longrightarrow b$ Let $x \in X$ and V be an open set containing x.Then V^c is closed and $x \notin V^c$. By (a) there exists disjoint $\widehat{D}$-open sets A and B such that $x \in A$ and $V^c \subset B$. That is $B^c \subset V$. Since every open set is $\widehat{D}$-open, V is $\widehat{D}$-open. Since B is $\widehat{D}$-open, B^c is $\widehat{D}$-closed. Therefore, $\widehat{D}cl(B^c) \subset V$. Since $A \cap B = \phi$, $A \subset B^c$. Therefore, $x \in A \subset \widehat{D}cl(A) \subset \widehat{D}cl(B^c) \subset V$. Hence, $x \in A \subset \widehat{D}cl(A) \subset V$.

$b \Longrightarrow a$. Let F be a closed set and $x \notin F$. This implies that F^c is an open set containing x. By (b) there exists an $\widehat{D}$-open set A such that $x \in A \subset \widehat{D}cl(A) \subset F^c$. That is, $F \subset (\widehat{D}cl(A))^c$. By Proposition 2.7 $\widehat{D}cl(A)$ is $\widehat{D}$-closed. Hence, $(\widehat{D}cl(A))^c$ is $\widehat{D}$-open. Therefore, A and $(\widehat{D}cl(A))^c$ are the required $\widehat{D}$-open sets. □

Theorem 3.12. *Assume that $\widehat{D}o(\tau)$ is closed under any union. If $f : X \to Y$ is a continuous $\widehat{D}$-closed surjective map and X is a regular space, then Y is ultra $\widehat{D}$-regular.*

Proof. Let $y \in Y$ and V be an open set containing y of Y. Let x be a point of X such that $y = f(x)$. Since f is continuous, $f^{-1}(V)$ is open in X. Since X is regular there exists an open set U such that $x \in U \subset cl(U) \subset f^{-1}(V)$. Hence, $y = f(x) \in f(U) \subset f(cl(U)) \subset V$. Since f is an $\widehat{D}$-closed map, $f(cl(U))$ is an $\widehat{D}$-closed set contained in the open set V. Since every open set is D-open, V is D-open. Hence, $spcl(f(cl(U))) \subset V$. Therefore $y \in f(U) \subset \widehat{D}cl(f(U)) \subset \widehat{D}cl(f(cl(U))) \subset spcl(f(cl(U))) \subset V$. This implies that $y \in f(U) \subset \widehat{D}cl(f(U) \subset V$. Since f is an open map and U is open in X, $f(U)$ is open in Y. Since every open set is $\widehat{D}$-open, $f(U)$ is $\widehat{D}$-open in Y. Thus for every point y of Y and every open set V containing y there exists an $\widehat{D}$-open set $f(U)$ such that $y \in f(U) \subset \widehat{D}cl(f(U)) \subset V$. Hence by theorem 7, Y is ultra $\widehat{D}$-regular. □

Definition 3.13. *A space X is said to be ultra $\widehat{D}$-normal if for disjoint closed sets A and B of X there exist disjoint $\widehat{D}$-open sets U and V such that $A \subset U$ and $B \subset V$.*

Theorem 3.14. *Assume that $\widehat{D}o(\tau)$ is closed under any union. If $f : X \to Y$ is a continuous $\widehat{D}$-closed surjective map and X is a normal space, then Y is ultra $\widehat{D}$-normal.*

Proof. Let A and B be disjoint closed sets of Y. Since X is normal there exist disjoint open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. By theorem 3.2, there exist $\widehat{D}$-open sets G and H such that $A \subset G$, $B \subset H$ and $f^{-1}(G) \subset U$, $f^{-1}(H) \subset V$. Then we have $f^{-1}(G) \cap f^{-1}(H) = \phi$ and hence $G \cap H = \phi$. Since G is $\widehat{D}$-open and A is closed, $A \subset G$ implies $A \subset spint(G) \subset \widehat{D}int(G)$. Similarly $B \subset \widehat{D}int(H)$. Therefore, $\widehat{D}int(G) \cap \widehat{D}int(H) = \phi$. Thus Y is ultra $\widehat{D}$-normal. □

Theorem 3.15. *If $f : X \to Y$ is a bijective $\widehat{D}$-closed map of a space X onto an $\widehat{D}$-connected space Y, then X is connected.*

Proof. Let us assume that X is not connected. Then there exist nonempty open sets U and V such that $U \cap V = \phi$ and $X = U \cup V$. Therefore U and V are clopen in X and $f(U)$ and $f(V)$ are $\widehat{D}$-closed. Moreover, we have $f(U) \cap f(V) = \phi$ and $f(U) \cup f(V) = Y$. Since f is bijective, $f(U)$ and $f(V)$ are nonempty. This indicates that Y is not $\widehat{D}$-connected. This is a contradiction. □

4. STRONGLY $\widehat{D}$-CLOSED AND QUASI $\widehat{D}$-CLOSED MAPS

Definition 4.1. *A map $f : X \to Y$ is said to be strongly $\widehat{D}$-closed if for each $\widehat{D}$-closed set F of X, $f(F)$ is $\widehat{D}$-closed in Y.*

Definition 4.2. *A map $f : X \to Y$ is said to be quasi $\widehat{D}$-closed if for each $\widehat{D}$-closed set F of X, $f(F)$ is closed in Y.*

Proposition 4.3. *Every quasi $\widehat{D}$-closed map is strongly $\widehat{D}$-closed.*

Proof. Obvious. □

Proposition 4.4. *Every quasi $\widehat{D}$-closed map is closed.*

Proof. Since every closed set is $\widehat{D}$-closed, we get the proof. □

Proposition 4.5. *Every strongly $\widehat{D}$-closed map is $\widehat{D}$-closed.*

Proof. Clearly follows from Definitions. □

Example 4.6. *Let $X = \{p,q,r\}$ and $Y = \{p,q,r\}$, $\tau = \{\phi,\{p\},\{q\},\{p,q\},X\}$ and $\sigma = \{\phi,\{p,q\},Y\}$. Clearly identity map $f : (X,\tau) \to (Y,\sigma)$ is strongly $\widehat{D}$-closed map but not quasi $\widehat{D}$-closed, Since $\{r\}$ is $\widehat{D}$-closed in X but $f(\{r\}) = \{r\}$ is not closed in Y.*

Example 4.7. *Let $X = \{p,q,r\}$ and $Y = \{p,q,r\}$, $\tau = \{\phi,\{p,q\},X\}$ and $\sigma = \{\phi,\{p\},\{p,q\},Y\}$. Clearly identity map $f : (X,\tau) \to (Y,\sigma)$ is closed map but not quasi $\widehat{D}$-closed, Since $\{q\}$ is $\widehat{D}$-closed in X but $f(\{q\}) = \{q\}$ is not closed in Y and not strongly $\widehat{D}$-closed, Since $\{p\}$ is $\widehat{D}$-closed in X but $f(\{p\}) = \{p\}$ is not $\widehat{D}$-closed in Y.*

Theorem 4.8. *A surjective mapping $f : X \to Y$ is quasi-$\widehat{D}$-closed if and only if for any subset B of Y and for each $\widehat{D}$-open set U of X containing $f^{-1}(B)$, there is an open set V of Y containing B such that $B \subset V$ and $f^{-1}(V) \subset U$.*

Proof. **Necessity** Suppose that f is quasi $\widehat{D}$-closed. Let S be any subset of Y and U an $\widehat{D}$-open set of X containing $f^{-1}(S)$. Put $V = (f(U^c))^c$. Then V is open in Y containing S and $f^{-1}(V) \subset U$.

Sufficiency. Let F be any $\widehat{D}$-closed set of X. Put $B = (f(F))^c$, then we have $f^{-1}(B) \subset F^c$ and F^c is $\widehat{D}$-open in X. By hypothesis there exists an open in V of Y such that $B \subset V$ and $f^{-1}(V) \subset F^c$ and so $F \subset (f^{-1}(V))^c = f^{-1}(V^c)$. Therefore, we obtain $f(F) = V^c$. Since V^c is closed, $f(F)$ is closed in Y. This gives f is quasi $\widehat{D}$-closed. □

Theorem 4.9. *In a topological space X, assume that $\widehat{D}o(\tau)$ is closed under any union. A map $f : X \to Y$ is quasi $\widehat{D}$-closed if and only if for every subset U of X, $cl(f(U)) \subset f(\widehat{D}cl(U))$.*

Proof. Let f be quasi $\widehat{D}$-closed. We have $U \subset \widehat{D}cl(U)$ and also $\widehat{D}cl(U)$ is an $\widehat{D}$-closed set. Hence we obtain $f(U) \subset f(\widehat{D}cl(U))$ and $f(\widehat{D}cl(U))$ is closed. Hence $cl(f(U)) \subset f(\widehat{D}cl(U))$.

Conversely, assume that the given condition holds. If U is an $\widehat{D}$-closed in X, then $cl(f(U)) \subset f(\widehat{D}cl(U)) = f(U)$. Consequently, $f(U) = cl(f(U))$ and hence f is quasi $\widehat{D}$-closed. □

Theorem 4.10. *In a topological space X, assume that $\widehat{D}o(\tau)$ is closed under any union. A map $f : X \to Y$ is strongly $\widehat{D}$-closed if and only if for every subset U of X, $\widehat{D}cl(f(U)) \subset f(\widehat{D}cl(U))$.*

Proof. Let f be strongly $\widehat{D}$-closed. We have $U \subset \widehat{D}cl(U)$ and also $\widehat{D}cl(U)$ is an $\widehat{D}$-closed set. Hence we obtain $f(U) \subset f(\widehat{D}cl(U))$ and $f(\widehat{D}cl(U))$ is closed. Hence $cl(f(U)) \subset f(\widehat{D}cl(U))$.

Conversely, assume that the given condition holds. If U is an $\widehat{D}$-closed in X, then $cl(f(U)) \subset f(\widehat{D}cl(U)) = f(U)$. Consequently, $f(U) = cl(f(U))$ and hence f is strongly $\widehat{D}$-closed. □

Proposition 4.11. *Let $f : X \to Y$ and $g : Y \to Z$ be two strongly $\widehat{D}$-closed mapping. Then $g \circ f : X \to Z$ is a strongly $\widehat{D}$-closed mapping.*

Proof. Obvious □

Theorem 4.12. *If $f : X \to Y$ and $g : Y \to Z$ be two mapping such that $g \circ f : X \to Z$ is strongly $\widehat{D}$-closed.*

i) If f is $\widehat{D}$-irresolute and surjective, then g is strong $\widehat{D}$-closed.
ii) If g is $\widehat{D}$-irresolute injection, then f is strongly $\widehat{D}$-closed.

Proof. i) Let A be a is $\widehat{D}$-closed set of Y. Since f is $\widehat{D}$-irresolute, $f^{-1}(A)$ is $\widehat{D}$-closed in X. Also since $g \circ f$ is strongly $\widehat{D}$-closed and f is surjective, $(g \circ f)f^{-1}(A) = g(A)$ is $\widehat{D}$-closed in Z. Hence g is strongly $\widehat{D}$-closed.

ii) Let B be a $\widehat{D}$-closed set of X. Since $g \circ f$ is $\widehat{D}$-closed, $(g \circ f)(B)$ is $\widehat{D}$-closed in Z. Also since g is $\widehat{D}$-irresolute, $g^{-1}(g \circ f)(B)$ is $\widehat{D}$-closed in Y. Since g is injective, $f(B)$ is $\widehat{D}$-closed in Y. Hence, f is strongly $\widehat{D}$-closed.

□

Theorem 4.13. *Assume that $\widehat{D}o(\tau)$ is closed under any union. If $f : X \to Y$ is a continuous strongly $\widehat{D}$-closed bijective map and X is a $\widehat{D}$-regular space, then Y is ultra $\widehat{D}$-regular.*

Proof. Let $y \in Y$ and V be an open set containing y of Y. Let x be a point of X such that $y = f(x)$. Since f is continuous, $f^{-1}(V)$ is open in X. By theorem 3.11, there exists an $\widehat{D}$-open set U such that $x \in U \subset \widehat{D}cl(U) \subset f^{-1}(V)$. Then, $y \in f(U) \subset f(\widehat{D}cl(U)) \subset V$. By proposition 2.7 $\widehat{D}cl(U)$ is $\widehat{D}$-closed. Since f is an strongly $\widehat{D}$-closed map, $f(\widehat{D}cl(U))$ is an $\widehat{D}$-closed set. Since every open set is D-open, V is D-open. Hence, $spcl(f(cl(U))) \subset V$. Therefore, we have $\widehat{D}cl(f(\widehat{D}cl(U))) \subset spcl(f(\widehat{D}cl(U))) \subset V$. This implies that $y \in f(U) \subset \widehat{D}cl(f(U)) \subset \widehat{D}cl(f(\widehat{D}cl(U))) \subset V$. That is, $y \in f(U) \subset \widehat{D}cl(f(U)) \subset V$. Now U is $\widehat{D}$-open implies U^c is $\widehat{D}$-closed in X. Since f is strongly $\widehat{D}$-closed, $f(U^c)$ is $\widehat{D}$-closed in Y. That is, $(f(U))^c$ is $\widehat{D}$-closed in Y. This implies that $f(U)$ is $\widehat{D}$-open in Y. Thus for every point y of Y and every open set V containing y there exists an $\widehat{D}$-open set $f(U)$ such that $y \in f(U) \subset \widehat{D}cl(f(U)) \subset V$. Hence by theorem 3.11, Y is ultra $\widehat{D}$-regular. □

Theorem 4.14. *If $f : X \to Y$ is a continuous quasi $\widehat{D}$-closed surjective map and X is an ultra $\widehat{D}$-normal space, then Y is normal.*

Proof. Let A and B be disjoint closed sets of Y. Since X is ultra $\widehat{D}$-normal there exist disjoint $\widehat{D}$-open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. By theorem 3.5, there exist open sets G and H of Y such that $A \subset G$, $B \subset H$ and $f^{-1}(G) \subset U$, $f^{-1}(H) \subset V$. Then we have $f^{-1}(G) \cap f^{-1}(H) = \phi$ and hence $G \cap H = \phi$. □

Theorem 4.15. *If $f : X \to Y$ be a bijective map. Then following hold:*

- *i) If f is strongly $\widehat{D}$-closed map and Y is an $\widehat{D}$-connected space, then X is $\widehat{D}$-connected.*
- *ii) If f is quasi $\widehat{D}$-closed map and Y is an $\widehat{D}$-connected space, then X is $\widehat{D}$-connected.*

Proof. i) Let us assume that X is not $\widehat{D}$-connected. Then there exist nonempty $\widehat{D}$-open sets U and V such that $U \cap V = \phi$ and $X = U \cup V$. Therefore U and V are $\widehat{D}$-clopen in X. Since f is strongly $\widehat{D}$-closed map, $f(U)$ and $f(V)$ are $\widehat{D}$-closed. Moreover, we have $f(U) \cap f(V) = \phi$ and $f(U) \cup f(V) = Y$. Since f is bijective, $f(U)$ and $f(V)$ are nonempty. This indicates that Y is not $\widehat{D}$-connected. This is a contradiction.

ii) Similar to that of (i)

□

Proposition 4.16. *Let $f : X \to Y$ from a space X to a $T_{\widehat{D}}Y$. Then the following are equivalent:*

i) f is strongly $\widehat{D}$-closed.

ii) f is quasi $\widehat{D}$-closed.

Proof. Follows by proposition 4.3 and by Definition 2.5. □

Conflict of Interests

The author(s) declare that there is no conflict of interests.

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