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COMMUTATIVITY RESULTS WITH DERIVATIONS ON SEMIPRIME RINGS

MEHSIN JABEL ATTEYA*

Department of Mathematics ,College of Education , Al-Mustansiriyah University ,Baghdad, Box: 46219,Iraq

Abstract: In this paper, let R be a 2-torsion free semiprime ring and U a non-zero ideal of R, d a derivation mapping. If R admitting

A derivation d satisfies one of the following .

(i) $[d(x),d(y)]=[x,y]$ for all x, y $\in$ U.

(ii) $[d(x)^2,d(y)^2]=[x,y]$ for all x, y $\in$ U.

(iii) $[d(x),d(y)]=[x^2,y^2]$ for all x, y $\in$ U.

(iv) $[d(x)^2,d(y)^2]=[x^2,y^2]$ for all x, y $\in$ U .

A non – zero derivation d satisfies one of the following:

(i)$d([d(x),d(y)])=[x,y]$ for all x, y $\in$ U.

(ii)$d([d(x),d(y)])=[d(x),d(y)]$ for all x, y $\in$ U.Then R contains a non-zero central ideal .

Keywords: derivation, prime ring, semiprime ring, central ideal.

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1.Introduction

Several authors have investigated about semiprime rings under derivations and give some results. In [5] M.N.Daif, proved that, let R be a semiprime ring and d a derivation of R with $d^3 \neq 0$. If $[d(x),d(y)]=0$ for all x, y $\in$R, then R contains a non-zero central ideal. M.N. Daif and H.E. Bell [4] proved that, let R be a semiprime ring admitting a derivation d for which either $xy+d(xy)=yx+d(yx)$ for all x,y $\in$R or $xy-d(xy)=yx-d(yx)$ for all x,y

*Corresponding author

E-mail address: mehsinatteya@yahoo.com(M. Atteya)

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$\in$R, then R is commutative. V. DeFilippis [6] proved that, when R be a prime ring let d a non-zero derivation of R, U$\neq$(0) a two-sided ideal of R, such that d([x,y])=[x,y] for all x,y $\in$U, then R is commutative. A.H. Majeed and Mehsin Jabel [11], then gave some results as, let R be a 2-torsion free semiprime ring and U a non-zero ideal of R.R admitting a non-zero derivation d satisfying d([d(x),d(y)])=[x,y] for all x, y$\in$U. If d acts as a homomorphism, then R contains a non-zero central ideal. Recently, Mehsin Jabel [12] proved, let R be a semiprime ring and U be a non-zero ideal of R. If R admits a generalized derivation D associated with a non-zero derivation d such that D(xy)-xy$\in$Z(R) for all x, y$\in$U, then R contains a non-zero central ideal. Where according to[3], Bresar defined the following notation, an additive mapping D:R→R is said to be a generalized derivation if there exists a derivation d:R→R such that D(xy)=D(x)y+xd(y) for all x,y$\in$R. Hence the concept of a generalized derivation covers both the concepts of a derivation and of a left multiplier (i.e.an additive map d satisfying d(xy)=d(x)y for all x,y $\in$ R,[13]). In this paper we shall study and investigate some results concerning a derivation d on semiprime ring R, we give some results about that.

2. Preliminaries

Throughout R will represent an associative ring and has a cancellation property with center Z(R),R is said to be n-torsion free,where n$\neq$0 is an integer, if whenever nx=0,with x $\in$ R then x=0.We recall that R is semiprime if xRx=(0) implies x= 0 and it is prime if xRy=(0) implies x = 0 or y = 0 . A prime ring is semiprime but the converse is not true in general**.**An additive mapping d:R→ R is called a derivation if d(xy)=d(x)y+xd(y) holds for all x,y $\in$R,and is said to be n-centralizing on U(resp.n– commuting on U), if $[x^n,d(x)]$ $\in$ Z(R) holds for all x $\in$U (resp.$[x^n,d(x)]=0$ holds for all x$\in$U), where n be a positive integer. We write [x,y] for xy–yx and make extensive use of basic commutator identities [xy,z]=x[y,z]+[x,z]y and [x,yz]=y[x,z]+[x,y]z.

To achieve our purposes , we mention the following results .

Lemma 2.1 ([8],Sublemma P.5). *Let R be a 2-torsion free semiprime ring. Suppose that a $\in$R , such that a commutes with every [a,x], x $\in$R , then a $\in$Z(R).*

Lemma 2.2 ([6]). *Let R be a prime ring and U is a non-zero left ideal.If R admits a derivation d with d(U)* $\neq$ *0,satisfies d is centralizing on U. Then R is commutative .*

Lemma 2.3 (10 ,Main Theorem) . *Let R be a semiprime ring, d a non-zero derivation of R, and U a non-zero left ideal of R. If for some positive integers* $t_0, t_1, \ldots, t_n$ *and all* $x \in U$ *,the identity* $[[\ldots[[d(x^{t0}), x^{t1}, x^{t2}], \ldots], x^{tn}] = o$ *holds, then either d(U)= o or else d(U) and d(R)U are contained in non-zero central ideal of R. In particular when R is a prime ring, R is commutative.*

Lemma 2.4(9,Lemma1.8). *Let R be a semiprime ring, and suppose that a* $\in$ *R centralizes all commutators [x,y], x,y* $\in$ *R. Then a* $\in$ *Z(R).*

Lemma 2.5 ([7]). *Let n be a fixed integer,let R be n!– torsion free semiprime ring and U be a non-zero left ideal of R. If R admits a derivation d which is non-zero on U and n-centralizing on U, then R contains a non-zero central ideal .*

Lemma 2.6 ([2]). *Let R be a prime ring with center Z(R),and let U be a non-zero ideal of R .If U is a commutative ideal, then R is commutative.*

Lemma 2.7([3],Theorem2.2). *Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R. If R admits a derivation d which is non-zero on U and [d(x),d(y)]=0 for all x,y* $\in$ *U, then R contains a non-zero central ideal.*

Lemma 2.8. *Let n be a fixed positive integer, R semiprime ring and some a* $\in$ *R.If* $a^n \in Z(R)$ *then a* $\in$ *Z(R).*

Proof. The result holds for n=1. If n $\geq$ 2,we have $a^n \in Z(R)$,then $a^{n-1} \in Z(R)$, inductively, we obtain $a \in Z(R)$.

Theorem 2.9. *Let R be a 2–torsion free semiprime ring and U a non-zero ideal of R.If R admitting to satisfying* $[x^2, y^2] = 0$ *for all U. Then R contains a non-zero central ideal .*

Proof. We have $[x^2,y^2]=0$ for all $x,y \in U$.The linearization (i.e. putting $x+y$ for x) in above relation gives

$[xy+yx,y^2]=0$ for all $x,y\in U$. (1)

$[x,y^2]y+y[x,y^2]=0$ for all $x,y\in U$. (2)

Also from (1) , we obtain

$[xy+ yx - yx + yx,y^2]=0$ for all $x , y \in U$. Then

$[[x,y]+ 2yx,y^2] = 0$ for all $x , y \in U$.

$[[x, y]y^2]+2[yx ,y^2]=0$ for all $x , y \in U$. Replacing x by x^2, we obtain

$[[x^2,y],y^2]+2y[x^2,y^2]=0$ for all $x,y \in U$.According to the relation $[x^2,y^2]=0$,then we obtain$[[x^2,y],y^2]= 0$ for all $x,y \in U$. Then

$[x^2,y]y^2=y^2[x^2,y]$ for all $x,y\in U$. (3)

From (2),we have

$y[x,y]y+[x,y]y^2+ y^2[x,y]+y[x,y]y= 0$ for all $x,y \in U$.Replacing x by x^2, we obtain

$y[x^2,y]y+[x^2,y]y^2+y^2[x^2,y]+[x^2,y]y=0$ for all $x,y\in U$. (4)

Substituting (3) in (4) , we obtain

$2(y[x^2,y]y+[x^2,y]y^2)=0$ for all $x,y \in U$.Since R is 2-torsion free , then

$y[x^2,y]y+[x^2,y]y^2=0$ for all $x,y\in U$. (5)

Left – multiplying (5) by y , we get

$y^2 [x^2,y]y+ y[x^2,y]y^2= 0$ for all $x , y \in U$.Then we set

$a=y[x^2,y]y$, $a \in R$,thus

$ya + ay = 0$ for all $y \in U$.Then

$[ya,r]+ [ay,r] = 0$for all $y \in U,r \in R$. Then

$y[a,r]+[y,r]a+a[y,r]+[a,r]y= 0$ for all $y\in U,r\in R$.Replacing r by a,we obtain

$[y,a]a+a[y,a]= 0$ for all $y \in U$.Then

$[y,a^2] = 0$ for all $y \in U$. Then

$[[y,a^2],r]= 0$ for all $y \in U$.Replacing r by a^2 and by using Lemma2.1, we obtain $a^2 \in Z(R)$,by Lemma 2.8,we get $a\in Z(R)$,i.e.,$y[x^2,y]y \in Z(R)$ for all $x , y \in U$.Then

$[y[x^2,y]y,r] = 0$ for all $x , y \in U,r \in R$.

Replacing r by y , we obtain

$[y[x^2,y]y,y] = 0$ for all $x , y \in U$. Then

$y[[x^2,y],y]y=0$ for all $x ,y \in U$. (6)

Right-multiplying (6) by $[[x^2,y],y]$, we get

$(y[[x^2,y],y])^2 = 0$ for all $x, y \in U$ .Left-multiplying by w with using the cancellation property of $w\, y[[x^2,y],y]$, $w \in R$,we obtain

$y[[x^2,y],y]=0$ for all $x,y \in U$. (7)

Left-multiplying (6) by $[[x^2,y],y]$, we obtain

$([[x^2,y],y]y)^2 = 0$ for all $x, y \in U$. Right-multiplying by w with using the cancellation property of $[[x^2,y],y]yw$, $w \in R$, we obtain

$[[x^2,y],y]y=0$ for all $x,y\in U$. (8)

Subtracting (7) and (8) , we obtain

$[[[x^2,y]y],y] = 0$ for all $x , y \in U$. (9)

We set $[[x^2,y],y] = b$, $b \in R$. Then

$[b,y] = 0$ for all $y \in U$. Then

$[[b,y],r]= 0$ for all $y \in U,r \in R$.

Replacing r by b and by using Lemma 2.1, we obtain

$b \in Z(R)$, i.e.$[[x^2,y] y] \in Z(R)$ for all $x,y \in U$,then

$[[[x^2,y],y],r]= 0$ for all $x,y \in U,r\in R$. (10)

Replacing r by $[x^2,y]$ and using Lemma 2.1, we obtain

$[x^2,y] \in Z(R)$ for all $x,y \in U$,then

$[[x^2,y],r]= 0$ for all $x,y \in U,r\in R$. (11)

Replacing r by $[x,z]^2$ and x by $[x,z]$ with using Lemma 2.1, we obtain $[x,z]^2 \in Z(R)$,by Lemma2.8,we get $[x,z] \in Z(R)$ for all $x \in U$, then U a non-zero central ideal .

3. Main results

Theorem 3.1. *Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R.If R admitting a derivation d satisfying $[d(x),d(y)] = [x,y]$ for all $x ,y \in U$. Then R contains a non-zero central ideal .*

Proof. When we have $d \neq 0$,then

$[d(x),d(y)]=[x,y]$ for all $x,y \in U$. Replacing x by xt , we obtain

$[d(x)t ,d(y)]+ [xd(t),d(y)]= [xt,y]$ for all $x,y, t \in U$.

$d(x)[t,d(y)]+[d(x),d(y)]t+x[d(t),d(y)]+[x,(y)]d(t)=x[t,y]+[x,y]t$ for all $x,y,t \in U$ Since $[d(x),d(y)]=[x,y]$,then we have

$d(x)[t,d(y)]+ [x,d(y)]d(t) =0$ for all $x,y,t \in U$.

Replacing t and y by x , we obtain

$d(x)[x,d(x)]+ [x,d(x)]d(x) = 0$ for all $x,y,t \in U$. Then

$[x,d(x)^2]=0$ for all $x \in U$. Replacing x by x+ y, with replacing y by x,we obtain

$8[d(x^2),x]=0$ for all $x \in U$. Since R is 2-torsion free with using Lemma 2.4,We get U is a non-zero central ideal .

We,now suppose that d=0,we obtain $[x,y]=0$ for all $x,y \in U$. Replacing y by r y ,we get

$r[x,y]+ [x,r]y= 0$ for all $x , y \in U, r \in R$.

Since $[x,y]= o$, then we obtain

$[x,r]y=0$ for all $x,y\in U,r\in R$. (12)

Replacing y by rx , we obtain

$[x,r]rx=0$ for all $x\in U,r \in R$. (13)

In(12) replacing y by xr , we get

$[x,r]xr=0$ for all $x\in U,r\in R$. (14)

From (13) and (14) , we obtain $[x,r]^2 =0$ for all $x \in U,r \in R$.

Right-multiplying by w with using the cancellation property of $[x,r]w,w \in R$.we obtain, R contains a non-zero central ideal .

Theorem 3.2. *Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R.If R admitting a derivation d satisfying $[d^2(x),d^2(y)]=[x,y]$ for all $x,y \in U$. Then R contains a non-zero central ideal .*

Proof . Suppose that $d\neq 0$, then we have

$[d^2 (x),d^2(y)]=[x,y]$ for all $x, y \in U$. Then

$[d^2(x)d^2(y),r]–[d^2(y)d^2(x),r]=[[x,y],r]$ for all $x,y\in U,r\in R$. (15)

Replacing r by $d^2 (y)d^2(x)$, we obtain

$[d^2(x)d^2(y), d^2(y)d^2(x)] = [[x,y], d^2(y)d^2(x)]$ for all $x, y \in U$. Then

$d^2(x)[d^2(y), d^2(y)d^2(x)]+[d^2(x), d^2(y)d^2(x)]d^2(y) = d^2(y)[[x,y], d^2(x)]+$

$[[x,y], d^2(y)]d^2(x)$ for all $x,y \in U$.

$d^2(x)d^2(y)[d^2(y), d^2(x)]+[d^2(x), d^2(y)]d^2(x)d^2(y) = d^2(y)[[x,y], d^2(x)]+[[x,y], d^2(y)]d^2(x)$

for all $x,y \in U$.

According to the relation $[d^2(x), d^2(y)]=[x,y]$, we have

$d^2(x)d^2(y)[y,x]+[x,y]d^2(x)d^2(y) = d^2(y)[[x,y], d^2(x)]+[[x,y], d^2(y)]d^2(x)$

for all $x,y \in U$.Then

$[x,y]d^2(x)d^2(y)-d^2(x)d^2(y)[x,y]=d^2(y)[[x,y], d^2(x)]+[[x,y], d^2(y)]d^2(x)$

for all $x,y \in U$. (16)

In (15),replacing r by $[x,y]$,we obtain

$[d^2(x)d^2(y),[x,y]]-[d^2(y)d^2(x),[x,y]]=o$ for all $x,y \in U$. (17)

Also from relation $[d^2(x), d^2(y)]=[x,y]$ for all $x,y \in U$, we have

$d^2(x)d^2(y)=[x,y]+d^2(y)d^2(x)$ for all $x,y \in U$. (18)

Now substituting (18) in (16) , we get

$[x,y]^2+[x,y]d^2(y)d^2(x)-[x,y]^2-d^2(y)d^2(x)[x,y]=d^2(y)[[x,y], d^2(x)]+[[x,y], d^2(y)] \quad d^2(x)$ for all $x,y \in U$. Thus

$[[x,y], d^2(y)d^2(x)]=d^2(y)[[x,y], d^2(x)]+[[x,y], d^2(y)]d^2(x)$ forallx,y $\in U$. (19)

Now from (19) and (17) , we get

$[d^2(x)d^2(y),[x,y]]+2[[x,y], d^2(y)d^2(x)]=d^2(y)[[x,y], d^2(x)]+[[x,y], d^2(y)]d^2(x)$

for all $x,y \in U$. (20)

By subtracting (17) from (19), we obtain

$3[[x,y], d^2(y)d^2(x)]=d^2(y)[[x,y], d^2(x)]+ [[x,y], d^2(y)]d^2(x)$ for all $x,y \in U$.Then

$2[[x,y], d^2(y)d^2(x)]=0$ for all $x,y \in U$.Since R is 2-torsion free, we obtain

$[[x,y], d^2(y)d^2(x)]= 0$ for all $x,y \in U$. By Lemma 2.4, we obtain

$d^2(y)\, d^2(x) \in Z(R)$ for all $x,y \in U$, then

$[t, d^2(y)d^2(x)]= 0$ for all $x,y,t \in U$. Replacing y by x , we obtain

$[t, d^2(x)^2] = 0$ for all $x,t \in U$.Then

$d^2(x)^2 \in Z(R)$, by Lemma 2.8 , we obtain $d^2(x) \in Z(R)$ i.e.

$[d^2(x),t]= 0$ for all $x,t \in U$ .The Linearization (i.e., putting x+ y for x),gives

$[d^2(x)y,t]+2[d(x)d(y),t]+[xd^2(y),t]=0$ for all $x,y,t \in U$.

According to the relation $[d^2(x),t]=0$, the precedence equation with replacing t and y by x,become $2[d(x)^2,x]=0$ for all $x \in U$. Since R is 2-torsion free,then $[d(x)^2,x]= 0$ for all $x\in U$. The Linearization (i.e., putting x+ y for x) with replacing y by x, gives

$8[d(x^2), x^2]= 0$ for all $x\in U$. Since R is 2-torsion free with using Lemma 2.3, we obtain U is a non –zero central ideal.

We have, when d=0,then $[x,y]= 0$ for $x,y \in U$.

Replacing y by ry , we obtain

$[x,r]y=0$ for all $x,y \in U,r \in R$.Replacing y by $w[x,r]$ for all $x\in U,r,w\in R$, we obtain

$[x,r]w[x,r]=0$ for all $x\in U,r,w\in R$.Then $[x,r]=0$ for all $x\in U,r\in R$.

Since R is semiprime ring ,then $[x,r] = 0$ for all $x \in U,r\in R$.

Then U is a non-zero central ideal .

Theorem 3.3. *Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R .If R admitting a derivation d satisfying* $[d(x),d(y)]=[x^2,y^2]$ *for all* $x,y \in U$. *Then R contains a non-zero central ideal* .

Proof. We suppose first that $d\neq 0$, then

$[d(x),d(y)]=[x^2,y^2]$ for all $x, y \in U$. Replacing x by x+y , we obtain

$[d(x),d(y)]=[x^2,y^2]+ [xy,y^2]+ [yx,y^2]$ for all $x , y \in U$.

According to the relation $[d(x),d(y)]=[x^2,y^2]$ for all $x,y \in U$,we obtain

$[xy,y^2]+ [yx,y^2]= 0$ for all $x , y \in U$. Replacing x by $d(y)y$, we get

$[d(y) y^2,y^2] + [yd(y) y,y^2] = 0$ for all $y \in U$. Then

$$[d(y^2)y,y^2] = 0 \text{ for all } y \in U . \quad (21)$$

Right-multiplying (21) by $y\, d(y^2)$, we get

$$[d(y^2), y^2]\, y^2d(y^2) = 0 \text{ for all } y \in U . \quad (22)$$

Also , by left-multiplying (21) by $y^2d(y^2)$ and right-multiplying by

$[d(y^2), y^2]$,we get

$y^2 d(y^2)[d(y^2),y^2] y^2 d(y^2)[d(y^2),y^2] = (y^2 d(y^2)[d(y^2),y^2])^2 = 0$ for all $y \in U$. Right-multiplying by w with using the cancellation property of $y^2 d(y^2)[d(y^2),y^2]w, w \in R$,we obtain

$y^2 d(y^2)[d(y^2),y^2] = 0$ for all $y \in U$. (23)

Left-multiplying (23) by $d(y^2)[d(y^2),y^2]d(y^2)$ and right-multiplying by $d(y^2)\, y^2$, we obtain

$d(y^2)[d(y^2),y^2]d(y^2)y^2 d(y^2)[d(y^2),y^2]d(y^2)y^2 = 0$ for all $y \in U$. Then

$(d(y^2)[d(y^2),y^2]d(y^2)y^2)^2 = 0$ for all $y \in U$. Left-multiplying by w with using the cancellation property of $d(y^2)[d(y^2),y^2]d(y^2)y^2 w, w \in R$, we obtain

$d(y^2)[d(y^2),y^2]d(y^2)y^2=0$ for all $y \in U$. (24)

Left-multiplying (24) by $[d(y^2),y^2]d(y^2)y^2$ and right-multiplying by $d(y^2)$,we obtain

$[d(y^2),y^2]d(y^2)y^2 d(y^2)[d(y^2),y^2]d(y^2)y^2 d(y^2) = 0$ for all $y \in U$. Then

$([d(y^2),y^2]d(y^2)y^2 d(y^2))^2 = 0$ for all $y \in U$. Right-multiplying by w with using the cancellation property of $([d(y^2),y^2]d(y^2)y^2 d(y^2)w, w \in R$ we obtain

$[d(y^2),y^2]d(y^2)y^2 d(y^2)=0$ for all $y \in U$. (25)

Left-multiplying (25) by $d(y^2)y^2$ with using from right the cancellation property on $d(y^2)$, we obtain

$d(y^2)y^2[d(y^2),y^2]d(y^2)y^2=0$ for all $y \in U$. (26)

From left on (26) by using the cancellation property on $d(y^2)\, y^2$, we obtain

$[d(y^2),y^2]d(y^2)y^2=0$ for all $y \in U$. (27)

Again from right on (27) by using the cancellation property on $d(y^2)\, y^2$,we obtain $[d(y^2),y^2] = 0$ for all $y \in U$. Then by Lemma 2.3,we obtain

R contains a non-zero central ideal .

When d=0, we obtain $[x^2,y^2] = 0$ for all $x,y \in U$. By Theorem 2.9, we complete the proof of theorem .

The following results can be proven in a similar way .

Theorem 3.4. *Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R.If R admitting a derivation d satisfying* $[d^2(x),d^2(y)] = [x^2,y^2]$ *for all* $x,y \in U$*.Then R contains* *a non-zero central ideal .*

Theorem 3.5. *Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R.If R admitting a non-zero derivation d to satisfying $d([d_{(x)},d_{(y)}])=[x,y]$ for all $x,y\in U$.Then R contains a non-zero central ideal .*

Proof. We have $d([d_{(x)},d_{(y)}])=[x,y]$ for all $x,y\in U$.

Then by replacing x by x^2,we obtain $d([d_{(x^2)},d_{(y)}])-[x^2,y]=0$ for all $x,y\in U$.

Then$d([d_{(x)}x,d_{(y)}])+d([xd_{(x)},d_{(y)}])-[x^2,y]=0$ for all $x,y\in U$.

$d(d_{(x)}[x,d_{(y)}])+d([d_{(x)},d_{(y)}]x)+d(x[d_{(x)},d_{(y)}])+d([x,d_{(y)}]d_{(x)})-[x^2,y]=0$ for all $x,y\in U$.Then $d^2_{(x)}[x,d_{(y)}]+d_{(x)}d([x,d_{(y)}])+d([d_{(x)},d_{(y)}])x+[d_{(x)},d_{(y)}]d_{(x)}+d_{(x)}[d_{(x)},d_{(y)}]+xd([d_{(x)},d_{(y)}])+d([x,d_{(y)}])d_{(x)}+[x,d_{(y)}]d^2_{(x)}-[x^2,y]=o$ for all $x,y\in U$.According to the relation $d([d_{(x)},d_{(y)}])=[x,y]$,then we obtain

$d^2_{(x)}[x,d_{(y)}]+d_{(x)}d([x,d_{(y)}])+[x,y]x+([d_{(x)},d_{(y)}])d_{(x)}+d_{(x)}[d_{(x)},d_{(y)}]+x[x,y]+d([x,d_{(y)}])d_{(x)}+[x,d_{(y)}]d^2_{(x)}-[x^2,y]=0$ for all $x,y\in U$. Replacing y by x , we obtain

$d^2_{(x)}[x,d_{(x)}]+d_{(x)}d([x,d_{(x)}])+d([x,d_{(x)}])d_{(x)}+[x,d_{(x)}]d^2_{(x)}=0$ for all $x\in U$. Then

$d^2_{(x)}[x,d_{(x)}]+[x,d_{(x)}]d^2_{(x)}+d_{(x)}(d(xd_{(x)})-d(d_{(x)}x))+(d(xd_{(x)})-d(d_{(x)}x))d_{(x)}=0$ for all $x\in U$. Then

$d^2_{(x)}xd_{(x)}-d^2_{(x)}d_{(x)}x+xd_{(x)}d^2_{(x)}-d_{(x)}xd^2_{(x)}+d_{(x)}^3+d_{(x)}xd^2_{(x)}-d_{(x)}d^2_{(x)}x-d_{(x)}^3+d_{(x)}^3+xd^2_{(x)}d_{(x)}-d^2_{(x)}xd_{(x)}-d_{(x)}^3=0$ for all $x\in U$. Then

$[x,d_{(x)}d^2_{(x)}]+[x,d^2_{(x)}d_{(x)}]=0$ for all $x\in U$ .Thus

$[x,d(d_{(x)}^2)]=o$ for all $x\in U$. We set $a=d_{(x)}^2$,then

$[x,d_{(a)}]=o$ for all $x\in U$.

$[[x,d_{(a)}],r]=0$ for all $x\in U,r\in R$.Replacing r by $d_{(a)}$,and using Lemma 2.1,we obtain $d_{(a)}\in Z(R)$ (i.e. $d^2_{(x)^2}\in Z(R)$ for all $x\in U$),then by Lemma 2.8,we get $d^2_{(x)}\in Z(R)$ for all $x\in U$,then $[d^2(x),r]=0$ for all $x\in U,r\in R$.Replacing x by xr and r by x, we obtain

$[d^2_{(xy)},x]=[d^2_{(x)}y+2d_{(x)}d_{(y)}+xd^2_{(y)},x]=0$ for all $x,y\in U$ .In the relation $[d^2_{(x)},r]=0$,replacing r by x ,we obtain $[d^2_{(x)},x]=0$ for all $x\in U$.Then according to this relation the equation $[d^2(x)y+2d(x)d(y)+x\,d^2(y),x]=0$ for all $x,y \in U$, with replacing y by x, become $2[d(x)^2,x]=0$ for all $x \in U$. Since R is 2-torsion free semiprime, then $[d(x)^2,x]=0$ for all $x \in U$. Thus $[[d(x)^2,x],d(x)^2]=0$ for all $x \in U$. By Lemma 2.1,we obtain $d(x)^2 \in Z(R)$ for all $x \in U$, then by Lemma 2.8,we get $d(x) \in Z(R)$ for all $x \in U$, then $[d(x),r]=0$ for all $x \in U$, $r \in R$. Replacing r by x, we obtain $[d(x),x]=0$ for all $x \in U$. By Lemma 2.3, R contains a non-zero central ideal.

Theorem 3.6. *Let R be a 2-torsion free semiprime ring. If R admitting a derivation d to satisfying $d([d(x),d(y)])=[x,y]$ for all $x, y \in R$. Then R is commutative.*

Proof. At first, when $d \neq 0$,by same method in Theorem 3.5, we obtain $d(x) \in Z(R)$ for all $x \in U$, then $[d(x),r]=0$ for all $x,r \in R$. Replacing r by $d(y)$,we get $[d(x),d(y)]=0$ for all $x,y \in R$. By substituting this relation in $d([d(x)+d(x)])=[x,y]$ for all $x,y \in R$,gives $[x,y]=0$ for all $x,y \in R$. Then R is commutative.

When d=0,it is clearly we obtain R is commutative.

Corollary 3.7. *Let R be a 2-torsion free prime ring and U a non-zero ideal of R. If R admitting a derivation d to satisfying $d([d(x),d(y)])=[x,y]$ for all $x,y \in U$. Then R is commutative.*

Proof. When $d \neq 0$,by using same method in Theorem 3.5,with Lemma 2.3,we get R is commutative.

When d=0,then $[x,y]=0$ for all $x,y \in U$.By Lemma 2.6,we obtain R is commutative.

Theorem 3.8. *Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R .If R admitting a non-zero derivation d to satisfying $d([d(x),d(y)])=[d(x),d(y)]$ for al $x,y \in U$.Then R contains a non-zero central ideal.*

Proof. We suppose first that a=[d(x),d(y)],a$\in$R.Then

d(a)=a. (28)

We set a=az ,z$\in$R, where z=[d(y),d(x)],then

d(az)=az .Thus

d(a)z+ad(z)=az. According to (28),we obtain

ad(z)=0. This implies

[d(x),d(y)]d([d(y),d(x)])=0 for all $\in$U.Since R has a cancellation property from right,we obtain

[d(x),d(y)]=0 for all x,y$\in$U.By Lemma 2.7,we get

R contains a non-zero central ideal.

Corollary 3.9. *Let R be a 2-torsion free prime ring and U a non-zero ideal of R.If R admitting a non-zero derivation d to satisfying d([d(x),d(y)])=[d(x),d(y)] for all x,y* $\in$ *U. Then R is commutative* .

We now have enough information to prove the following result .

Theorem 3.10. *Let R be a 2-torsion free semiprime ring and U a non-zero ideal of R . If R admitting a non-zero derivation d to satisfying one of the following conditions .*

(i) [d(x),d(y)]=[x,y] for all x,y $\in$ *U.*

(ii)$[d^2(x),d^2(y)]=[x,y]$ *for all x,y* $\in$ *U.*

(iii)$[d(x),d(y)]=[x^2,y^2]$ *for all x,y*$\in$ *U.*

(iv) $[d^2(x),d^2(y)]=[x^2,y^2]$ *for all x,y* $\in$ *U .*

(v)d([d(x) ,d(y)])= [x,y] for all x,y $\in$ *U .*

(vi) d([d(x),d(y)])=[d(x) ,d(y)] for all x,y $\in$ *U. Then d(U) centralizes U* **.**

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