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MATRIX REPRESENTATION OF OPERATIONS OF SOFT PARTIAL ORDERINGS ON A GENERALIZED SOFT POSET

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Abstract. A soft set is a parametrized family of subsets of an initial universal set. Soft set theory is a generalization of fuzzy set theory and is a mathematical tool for dealing with uncertainty and vagueness. Posets are used in many applications of Mathematics and Computer Science. Matrix representations are more applicable for handling data in computer programs. In this paper, we introduce the ordinary matrix representation of a soft relation, soft matrix representation of soft partial ordering and operations of soft partial orderings on a generalized soft poset.

Keywords: gs-poset; soft partial ordering; soft matrix representation.

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1. Introduction

Classical set theory is not enough to solve many complicated problems in different fields which containing various types of uncertainties. The concept of soft set was introduced by D Molodstov in [7] as a generalization of fuzzy set for dealing with uncertain data. He defined some basic notions and showed that it can be applied to more fields of mathematics, economics, engineering, physics and so on.

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A partial ordering is a binary relation which is reflexive, antisymmetric and transitive. A poset is a set together with a partial ordering on it. Poset is actually the generalization of the concept of ordering or sequencing or arranging the elements of a set. S.Das and S.K.Samanta in [2] introduced the notion of soft elements, soft real numbers and their properties. In this paper, we study the concept of soft matrix representation of a soft partial ordering and operations of soft partial orderings on a generalized soft poset(gs poset) in terms of soft elements. Here we uses zero-one matrix representation for a gs poset which are more appropriate to computer operations.

This paper is organized as follows. In section 2 we recall some preliminaries. In section 3, we explain the concept of generalized soft poset(gs poset) with some properties. Also, we describe the ordinary matrix representation of a soft relation and some of its properties. Here we also give different operations of soft relations on a gs poset. In section 4 we describe the soft matrix representation of a gs poset with example and give a relation between soft matrix representation and ordinary matrix representation. Ihe last section deals with operations of soft partial orderings on a gs poset and their soft matrix representations with sufficient examples.

2. PRELIMINARIES

Definition 2.1. [7] *Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and $A \subseteq E$. Then a soft set over X is a pair (F,A), where F is a mapping from A to P(X).*

Definition 2.2. [5]*Let E be the set of parameters and $A \subseteq E$. Then for each soft set (F,A) over U a soft set (H,E) is constructed over X,where for all $\alpha \in E$,*

$$H(\lambda) = \begin{cases} F(\lambda) & \text{if } \lambda \in A \\ \phi & \text{if } \lambda \in E \setminus A \end{cases}$$

Thus the soft sets (F,A) and (H,E) are equivalent to each other and the usual set operations on both are same. So we have to consider soft sets over the same parameter set A.

Definition 2.3. [3] *Let (F,A) and (G,B) be two soft sets over a non empty universal set X . Then (a) (F,A) is a soft subset of (G,B) if (i) $A \subseteq B$ and* *(ii)* $F(a) \subseteq G(a), \forall a \in A$.

We write $(F,A)\tilde{\subset}(G,B)$.

(b) (F,A) *and* (G,B) *are soft equal if* (F,A) *is a soft subset of* (G,B) *and* (G,B) *is a soft subset of* (F,A).

Definition 2.4. [6]*Let X be a nonempty universal set and A be a nonempty parameter set. Then a soft set* (F,A) *over X is said to be a null soft set if* $F(\lambda) = \phi$, $\forall \lambda \in A$ *and absolute soft set if* $F(\lambda) = X$, $\forall \lambda \in A$.

The absolute soft set over X with parameter set A is denoted by $\tilde{X}$ *or* $(\tilde{X},A)$ *and the null soft set is denoted by* $\tilde{\Phi}$ *or* $(\tilde{\Phi},A)$.

Definition 2.5. [2]*Let X be a nonempty universal set and A be a nonempty parameter set. Then a function* $\tilde{x} : A \to X$ *is called a soft element of* $\tilde{X}$ *and we write* $\tilde{x}\ \tilde{\in}\ \tilde{X}$.

A soft element $\tilde{x}$ *is said to be belongs to a soft set* (F,A) *over X if* $\tilde{x}(\lambda) \in F(\lambda)$, $\forall \lambda \in A$ *and we write* $\tilde{x}\ \tilde{\in}\ (F,A)$.

Thus for a soft set (F,A) *over X with the parameter set A with* $F(\lambda) \neq \phi$, $\forall \lambda \in A$, *we have* $F(\lambda) = \{\tilde{x}(\lambda) : \tilde{x}\ \tilde{\in}\ (F,A)\}$. *Soft elements are usually denoted by* $\tilde{x},\tilde{y},\tilde{z}$, *etc.*

Definition 2.6. [1]*Let X be a nonempty set. The collection of all soft sets* (F,A) *over X for which* $F(\lambda) \neq \phi$, $\forall \lambda \in A$ *together with the null soft set* $(\tilde{\Phi},A)$ *is denoted by* $S(\tilde{X})$.

For any non null soft set in $S(\tilde{X})$, *the collection of all soft elements of* (F,A) *is denoted by* $SE((F,A)$.

Let $\mathscr{B}$ *be a collection of soft elements of* (F,A). *Then the soft set generated by* $\mathscr{B}$ *is given by* $SS(\mathscr{B}) = (G,A)$, *where* $G(\lambda) = \{\tilde{x}(\lambda) : \tilde{x} \in \mathscr{B}\}$.

Definition 2.7. [4]*Let* (F,A) *be a soft set in* $S(\tilde{X})$ *with a nonempty parameter set A. Then a soft relation* $\tilde{\leq}$ *on* (F,A) *is a binary relation on* $SE(F,A)$. *Then it is said to be*

(i)soft reflexive if $\tilde{x}\ \tilde{\leq}\ \tilde{x},\ \forall \tilde{x} \in SE(F,A)$

(ii)soft symmetric if $\tilde{x}\ \tilde{\leq}\ \tilde{y} \Rightarrow \tilde{y}\ \leq\ \tilde{x},\ \forall \tilde{x},\tilde{y} \in SE(F,A)$

(iii)soft antisymmetric if $\tilde{x}\ \leq\ \tilde{y}$ *and* $\tilde{y}\ \leq\ \tilde{x} \Rightarrow \tilde{x} = \tilde{y},\ \forall \tilde{x},\tilde{y} \in SE(F,A)$ *and*

(iv)soft transitve if $\tilde{x}\ \leq\ \tilde{y}$ *and* $\tilde{y}\ \leq\ \tilde{z} \Rightarrow \tilde{x} = \tilde{z},\ \forall \tilde{x},\tilde{y},\tilde{z} \in SE(F,A)$.

A soft partial ordering $\tilde{\leq}$ is a soft relation on (F,A) if it is soft reflexive, soft antisymmetric and soft transitive.

Definition 2.8. [8]*Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two $m \times n$ zero-one matrices. Then the Boolean operations join($\vee$) and meet($\wedge$) of the matrices A and B are given by*
$A \vee B = [c_{ij}]$, where $c_{ij} = a_{ij} \vee b_{ij}, \forall i$ and j
and
$A \wedge B = [d_{ij}]$, where $d_{ij} = a_{ij} \wedge b_{ij}, \forall i$ and j.

Definition 2.9. [8]*Let $A = [a_{ij}]$ be an $m \times k$ zero-one matrix and $B = [b_{ij}]$ be a $k \times n$ zero-one matrix. Then the Boolean product of A and B is given by*
$A \odot B = [c_{ij}]$ which is the $m \times n$ matrix, where $c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee ... \vee (a_{ik} \wedge b_{kj})$.

3. Gs Posets and Its Ordinary Matrix Representation

Throughout this section X denotes a non empty universal set with a non empty parameter set A. Let (F,A) be a soft set in $S(\tilde{X})$ with a non empty parameter set A .Then a soft relation $\tilde{\leq}$ on (F,A) is a binary relation on $SE(F,A)$.

Definition 3.1. *Let (F,A) be a soft set over a poset $(P,\leq)$. Then (F,A) is said to be a soft poset over P if $F(\lambda)$ is a subposet of P. Since a subset of a poset is itself a poset, every soft set over P is a soft poset. Hence any soft set over a poset is called a soft poset.*

A generalized soft poset (gs poset) is a soft set (F,A) in $S(\tilde{X})$ together with a soft partial ordering on it and is denoted by $((F,A),\tilde{\leq})$.

Proposition 3.2. *Any soft set (F,A) in $S(\tilde{P})$ over a poset $(P,\leq)$ is a gs poset. In particular, $(\tilde{P},A)$ is a gs-poset.*

Proof. *Let $\tilde{P}$ be the absolute soft set over a poset $(P,\leq)$.*
Define the relation $\tilde{\leq}$ on $SE(F,A)$ by $\tilde{x} \;\tilde{\leq}\; \tilde{y}$ if and only if $\tilde{x}(\lambda) \leq \tilde{y}(\lambda), \forall \lambda \in A$
(i) Here $\tilde{x} \;\tilde{\in}\; (F,A) \Rightarrow \tilde{x}(\lambda) \in F(\lambda) \subseteq P \Rightarrow \tilde{x}(\lambda) \in P, \forall \lambda \in A$.
Since $(P,\leq)$ is a poset and $\tilde{x}(\lambda) \in P, \;\forall \lambda \in A$, we have $\tilde{x}(\lambda) \leq \tilde{x}(\lambda), \;\forall \lambda \in A$.
Hence $\tilde{x} \;\tilde{\leq}\; \tilde{x}, \;\forall \tilde{x} \;\tilde{\in}\; (F,A)$ and the relation $\tilde{\leq}$ on (F,A) is soft reflexive .

(ii)Let $\tilde{x},\tilde{y} \tilde{\in} (F,A)$such that $\tilde{x} \tilde{\leq} \tilde{y}$ and $\tilde{y} \tilde{\leq} \tilde{x}$.

Now $\tilde{x} \tilde{\leq} \tilde{y}$ and $\tilde{y} \tilde{\leq} \tilde{x} \Rightarrow \tilde{x}(\lambda) \leq \tilde{y}(\lambda)$ and $\tilde{y}(\lambda) \leq \tilde{x}(\lambda) \Rightarrow \tilde{x}(\lambda) = \tilde{y}(\lambda)$, $\forall \lambda \in A$.

$\Rightarrow \tilde{x} = \tilde{y}$.

Hence $\tilde{\leq}$ is soft antisymmetric on (F,A) .

(iii)Let $\tilde{x},\tilde{y},\tilde{z} \tilde{\in} (F,A)$such that $\tilde{x} \tilde{\leq} \tilde{y}$ and $\tilde{y} \tilde{\leq} \tilde{z}$.

Now $\tilde{x} \tilde{\leq} \tilde{y}$ and $\tilde{y} \tilde{\leq} \tilde{z} \Rightarrow \tilde{x}(\lambda) \leq \tilde{y}(\lambda)$ and $\tilde{y}(\lambda) \leq \tilde{z}(\lambda) \Rightarrow \tilde{x}(\lambda) \leq \tilde{z}(\lambda)$, $\forall \lambda \in A$.

$\Rightarrow \tilde{x} \tilde{\leq} \tilde{z}$.

Hence $\tilde{\leq}$ is soft transitive on (F,A) .

Hence (F,A) is a gs poset.

In particular, taking $\tilde{P}$ instead of (F,A), we get $\tilde{P}$ is a gs poset .

Here $\tilde{P}$ is called the gs poset generated by the poset $(P,\leq)$.

Proposition 3.3. *Any family of partial orderings $\{\leq_\lambda : \lambda \in A\}$ on a set X generate a gs-poset.*

Proof. *Consider $(\tilde{X},A)$ with $\tilde{\leq}$ defined on $SE(\tilde{X})$ by $\tilde{x} \tilde{\leq} \tilde{y}$ iff $\tilde{x}(\lambda) \leq_\lambda \tilde{y}(\lambda)$, $\forall \lambda \in A$. Then as in the proof of proposition 3.2, we get $(\tilde{X},A)$ with $\tilde{\leq}$ is a gs poset.*

Note. *The gs poset defined in the above proposition is said to be the gs poset generated by the family of partial orderings $\{\leq_\lambda : \lambda \in A\}$ on P.*

Definition 3.4. *Let (F,A) be a gs poset. Then a soft subset (G,A) of (F,A) is a gs subposet of (F,A) if (G,A) itself a gs poset under the same soft relation.*

Example 3.5. *Consider $(\tilde{R},A)$, the absolute soft set over R, the set of all real numbers wth the partial ordering $\leq$ (with the usual meaning) on R. Define the soft relation $\tilde{\leq}$ on $\tilde{R}$ by $\tilde{r} \tilde{\leq} \tilde{s}$ if and only if $\tilde{r}(\lambda) \leq \tilde{s}(\lambda)$, $\forall \lambda \in A$. Then $\tilde{R}$ is a gs poset. Also, $(\tilde{Q},A)$ is a gs subposet of $(\tilde{R},A)$, where Q is the set of rational numbers.*

Definition 3.6. *The soft set (F,A) in $S(\tilde{X})$ is said to be soft finite if $|(F,A)|$, the number of soft elements in (F,A) is finite. If $|(F,A)|$ is infinite then (F,A) is said to be soft infinite.*

Definition 3.7. *Let (F,A) be a soft finite soft set in $S(\tilde{X})$ with the soft relation $\tilde{\leq}$ on (F,A) and $|(F,A)| = n$. Then the ordinary matrix represenation of the soft relation $\tilde{\leq}$ on the soft set*

(F,A) is the ordinary matrix representation of $\tilde{\leq}$ on $SE(F,A)$. It is given by

$$M_{(F,A),\tilde{\leq}} = [m_{ij}]_n, \text{ where } m_{ij} = \begin{cases} 1 & \text{if } \tilde{x}_i \ \tilde{\leq} \ \tilde{x}_j \\ 0 & \text{otherwise.} \end{cases}$$

Using matrix representation of soft relations we can prove the following theorem.

Theorem 3.8. *If a soft set (F,A) in $S(\tilde{X})$ have n soft elements then it have exactly $3^{\frac{n^2-n}{2}}$ different ways of soft relations which are both soft reflexive and soft anti symmetric.*

Proof. *Each gs-poset (F,A) with n soft elements have a unique matrix representation which have the following properties.*
(i)diagonal elements must be 1.[By reflexivity]
(ii)In an $n \times n$ matrix $[m_{ij}]$, for $i \neq j$ there are $\frac{n^2-n}{2}$ number of pairs (m_{ij}, m_{ji}) in the matrix and each pair have three possibilities $(1,0),(0,1)$ and $(0,0)$.
$[(1,1)$ is not possible in a gs-poset since $m_{ij} = 1$ and $m_{ji} = 1 \Rightarrow \tilde{x}_i \ \tilde{\leq} \ \tilde{x}_j$ and $\tilde{x}_j \ \tilde{\leq} \ \tilde{x}_i \Rightarrow \tilde{x}_i \ \tilde{=} \ \tilde{x}_j$.(By antisymmetry)]
Hence there are exactly $3^{\frac{n^2-n}{2}}$ different ways of soft relations which are both soft reflexive and soft anti symmetric.

Example 3.9. *If (F,A) have two soft elements then it have exactly $3^{\frac{2^2-2}{2}} = 3$ different ways of soft partial orderings. Their matrix representations are $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$,and $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.*
Similarly, if (F,A) have three soft elements then it have exactly $3^{\frac{3^2-3}{2}} = 27$ different ways of soft partial orderings.

Definition 3.10. *Soft sets over set of matrices with a nonempty parameter set is called a soft matrix set. If (F,A) is a soft matrix set, then if $F(\lambda)$ is a singleton set of matrices for each $\lambda \in A$ then identifying it with that matrix is called a soft matrix element.*

Example 3.11. *Let (F,A) be a soft set over $M_2(Z_2)$, the set of all 2×2 matrices with entries zeros and ones. Let $A = \{e_1, e_2\}$. Define $\tilde{x}(e_1) = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$, $\tilde{x}(e_2) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. Here $\tilde{x}$ is a soft matrix element.*

4. Soft Matrix Representation of a gs Poset

Matrix representation is an efficient tool in computer programming. In this section we discuss the soft matrix representation of a gs-poset and is given as a soft matrix element of a soft set over $M_n(Z_2)$, the set of all $n \times n$ matrices with entries zeros and ones with parameter set A.

Definition 4.1. *Let $\tilde{P}$ be a gs poset generated by a poset $(P, \leq)$ with a non empty parameter set A. Let (F,A) be any soft finite soft set in $\tilde{P}$ and $SE(F,A) = \{\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n\}$. Here the soft partial ordering $\tilde{\leq}$ on $\tilde{P}$ is given by $\tilde{x}_i \ \tilde{\leq} \ \tilde{x}_j$ iff $\tilde{x}_i(\lambda) \leq \tilde{x}_j(\lambda)$, $\forall \lambda \in A$ for $\tilde{x}_i, \tilde{x}_j \ \tilde{\in} \ \tilde{P}$. Then the soft relation $\tilde{\leq}$ on (F,A) can be represented by the soft matrix element $\tilde{M}_{(F,A),\leq}$ over $M_n(Z_2)$. It is given by*

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$\tilde{M}_{(F,A),\tilde{\leq}}(\lambda) = [\tilde{m}_{\tilde{x}_i,\tilde{x}_j}(\lambda)]_n$, *a matrix of order n, where* $\tilde{m}_{\tilde{x}_i,\tilde{x}_j}(\lambda) = [\tilde{m}_{\tilde{x}_i(\lambda),\tilde{x}_j(\lambda)}]_n$, *a matrx of order n, where* $\tilde{m}_{\tilde{x}_i(\lambda),\tilde{x}_j(\lambda)} = \begin{cases} 1 & \text{if } \tilde{x}_i(\lambda) \leq \tilde{x}_j(\lambda) \\ 0 & \text{otherwise} \end{cases}$

Note. *By proposition 3.2, any non null soft finite soft set (F,A) in $S(\tilde{P})$, where $\tilde{P}$ is the gs poset generated by a poset $(P, \leq)$ is a gs-poset and have a soft matrix representation.*

Proposition 4.2. *If (F,A) is any gs poset in $S(\tilde{P})$, where $\tilde{P}$ is generated by a poset $(P, \leq)$ then the ordinary matrix representation of the gs poset (F,A),*
$M_{(F,A),\tilde{\leq}} = \wedge \left\{ \tilde{M}_{(F,A),\tilde{\leq}}(\lambda) \right\}$, *where $\wedge$ is taken over all $\lambda \in A$.*

Proof. *It follows from the definitions 3.7 and 4.1.*

Example 4.3. *Let $A = \{e_1, e_2\}$, $P = \{1,2,3,4\}$. Define (F,A) by $F(e_1) = \{1,2\}$, $F(e_2) = \{1,4\}$. Then its soft elements are $\tilde{x}_1(e_1) = 1$, $\tilde{x}_1(e_2) = 1$, $\tilde{x}_2(e_1) = 1$, $\tilde{x}_2(e_2) = 4$, $\tilde{x}_3(e_1) = 2, \tilde{x}_3(e_2) = 1$, $\tilde{x}_4(e_1) = 2$, $\tilde{x}_4(e_2) = 4$.*
The ordinary matrix representation of the gs poset (F,A) induced from the gs poset $(\tilde{P}, \tilde{\leq})$ generated by the poset $(P, \leq)$ is given by

$$M_{(F,A),\leq} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The soft matrix representation of the soft partial ordering $\tilde{\leq}$ *on* (F,A) *generated by the partial ordering* $\leq$ *on P,* $\tilde{M}_{(F,A),\leq}$ *is given by* $(\tilde{M}_{(F,A),\leq})$*, where*

$$\tilde{M}_{(F,A),\tilde{\leq}}(e_1) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \text{ and } \tilde{M}_{(F,A),\tilde{\leq}}(e_2) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Here $M_{(F,A),\leq} = \tilde{M}_{(F,A),\tilde{\leq}}(e_1) \wedge \tilde{M}_{(F,A),\tilde{\leq}}(e_2)$

Note. *In the above example ordinary matrix representation of* $\tilde{P}$ *is a square matrix of order 16 and its soft matrix represetation* $\tilde{M}_{\tilde{P},\tilde{\leq}}$ *is a soft matrix element of a soft set over* $M_{16}(Z_2)$ *with the same parameter set A.*

5. Operations of Soft Partial Orderings on a gs Poset and Its Matrix Representations

If we are given two soft partial ordering in a gs poset then their operations union, intersection and composition can also be represented in terms of ordinary and soft matrix representation by means of Boolean operations meet, join and product of matrices.

Definition 5.1. *Let* $\tilde{\leq}_1, \tilde{\leq}_2$ *be two soft partial ordering on a gs poset* (F,A) *over a nonempty universal set X with the parameter set A. Then their union, intersection, composition, difference and symmetric difference are given by for* $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} (F,A)$ *,*

(1)$\tilde{x} \ \tilde{\leq}_1 \bigcup \tilde{\leq}_2 \ \tilde{y} \Rightarrow \tilde{x} \ \tilde{\leq}_1 \ \tilde{y}$ *or* $\tilde{x} \ \tilde{\leq}_2 \ \tilde{y}$

(2)$\tilde{x} \ \tilde{\leq}_1 \bigcap \tilde{\leq}_2 \ \tilde{y} \Rightarrow \tilde{x} \ \tilde{\leq}_1 \ \tilde{y}$ *and* $\tilde{x} \ \tilde{\leq}_2 \ \tilde{y}$

(3)$\tilde{x} \ \tilde{\leq}_1 \circ \tilde{\leq}_2 \ \tilde{y} \Rightarrow$ *if there exists an* $\tilde{z} \ \tilde{\in}(F,A)$ *such that* $\tilde{x} \ \tilde{\leq}_1 \ \tilde{z}$ *or* $\tilde{z} \ \tilde{\leq}_2 \ \tilde{y}$

(4)$\tilde{x} \ \tilde{\leq}_1 - \tilde{\leq}_2 \ \tilde{y} \Rightarrow \tilde{x} \ \tilde{\leq}_1 \tilde{y}$ *and* $\tilde{x} \ \tilde{\nleq}_2 \ \tilde{y}$

(5)$\tilde{x} \ \tilde{\leq}_1 \oplus \tilde{\leq}_2 \ \tilde{y} \Rightarrow \tilde{x}(\tilde{\leq}_1 \bigcup \tilde{\leq}_2)\tilde{y}$ *but* $\tilde{x} \ \tilde{\nleq}_1 \ \tilde{y}$ *and* $\tilde{x} \ \tilde{\nleq}_2 \ \tilde{y}$*.*

Theorem 5.2. *If* $\tilde{\leq}_1, \tilde{\leq}_2$ *be two soft partial ordering on a gs poset* (F,A) *over a nonempty universal set X with the parameter set A. Then the ordinary matrix representations of their union, intersection and composition are given by*

(1)$M_{\tilde{\leq}_1 \bigcup \tilde{\leq}_2} = M_{\tilde{\leq}_1} \vee M_{\tilde{\leq}_2}$

(2)$M_{\tilde{\leq}_1 \bigcap \tilde{\leq}_2} = M_{\tilde{\leq}_1} \wedge M_{\tilde{\leq}_2}$

(3)$M_{\tilde{\leq}_1 \circ \tilde{\leq}_2} = M_{\tilde{\leq}_1} \odot M_{\tilde{\leq}_2}$, *where* $\odot$ *denotes the Boolean product of* $M_{\tilde{\leq}_1}$ *and* $M_{\tilde{\leq}_2}$

Proof. *Let* $\tilde{\leq}_1$, $\tilde{\leq}_2$ *be two soft partial ordering on a gs poset* (F,A) *over a nonempty universal set X with the parameter set A.*

(1)Let $M_{\tilde{\leq}_1}$ *and* $M_{\tilde{\leq}_2}$ *be the ordinary matrix representations of* $\tilde{\leq}_1$ *and* $\tilde{\leq}_2$ *respectively.*

The matrix representing their union $M_{\tilde{\leq}_1 \bigcup \tilde{\leq}_2}$ *has value 1 in the position where either* $M_{\tilde{\leq}_1}$ *or* $M_{\tilde{\leq}_2}$ *has value 1. Also, it has value 0 if both the positions of* $M_{\tilde{\leq}_1}$ *and* $M_{\tilde{\leq}_2}$ *are 0.*

Hence $M_{\tilde{\leq}_1 \bigcup \tilde{\leq}_2} = M_{\tilde{\leq}_1} \vee M_{\tilde{\leq}_2}$.

Similarly we get (2) and (3).

Remark. *In the above theorem, if* (F,A) *is a gs poset in* $S(\tilde{P})$, *where* $\tilde{P}$ *is generated by a poset* $(P,\leq)$, *then the soft matrix representation of union,intersection and composition of* $\tilde{\leq}_1$ *and* $\tilde{\leq}_2$ *are given by for* $\lambda \in A$,

(1)$\tilde{M}_{\tilde{\leq}_1 \bigcup \tilde{\leq}_2}(\lambda) = \tilde{M}_{\tilde{\leq}_1}(\lambda) \vee \tilde{M}_{\tilde{\leq}_2}(\lambda)$

(2)$\tilde{M}_{\tilde{\leq}_1 \bigcap \tilde{\leq}_2}(\lambda) = \tilde{M}_{\tilde{\leq}_1}(\lambda) \wedge \tilde{M}_{\tilde{\leq}_2}(\lambda)$

(3)$\tilde{M}_{\tilde{\leq}_1 \circ \tilde{\leq}_2}(\lambda) = \tilde{M}_{\tilde{\leq}_1}(\lambda) \odot \tilde{M}_{\tilde{\leq}_2}(\lambda)$

The ordinary matrix representations are given by

(1)$M_{\tilde{\leq}_1 \bigcup \tilde{\leq}_2} = \wedge(\tilde{M}_{\tilde{\leq}_1 \bigcup \tilde{\leq}_2}(\lambda))$

(2)$M_{\tilde{\leq}_1 \bigcap \tilde{\leq}_2} = \wedge(\tilde{M}_{\tilde{\leq}_1 \bigcap \tilde{\leq}_2}(\lambda))$

(3)$M_{\tilde{\leq}_1 \circ \tilde{\leq}_2} = \wedge(\tilde{M}_{\tilde{\leq}_1 \circ \tilde{\leq}_2}(\lambda))$

Example 5.3. *Let* $\tilde{\leq}_1$, $\tilde{\leq}_2$ *be two soft partial ordering on a absolute gs poset* $\tilde{P}$, *where* $P = \{1,2,4\}$ *with the parameter set* $A = \{e_1, e_2\}$, *where* $\tilde{\leq}_1$ *defined by*

$\tilde{x} \ \tilde{\leq}_1 \ \tilde{y}$ *iff* $\tilde{x}(\lambda) \ \leq \ \tilde{x}(\lambda), \ \forall \lambda \in A$

and $\tilde{\leq}_2$ *defined by*

$\tilde{x} \ \tilde{\leq}_2 \ \tilde{y}$ *iff* $\tilde{y}(\lambda) \ | \ \tilde{x}(\lambda), \ \forall \lambda \in A$, $' \leq '$ *and* $'|'$ *are the usual relations 'less than or equal to' and 'divides' respectively.*

Soft elements of $\tilde{P}$ *are* $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{x}_3$, $\tilde{x}_4$, $\tilde{x}_5$, $\tilde{x}_6$, $\tilde{x}_7$, $\tilde{x}_8$ *and* $\tilde{x}_9$ *and are given by*

$\tilde{x}_1(e_1) = 1, \ \tilde{x}_1(e_2) = 1, \ \tilde{x}_2(e_1) = 1, \ \tilde{x}_2(e_2) = 2, \ \tilde{x}_3(e_1) = 1, \ \tilde{x}_3(e_2) = 4,$

$\tilde{x}_4(e_1)=2,\ \tilde{x}_4(e_2)=1,\ \tilde{x}_5(e_1)=2,\ \tilde{x}_5(e_2)=2,\ \tilde{x}_6(e_1)=2,\ \tilde{x}_6(e_2)=4,$

$\tilde{x}_7(e_1)=4,\ \tilde{x}_7(e_2)=1,\ \tilde{x}_8(e_1)=4,\ \tilde{x}_8(e_2)=2,\ \tilde{x}_9(e_1)=4,\ \tilde{x}_9(e_2)=4.$

Then the soft matrix representations of $(\tilde{P},\tilde{\leq}_1)$ *and* $(\tilde{P},\tilde{\leq}_2)$ *are given by*

$$M_{\tilde{P},\tilde{\leq}_1}(e_1)=\begin{pmatrix}1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\0&0&0&1&1&1&1&1&1\\0&0&0&1&1&1&1&1&1\\0&0&0&1&1&1&1&1&1\\0&0&0&0&0&0&1&1&1\\0&0&0&0&0&0&1&1&1\\0&0&0&0&0&0&1&1&1\end{pmatrix},\quad M_{\tilde{P},\tilde{\leq}_1}(e_2)=\begin{pmatrix}1&1&1&1&1&1&1&1&1\\0&1&1&0&1&1&0&1&1\\0&0&1&0&0&1&0&0&1\\1&1&1&1&1&1&1&1&1\\0&1&1&0&1&1&0&1&1\\0&0&1&0&0&1&0&0&1\\1&1&1&1&1&1&1&1&1\\0&1&1&0&1&1&0&1&1\\0&0&1&0&0&1&0&0&1\end{pmatrix}\ \textit{and}$$

$$M_{\tilde{P},\tilde{\leq}_2}(e_1)=\begin{pmatrix}1&1&1&0&0&0&0&0&0\\1&1&1&0&0&0&0&0&0\\1&1&1&0&0&0&0&0&0\\0&0&0&1&1&1&1&1&1\\0&0&0&1&1&1&1&1&1\\0&0&0&1&1&1&1&1&1\\0&0&0&0&0&0&1&1&1\\0&0&0&0&0&0&1&1&1\\0&0&0&0&0&0&1&1&1\end{pmatrix},\quad M_{\tilde{P},\tilde{\leq}_2}(e_2)=\begin{pmatrix}1&0&0&1&0&0&1&0&0\\1&1&0&1&1&0&1&1&0\\1&1&1&1&1&1&1&1&1\\1&0&0&1&0&0&1&0&0\\1&1&0&1&1&0&1&1&0\\1&1&1&1&1&1&1&1&1\\1&0&0&1&0&0&1&0&0\\1&1&0&1&1&0&1&1&0\\1&1&1&1&1&1&1&1&1\end{pmatrix}$$

Also $M_{\tilde{\leq}_1\bigcup\tilde{\leq}_2}, M_{\tilde{\leq}_1\bigcap\tilde{\leq}_2} and M_{\tilde{\leq}_1\circ\tilde{\leq}_2}$ *are given by*

$$M_{\tilde{\leq}_1\bigcup\tilde{\leq}_2}(e_1)=\begin{pmatrix}1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\0&0&0&1&1&1&1&1&1\\0&0&0&1&1&1&1&1&1\\0&0&0&1&1&1&1&1&1\\0&0&0&0&0&0&1&1&1\\0&0&0&0&0&0&1&1&1\\0&0&0&0&0&0&1&1&1\end{pmatrix}, M_{\tilde{\leq}_1\bigcup\tilde{\leq}_2}(e_2)=\begin{pmatrix}1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1&1\end{pmatrix},$$

$$M_{\tilde{\leq}_1 \bigcap \tilde{\leq}_2}(e_1)=\begin{pmatrix} 1&1&1&0&0&0&0&0&0\\ 1&1&1&0&0&0&0&1&1\\ 1&1&1&0&0&0&0&0&0\\ 0&0&0&1&1&1&1&1&1\\ 0&0&0&1&1&1&1&1&1\\ 0&0&0&1&1&1&1&1&1\\ 0&0&0&0&0&0&1&1&1\\ 0&0&0&0&0&0&1&1&1\\ 0&0&0&0&0&0&1&1&1 \end{pmatrix}, M_{\tilde{\leq}_1 \bigcap \tilde{\leq}_2}(e_2)=\begin{pmatrix} 1&0&0&1&0&0&1&0&0\\ 0&1&0&0&1&0&0&1&0\\ 0&0&1&0&0&1&0&0&1\\ 1&0&0&1&0&0&1&0&0\\ 0&1&0&0&1&0&0&1&0\\ 0&0&1&0&0&1&0&0&1\\ 1&0&0&1&0&0&1&0&0\\ 0&1&0&0&1&0&0&1&0\\ 0&0&1&0&0&1&0&0&1 \end{pmatrix} \textit{ and}$$

$$M_{\tilde{\leq}_1 \circ \tilde{\leq}_2}(e_1)=\begin{pmatrix} 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1\\ 0&0&0&1&1&1&1&1&1\\ 0&0&0&1&1&1&1&1&1\\ 0&0&0&1&1&1&1&1&1\\ 0&0&0&0&0&0&1&1&1\\ 0&0&0&0&0&0&1&1&1\\ 0&0&0&0&0&0&1&1&1 \end{pmatrix}, M_{\tilde{\leq}_1 \circ \tilde{\leq}_2}(e_2)=\begin{pmatrix} 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1 \end{pmatrix}$$

Here $M_{\tilde{\leq}_1 \bigcup \tilde{\leq}_2} = M_{\tilde{\leq}_1} \vee M_{\tilde{\leq}_2}$, $M_{\tilde{\leq}_1 \bigcap \tilde{\leq}_2} = M_{\tilde{\leq}_1} \wedge M_{\tilde{\leq}_2}$ *and* $M_{\tilde{\leq}_1 \circ \tilde{\leq}_2} = M_{\tilde{\leq}_1} \odot M_{\tilde{\leq}_2}$.

6. Conclusion

Posets play a vital role in algebra, topology, probability theory, computer science, economics, etc. In this paper, we discuss the concept of gs-poset in terms of soft elements. Also we explain the ordinary matrix representation and the soft matrix representation of a gs-poset as well as operations of soft partial orderings on it. Matrix representations are most appropriate in computer programs. We hope that this theory can be extended in more applicable ways.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

[1] S. Das, S.K. Samanta, On soft metric spaces, J. Fuzzy Math. 21(3) (2013), 707-734.

[2] S. Das, S.K. Samanta, Soft real sets, soft real numbers and their properties, J. Fuzzy Math. 20(3) (2013), 551-576.

[3] F. Feng, C. Li, B. Davvaz, M.I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Comput. 14 (2010), 899–911.

[4] D. Susha, M. John, Soft valuation on a generalized soft lattice, J. Math. Comput. Sci. 10 (2020), 339-358

[5] Z. Ma, W. Yang, B.-Q. Hu, Soft set theory based on its extension, Fuzzy Inform. Eng. 2 (2010), 423–432.

[6] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Computers Math. Appl. 45 (2003), 555–562.

[7] D. Molodtsov, Soft set theory—First results, Computers Math. Appl. 37 (1999), 19–31.

[8] K.H. Rosen, Discrete mathematics and its applications with combinatorics and graph theory, Colloquium Publications, 2014.