Available online at http://scik.org
J. Math. Comput. Sci. 11 (2021), No. 5, 5252-5266
https://doi.org/10.28919/jmcs/5943
ISSN: 1927-5307

INVERSE DOMINATION ON MULTI FUZZY GRAPH

R. MUTHURAJ[1,*], S. REVATHI[2]

[1]PG and Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai,

Affiliated to Bharathidasan University, Tiruchirappalli, Tamilnadu, India

[2]Department of Mathematics, Saranathan College of Engineering, Trichy – 620012, Tamilnadu, India

Abstract: In this paper, inverse domination on multi fuzzy graph, inverse total domination of a multi fuzzy graph are introduced. The properties of inverse domination number and inverse total domination number on multi fuzzy graph are discussed. The necessary and sufficient condition for the existence of an inverse dominating set and an inverse total dominating set are established. Some basic theorems related to the stated dominations have also been presented.

Keywords: multi fuzzy graph; inverse dominating set; inverse total dominating set; inverse domination number; inverse total domination number.

AMS Subject Classification (2010): 03E72, 03B20, 03F55, 05C72, 05C07, 05C62.

1. INTRODUCTION

The study of domination in graphs was established by O. Ore [13] and C. Berge [1]. T.W. Haynes, S.T. Hedetniemi and P.J. Slater [5] was introduced the Fundamentals of domination

*Corresponding author
E-mail address: rmr1973@gmail.com
Received April 28, 2021

in graphs. E.J. Cockayne and S.T. Hedetniemi [2] initiated Total domination in graphs. The inverse domination in graphs concept proposed by V.R. Kulli and S.C. Sigarkanti [8]. V.R. Kulli and R.R. Iyer [7] discussed the concept of Inverse total domination in graphs. The concept of fuzzy graph was first introduced by Kaufmann [6] from the concept fuzzy relation introduced by L.A. Zadeh [17]. A. Nagoorgani and V.T. Chandrasekaran [12] was introduced the concept of domination in fuzzy graph using strong arcs. C.V.R. Hari Narayanan, S. Revathi and R. Muthuraj [4] was discussed connected total perfect dominating set in fuzzy graph. S. Ghobadi, N.D. Soner and Q.M. Mahyoub [3] was discussed inverse dominating set in fuzzy graph. A. Somasundaram and S. Somasundaram [15,16] discussed about domination in fuzzy graph with effective edges and also discussed Total domination in fuzzy graph using effective edges. R. Muthuraj et al. [10, 9] was discussed On Anti fuzzy graph and Total Strong (Weak) Domination on Anti fuzzy graph. S. Sabu and T.V. Ramakrishnan [14] proposed the theory of multi-fuzzy sets and it is used to characterize the problems in the field of taste recognition, decision making, pattern recognition, image processing and approximation of vague data. R. Muthuraj and S. Revathi [11] was defined the concept of Multi fuzzy graph. In this paper we obtain the concept of Inverse domination on Multi fuzzy graph and Inverse total domination of a Multi fuzzy graph. We defined some properties and the necessary and sufficient condition for the inverse and inverse total domination number of a Multi fuzzy graph.

2. PRELIMINARIES: MULTI FUZZY GRAPH

In this section, the concept of multi fuzzy graph is introduced and discussed its related concepts. Throughout this paper, denote the edge between two vertices u and v as uv.

2.1 Definition [12]

A fuzzy graph $G = (\sigma,\mu)$ defined on the underlying crisp graph $G^* = (V, E)$, where $E \subseteq V \times V$, is a pair of functions $\sigma: V \rightarrow [0,1]$ and $\mu: V\times V \rightarrow [0,1]$, μ is a symmetric fuzzy relation on σ such that $\mu(uv) \leq \min\{\sigma(u), \sigma(v)\}$ for all $u, v \in V$.

2.2 Definition [14]

Let X be a non-empty set. A multi-fuzzy set A in X is defined as a set of ordered sequences: $A = \{(x, \mu_1(x), \mu_2(x),.....,\mu_i(x),....) : x \in X\}$, where $\mu_i : X \rightarrow [0,1]$ for all i .

2.3 Remark:

i. If the sequences of the membership functions have only k-terms (finite number of terms), k is called the dimension of A.

2.4 Definition [11]

A multi fuzzy graph (MFG) of dimension m defined on the underlying crisp graph $G^* = (V, E)$, where $E \subseteq V \times V$, is denoted as $G = ((\sigma_1, \sigma_2, ...,\sigma_m), (\mu_1, \mu_2, ..., \mu_m))$, and $\sigma_i : V \rightarrow [0, 1]$ and $\mu_i: V \times V \rightarrow [0, 1]$, μ_i is a symmetric fuzzy relation on σ such that $\mu_i(uv) \leq \min \{ \sigma_i(u) , \sigma_i(v) \}$, for all i = 1,2, 3, …,m; u, v ∈V and uv∈ E.

2.5 Definition [11]

Let $G = ((\sigma_1, \sigma_2, ...,\sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m. Then the cardinality of a MFG G is denoted as |G| and is defined as

$$|G| = \sum_{v_i \in V} \frac{1+\sigma_1(v_i)+\sigma_2(v_i)+...+\sigma_m(v_i)}{m} + \sum_{v_i v_j \in E} \frac{1+\mu_1(v_i v_j)+\mu_2(v_i v_j)+...+\mu_m(v_i v_j)}{m} .$$

2.6 Definition [11]

Let $G = ((\sigma_1, \sigma_2, ...,\sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m. Then the vertex cardinality of a MFG G or the order of a MFG is denoted as |V| or O(G) or p and is defined as

$$|V| = \sum_{v_i \in V} \frac{1+\sigma_1(v_i)+\sigma_2(v_i)+...+\sigma_m(v_i)}{m} .$$

2.7 Definition [11]

Let $G = ((\sigma_1, \sigma_2, ...,\sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m. Let $D \subseteq V$. Then the cardinality of D of G or the fuzzy cardinality of D of G is denoted as |D| and is defined as

$$|D| = \sum_{v_i \in D} \frac{1+\sigma_1(v_i)+\sigma_2(v_i)+...+\sigma_m(v_i)}{m} .$$

2.8 Definition [11]

Let G = ((σ_1, σ_2, ….,σ_m), (μ_1, μ_2, …., μ_m)) be a MFG of dimension m. Then the degree of a vertex u ∈ V in G is defined as $d_G(u) = \sum_{v\in V} \frac{1+\mu_1(uv)+\mu_2(uv)+...+\mu_m(uv)}{m}$.

2.9 Definition [11]

Let G = ((σ_1, σ_2, ….,σ_m), (μ_1, μ_2, …., μ_m)) be a MFG of dimension m. An edge uv = e is said to be an effective edge if $\mu_i(uv) = \min\{\sigma_i(u), \sigma_i(v)\}$, for all i = 1,2, 3, …,m.

2.10 Definition [11]

Let G = ((σ_1, σ_2, ….,σ_m), (μ_1, μ_2, …., μ_m)) be a MFG of dimension m. Let u ∈ V of G. Then the neighbors (neighborhood) of u or an open neighbors of u∈ V of G is denoted by N(u) and is defined as N(u) = {v ∈V / $\mu_i(uv) = \min\{\sigma_i(u), \sigma_i(v)\}$, for all i = 1,2, 3, …,m}.

The closed neighbors of u ∈ V of G is denoted by N[u] and is defined as N[u] = N(u) ∪ {u}.

2.11 Definition [11]

Let G = ((σ_1, σ_2, ….,σ_m), (μ_1, μ_2, …., μ_m)) be a MFG of dimension m. The vertex u ∈ V of G is said to be isolated vertex if $\mu_i(uv) < \min\{\sigma_i(u), \sigma_i(v)\}$, for some i = 1,2, 3, …,m and for all v ∈V – {u}.

In other words, the vertex u ∈ V of G is said to be isolated vertex if N(u) = ϕ or | N(u) | = 0.

2.12 Definition [11]

Let G = ((σ_1, σ_2, ….,σ_m), (μ_1, μ_2, …., μ_m)) be a MFG of dimension m. The vertex u ∈ V of G is said to be pendant vertex if | N(u) | = 1. An edge incident on the pendant vertex is called a pendant edge.

2.13 Definition [11]

Let G = ((σ_1, σ_2, ….,σ_m), (μ_1, μ_2, …., μ_m)) be a MFG of dimension m. Let u, v ∈ D. We say that u dominates v in G if $\mu_i(uv) = \min\{\sigma_i(u), \sigma_i(v)\}$, for all i = 1,2, 3, …,m ; uv∈ E.

A subset D of V is said to be a dominating set in G if for every v ∈V – D there exist u ∈ D such that u dominates v.

A dominating set D of V is said to be a minimal dominating set if no proper subset of D is a dominating set of G.

The minimum fuzzy cardinality of a minimal dominating set in G is called the domination number of G and is denoted by γ(G) or simply γ and the corresponding minimal dominating set is called the minimum dominating set of G.

2.14 Definition [11]

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Then a dominating set T of V is said to be a total dominating set T of G if the induced sub graph of T has no isolated vertex.

A total dominating set T of a MFG G is called minimal total dominating set on MFG G, if no proper subset of T is a total dominating set of G.

The minimum fuzzy cardinality among all minimal total dominating set on MFG G is called total dominating number of a MFG G and is denoted by $\gamma_T(G)$ and the corresponding minimal dominating set is called is called the minimum dominating set of G.

2.15 Example

In the following MFG with dimension 3,

$D_1 = \{a\}$; $D_2 = \{b, c\}$; $D_3 = \{b, d\}$, $D_3 = \{d\}$ are the minimal dominating sets of G with cardinality 1.13, 0.94, 1.4, 0.93 respectively and the domination number is $\gamma = 0.93$.

$D_1 = \{a, b\}$; $D_2 = \{a, c\}$; $D_3 = \{a, d\}$ are the minimal total dominating sets of G with cardinality 1.6, 1.6, 2.06 respectively and the total domination number is $\gamma = 1.6$.

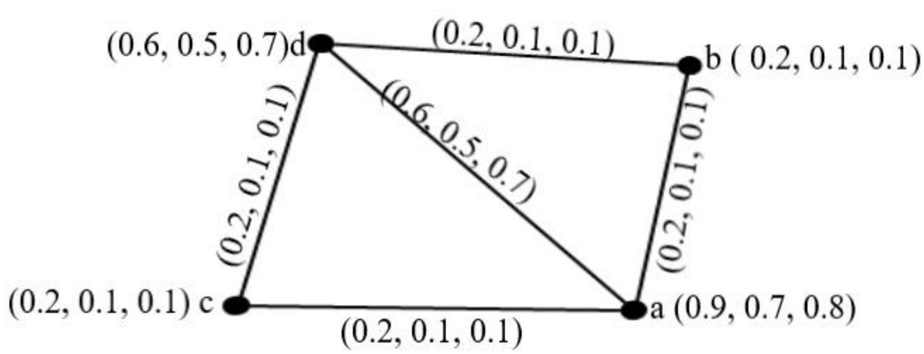

Fig. 2.1

MFG of dimension 3

3. MAIN RESULTS: INVERSE DOMINATION ON MFG

In this section, the concept of inverse domination on MFG is defined and its domination number is obtained for MFG with examples. Throughout this section, domination on MFG using effective edges only considered.

3.1 Definition

Let $G = ((\sigma_1, \sigma_2, ..., \sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m. Let I be a minimum dominating set of a MFG G. If the subset $I' \subset V - I$ is a dominating set of G then I′ is called an inverse dominating set with respect to I.

The minimum fuzzy cardinality taken over all minimal inverse dominating sets in G is called inverse domination number of G and it is denoted by γ'.

3.2 Example

Consider the following MFG G.

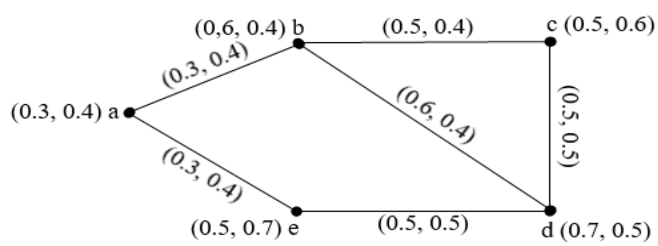

Fig. 3.1

Let I = {a, b} be a minimum dominating γ - set of a MFG G and in V – I, the sets, $I_1' = \{c, e\}$ and $I_2' = \{d, e\}$ and $I_3' = \{c, d, e\}$ are the dominating sets of G. By definition 3.1, $I_1' = \{c, e\}$ is the minimum fuzzy cardinality of all inverse dominating sets of G with respect to I. Hence, $\gamma(G) = 1.85$ and $\gamma'(G) = 2.15$.

3.3 Observation

Let $G = ((\sigma_1, \sigma_2, ..., \sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m with at least one inverse dominating set, then $\gamma(G) \leq \gamma'(G)$.

3.4 Theorem

Let G = $((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m with at least one inverse dominating set. Then, $\gamma(G) + \gamma'(G) \leq p$.

Proof

Let G = $((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Let I′ be the inverse dominating set with respect to I in V of G. Then, $I' \subset V - I$. Therefore, $\gamma'(G) \leq p - \gamma(G)$. Hence, $\gamma(G) + \gamma'(G) \leq p$.

3.5 Theorem

Let G = $((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Then, $\gamma'(G) < p$.

Proof

Let G = $((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Then by Theorem 3.4, $\gamma(G) + \gamma'(G) \leq p$. Hence, $\gamma'(G) < p$.

3.6 Theorem

Let G = $((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m with at least one isolated vertex. Then, $\gamma'(G) = 0$.

Proof

Let G = $((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Let I be the minimum dominating γ - set of a MFG G and $u \in I$ be the isolated vertex of G. Then, for all $v \in V - I$, $\mu_i(uv) < \min \{ \sigma_i(u), \sigma_i(v) \}$, for some $i = 1, 2, 3, \ldots, m$.

That is, there is no vertex in V – I dominate u. Hence, there is no dominating set of G in V – I. Hence, $\gamma'(G) = 0$.

3.7 Remark

The following theorem is the necessary and sufficient conditions for the existence of at least one inverse dominating set of a MFG G.

3.8 Theorem

Let G = $((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Let I be a minimal dominating set of G. Then G has an inverse dominating set if and only if the following conditions hold.

i. G has no isolated vertices,

ii. $n(I) \leq n(V - I)$, where n(I), is the number of elements in the set I and vice versa.

Proof

Let $G = ((\sigma_1, \sigma_2, ..., \sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m. Let I be a minimal dominating set of G. Let $I' \subset V - I$ be the inverse dominating set of G. Then, by theorem 3.6, G has no isolated vertex as $I' \subset V - I$ be the inverse dominating set of G.

Suppose $n(I) > n(V - I)$. Let I be a minimal dominating set of G. If every vertex of V – I is dominated by exactly one vertex of I, then there exists at least one vertex $u \in I$ such that either u dominates a vertex in V – I which is also dominated by some other vertex of I or u does not dominate any vertex of V – I. This is a contradiction to I is a minimum dominating set of G. Hence, $n(I) \leq n(V - I)$.

Conversely, let the conditions hold for any MFG G. Let I be the minimum dominating set of G. Then by condition (i), every vertex of V – I can dominate its neighbor in I. By condition (ii), V – I contains a dominating set with respect to I. Hence, inverse dominating set $I' \subset V - I$ exists in G.

4. MAIN RESULTS: RELATED CONCEPTS OF INVERSE TOTAL DOMINATION ON MFG

In this section, the related concepts of inverse total domination on MFG and its properties are discussed.

4.1 Definition

Let $G = ((\sigma_1, \sigma_2, ..., \sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m. Let T be a minimum total dominating set of a MFG G. If the subset $T_I \subset V - T$ is a total dominating set of G, then T_I is called an inverse total dominating set with respect to T.

The minimum fuzzy cardinality taken over all minimal inverse total dominating set in G is called inverse total domination number of G and is denoted by $\gamma_{T_I}(G)$.

4.2 Example

Consider a MFG G with dimension 3.

Minimal total dominating sets of G are $T_1 = \{a, b\}$, $T_2 = \{c, d\}$, $T_3 = \{e, f\}$ and the minimum total dominating set of G is $T_1 = \{a, b\}$. The minimal total dominating sets in V – T are $T_{I_1} = \{c, d\}, T_{I_2} = \{e, f\}$ and inverse total dominating set of G is $T_{I_2} = \{e, f\} \subset V - T = \{c, d, e, f\}$, by the usual computations, the total domination number, and an inverse total domination number respectively as $\gamma_T = 1.6$, $\gamma_{T_I} = 1.7$.

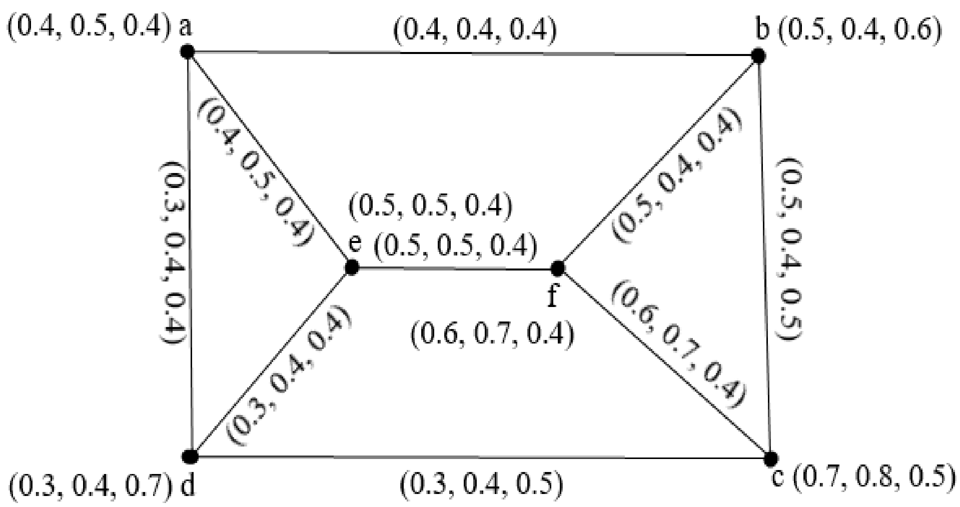

Fig. 4.1

4.3 Theorem

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Then, for any inverse total dominating set in G, $\gamma_T(G) \leq \gamma_{T_I}(G)$.

Proof

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m and T be the minimum total dominating set of V in G. Let $\gamma_T(G)$ be the total domination number of a G and let T_I be an inverse total dominating set of V – T of G. Then $\gamma_T(G)$ be the inverse total domination number of G. Then, $\gamma_T(G) \leq \gamma_{T_I}(G)$.

4.4 Theorem

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m with pendant vertex (end vertex). Then $\gamma_T(G) = 0$.

Proof

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m with pendant vertex (end vertex).Let T be the minimum total dominating set of V in G. Let $u \in V$ be an end vertex of G and let u is adjacent to the vertex $v \in V$. Then $u \in V - T$ and $v \in T$. Since, $u, v \notin T$ as T is the minimum total dominating set of T. Hence, $u \in V - T$ and $v \in T$. The vertex $u \in V - T$ has no neighbors in $V - T$. Hence, total domination set in $V - T$ does not exist. Hence, $\gamma_T(G) = 0$.

4.5 Theorem

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m with at least one isolated vertex. Then $\gamma_T(G) = 0$.

Proof

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m and let $u \in V$ be an isolated vertex. Then, it is not possible to have a total dominating set of V in G. Suppose, let T be a total dominating set of V in G. If $u \in T$, then $\langle T \rangle$ has an isolated vertex. Hence, $u \notin T$. If $u \in V - T$, then there is no vertex in T has a neighbor as u. That is, there is no vertex in T dominate u. Hence, it is not possible to have a total dominating set in V of G. Hence, $\gamma_T(G) = 0$.

4.6 Theorem

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m with at least one inverse total dominating set. Then $\gamma_T(G) + \gamma_{T_I}(G) \leq p$.

Proof

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Let T_I be the inverse total dominating set with respect to T of V in G. Then, $T_I \subset V - T$. Therefore, $\gamma_{T_I}(G) \leq p - \gamma_T(G)$.

Hence, $\gamma_T(G) + \gamma_{T_I}(G) \leq p$.

4.7 Theorem

Let $G = ((\sigma_1, \sigma_2, ..., \sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m, then $\gamma_{T_I}(G) < p$.

Proof

Let $G = ((\sigma_1, \sigma_2, ..., \sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m. Then by Theorem 4.6, $\gamma_T(G) + \gamma_{T_I}(G) \leq p$. Hence, $\gamma_{T_I}(G) < p$.

4.8 Remark

The following theorem is the necessary and sufficient conditions for the existence of at least one inverse total dominating set of a MFG G.

4.9 Theorem

Let $G = ((\sigma_1, \sigma_2, ..., \sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m. Let T be a minimal total dominating set of G. Then G has an inverse total dominating set if and only if the following conditions hold.

i. G has no isolated vertex,

ii. G has no pendant vertex (end vertex),

iii. For any $u \in T$, $N(u) \not\subset T$.

iv. $n(T) \leq n(V - T)$, where n(T), is the number of elements in the set T and vice versa.

v. At least n(T) vertices of $V - T$ is not isolated.

Proof

Let $G = ((\sigma_1, \sigma_2, ..., \sigma_m), (\mu_1, \mu_2, ..., \mu_m))$ be a MFG of dimension m and Let T be the minimum total dominating set in V of G. Let $T_I \subset V - T$ be an inverse total dominating set of G.

By theorems 4.4 and 4.5, G has no isolated and pendant vertex as $T_I \subset V - T$ is an inverse total dominating set of G.

If $N(u) \subset T$, for some $u \in T$, then does not exist any $v \in V - T$ such that v dominates u. Hence, $N(u) \not\subset T$, for all $u \in T$ as $T_I \subset V - T$ is an inverse total dominating set of G.

Suppose $n(T) > n(V - T)$. Let T be a minimal total dominating set of G and $T_I \subset V - T$ be an inverse total dominating set of G. If every vertex of $V - T$ is dominated by exactly one vertex of T, then there exists at least one vertex $u \in T$ such that either u dominates a vertex in $V - T$ which is also dominated by some other vertex of T or u does not dominate any vertex of V – T. This is a contradiction to T is a minimum total dominating set of G. Hence, n(I) ≤n(V – I).

Given T is the minimum total dominating set in V of G and $T_I \subset V - T$ be an inverse total dominating set of G. Then, $\langle T_I \rangle$ sub graph of $\langle V - T \rangle$ has no isolated vertices. There should be at least n(T) vertices in V – T are not isolated to dominate the vertices of T of G.

Conversely, let the conditions hold for any MFG G. Then by condition (i), the minimum total dominating set T of G exists. Condition (ii) assertion that it is possible to have T_I in V – T. By condition (iii), every vertex of T can be dominated by its neighbor vertex in V – T. By condition (iv), T is the minimum total dominating set of G and hence V – T has equal or more number of vertices as T. By condition (v), there exists $T_I \subset V - T$ such that $\langle T_I \rangle$ has no isolated vertices. Hence, V – T contains an inverse total dominating set with respect to T. Hence, inverse total dominating set $T_I \subset V - T$ exists in G.

4.10 Remark

Consider the Let G = $((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension mwith no isolated and pendant vertex and having n vertices. Then,

Case i: Suppose n = 1, then obviously there is no total dominating set in G. This results that inverse total dominating set does not exist.

Case ii: Suppose n = 2, then there exists a total dominating set in G. By definition of total dominating set in multi fuzzy graph, there exists no vertices belongs to V −T. Hence, inverse total dominating set does not exist.

Case iii: Let n = 3. Let T be a minimum total dominating set of V in G with n(T) = 2. Then $\langle V - T \rangle$ has an isolated vertex. Hence it is not possible to have a total dominating set in V – T. Hence, inverse total dominating set does not exist.

Case iv: Let n = 4. Let T be a minimum total dominating set of V in G. Let n(T) = 2. If n(T) > 2, then either V – T = ϕ or $\langle V - T \rangle$ has an isolated vertex. Hence, it is not possible to have a total dominating set in V – T. So, if n(T) = 2 and $\langle V - T \rangle$ has no isolated vertex then only V – T contains a total dominating set in G.

In general, if

i. T is a total dominating set of V in G then, by the definition 2.23, n(T)≥ 2.

ii. T_I is an inverse total dominating set of G then n(G) ≥ 4.

4.11 Remark

Let G = ((σ_1, σ_2, ...,σ_m), (μ_1, μ_2 , ..., μ_m)) be a MFG of dimension m with n vertices and n edges, where n = 4, 8, 12, …. Let T be a minimum total dominating set of G. Then, an inverse total dominating set T_I exists with respect to T and $n(T)=n(T_I)=n(V-T)=\frac{n}{2}$.

4.12 Theorem

Let G = ((σ_1, σ_2, ...,σ_m), (μ_1, μ_2 , ..., μ_m)) be a complete MFG of dimension m. Let T_I be an inverse total dominating set with respect to the minimum total dominating set T of G. Then both ⟨ T ⟩ and ⟨ T_I⟩ are connected.

Proof

Let G = ((σ_1, σ_2, ...,σ_m), (μ_1, μ_2 , ..., μ_m)) be a complete MFG of dimension m. Let T_I be an inverse total dominating set with respect to the minimum total dominating set T of G.

Let u∈T, then u is adjacent to all vertices of V in G and also adjacent to all vertices of T. That is, there is a fuzzy path between all vertices of T in G. Hence, ⟨ T ⟩ is connected.

Let v∈T_I, then v is adjacent to all vertices of V in G and also adjacent to all vertices of T_I. That is, there is a fuzzy path between all vertices of T_I in G. Hence, ⟨ T_I⟩ is connected.

4.13 Theorem

Let G = ((σ_1, σ_2, ...,σ_m), (μ_1, μ_2 , ..., μ_m)) be any complete bipartite MFG with dimension m. Then, a minimum total dominating set T and an inverse total dominating set T_I of G exists.

Proof

Let G = ((σ_1, σ_2, ...,σ_m), (μ_1, μ_2 , ..., μ_m)) be any complete bipartite MFG with dimension m. Then, V = V_1∪ V_2 and V_1∪ V_2 = ϕ. Let T = {u, v}, where $u \in V_1$ and $v \in V_2$ be a total dominating set of G and T_I = {x, y} ⊂ V – T, where $x \in V_1$- {u} and $y \in V_2 - \{v\}$ be the total dominating set of G. Hence, both T and T_I exist for any complete multi fuzzy graph.

5. Conclusion

The authors introduced the concept of multi fuzzy graph and its based definitions and the concept of domination on multi fuzzy graph. The application of multi fuzzy graph and operations and other dominations on multi fuzzy graph will be reported in forth coming papers.

Acknowledgement

The authors are more grateful for your valuable comments and suggestions in improving this paper.

Conflict of Interests

The author(s) declare that there is no conflict of interests.

References

[1] C. Berge, Graphs and hyper graphs, North-Holland, Amsterdam, 1973.

[2] E.J. Cockayne, S.T. Hedetniemi, Towards a theory of domination in graphs, Networks, 7 (1977), 247-261.

[3] S. Ghobadi, N.D. Soner, Q.M. Mahyoub, Inverse dominating set in fuzzy graphs, J. Proc. Jangjeon Math. Soc. 11 (2008), 75-81.

[4] S. Revathi, C.V.R. Harinarayanan, R. Muthuraj, Connected total perfect dominating set in fuzzy graph, Int. J. Comput. Appl. Math. 12 (2017), 84-98.

[5] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, Fundamentals of domination in graphs, Marcel Dekker, Inc., New York, 1998.

[6] A. Kaufmann, Introduction to the theory of fuzzy subsets, Academic Press, New York, (1975).

[7] V.R. Kulli, R.R. Iyer, Inverse total domination in graphs, J. Discrete Math. Sci. Cryptography. 10 (2007), 613–620.

[8] V.R. Kulli, S.C. Sigarkanti, Inverse domination in graphs, Nat. Acad. Sci. Lett. 14(1991), 473-475.

[9] R. Muthuraj, P.J. Jayalakshmi, S. Revathi, Total strong (weak) domination on anti fuzzy graph, Adv. Math.: Sci. J. 8 (2019), 93-97.

[10] R. Muthuraj, A. Sasireka, On Anti fuzzy graph, Adv. Fuzzy Math. 12 (2017), 1123-1135.

[11] R. Muthuraj, S. Revathi, Multi fuzzy graph, J. Critic. Rev. 7 (2020), 4122-4131.

[12] R. Muthuraj, A. Sasireka, Connected domination number on cartesian product of simple fuzzy graphs, J. Math. Comput. Sci. 7 (2017), 189-200.

[13] R. Muthuraj, A. Sasireka, Edge domination number on cartesian product of simple fuzzy graphs, Res. Rev.: Discrete Math. Struct. 4 (2017), 8- 17.

[14] R. Muthuraj, A. Sasireka, Total domination number on Cartesian product of simple fuzzy graphs, Int. J. Comput. Appl. Math. 12 (2017), 168-185.

[15] R. Muthuraj, A. Sasireka, Connected domination on anti fuzzy graph, J. Appl. Sci. Comput. 5 (2018), 18-28.

[16] R. Muthuraj, A. Sasireka, Total domination on anti fuzzy graph, New Trends Math. Sci. 6 (2018), 28-39.

[17] A. Nagoorgani, V.T. Chandrasekaran, Domination in fuzzy graph, Adv. Fuzzy Sets Syst. 1 (2006), 17-26.

[18] O. Ore, Theory of graphs, Amer. Math. Soc. Colloq. Publi. Vol. 38, Amer. Math. Soc. Providence. RI, 1962.

[19] S. Sebastian, T.V. Ramakrishnan, Multi-fuzzy sets, Int. Math. Forum, 50 (2010), 2471-2476.

[20] A. Somasundaram, S. Somasundaram, Domination in fuzzy graphs – I, Pattern Recognit. Lett. 19 (1998), 787-791.

[21] A. Somasundaram, Domination in fuzzy graphs-II, J. Fuzzy Math. 13 (2005), 281-288.

[22] L.A. Zadeh, Fuzzy sets, Inform. Control, 8 (1965), 338-353.