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NANO BINARY CONTRA CONTINUOUS FUNCTIONS IN NANO BINARY TOPOLOGICAL SPACES

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Abstract: The purpose of this paper, we introduce and study the nano binary contra continuous function in nano binary topological spaces. Also we introduce some nano binary contra continuous functions and their characterizations are also studied. We introduce and discuss the nano binary D-continuous function in nano binary topological spaces.

Keywords: N_B -contra continuous; N_B contra α -continuous; N_B contra semi-continuous; N_B contra pre-continuous; N_B contra β-continuous; N_B perfectly continuous; N_B strongly continuous; N_B D- continuous.

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1. INTRODUCTION

M. Lellis Thivagar [1] introduced the concept of nano topological space with respect to a subset

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X of a universe U. S. Nithyanantha Jothi and P. Thangavelu [2] introduced the concept of binary topological spaces. By combining these two concepts G. Hari Siva Annam and J. Jasmine Elizabeth [3] introduced nano binary topological spaces. J. Jasmine Elizabeth and G. Hari Siva Annam [4] introduced nano binary continuous function in nano binary topological spaces. In this paper we have introduced a new class of functions on nano binary topological spaces called nano binary contra continuous functions and derived their characterizations in terms of nano binary strongly continuous and nano binary perfectly continuous. Also the relationships between some nano binary contra continuous functions are studied. Also we have introduced the nano binary D-continuous function.

2. PRELIMINARIES

The concepts given here help us to recall our memories regarding the basic concepts of nano binary topological spaces.

Definition 2.1: [3] Let (U_1, U_2) be a non-empty finite set of objects called the universe and R be an equivalence relation on (U_1, U_2) named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U_1, U_2, R) is said to be the approximation space. Let $(X_1, X_2) \subseteq (U_1, U_2)$

1. The lower approximation of (X_1, X_2) with respect to R is the set of all objects, which can be for certain classified as (X_1, X_2) with respect to R and it is denoted by $L_R(X_1, X_2)$.

That is, $L_R(X_1, X_2) = \cup_{(x_1,x_2)\epsilon(U_1,U_2)}\{R\,(x_1, x_2): R(x_1, x_2) \subseteq (X_1, X_2)\}$

Where $R(x_1, x_2)$ denotes the equivalence class determined by (x_1, x_2)

2. The upper approximation of (X_1, X_2) with respect to R is the set of all objects, which can be possibly classified as (X_1, X_2) with respect to R and it is denoted by $U_R(X_1, X_2)$.

That is, $U_R(X_1, X_2) = \cup_{(x_1,x_2)\epsilon(U_1,U_2)}\{R\,(x_1, x_2): R(x_1, x_2) \cap (X_1, X_2) \neq \phi\}$

3. The boundary region of (X_1, X_2) with respect to R is the set of all objects, which can be classified neither as (X_1, X_2) nor as not $-(X_1, X_2)$ with respect to R and it is denoted by $B_R(X_1, X_2)$.

That is, $B_R(X_1,X_2) = U_R(X_1,X_2) - L_R(X_1,X_2)$

Proposition 2.2: [3] If (U_1,U_2,R) is an approximation space and(X_1,X_2), $(Y_1,Y_2) \subseteq (U_1,U_2)$, then

1. $L_R(X_1,X_2) \subseteq (X_1,X_2) \subseteq U_R(X_1,X_2)$
2. $L_R(\phi,\phi) = U_R(\phi,\phi) = (\phi,\phi)$ and $L_R(U_1,U_2) = U_R(U_1,U_2) = (U_1,U_2)$
3. $U_R\big((X_1,X_2)\cup(Y_1,Y_2)\big) = U_R(X_1,X_2) \cup U_R(Y_1,Y_2)$
4. $U_R\big((X_1,X_2)\cap(Y_1,Y_2)\big) \subseteq U_R(X_1,X_2) \cap U_R(Y_1,Y_2)$
5. $L_R\big((X_1,X_2)\cup(Y_1,Y_2)\big) \supseteq L_R(X_1,X_2) \cup L_R(Y_1,Y_2)$
6. $L_R\big((X_1,X_2)\cap(Y_1,Y_2)\big) \subseteq L_R(X_1,X_2) \cap L_R(Y_1,Y_2)$
7. $L_R(X_1,X_2) \subseteq L_R(Y_1,Y_2)$ and $U_R(X_1,X_2) \subseteq U_R(Y_1,Y_2)$ whenever $(X_1,X_2) \subseteq (Y_1,Y_2)$
8. $U_R(X_1,X_2)^C = [L_R(X_1,X_2)]^C \; and \; L_R(X_1,X_2)^C = [U_R(X_1,X_2)]^C$
9. $U_RU_R(X_1,X_2) = L_RU_R(X_1,X_2) = U_R(X_1,X_2)$
10. $L_RL_R(X_1,X_2) = U_RL_R(X_1,X_2) = L_R(X_1,X_2)$

Definition 2.3: [3] Let (U_1,U_2) be the universe, R be an equivalence on(U_1,U_2) and $\tau_R(X_1,X_2) = \{(U_1,U_2),(\phi,\phi),L_R(X_1,X_2),U_R(X_1,X_2),B_R(X_1,X_2)\}$ where $(X_1,X_2) \subseteq (U_1,U_2)$. Then by the property R(X_1,X_2) satisfies the following axioms

1. (U_1,U_2) and $(\phi,\phi) \in (X_1,X_2)$
2. The union of the elements of any sub collection of $\tau_R(X_1,X_2)$ is in $\tau_R(X_1,X_2)$
3. The intersection of the elements of any finite sub collection of $\tau_R(X_1,X_2)$ is in $\tau_R(X_1,X_2)$.

That is, $\tau_R(X_1,X_2)$ is a topology on (U_1,U_2) called the nano binary topology on (U_1,U_2) with respect to (X_1,X_2).

We call $(U_1,U_2,\tau_R(X_1,X_2))$ as the nano binary topological spaces. The elements of $\tau_R(X_1,X_2)$ are called as nano binary open sets and it is denoted by N_B open sets. Their complement is called N_B closed sets.

Definition 2.4: [3] If $\big(U_1,U_2,\tau_R(X_1,X_2)\big)$ is a nano binary topological spaces with respect to (X_1,X_2) and if $(H_1,H_2) \subseteq (U_1,U_2)$, then the nano binary interior of (H_1,H_2) is defined as the union of all N_B open subsets of (A_1, A_2) and it is defined by $N_B{}^\circ(H_1, H_2)$

That is, $N_B{}^\circ$ (H_1, H_2) is the largest N_B open subset of (H_1, H_2). The nano binary closure of (H_1, H_2) is defined as the intersection of all N_B closed sets containing (H_1, H_2) and it is denoted by $\overline{N_B}(H_1, H_2)$.

That is, $\overline{N_B}(H_1, H_2)$ is the smallest N_B closed set containing (H_1, H_2).

Proposition 2.5: [3] Let $(U_1, U_2, \tau_R(X_1, X_2))$ be a nano binary topological space and $(A_1, A_2), (B_1, B_2) \in P(X_1) \times P(X_2)$ then

i) $N_B^\circ(\varphi, \varphi) = (\varphi, \varphi)$

$$\overline{N_B}((\varphi, \varphi)) = ((\varphi, \varphi))$$

ii) $N_B^\circ\ (U_1, U_2) = (U_1, U_2)$

$$\overline{N_B}(U_1, U_2) = (U_1, U_2)$$

iii) $N_B^\circ\ (A_1, A_2) \subseteq (A_1, A_2) \subseteq \overline{N_B}\ (A_1, A_2)$

iv) $(A_1, A_2) \subseteq (B_1, B_2)$ implies $N_B^\circ\ (A_1, A_2) \subseteq N_B^\circ\ (B_1, B_2)$and

$\overline{N_B}(A_1, A_2) \subseteq \overline{N_B}\ (B_1, B_2)$

v) $N_B^\circ\ ((A_1, A_2) \cap (B_1, B_2)) \subseteq N_B^\circ\ (A_1, A_2) \cap N_B^\circ(B_1, B_2)$

vi) $\overline{N_B}((A_1, A_2) \cap (B_1, B_2)) \subseteq \overline{N_B}\ (A_1, A_2) \cap \overline{N_B}\ (B_1, B_2)$

vii) $N_B^\circ\ ((A_1, A_2) \cup (B_1, B_2)) \supseteq N_B^\circ\ (A_1, A_2) \cup N_B^\circ\ (B_1, B_2)$

viii) $\overline{N_B}\ ((A_1, A_2) \cup (B_1, B_2)) \supseteq \overline{N_B}\ (A_1, A_2) \cup \overline{N_B}(B_1, B_2)$

ix) $N_B^\circ(N_B^\circ(A_1, A_2)) \subseteq N_B^\circ\ (A_1, A_2)$

x) $\overline{N_B}\ (\overline{N_B}\ (A_1, A_2)) \supseteq \overline{N_B}\ (A_1, A_2)$

xi) $N_B^\circ(\overline{N_B}\ (A_1, A_2)) \supseteq N_B^\circ(A_1, A_2)$

xii) $\overline{N_B}\ (N_B^{\circ}(A_1, A_2)) \subseteq \overline{N_B}\ (A_1, A_2)$

Definition 2.6: [4] A subset (H_1, H_2) of a nano binary topological spaces $(U_1, U_2, \tau_R(X_1, X_2))$ is called

1. N_B α-open if $(H_1, H_2) \subseteq N_B{}^{\circ}\ (\overline{N_B}\ (N_B{}^{\circ}\ (H_1, H_2)))$.

2. N_B semi- open set if $(H_1, H_2) \subseteq \overline{N_B}\ (N_B{}^{\circ}\ (H_1, H_2))$

3. N_B pre - open set if $(H_1, H_2) \subseteq N_B{}^{\circ}\ (\overline{N_B}(H_1, H_2))$

4. N_B β-open if $(H_1, H_2) \subseteq \overline{N_B}\ (N_B{}^{\circ}\ (\overline{N_B}\ (H_1, H_2)))$.

The complements of the above mentioned sets are called their respective N_B closed sets.

Results 2.7: [4]

1. Every N_B open sets is N_B α-open.
2. Every N_B α-open is N_B semi -open.
3. Every N_B α-open is N_B pre -open.
4. Every N_B pre –open is N_B β-open.
5. Every N_B semi –open is N_B β-open.
6. Every N_B open is N_B semi -open.
7. Every N_B open is N_B pre -open.
8. Every N_B α-open is N_B β-open.
9. Every N_B open is N_B β-open.

Note 2.8: The above result is also true for every N_B closed sets.

Note 2.9: The converse of the above result is not true.

Definition 2.10: [4] Let $(U_1, U_2, \tau_R(X_1, X_2))$ and $(V_1, V_2, \tau_{R'}(Y_1, Y_2))$ be nano binary topological spaces. Then a mapping f: $(U_1, U_2, \tau_R(X_1, X_2)) \rightarrow (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ is nano binary continuous on (U_1, U_2) if the inverse image of every N_B open in (V_1, V_2) is N_B open in (U_1, U_2) and it is denoted by N_B- continuous.

Definition 2.11: [4] Let $(U_1, U_2, \tau_R(X_1, X_2))$ and $(V_1, V_2, \tau_{R'}(Y_1, Y_2))$ be nano binary topological spaces. A mapping f: $(\mathrm{U}_1, \mathrm{U}_2, \tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2)) \to (\mathrm{V}_1, V_2, \tau_{R'}(\mathrm{Y}_1, \mathrm{Y}_2))$ is said to be

1. N_B α-continuous if $(f^{-1}$ $(\mathrm{B}_1, \mathrm{B}_2))$ is N_B α-open in $(\mathrm{U}_1, \mathrm{U}_2)$ for every N_B open $(\mathrm{B}_1, \mathrm{B}_2)$ in (V_1, V_2).
2. N_B semi -continuous if $(f^{-1}$ $(\mathrm{B}_1, \mathrm{B}_2))$ is N_B semi-open in $(\mathrm{U}_1, \mathrm{U}_2)$ for every N_B open $(\mathrm{B}_1, \mathrm{B}_2)$ in(V_1, V_2).
3. N_B pre -continuous if $(f^{-1}$ $(\mathrm{B}_1, \mathrm{B}_2))$ is N_B pre-open in $(\mathrm{U}_1, \mathrm{U}_2)$ for every N_B open $(\mathrm{B}_1, \mathrm{B}_2)$ in(V_1, V_2).
4. N_B β -continuous if $(f^{-1}$ $(\mathrm{B}_1, \mathrm{B}_2))$ is N_B β -open in $(\mathrm{U}_1, \mathrm{U}_2)$ for every N_B open $(\mathrm{B}_1, \mathrm{B}_2)$ in (V_1, V_2).

Results 2.12: [4]

1. Every N_B- continuous is N_B α-continuous.
2. Every N_B α- continuous is N_B semi - continuous.
3. Every N_B α- continuous is N_B pre - continuous.
4. Every N_B pre – continuous is N_B β- continuous.
5. Every N_B semi – continuous is N_B β- continuous.

3. NANO BINARY CONTRA CONTINUITY

Definition 3.1: Let $(\mathrm{U}_1, \mathrm{U}_2, \tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2))$ and $(\mathrm{V}_1, V_2, \tau_{R'}(\mathrm{Y}_1, \mathrm{Y}_2))$ be nano binary topological spaces. Then a mapping f: $\left(\mathrm{U}_1, \mathrm{U}_2, \tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2)\right) \to (\mathrm{V}_1, V_2, \tau_{R'}(\mathrm{Y}_1, \mathrm{Y}_2))$ is nano binary contra continuous function if the inverse image of every N_B open in $(\mathrm{V}_1, \mathrm{V}_2)$ is N_B closed in $(\mathrm{U}_1, \mathrm{U}_2)$ and it is denoted by N_B- contra continuous.

Example 3.2: Let $\mathrm{U}_1 = \{a, b, c\}, \mathrm{U}_2 = \{1, 2\}$ with ${(\mathrm{U}_1, \mathrm{U}_2)}/{\mathrm{R}} = \{(\{\mathrm{a}, \mathrm{b}\}, \{2\}), (\{\mathrm{c}\}, \{1\})\}$ and $(\mathrm{X}_1, \mathrm{X}_2) = (\{\mathrm{a}, \mathrm{c}\}, \{1\})$. Then $\tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2) = \{(\Phi, \Phi), (\mathrm{U}_1, \mathrm{U}_2), (\{\mathrm{c}\}, \{1\}), (\{\mathrm{a}, \mathrm{b}\}, \{2\})\}$. The N_B closed sets are $(\mathrm{U}_1, \mathrm{U}_2), (\Phi, \Phi), (\{\mathrm{a}, \mathrm{b}\}, \{2\}), (\{\mathrm{c}\}, \{1\})$. Let $\mathrm{V}_1 = \{x, y, z\}, \mathrm{V}_2 = \{e, f\}$ with ${(\mathrm{V}_1, V_2)}/{R'} = \{(\{\mathrm{x}, \mathrm{z}\}, \{\mathrm{e}\}), (\{\mathrm{y}\}, \{\mathrm{f}\})\}$ and $(\mathrm{Y}_1, Y_2) = (\{x, y\}, \{\mathrm{f}\})$. Then $\tau_{R'}(\mathrm{Y}_1, Y_2) = \{(\Phi, \Phi), (\mathrm{V}_1, \mathrm{V}_2), (\{\mathrm{x}, \mathrm{z}\}, \{\mathrm{e}\}), (\{\mathrm{y}\}, \{\mathrm{f}\})\}$.

Define f: $(\mathrm{U}_1, \mathrm{U}_2, \tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2)) \to (\mathrm{V}_1, V_2, \tau_{R'}(\mathrm{Y}_1, Y_2))$ as f $(\{\mathrm{a}\}, \{1\}) = (\{\mathrm{x}\}, \{\mathrm{f}\}), \mathrm{f}(\{\mathrm{a}\}, \{2\}) =$

$(\{x\},\{e\})$, $f(\{b\},\{1\}) = (\{z\},\{f\})$, $f(\{b\},\{2\}) = (\{z\},\{e\})$, $f(\{c\},\{1\}) = (\{y\},\{f\})$, $f(\{c\},\{2\}) = (\{y\},\{e\})$. Then $f^{-1}(\{y\},\{f\}) = (\{c\},\{1\})$ and $f^{-1}(\{x,z\},\{e\}) = (\{a,b\},\{2\})$, which is N_B closed in (U_1,U_2). That is the inverse image of every N_B open in (V_1,V_2) is N_B closed in (U_1,U_2). Therefore, f is N_B- contra continuous.

Theorem 3.3: For a function f: $(U_1,U_2,\tau_R(X_1,X_2)) \rightarrow (V_1,V_2,\tau_{R'}(Y_1,Y_2))$ the following conditions are equivalent:

(i) f is N_B- contra continuous.

(ii) The inverse image of each N_B closed set in (V_1,V_2) is N_B open in (U_1,U_2) .

(iii) For each $(x_1,x_2) \in (U_1,U_2)$ and each N_B closed set (B_1,B_2) in (V_1,V_2) with $f(x_1,x_2) \in (B_1,B_2)$, there exists an N_B open set (A_1,A_2) in (U_1,U_2) such that $f(A_1,A_2) \subseteq (B_1,B_2)$.

Proof: (i) ⇒(ii) Let f be N_B- contra continuous. Let (B_1,B_2) be N_B closed in (V_1,V_2). That is $(V_1,V_2)-(B_1,B_2)$ is N_B open in (V_1,V_2). Since f is N_B- contra continuous, $f^{-1}\left((V_1,V_2)-(B_1,B_2)\right)$ is N_B closed in (U_1,U_2). But $f^{-1}\left((V_1,V_2)-(B_1,B_2)\right) = (U_1,U_2)-f^{-1}(B_1,B_2)$. Therefore, $f^{-1}(B_1,B_2)$ is N_B open in (U_1,U_2). Thus the inverse image of each N_B closed set in (V_1,V_2) is N_B open in (U_1,U_2).

(ii) ⇒(i) Let (B_1,B_2) be a N_B open in (V_1,V_2). Then $(V_1,V_2)-(B_1,B_2)$ is N_B closed in (V_1,V_2). By assumption, $f^{-1}\left((V_1,V_2)-(B_1,B_2)\right)$ is N_B open in (U_1,U_2). Therefore, $f^{-1}(B_1,B_2)$ is N_B closed in (U_1,U_2). Hence f is N_B- contra continuous.

(ii) ⇒ (iii) Let (B_1,B_2) be a N_B closed set such that $f(x_1,x_2) \in (B_1,B_2)$. By assumption, $(x_1,x_2) \in f^{-1}(B_1,B_2)$, which is N_B open. Let $(A_1,A_2) = f^{-1}(B_1,B_2)$. Then $f(A_1,A_2) \subseteq (B_1,B_2)$.

(iii) ⇒ (ii) Let (B_1,B_2) be a N_B closed set in (V_1,V_2) with $f(x_1,x_2) \in (B_1,B_2)$. Then there exists a N_B open set (A_1,A_2) in (U_1,U_2) such that $f(A_1,A_2) \subseteq (B_1,B_2)$. Therefore, $f^{-1}(B_1,B_2) = \cup\{(A_1,A_2): (A_1,A_2) \subseteq f^{-1}(B_1,B_2)\}$ is N_B open in (U_1,U_2)

Remark 3.4: The concept of N_B - continuity and N_B - contra continuity are independent as shown in the following example

Example 3.5: Let $U_1 = \{a, b, c\}, U_2 = \{1, 2\}$ with $(U_1, U_2)/_R = \{(\{a, b\}, \{2\}), (\{c\}, \{1\})\}$ and $(X_1, X_2) = (\{b\}, \{2\})$. Then $\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), (\{a, b\}, \{2\})\}$. Let $V_1 = \{x, y, z\}, V_2 = \{e, f\}$ with $(V_1, V_2)/_{R'} = \{(\{x, z\}, \{e\}), (\{y\}, \{f\})\}$ and $(Y_1, Y_2) = (\{z\}, \{e\})$. Then $\tau_{R'}(Y_1, Y_2) = \{(\Phi, \Phi), (V_1, V_2), (\{x, z\}, \{e\})\}$. Define f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ as f $(\{a\}, \{1\}) = (\{x\}, \{f\})$, $f(\{a\}, \{2\}) = (\{x\}, \{e\})$, $f(\{b\}, \{1\}) = (\{z\}, \{f\})$, $f(\{b\}, \{2\}) = (\{z\}, \{e\})$, f $(\{c\}, \{1\}) = (\{y\}, \{f\})$, f $(\{c\}, \{2\}) = (\{y\}, \{e\})$. Here $(B_1, B_2) = (\{x, z\}, \{e\})$. Then $f^{-1}(\{x, z\}, \{e\}) = (\{a, b\}, \{2\})$. Here f is N_B-continuous function, but not N_B- contra continuous. Because $(\{a, b\}, \{2\})$ is not N_B closed in (U_1, U_2), where $(\{x, z\}, \{e\})$ is N_B open in (V_1, V_2). Therefore, f is N_B- continuous but not N_B- contra continuous.

Example 3.6: Let $U_1 = \{a, b, c\}, U_2 = \{1, 2\}$ with $(U_1, U_2)/_R = \{(\{a, b\}, \{2\}), (\{c\}, \{1\})\}$ and $(X_1, X_2) = (\{b\}, \{2\})$. Then $\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), (\{a, b\}, \{2\})\}$. The N_B closed sets are $(\Phi, \Phi), (U_1, U_2), (\{c\}, \{1\})$ Let $V_1 = \{x, y, z\}, V_2 = \{e, f\}$ with $(V_1, V_2)/_{R'} = \{(\{x, z\}, \{e\}), (\{y\}, \{f\})\}$ and $(Y_1, Y_2) = (\{y\}, \{f\})$. Then $\tau_{R'}(Y_1, Y_2) = \{(\Phi, \Phi), (V_1, V_2), (\{y\}, \{f\})\}$. Define f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ as f $(\{a\}, \{1\}) = (\{x\}, \{f\})$, $f(\{a\}, \{2\}) = (\{x\}, \{e\})$, f $(\{b\}, \{1\}) = (\{z\}, \{f\})$, f $(\{b\}, \{2\}) = (\{z\}, \{e\})$, f $(\{c\}, \{1\}) = (\{y\}, \{f\})$, f $(\{c\}, \{2\}) = (\{y\}, \{e\})$. Therefore, $f^{-1}(\{y\}, \{f\}) = (\{c\}, \{1\})$, which is N_B closed in (U_1, U_2) but not N_B open in (U_1, U_2). Hence f is N_B- contra continuous but not N_B- continuous.

Note 3.7: Every N_B-continuous is N_B- contra continuous if every N_B open set is N_B closed.

Definition 3.8: A function f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ is said to be

(i) N_B perfectly continuous, if $f^{-1}(A_1, A_2)$ is N_B clopen in (U_1, U_2) for every N_B open set (A_1, A_2) in (V_1, V_2).

(ii) N_B strongly continuous, if $f^{-1}(A_1, A_2)$ is N_B clopen in (U_1, U_2) for every subset (A_1, A_2) in (V_1, V_2).

Definition 3.9: A function f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ is said to be

(i) N_B contra α- continuous, if $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B α-closed in $(\mathrm{U}_1, \mathrm{U}_2)$ for every N_B open $(\mathrm{B}_1, \mathrm{B}_2)$ in $(\mathrm{V}_1, \mathrm{V}_2)$.

(ii) N_B contra pre- continuous, if $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B pre-closed in $(\mathrm{U}_1, \mathrm{U}_2)$ for every N_B open $(\mathrm{B}_1, \mathrm{B}_2)$ in$(\mathrm{V}_1, \mathrm{V}_2)$.

(iii) N_B contra semi- continuous, if $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B semi-closed in $(\mathrm{U}_1, \mathrm{U}_2)$ for every N_B open $(\mathrm{B}_1, \mathrm{B}_2)$ in $(\mathrm{V}_1, \mathrm{V}_2)$.

(iv) N_B contra β- continuous, if $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B β-closed in $(\mathrm{U}_1, \mathrm{U}_2)$ for every N_B open $(\mathrm{B}_1, \mathrm{B}_2)$ in $(\mathrm{V}_1, \mathrm{V}_2)$.

Result 3.10:

i) Every N_B- contra continuous is N_B contra α- continuous.

ii) Every N_B contra α -continuous is N_B contra pre- continuous.

iii) Every N_B contra α -continuous is N_B contra semi- continuous.

iv) Every N_B- contra continuous is N_B contra pre- continuous

v) Every N_B- contra continuous is N_B contra semi- continuous.

vi) Every N_B contra pre- continuous is N_B contra β- continuous.

vii) Every N_B contra semi- continuous is N_B contra β- continuous.

viii) Every N_B- contra continuous is N_B contra β- continuous.

ix) Every N_B contra α -continuous is N_B contra β- continuous.

Proof: (i) Given f is N_B- contra continuous. Let $(\mathrm{B}_1, \mathrm{B}_2)$ be N_B open in $(\mathrm{V}_1, \mathrm{V}_2)$. Since f is N_B- contra continuous, $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B closed in $(\mathrm{U}_1, \mathrm{U}_2)$. Since every N_B closed is N_B α –closed, $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B α -closed in $(\mathrm{U}_1, \mathrm{U}_2)$. Therefore, f is N_B contra α-continuous.

Similarly we can prove (ii), (iii), (iv), (v), (vi), (vii), (viii) and (ix).

Remark 3.11: The concept of N_B contra pre- continuous and N_B contra semi- continuous are independent as shown in the following examples.

Example 3.12: Let $\mathrm{U}_1 = \{a, b, c\}, \mathrm{U}_2 = \{1, 2\}$ with ${(\mathrm{U}_1, \mathrm{U}_2)}/_{\mathrm{R}} = \{(\{\mathrm{a}, \mathrm{b}\}, \{2\}), (\{\mathrm{c}\}, \{1\})\}$ and $(\mathrm{X}_1, \mathrm{X}_2)$ =$(\{\mathrm{b}\}, \{2\})$. Then $\tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2)$ =$\{(\Phi, \Phi), (\mathrm{U}_1, \mathrm{U}_2), (\{\mathrm{a}, \mathrm{b}\}, \{2\})\}$. Let $\mathrm{V}_1 = \{x, y, z\}, \mathrm{V}_2 =$

$\{e, f\}$ with $(V_1, V_2)/_{R'} = \{(\{x,z\},\{e\}), (\{y\},\{f\})\}$ and $(Y_1, Y_2) = (\{z\},\{e\})$. Then $\tau_{R'}(Y_1, Y_2) = \{(\Phi,\Phi), (V_1, V_2), (\{x,z\},\{e\})\}$. Define f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ as f $(\{a\},\{1\}) = (\{x\},\{e\})$, f$(\{a\},\{2\}) = (\{x\},\{f\})$, f $(\{b\},\{1\}) = (\{y\},\{e\})$, f $(\{b\},\{2\}) = (\{y\},\{f\})$, f $(\{c\},\{1\}) = (\{z\},\{e\})$, f $(\{c\},\{2\}) = (\{z\},\{f\})$. Here $(B_1, B_2) = (\{x,z\},\{e\})$. Then $f^{-1}(\{x,z\},\{e\}) = (\{a,c\},\{1\})$. Here f is N_B contra pre-continuous, but not N_B contra semi-continuous. Because $(\{a,c\},\{1\})$ is N_B pre-closed and not N_B semi-closed in(U_1, U_2), where $(\{x,z\},\{e\})$ is N_B open in (V_1, V_2). Therefore, f is N_B contra pre-continuous but not N_B contra semi-continuous.

Example 3.13: Let $U_1 = \{a, b, c, d, e\}, U_2 = \{1, 2,3,4\}$ with $(U_1, U_2)/_R = \{(\{a,b\},\{2\}), (\{c\},\{4\}), (\{d\},\{3\}), (\{e\},\{1\})\}$. Let $(X_1, X_2) = (\{a,c,d\},\{2,3,4\})$.

Then $\tau_R(X_1, X_2) = \{(\Phi,\Phi), (U_1, U_2), (\{c,d\},\{3,4\}), (\{a,b,c,d\},\{2,3,4\}), (\{a,b\},\{2\})\}$.

Let $V_1 = \{x, y, z\}$, $V_2 = \{e, f\}$ with $(V_1, V_2)/_{R'} = \{(\{x,z\},\{e\}), (\{y\},\{f\})\}$.

Let $(Y_1, Y_2) = (\{x\},\{e\})$. Then $\tau_{R'}(Y_1, Y_2) = \{(\Phi,\Phi), (V_1, V_2), (\{x,z\},\{e\})\}$. Define f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ as f $(\{a\},\{1\}) = (\{x\},\{e\})$, f $(\{a\},\{2\}) = (\{x\},\{e\})$, f $(\{a\},\{3\}) = (\{x\},\{f\})$, f$(\{a\},\{4\}) = (\{x\},\{f\})$, f$(\{b\},\{1\}) = (\{z\},\{e\})$, f $(\{b\},\{2\}) = (\{z\},\{e\})$, f $(\{b\},\{3\}) = (\{z\},\{f\})$, f $(\{b\},\{4\}) = (\{z\},\{f\})$, f $(\{c\},\{1\}) = (\{y\},\{e\})$, f $(\{c\},\{2\}) = (\{y\},\{e\})$, f $(\{c\},\{3\}) = (\{y\},\{f\})$, f $(\{c\},\{4\}) = (\{y\},\{f\})$, f $(\{d\},\{1\}) = (\{y\},\{e\})$, f $(\{d\},\{2\}) = (\{y\},\{e\})$, f $(\{d\},\{3\}) = (\{y\},\{f\})$, f $(\{d\},\{4\}) = (\{y\},\{f\})$,f $(\{e\},\{1\}) = (\{y\},\{e\})$, f $(\{e\},\{2\}) = (\{y\},\{e\})$, f $(\{e\},\{3\}) = (\{y\},\{f\})$, f $(\{e\},\{4\}) = (\{y\},\{f\})$. $f^{-1}(\{x,z\},\{e\}) = (\{a,b\},\{1,2\})$. Here f is N_B contra semi-continuous, but not N_B contra pre-continuous. Because $(\{a,b\},\{1,2\})$ is N_B semi-closed and not N_B pre-closed in (U_1, U_2), where $(\{x,z\},\{e\})$ is N_B open in (V_1, V_2). Therefore, f is N_B contra semi-continuous but not N_B contra pre-continuous.

Theorem 3.14: Let f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ be the function. If f is N_B strongly continuous then f is N_B perfectly continuous.

Proof: Given f is N_B strongly continuous. Let (B_1, B_2) be N_B open in (V_1, V_2). Since f is N_B

strongly continuous, $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B clopen in $(\mathrm{U}_1, \mathrm{U}_2)$. Hence f is N_B perfectly continuous.

Remark 3.15: The converse of the above theorem need not be true by the following example.

Example 3.16: Let $\mathrm{U}_1 = \{a, b, c\}, \mathrm{U}_2 = \{1, 2\}$ with $(\mathrm{U}_1, \mathrm{U}_2)/_{\mathrm{R}} = \{(\{\mathrm{a}, \mathrm{b}\}, \{2\}), (\{\mathrm{c}\}, \{1\})\}$ and $(\mathrm{X}_1, \mathrm{X}_2) = (\{\mathrm{a}, \mathrm{c}\}, \{1\})$. Then $\tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2) = \{(\Phi, \Phi), (\mathrm{U}_1, \mathrm{U}_2), (\{\mathrm{c}\}, \{1\}), (\{\mathrm{a}, \mathrm{b}\}, \{2\})\}$. The N_B closed sets are $(\mathrm{U}_1, \mathrm{U}_2), (\Phi, \Phi), (\{\mathrm{a}, \mathrm{b}\}, \{2\}), (\{\mathrm{c}\}, \{1\})$. Let $\mathrm{V}_1 = \{x, y, z\}, \mathrm{V}_2 = \{e, f\}$ with $(\mathrm{V}_1, V_2)/_{R'} = \{(\{\mathrm{x}, \mathrm{z}\}, \{\mathrm{e}\}), (\{\mathrm{y}\}, \{\mathrm{f}\})\}$. Let $(\mathrm{Y}_1, Y_2) = (\{z\}, \{\mathrm{e}\})$. Then $\tau_{R'}(\mathrm{Y}_1, Y_2)$ $= \{(\Phi, \Phi), (\mathrm{V}_1, \mathrm{V}_2), (\{\mathrm{x}, \mathrm{z}\}, \{\mathrm{e}\})\}$. Define f: $(\mathrm{U}_1, \mathrm{U}_2, \tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2)) \rightarrow (\mathrm{V}_1, V_2, \tau_{R'}(\mathrm{Y}_1, \mathrm{Y}_2))$ as f $(\{\mathrm{a}\}, \{1\}) = (\{\mathrm{x}\}, \{\mathrm{e}\})$, f $(\{\mathrm{a}\}, \{2\}) = (\{\mathrm{x}\}, \{\mathrm{e}\})$, f $(\{\mathrm{b}\}, \{1\}) = (\{\mathrm{z}\}, \{\mathrm{e}\})$, f $(\{\mathrm{b}\}, \{2\}) = (\{\mathrm{z}\}, \{\mathrm{e}\})$, f $(\{\mathrm{c}\}, \{1\}) = (\{\mathrm{y}\}, \{\mathrm{e}\})$, f $(\{\mathrm{c}\}, \{2\}) = (\{\mathrm{y}\}, \{e\})$. $f^{-1}(\{\mathrm{y}\}, \{\mathrm{e}\}) = (\{\mathrm{c}\}, \{2\})$, which is not N_B clopen in $(\mathrm{U}_1, \mathrm{U}_2)$ and $f^{-1}(\{\mathrm{x}, \mathrm{z}\}, \{\mathrm{e}\}) = (\{a, b\}, \{2\})$, which is N_B clopen in $(\mathrm{U}_1, \mathrm{U}_2)$. Therefore, f is N_B perfectly continuous but not N_B strongly continuous.

Theorem 3.17: Let f: $\left(\mathrm{U}_1, \mathrm{U}_2, \tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2)\right) \rightarrow (\mathrm{V}_1, V_2, \tau_{R'}(\mathrm{Y}_1, \mathrm{Y}_2))$ be the function. Then f is N_B perfectly continuous if and only if f is N_B- contra continuous and N_B- continuous.

Proof: Let $(\mathrm{B}_1, \mathrm{B}_2)$ be N_B open in $(\mathrm{V}_1, \mathrm{V}_2)$. Since f is N_B perfectly continuous, $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B clopen in $(\mathrm{U}_1, \mathrm{U}_2)$. Hence $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is both N_B closed and N_B open in $(\mathrm{U}_1, \mathrm{U}_2)$. Hence f is both N_B- contra continuous and N_B- continuous. Conversely, let f be N_B- contra continuous and N_B- continuous. Let $(\mathrm{B}_1, \mathrm{B}_2)$ be N_B open in $(\mathrm{V}_1, \mathrm{V}_2)$. Since f is N_B- contra continuous, $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B closed in $(\mathrm{U}_1, \mathrm{U}_2)$. Since f is N_B- continuous, $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is N_B open in $(\mathrm{U}_1, \mathrm{U}_2)$. Therefore, $f^{-1}(\mathrm{B}_1, \mathrm{B}_2)$ is both N_B closed and N_B open in $(\mathrm{U}_1, \mathrm{U}_2)$. Hence f is N_B perfectly continuous.

Remark 3.18: The above theorem is true if f is both N_B -continuous and N_B -contra continuous otherwise it is not true by the following example.

Example 3.19: Let $\mathrm{U}_1 = \{a, b, c\}, \mathrm{U}_2 = \{1, 2\}$ with $(\mathrm{U}_1, \mathrm{U}_2)/_{\mathrm{R}} = \{(\{\mathrm{a}, \mathrm{b}\}, \{2\}), (\{\mathrm{c}\}, \{1\})\}$ and $(\mathrm{X}_1, \mathrm{X}_2) = (\{\mathrm{b}\}, \{2\})$. Then $\tau_{\mathrm{R}}(\mathrm{X}_1, \mathrm{X}_2) = \{(\Phi, \Phi), (\mathrm{U}_1, \mathrm{U}_2), (\{\mathrm{a}, \mathrm{b}\}, \{2\})\}$. The N_B closed sets are $(\Phi, \Phi), (\mathrm{U}_1, \mathrm{U}_2), (\{\mathrm{c}\}, \{1\})$ Let $\mathrm{V}_1 = \{x, y, z\}, \mathrm{V}_2 = \{e, f\}$ withv $(\mathrm{V}_1, V_2)/_{R'} =$

$\{(\{x, z\}, \{e\}), (\{y\}, \{f\})\}$ and $(Y_1, Y_2) = (\{y\}, \{f\})$. Then $\tau_{R'}(Y_1, Y_2) = \{(\Phi, \Phi), (V_1, V_2), (\{y\}, \{f\})\}$. Define f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ as $f(\{a\}, \{1\}) = (\{x\}, \{f\})$, $f(\{a\}, \{2\}) = (\{x\}, \{e\})$, $f(\{b\}, \{1\}) = (\{z\}, \{f\})$, $f(\{b\}, \{2\}) = (\{z\}, \{e\})$, $f(\{c\}, \{1\}) = (\{y\}, \{f\})$, $f(\{c\}, \{2\}) = (\{y\}, \{e\})$. Therefore, $f^{-1}(\{y\}, \{f\}) = (\{c\}, \{1\})$, which is N_B closed in (U_1, U_2) but not N_B open in (U_1, U_2). Hence f is N_B- contra continuous but not N_B -continuous. Also f is not N_B perfectly continuous.

Result 3.20: Let f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ be the function.

i) If f is N_B perfectly continuous then f is N_B contra α-continuous and N_B α -continuous.

ii) If f is N_B perfectly continuous then f is N_B contra pre-continuous and N_B pre – continuous.

iii) If f is N_B perfectly continuous then f is N_B contra semi-continuous and N_B semi- continuous.

iv) If f is N_B perfectly continuous then f is N_B contra β-continuous and N_B β – continuous.

By theorem 3.17 and definition 2.6, the above result is true but none of these implications is reversible as example 3.19.

Theorem 3.21: Every N_B strongly continuous function is both N_B- continuous and N_B- contra continuous.

Proof: Let (B_1, B_2) be a subset of (V_1, V_2). Since f is N_B strongly continuous, $f^{-1}(B_1, B_2)$ is N_B clopen in (U_1, U_2). That is, $f^{-1}(B_1, B_2)$ is both N_B open and N_B closed in (U_1, U_2). Since it holds for every subset of (V_1, V_2), it is also true for all the N_B open sets in (V_1, V_2). Therefore, f is both N_B- continuous and N_B- contra continuous.

Remark 3.22: The converse of the above theorem need not be true by the following example.

Example 3.23: In example 3.19, the function f: $(U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ is N_B – contra continuous. Consider $(\{x, y\}, \{e\})$ is any subset of (V_1, V_2). Then $f^{-1}(\{x, y\}, \{e\}) = (\{a, c\}, \{2\})$, which is not N_B open in (U_1, U_2). Therefore, f is N_B –contra continuous. But f is neither N_B strongly continuous nor N_B- continuous.

Result 3.24: Let f: $(U_1, U_2, \tau_R(X_1, X_2)) \rightarrow (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ be the function.

i) If f is N_B strongly continuous then f is N_B contra α-continuous and N_B α - continuous.

ii) If f is N_B strongly continuous then f is N_B contra pre-continuous and N_B pre – continuous.

iii) If f is N_B strongly continuous then f is N_B contra semi-continuous and N_B semi- continuous.

iv) If f is N_B strongly continuous then f is N_B contra β-continuous and N_B β – continuous.

By theorem 3.21 and definition 2.6, the above result is true but none of these implications is reversible as example 3.23.

Remark 3.25: Composition of two N_B- contra continuous functions need not be N_B- contra continuous as shown in the following example.

Example 3.26: Let $U_1 = \{a, b, c\}, U_2 = \{1, 2\}$ with $(U_1, U_2)/_R = \{(\{a, b\}, \{2\}), (\{c\}, \{1\})\}$ and (X_1, X_2) =$(\{b\}, \{2\})$. Then $\tau_R(X_1, X_2)$ =$\{(\Phi, \Phi), (U_1, U_2), (\{a, b\}, \{2\})\}$. The N_B closed sets are $(\Phi, \Phi), (U_1, U_2), (\{c\}, \{1\})$ Let $V_1 = \{a, b, c\}, V_2 = \{1, 2\}$ with $(V_1, V_2)/_{R'} = \{(\{a, c\}, \{1\}), (\{b\}, \{2\})\}$ and (Y_1, Y_2) =$(\{b\}, \{2\})$. Then $\tau_{R'}(Y_1, Y_2)$ =$\{(\Phi, \Phi), (V_1, V_2), (\{b\}, \{2\})\}$. The N_B closed sets are $(\Phi, \Phi), (V_1, V_2), (\{a, c\}, \{1\})$ Define f: $(U_1, U_2, \tau_R(X_1, X_2)) \rightarrow (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ as f $(\{a\}, \{1\}) = (\{a\}, \{2\})$, f$(\{a\}, \{2\}) = (\{a\}, \{1\})$, f $(\{b\}, \{1\}) = (\{c\}, \{2\})$, f $(\{b\}, \{2\}) = (\{c\}, \{1\})$, f $(\{c\}, \{1\}) = (\{b\}, \{2\})$, f $(\{c\}, \{2\}) = (\{b\}, \{1\})$. Therefore, $f^{-1}(\{b\}, \{2\}) = (\{c\}, \{1\})$, which is N_B closed in (U_1, U_2). Hence f is N_B- contra continuous. Let $W_1 = \{a, b, c\}, W_2 = \{1, 2\}$ with $(W_1, W_2)/_{R''} = \{(\{a\}, \{2\}), (\{b, c\}, \{1\})\}$ and (Z_1, Z_2) = $(\{b\}, \{1\})$. Then $\tau_{R''}(Z_1, Z_2)$ = $\{(\Phi, \Phi), (W_1, W_2), (\{b, c\}, \{1\})\}$. The N_B closed sets are $(\Phi, \Phi), (W_1, W_2), (\{a\}, \{2\})$ Define $g: (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \rightarrow (W_1, W_2, \tau_{R''}(Z_1, Z_2))$ as $g(\{a\}, \{1\}) = (\{b\}, \{2\}), g(\{a\}, \{2\}) = (\{c\}, \{1\}), g(\{b\}, \{1\}) = (\{a\}, \{2\}), g(\{b\}, \{2\}) =$

$(\{a\},\{1\}), g(\{c\},\{1\}) = (\{a\},\{2\}), g(\{c\},\{2\}) = (\{a\},\{1\})$. Therefore, $g^{-1}(\{b,c\},\{1\}) = (\{a\},\{2\})$. Hence $f^{-1}\left(g^{-1}(\{b,c\},\{1\})\right) = f^{-1}(\{a\},\{2\}) = (\{a\},\{1\})$, which is not a N_B closed set. Hence composition of two N_B –contra continuous functions is not a N_B –contra continuous function.

Theorem 3.27: Let $f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ and $g: (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \to (W_1, W_2, \tau_{R''}(Z_1, Z_2))$ be the functions then gof is N_B –contra continuous if g is N_B –continuous and f is N_B –contra continuous.

Proof: Given that g is N_B –continuous and f is N_B –contra continuous. Let (B_1, B_2) be N_B open in (W_1, W_2). Since g is N_B –continuous, $g^{-1}(B_1, B_2)$ be N_B open in (V_1, V_2). Since f is N_B –contra continuous, $f^{-1}(g^{-1}(B_1, B_2))$ is N_B closed in (U_1, U_2). That is, $(gof)^{-1}(B_1, B_2)$ is N_B closed in (U_1, U_2). Hence gof is N_B –contra continuous.

Remark 3.28: Let $f: (U_1, U_2, \tau_R(X_1, X_2)) \to (V_1, V_2, \tau_{R'}(Y_1, Y_2))$ and $g: (V_1, V_2, \tau_{R'}(Y_1, Y_2)) \to (W_1, W_2, \tau_{R''}(Z_1, Z_2))$ be the functions then

i) gof is N_B perfectly continuous if g is N_B perfectly continuous and f is N_B –contra continuous.

ii) gof is N_B strongly continuous if g is N_B strongly continuous and f is N_B –contra continuous.

iii) gof is N_B perfectly continuous if g is N_B-continuous and f is N_B perfectly continuous.

iv) gof is N_B perfectly continuous if g is N_B -continuous and f is N_B strongly continuous.

v) gof is N_B -continuous if g is N_B –contra continuous and f is N_B -continuous.

vi) gof is N_B -continuous if g is N_B -continuous and f is N_B -continuous.

vii) gof is N_B perfectly continuous if g is N_B perfectly continuous and f is N_B strongly continuous.

viii) gof is N_B strongly continuous if g is N_B strongly continuous and f is N_B perfectly continuous.

Note 3.29: The above remark follows from theorems 3.14, 3.17, 3.21 and 3.27.

4. Nano Binary D – Continuous

Definition 4.1: A nano binary subset $(\mathrm{A}_1, \mathrm{A}_2)$ of a nano binary topological space $(\mathrm{U}_1, \mathrm{U}_2, \tau_\mathrm{R}(\mathrm{X}_1, \mathrm{X}_2))$ is called nano binary dense if $\overline{N_B}\,(\mathrm{A}_1, \mathrm{A}_2) = (\mathrm{U}_1, \mathrm{U}_2)$ and it is denoted by N_B-dense.

Definition 4.2: Let $(\mathrm{U}_1, \mathrm{U}_2, \tau_\mathrm{R}(\mathrm{X}_1, \mathrm{X}_2))$ and $(\mathrm{V}_1, V_2, \tau_{R'}(\mathrm{Y}_1, \mathrm{Y}_2))$ be nano binary topological spaces. Then a mapping f: $\big(\mathrm{U}_1, \mathrm{U}_2, \tau_\mathrm{R}(\mathrm{X}_1, \mathrm{X}_2)\big) \rightarrow (\mathrm{V}_1, V_2, \tau_{R'}(\mathrm{Y}_1, \mathrm{Y}_2))$ is nano binary

D - continuous function if the inverse image of every N_B open in $(\mathrm{V}_1, \mathrm{V}_2)$ is N_B -dense in $(\mathrm{U}_1, \mathrm{U}_2)$ and it is denoted by N_B D -continuous.

Example 4.3: Let $\mathrm{U}_1 = \{a, b, c\}, \mathrm{U}_2 = \{1, 2\}$ with $(\mathrm{U}_1, U_2)/_\mathrm{R} = \{(\{\mathrm{a}, \mathrm{b}\}, \{2\}), (\{\mathrm{c}\}, \{1\})\}$ and $(\mathrm{X}_1, X_2) =(\{\mathrm{b}\}, \{2\})$. Then $\tau_\mathrm{R}(\mathrm{X}_1, X_2) =\{(\Phi, \Phi), (\mathrm{U}_1, \mathrm{U}_2), (\{\mathrm{a}, \mathrm{b}\}, \{2\})\}$. Let $\mathrm{V}_1 = \{x, y, z\}, \mathrm{V}_2 = \{e, f\}$ with $(\mathrm{V}_1, V_2)/_{R'} = \{(\{\mathrm{x}, \mathrm{z}\}, \{\mathrm{e}\}), (\{\mathrm{y}\}, \{\mathrm{f}\})\}$ and $(\mathrm{Y}_1, Y_2) =(\{x, y\}, \{\mathrm{f}\})$. Then $\tau_{R'}(\mathrm{Y}_1, Y_2) =\{(\Phi, \Phi), (\mathrm{V}_1, \mathrm{V}_2), (\{\mathrm{x}, \mathrm{z}\}, \{\mathrm{e}\}), (\{\mathrm{y}\}, \{\mathrm{f}\})\}$. Define f: $\big(\mathrm{U}_1, \mathrm{U}_2, \tau_\mathrm{R}(\mathrm{X}_1, \mathrm{X}_2)\big) \rightarrow \big(\mathrm{V}_1, V_2, \tau_{R'}(\mathrm{Y}_1, Y_2)\big)$ as $f(\{\mathrm{a}\}, \{1\}) = (\{\mathrm{y}\}, \{\mathrm{f}\})$, $f(\{\mathrm{a}\}, \{2\}) = (\{\mathrm{y}\}, \{\mathrm{e}\}), f(\{\mathrm{b}\}, \{1\}) = (\{\mathrm{x}\}, \{\mathrm{f}\})$, $f(\{\mathrm{b}\}, \{2\}) = (\{\mathrm{x}\}, \{\mathrm{e}\})$, $f(\{\mathrm{c}\}, \{1\}) = (\{\mathrm{z}\}, \{\mathrm{f}\})$, $f(\{\mathrm{c}\}, \{2\}) = (\{\mathrm{z}\}, \{e\})$. Then $f^{-1}(\{\mathrm{y}\}, \{\mathrm{f}\}) = (\{\mathrm{a}\}, \{1\})$ and $f^{-1}(\{\mathrm{x}, \mathrm{z}\}, \{\mathrm{e}\}) = (\{\mathrm{b}, \mathrm{c}\}, \{2\})$, which is N_B -dense in $(\mathrm{U}_1, \mathrm{U}_2)$. That is, the inverse image of every N_B open in $(\mathrm{V}_1, \mathrm{V}_2)$ is N_B -dense in $(\mathrm{U}_1, \mathrm{U}_2)$. Therefore, f is N_B D-continuous.

Definition 4.4: A nano binary topological space $(\mathrm{U}_1, \mathrm{U}_2, \tau_\mathrm{R}(\mathrm{X}_1, \mathrm{X}_2))$ is called a nano binary submaximal space if every N_B-dense subset of $(\mathrm{U}_1, \mathrm{U}_2)$ is N_B open in $(\mathrm{U}_1, \mathrm{U}_2)$ and it is denoted by N_B submaximal space.

Definition 4.5: A nano binary topological space $(\mathrm{U}_1, \mathrm{U}_2, \tau_\mathrm{R}(\mathrm{X}_1, \mathrm{X}_2))$ is called a nano binary hyperconnected space if every N_B open subset of $(\mathrm{U}_1, \mathrm{U}_2)$ is N_B –dense in $(\mathrm{U}_1, \mathrm{U}_2)$ and it is denoted by N_B hyperconnected space.

Remark 4.6: Every N_B D-continuous is not N_B-continuous as shown in the following example.

Example 4.7: In example 4.3, f is N_B D-continuous. But $f^{-1}(\{y\},\{f\}) = (\{a\},\{1\})$ and $f^{-1}(\{x,z\},\{e\}) = (\{b,c\},\{2\})$, which is not N_B open in (U_1,U_2). Therefore, f is not N_B – continuous. Hence f is N_B D-continuous but not N_B –continuous.

Note 4.8: Similarly we can say the following:

i) Every N_B D-continuous is not N_B α – continuous.

ii) Every N_B D-continuous is not N_B semi-continuous.

iii) Every N_B D-continuous is not N_B pre –continuous

iv) Every N_B D-continuous is not N_B β – continuous.

Remark 4.9: In N_B submaximal space, every N_B D-continuous is N_B –continuous.

Note 4.10: In N_B submaximal space,

i) Every N_B D-continuous is N_B α –continuous.

ii) Every N_B D-continuous is N_B semi–continuous.

iii) Every N_B D-continuous is N_B pre–continuous.

iv) Every N_B D-continuous is N_B β –continuous.

Remark 4.11: Every N_B –continuous is not N_B D-continuous as shown in the following example.

Example 4.12: Let $U_1 = \{a,b,c\}, U_2 = \{1,2\}$ with $(U_1,U_2)/_R = \{(\{a,b\},\{2\}),(\{c\},\{1\})\}$ and $(X_1,X_2) = (\{a,c\},\{1\})$. Then $\tau_R(X_1,X_2) = \{(\Phi,\Phi),(U_1,U_2),(\{c\},\{1\}),(\{a,b\},\{2\})\}$. The N_B closed sets are $(U_1,U_2),(\Phi,\Phi),(\{a,b\},\{2\}),(\{c\},\{1\})$. Let $V_1 = \{x,y,z\}, V_2 = \{e,f\}$ with $(V_1,V_2)/_{R'} = \{(\{x,z\},\{e\}),(\{y\},\{f\})\}$ and $(Y_1,Y_2) = (\{x,y\},\{f\})$. Then $\tau_{R'}(Y_1,Y_2) = \{(\Phi,\Phi),(V_1,V_2),(\{x,z\},\{e\}),(\{y\},\{f\})\}$.

Define f: $(U_1,U_2,\tau_R(X_1,X_2)) \to (V_1,V_2,\tau_{R'}(Y_1,Y_2))$ as f $(\{a\},\{1\}) = (\{x\},\{f\})$, f$(\{a\},\{2\}) = (\{x\},\{e\})$, f $(\{b\},\{1\}) = (\{z\},\{f\})$, f $(\{b\},\{2\}) = (\{z\},\{e\})$, f $(\{c\},\{1\}) = (\{y\},\{f\})$, f $(\{c\},\{2\}) = (\{y\},\{e\})$. Then $f^{-1}(\{y\},\{f\}) = (\{c\},\{1\})$ and $f^{-1}(\{x,z\},\{e\}) = (\{a,b\},\{2\})$, which is N_B open but not N_B dense in (U_1,U_2). Therefore, f is N_B-continuous but not N_B D-continuous.

Remark 4.13: In N_B hyperconnected space, every N_B –continuous is N_B D-continuous.

Note 4.14: i) Every N_B α – continuous is not N_B D-continuous.

ii) Every N_B semi-continuous is not N_B D-continuous.

iii) Every N_B pre-continuous is not N_B D–continuous.

iv) Every N_B β D-continuous is not N_B D– continuous.

5. CONCLUSION

In this paper, we have defined N_B- contra continuous function in nano binary topological spaces and their characterizations were studied. Also we have explored some nano binary contra continuous functions in nano binary topological spaces and their features were discussed. Also we have defined N_B D– continuous and some properties are discussed.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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